This question paper consists of a cover page, 18 pages, 3 diagram sheets and a formula sheet.

INSTRUCTION
This diagram sheet must be handed in with the answer book. Please ensure that your details are complete.

INSTRUKSIE
EXAMINATION NUMBER
CENTRE NUMBER

QUESTION 4.2

QUESTION 6.1

QUESTION 6.2

QUESTION 7

QUESTION 8.1
INSTRUCTIONS
1. Answer ALL the questions.

2. A formula sheet is included in the question paper.

3. Show ALL the necessary calculations.

4. Number ALL the answers clearly and correctly.
5. The diagrams are not drawn to scale.
6. Three diagram sheets are included. Place it in the ANSWER BOOK.
7. Non-programmable calculators may be used, unless the question states otherwise.

ANALYTICAL GEOMETRY
NOTE: IN THIS SECTION ONLY ANALYTICAL METHODS MAY BE USED. ACCURATE CONSTRUCTIONS AND MEASUREMENTS MAY NOT BE USED.

QUESTION 1
1.1 P(-3; 2) and Q(5; 8) are two points in a Cartesian plane.
1.1.1 Calculate the length of PQ. (2)
1.1.2 Calculate the angle that PQ forms with the x-axis, rounded off to one decimal digit. (3)
1.1.3 Determine the equation of the perpendicular bisector of PQ in the form \( ax + by + c = 0 \) (5)
1.2 In the diagram below A, B, C and D(3; 9) are the vertices of a rhombus. The equation of AC is \( x + 3y = 13 \)
1.2.1 Show that the equation of BD is \( 3x - y = 0 \) (3)
1.2.2 Calculate the coordinates of K if the diagonals of the rhombus intersect at point K. (4)
1.2.3 Determine the coordinates of B. (2)
1.2.4 Calculate the coordinates of A and C if \( AD = \sqrt{13} \) units. (8)

QUESTION 2
2.1 Write down the equation of the circle with the centre at (-2; 3) and a radius of \( \sqrt{13} \) units. (2)
2.2 \( x^2 + y^2 + 4x - 12y + 4 = 0 \) is the equation of a circle with centre M and radius \( r \).
2.2.1 Calculate the coordinates of the centre M and the length of the radius r. (5)
2.2.2 Write down the coordinates of the point(s) where this circle intersects the x-axis without any further calculations. (2)
2.2.3 Determine the equation(s) of the tangent(s) to this circle which are parallel to the y-axis. (2)

2.3 A circle with centre P(x; y) passes through A(4; -1) and touches the line y = 3
2.3.1 Determine the equation of the locus of P. (4)
2.3.2 Calculate the gradient of this locus at the point where x = 1 (3)
2.3.3 Determine the equation of the tangent to the locus of P where x = 1 (3)

TRIGONOMETRY

QUESTION 3

3.1 If \( \sin 161^\circ = t \), express the following in terms of t:
3.1.1 \( \cos 19^\circ \) (3)
3.1.2 \( \tan 71^\circ \) (3)
3.1.3 \( \sec (-341^\circ) \) (2)

3.2 If \( p \sin \theta = -3 \) and \( p \cos \theta = 3 \), \( p > 0 \), determine the value of the following:
3.2.1 \( \theta \) for \( \theta \in [0^\circ; 360^\circ] \) (4)
3.2.2 \( p \) (Leave the answer in surd form if necessary.) (3)

3.3 Prove that:
\[
\frac{\csc(-0) + \sec(180^\circ + 0)}{\cot(90^\circ - 0) - \cot(-0 - 180^\circ)} = -\left( \cos \theta + \sin \theta \right)
\]

3.4 Determine the value of 
\[
\frac{3}{2} \cot^2 \left(-60^\circ\right) - \frac{3}{2} \cos 330^\circ - 2 \sin^2 \left(-1035^\circ\right)
\]
without using a calculator. (7)

QUESTION 4

4.1 If \( 1 + \tan \theta = 2 \theta \) and \( \cos \theta \), show that \( \sin \theta = 0 \) if \( \sin 2\theta = -1 \) (5)
4.1.2 Determine the value(s) of \( \theta \in [-180^\circ; 90^\circ] \) for which \( 1 + \tan \theta = \cos 2\theta \) (3)

4.2.1 Make sketch graphs of \( f(\theta) = 1 + \tan \theta \) and \( g(\theta) = \cos 2\theta \) for \( \theta \in [-180^\circ; 90^\circ] \) on the set of axes provided on the diagram sheet. (6)
4.2.2 Write down the period of \( \cos 2\theta \) (1)
4.2.3 Determine, by using the graphs, the value(s) of \( \theta \) for which \( \cos 2\theta - 1 < \tan \theta \), for \( \theta \in [-180^\circ; 90^\circ] \) (4)
QUESTION 5

5.1 Using the formulae for \( \cos(A + B) \) and \( \sin(A + B) \), prove that:
\[
\tan \left( A + B \right) = \frac{\tan A + \tan B}{1 - \tan A \tan B}
\]  
(3)

5.1.2 Determine the general solution of
\[
\frac{\tan 2x + \tan 40^\circ}{1 - \tan 2x \tan 40^\circ} = 1
\]  
(4)

5.2 Prove that
\[
\frac{1}{2} \left( \cot \theta - \frac{\sec \theta}{\csc \theta} \right) = 2 \theta
\]  
(5)

5.2.1 Using fundamental identities.

5.2.2 Give the general solution of \( \theta \), for which the identity is undefined.

5.2.3 Hence, solve for \( \theta \) in
\[
\frac{1}{2} \left( \cot \theta - \frac{\sec \theta}{\csc \theta} \right) = \tan \theta
\]  
(6)

if \( \theta \in [-90^\circ; 180^\circ] \).

QUESTION 6

6.1 In the diagram alongside, B of \( \triangle ABC \) is obtuse. Use the diagram to prove that:
\[
\frac{\sin A}{a} = \frac{\sin B}{b}
\]  
(4)

6.2 In the diagram alongside, SP is a vertical tower and the points R and Q are in the same horizontal plane as S, the foot of the tower.

\( \hat{SPR} = x \), \( \hat{SQR} = 90^\circ + x \),

\( SQR = 2x \) and \( RP = 2 \) units.

6.2.1 Given that \( \sin(90^\circ + x) = \sin(90^\circ - x) \), prove that \( \text{RQ} = 1 \) unit.  
(6)

6.2.2 Prove that:
\[
\text{SQ} = 2 \cos 2x - 1 \text{ vir } x \in (0^\circ; 30^\circ)
\]  
(9)

[19]
**GEOMETRY**

**NOTE:** DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEET OR REDRAWN IN THE ANSWER BOOK. DETACH THE DIAGRAM SHEET FROM THE QUESTION PAPER AND PLACE IT IN THE ANSWER BOOK. GIVE A REASON FOR EACH STATEMENT.

**QUESTION 7**

7.1 In the diagram below, A, B and C are on a circle with centre S. Chords BA and CD are produced to meet at E. AC and BD intersect at F and SB and SC are drawn.

\[ \angle BAC = x \quad \text{and} \quad \angle BCA = y \]

7.1.1 Express \( \angle DFE \) in terms of \( x \) and \( y \).

7.1.2 Prove that:

\[ \angle BSC = \angle DFE + \hat{E} \]

**QUESTION 8**

8.1 In the diagram alongside, KM is a tangent to circle O at L. Use the diagram to prove the theorem which states that:

\[ \angle HIP = \angle PNL \]

\[ \angle KLP = \angle PNL \]
8.2  In the diagram alongside, GF is a tangent to the circle at A. AB is a chord and BD \perp AF intersects the circle at C. E is on AB such that DE = DA. EC produced meets AF at F. BF is joined but is not a tangent. AC is produced to meet BF at H.

Prove that:

8.2.1  \( \angle DAC = \angle BAD \)  

8.2.2  ADCE is a cyclic quadrilateral  

8.2.3  AH \perp BF  

8.2.4  CD is the bisector of \( \angle ACF \)  

[23]

QUESTION 9

In the diagram alongside, XP is the perpendicular bisector of side WY of \( \triangle WXY \).

Q is a point on WX such that WQ : WX = 3 : 5. XP and YQ intersect at T. QR is drawn parallel to XP.

Determine:

9.1  \( \frac{YP}{YR} \)  

9.2  \( \frac{QR}{TP} \)  

9.3  \( \frac{\text{Area } \triangle TPY}{\text{Area } \triangle QRY} \)  

[8]
QUESTION 10

10.1 In the diagram alongside, \( \angle ABC = 90^\circ \) and BD is drawn perpendicular to AC. Use the diagram to prove the theorem which states that:

\[ \triangle ABC \parallel \triangle ABD \parallel \triangle BDC \]  (5)

10.2 In the diagram alongside, two circles touch internally at S. O is the centre of the bigger circle and OS is a diameter of the smaller circle. PR is a double chord such that PT = TR intersecting the smaller circle at W. SW is produced to meet the bigger circle at V. VOR is a straight line. WO and PS are drawn.

Prove that:

10.2.1 \( SW = WV \)  
10.2.2 \( SW^2 = PW \cdot WR \)  
10.2.3 \( SW^2 = WT \cdot WR \)  
10.2.4 \( PW : WR = 1 : 3 \)  

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ T_n = a + (n-1)d \quad S_n = \frac{n}{2} (a + 1) \quad S_n = \frac{n}{2} [2a + (n-1)d] \]

\[ T_n = a r^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \quad S_n = \frac{a(1 - r^n)}{r - 1} \quad S_n = \frac{a}{1 - r} \]

\[ A = P \left( 1 + \frac{r}{100} \right)^n \quad A = P \left( 1 - \frac{r}{100} \right)^n \]

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ y = mx + c \]

\[ y - y_1 = m(x - x_1) \]

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ m = \tan \theta \]

\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

\[ x^2 + y^2 = r^2 \quad (x - p)^2 + (y - q)^2 = r^2 \]

In \( \triangle ABC \):

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

\[ a^2 - b^2 + c^2 - 2bc \cos A \]

\[ \text{area } \triangle ABC = \frac{1}{2} ab \sin C \]
1.1.1 \[ PQ^2 = (5 + 3)^2 - (3 - 2)^2 \]
\[ = 8^2 + 6^2 \]
\[ PQ = 10 \]

1.1.2 \[ m_{\overline{PQ}} = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{6}{8} \]
\[ \therefore \theta = 36.9^\circ \]

1.1.3 Medpt PQ: \((1;5)\)
Grad line \(\perp\): \(m = -\frac{4}{3}\)
Eq line \(y = (-4/3)x + c\)
subst: \((1;5)\) \(5 = (-4/3)(1) + c\)
\[ c = 19/3 \]

Eq \[ y = (-4/3)x + 19/3 \]
and \[ 3y + 4x - 19 = 0 \]

1.2.1 Eq \(AC: x + 3y = 13\) and \(y = -(1/3)x + 13/3\)
BD \(\perp\) AC and through \((3;9)\).
Eq BD: \[ y = 3x + c \]
\[ \therefore 9 = 3(3) + c \]
\[ \therefore c = 0 \]
Eq \[ y = 3x + 0 \] or \(3x - y = 0\)

1.2.2 Solve equations for AC and BD simultaneously.
\[-(1/3)x + 13/3 = 3x\]
\[-x + 13 = 9x\]
\[x = 1.3 \quad [\text{subst.}] \]
\[y = 3,9\]

1.2.3 \[ B(x;y) \]
\[ \frac{3+x}{2} \quad \text{AND} \quad \frac{9+y}{2} = \frac{39}{10} \]
1.2.4

\[(3 - x)^2 + (9 - y)^2 = 73\]  Points equidistant from D.
\[[3 - (13 - y)]^2 + (9 - y)^2 = 73\]  Solve equations for AC and circle simultaneously.
\[(100 - 60y + 9y^2 + 81 - 18y + y^2 = 73\]
\[10y^2 - 78y + 108 = 0\]
\[5y^2 - 39y + 54 = 0\]
Discriminant a square, factorise.
\[(5y - 9)(y - 6) = 0\]
Thus \(y = 9/5\) OR \(y = 6\)
and \(x = 7,6\) \(x = -5\)
Thus A(-5;6) and C(7,6; 1,8)

2.1

\[(x + 2)^2 + (y - 3)^2 = 13\]

2.2.1

\[x^2 + 4x + y^2 - 12y + 4 = 0\]
\[x^2 + 4x + 4 + y^2 - 12y + 36 = -4 + 36 + 4\]  [Complete the square]
\[(x + 4)^2 + (y - 6)^2 = 36\]
Thus: M(-2; 6) en \(r = 6\)

2.2.2 X-intercept: \(x = -2\)
\(r = 6\) and M(-2;6)
X-axis touches the circle at \(x = -2\).

2.2.3 Tangent A is 6 units to the left of M.
Tangent B is 6 units to the right of M.
Thus: A: \(x = -8\) and B: \(x = 4\).

2.3.1

\[(radius)^2 = (\text{distance from mdpt. to tangent})^2\]
\[(x - 4)^2 + (y + 1)^2 = (y - 3)^2\]
\[x^2 - 8x + 16 + y^2 + 2y + 1 = y^2 - 6y + 9\]
\[x^2 - 8x + 8 = -8y\]
y = \((-1/8)x^2 + x - 1\)

2.3.2 Gradient: \(Dx\)
\[= (-1/8)2x + 1\]
\[= (-1/8)(2) + 1\]  [subst. \(x = 1\)]
\[= 3/4\]

2.3.3 Point of tangency: \((1; -1/8)\)
Eq. tangent: \(y = mx + c\)
\[= (3/4)x + c\]
\[= (3/4)(1) + c\]  [subst.]
c = \(-7/8\)
Eq. tangent: \(y = (3/4)x - 7/8\)

3.1.1 \(\cos 19^\circ = \sqrt{1 - \sin^2 19^\circ} = \sqrt{1 - \sin^2 161^\circ} = \sqrt{1 - t^2}\)
3.1.2 \[ \tan 71^\circ = \frac{\sin 71^\circ}{\cos 71^\circ} = \frac{\cos 19^\circ}{\sin 19^\circ} = \frac{\sqrt{1 - t^2}}{t} \]

3.1.3 \[ \sec(-341^\circ) = \sec 341^\circ = \sec 19^\circ = \frac{1}{\cos 19^\circ} = \frac{1}{\sqrt{1 - t^2}} \]

3.2.1 \[
\begin{align*}
\text{psin} \theta &= -3 \quad \text{(i)} \\
\text{pcos} \theta &= 3 \quad \text{(ii)} \\
\theta \tan \theta &= -1 \quad \text{(i)/(ii)} \\
\theta &= 180^\circ - 45^\circ \quad \text{OR} \quad 2 = 360^\circ - 45^\circ \\
&= 135^\circ = 315^\circ \\
\text{But sin} \theta \text{ is negative and cos} \theta \text{ is positive; only one answer: } \theta = 315^\circ
\end{align*}
\]

3.2.2 \[ \text{pcos}315^\circ = 3 \text{[given]} \]
\[
\frac{1}{\sqrt{2}} = \frac{3}{p}
\]

Thus: \[ p = 3\sqrt{2} \]

3.3 \[
\text{LHS} = \frac{\cosec(-\theta) + \sec(180^\circ + \theta)}{\cot(90^\circ - \theta) - \cot(-\theta - 180^\circ)}
\]
\[
= \frac{-\cosec \theta - \sec \theta}{\tan \theta + \cot(180^\circ + \theta)}
\]
\[
= \frac{-1}{\sin \theta} - \frac{1}{\cos \theta}
\]
\[
= \frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}
\]
\[
= \frac{-\cos \theta - \sin \theta}{\sin \theta + \cos \theta}
\]
\[
= -\cos \theta + \sin \theta
\]
\[
= \text{RHS}
\]

3.4 \[
(3/2)\cot^2(-60) - (3/2)\cos330^\circ - 2\sin^2(1035^\circ)
\]
\[
= \frac{3}{2} \left( -1 \right)^2 - \frac{3}{2} \left( \frac{\sqrt{3}}{2} \right)^2 - 2 \left( \frac{1}{\sqrt{2}} \right)^2
\]
\[
= \frac{1}{2} - \frac{3\sqrt{3}}{4} - 1
\]
\[
= -3\sqrt{3} - \frac{1}{2}
\]

4.1.1 \[
\begin{align*}
1 + \tan \theta &= \cos 2 \theta \\
1 + \sin \theta /\cos \theta &= 1 - 2\sin^2 \theta \quad \text{[cos2 nie nul]} \\
\sin \theta &= -2\sin^2 \theta \cos \theta \\
\sin \theta (1 + 2\sin \theta \cos \theta) &= 0 \\
\therefore \sin \theta &= 0 \\
\text{OR} \quad 1 + 2\sin \theta \cos \theta &= 0 \\
2\sin \theta \cos \theta &= -1 \\
\therefore \sin 2\theta &= -1
\end{align*}
\]
4.1.2 \[ \sin \theta = 0 \text{ OR } \sin 2\theta = -1 \]
\[ \therefore \theta = 0^\circ \text{ of } -180^\circ : 2\theta = 270^\circ + k.360^\circ \text{.. } k \in \mathbb{Z} \]
\[ \therefore 2\theta = 135^\circ + k.180^\circ \]
\[ \therefore \theta = -45^\circ \]

4.2.1

4.2.2 Period of \( \cos 2\theta \) is 180°.

4.2.3 \( \cos 2\theta - 1 \leq \tan \theta \)
\( \cos 2\theta \leq \tan \theta + 1 \)
Thus: \(-180^\circ \leq \theta \leq -90^\circ \) or \( \theta = -45^\circ \) or \( 0^\circ \leq \theta < 90^\circ \).

5.1.1 Theory.

5.1.2 \( \text{LHS} = \tan(2x + 40^\circ) \) from 5.1.1.
\[ \therefore \tan(2x + 40^\circ) = 1 \]
\[ \therefore 2x + 40^\circ = 45^\circ + k.180^\circ \text{.. } k \in \mathbb{Z} \]
\[ 2x = 5^\circ + k.180^\circ \]
\[ x = 2.5^\circ + 90^\circ.k \text{.. } k \in \mathbb{Z} \]

5.2.1
\[ \text{LHS} = \frac{1}{2} \left( \cot \theta - \sec \theta \right) \]
\[ = \frac{1}{2} \left( \frac{\cos \theta - \sin \theta}{\sin \theta \cos \theta} \right) \]
\[ = \frac{1}{2} \left( \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} \right) \]
\[ = \frac{\cos^2 \theta - \sin^2 \theta}{2 \sin \theta \cos \theta} \]
\[ = \frac{\cos 2\theta}{\sin 2\theta} \]
\[ = \cot 2\theta = \text{RHS} \]
5.2.2 The identity is not defined for $\sin \theta = 0$ OR $\sin 2\theta = 0$ OR $\cos \theta = 0$ i.e. not defined for $\theta = 0^\circ + k.90^\circ \ldots k \in \mathbb{Z}$

5.2.3 $\cot 2\theta = \tan \theta$
$\tan(90 - 2\theta) = \tan \theta$
$90 - 2\theta = 0 + k.180^\circ \ldots k \in \mathbb{Z}$
$-3\theta = -90 + k.180^\circ \ldots k \in \mathbb{Z}$
$-\theta = -30^\circ + k.60^\circ \ldots k \in \mathbb{Z}$
$\theta = 30 + k.60^\circ \ldots k \in \mathbb{Z}$

Solution: $\theta = -30^\circ, 30^\circ, 150^\circ$. [\theta not defined for 90°]

6.1 Proof of Sine Rule.

6.2.1 In triangle PRS:
$\frac{SR}{2} = \sin x$
$\therefore SR = 2\sin x$

In triangle RSQ:
$\frac{SR}{\sin 2x} = \frac{RQ}{\sin(90 + x)}$
$\therefore RQ = \frac{2 \sin x \sin(90 + x)}{\sin 2x} = \frac{2 \sin x \sin(90 + x)}{2 \sin x \cos x} = \frac{\sin(90 + x)}{\cos x} = \frac{\sin(90 + x)}{\sin(90 - x)}$

6.2.2 In triangle RSQ: $\angle R = 90^\circ - 3x$
Therefor : $\frac{SQ}{\sin(90 - 3x)} = \frac{1}{\sin(90 + x)}$
$\therefore SQ = \frac{\sin(90 - 3x)}{\sin(90 + x)}$
$= \frac{\cos 3x}{\sin(90 - x)}$
$= \frac{\cos 3x}{\cos x}$
$= \frac{\cos 2x \cos x - \sin 2x \sin x}{\cos x}$
$= \frac{\cos 2x \cos x - 2 \sin x \cos x \sin x}{\cos x}$
$= \cos 2x - 2 \sin^2 x$
$= (2 \cos^2 x - 1) - 2 \sin^2 x$
$= 2(\cos^2 x - \sin^2 x) - 1$
$= 2 \cos 2x - 1$

7.1.1 $\angle DFA = x + y$ [ext $\angle$ = sum of int. opp. angles]
7.1.2 \( \angle BSC = 2y \) \( [\angle \text{at centre} = 2\angle \text{angle at circum.}] \)
\[ = y + y \]
\[ = y + (x + \angle E) \] \( [\angle C2 = x \text{ and } \angle A1 \text{ ext. angle of triangle}] \)
\[ = y + x + \angle E \]
\[ = (y + x) + \angle E \]
\[ = \angle DFA + \angle E [\text{proved in 7.1.1}] \]

8.1 Proof of tangent-chord theorem.

8.2.1 \( \angle \pi C2 = \angle B1 + \angle \pi A2 \) \( [\text{ext.} \square \text{ of triangle}] \)

but \( \angle B1 = \angle A3 \) \( [\text{tangent; chord}] \)

\[ \therefore \angle C2 = \angle A3 + \angle A2 \]
\[ \therefore \angle DCA = \angle BAD \]

8.2.2 \( \angle E1 = \angle A2 + \angle A3 \) \( [DA = DE] \)
\[ = \angle C2 \] \( [\text{proved in 8.2.1}] \)

but \( \angle E1 \) and \( \angle C2 \) are subtended by \( DA \)
\[ \therefore \text{ADCE a cyclic quad.} \]

8.2.3 \( \angle E3 = \angle D3 = 90° \) \( [\text{ext.} \angle = \text{int. opp. angle of cyclic quad}] \)
\[ \therefore \text{AH} \perp \text{BF} \] \( [\text{Three altitudes concurrent}] \)

8.2.4 \( \angle C2 = \angle A2 + \angle A3 \) \( [\text{proved in 8.2.1}] \)
\[ = \angle C3 \] \( [\text{ext.} \angle = \text{int. opp angle of cyclic quad ADCE}] \)
\[ \therefore \text{CD bisects } \angle ACF. \]

9.1 \[ \frac{WQ}{QR} = \frac{3}{2} \] \( [\text{given}] \)
\[ \frac{WR}{RP} = \frac{3}{2} \] \( [RQ // PX] \)
\[ \frac{WP}{FY} = \frac{5}{5} \] \( [\text{XP perpendicular bisector}] \)
\[ \frac{YP}{YR} = \frac{5}{7} \] \( [RQ // PT] \)

9.2 In triangle YPT and triangle YRQ:
\[ \angle Y = \angle Y \]
\[ \angle P = \angle R = 90° \] \( [\text{corresponding angles}] \)

and \( \angle YTP = \angle YQR \) \( [\text{angles of triangle } = 180°] \)
\[ \therefore \triangle YPT // \triangle YRQ \]
\[ \frac{QR}{TP} = \frac{RY}{PY} = \frac{5}{7} \]
9.3 \[\triangle TPY/\triangle QRY\]
\[
= \frac{0.5 PT \cdot PY}{0.5 QR \cdot RY}
\]
\[
= \frac{5 \cdot 5}{7 \cdot 7}
\]
\[
= \frac{25}{49}
\]

10.1 Theorem.

10.2.1 \[\angle W_1 + \angle W_2 = 90^\circ \quad [\angle \text{in semi circle}]\]
\[\therefore SW = WV \quad \text{[chord perpendicular to radius]}\]

10.2.2 In \(\triangle PWS\) en \(\triangle VWS\) is
\[\angle S_1 = \angle R_2 \quad \text{[subtended by} \ PV]\]
\[\angle P = \angle PV \quad [\angle \text{SR}]\]
\[\therefore \angle PWS \parallel \angle VWS\]
\[
\frac{SW}{WR} = \frac{PW}{WV}
\]
SW \cdot WV = PW \cdot WR
SW^2 = PW \cdot WR \quad \text{[SW = WV]}

10.2.3 SO \perp PR \quad [PT = TR, \text{given}]\]
\[\angle S_2 + \angle S_3 = 90^\circ \quad [\angle \text{in semi circle}]\]
\[\therefore \triangle SWT \parallel \triangle RWS \quad \text{[Theorem in 10.1]}\]
\[\therefore SW^2 = WT \cdot WR\]

10.2.4 PW \cdot WR = WT \cdot WR \quad \text{[proved in 10.2.2 and 10.2.3]}\]
\[\therefore PW = WT \quad \text{[divide by} \ WR]\]
and PT = TR \quad [\text{given}]\]
\[\therefore PW : WR = 1:3\]