0048.1

MATHEMATICS PAPER 1
WISKUNDE VRAESTEL 1

HIGHER GRADE
HOëRGRAAD

NOVEMBER 2002

MARKS: 200
PUNTE: 200
3 HOURS
3 URE

DEPARTMENT OF EDUCATION
DEPARTEMENT VAN ONDERWYS

NATIONAL SENIOR CERTIFICATE EXAMINATION – 2002
NASIONALE SENIOR SERTIFIKAAT EKSAMEN – 2002

This question paper consists of a cover page and 10 pages.
Hierdie vraestel bestaan uit 'n voorblad en 10 bladsye.

INSTRUCTIONS

1. This paper consists of 9 questions. Answer **ALL** the questions.

2. Clearly show all calculations, diagrams, graphs, etc. you have used in determining the answers.

3. An approved calculator (non-programmable and/or non-graphical) may be used, unless stated otherwise.

4. If necessary, answers should be rounded off to **TWO** decimal digits unless otherwise stated.

5. The attached diagram sheet must be used in QUESTION 9. Detach it from the question paper, fill in your examination number and the centre number and insert it in the **FRONT** of the answer book.

6. Number the answers **EXACTLY** as the questions are numbered.

7. Diagrams are not necessarily drawn to scale.

8. It is in your own interest to write legibly and to present the work neatly.

9. An information sheet with formulae is included.

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x+3)(3-x)=4$ (4)

1.1.2 $x+2\sqrt{x}-8=0$ (5)

1.1.3 $(x^2+1)(x-1)=0$ (2)

1.1.4 $|4-x| \leq 20$ (4)

1.2 For which values of x will the expression:

$$\sqrt{\frac{3-x}{x+7}}$$

be a real number? (5)

1.3 Determine all values of k for which the equation
$kx^2 + kx + 2 = 0$ has non-real roots. (5)

[25]

QUESTION 4

4.1 Given: $f(x)=2x^3+ax^2+ax-2$
When $f(x)$ is divided by $2x+1$ the remainder is b. Determine a in terms of b. (5)

4.2 $x-2$ is a factor of $f(x)=(x-8).p(x)+k$, where k is a constant. If $p(x)$ is divided by $x-2$, the remainder is 5. Determine the value of k. (5)
[10]

QUESTION 5

5.1 Show, **without using a calculator**, that:

$$1+4\left(\log_4 3\right)\left(\log_9 \frac{1}{2}\right)=0$$ (3)

5.2 Solve for x, **without using a calculator**:

5.2.1 $2^{x+1}+7=2^{2-x}$ (6)

5.2.2 $x\log 5 = \log\frac{3}{5}+x\log 3$ (4)

5.2.3 $\log(2x-3) \geq -\log(x-2)$ (10)

5.3 Given: $f(x)=3^{-x}$

5.3.1 Determine the equation which defines f^{-1} in the form $y = \ldots$. (2)

5.3.2 Draw sketch graphs of f and f^{-1}, on the same set of axes. Clearly label the graphs and indicate all intercepts with the axes. (4)

5.3.3 By means of a dotted line and the letter Q, indicate on the graph where you would read off the value of x for the solution of the equation $f(x) = f^{-1}(x)$. (1)

5.4 If $3^{3{,}096}=30$, determine $\log_3 90$ **without using a calculator**. Show ALL your calculations. (3)
[33]

QUESTION 6

6.1 The sum to n terms of an arithmetic series is:

$$S_n = \frac{n}{2}(7n + 15)$$

6.1.1 How many terms of the series must be added to give a sum of 425? (5)

6.1.2 Determine the sixth term of the series. (4)

6.2 Michael saved R400 during the first month of his working life. In each subsequent month, he saved 10% more than what he had saved in the previous month.

6.2.1 How much did he save in the 7^{th} working month? (4)

6.2.2 How much did he save altogether in his first 7 working months? (3)

6.2.3 In which month of his working life did he save more than R1 500 for the first time? (5)

6.3 If $a + 1;\ a - 1;\ 2a - 5$ are the first 3 terms of a convergent geometric series, calculate:

6.3.1 The value of a. (7)

6.3.2 The sum to infinity of the series. (4)

[32]

QUESTION 7

7.1 If $f(x) = 3x - x^2$, determine $f^{\prime}(x)$ **from first principles.** (6)

7.2 Determine $\frac{dy}{dx}$, if:

7.2.1 $xy = 5$ (2)

7.2.2 $y = \dfrac{1 - 2x + \sqrt{x}}{x^2}$ (4)

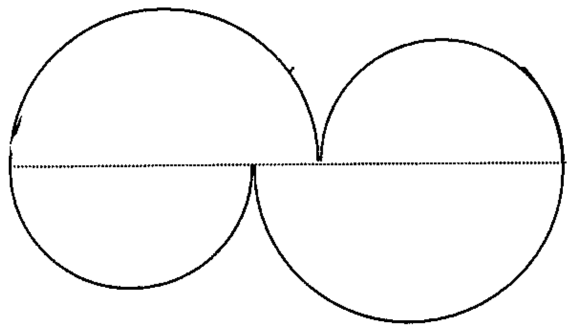
7.3 Given $f(x) = 2x^2 + x - 1$, calculate the equation of the tangent to the curve of f where the gradient equals -3. (6)

7.4 Draw a neat sketch graph of the function $y = x^3 - x^2 - 5x - 3$.
Clearly show all intercepts with the axes as well as the coordinates of the turning points. Show ALL your calculations. (17)

[35]

QUESTION 8

8.1 An advertising company has asked you to design an advertising disc that consists of four semicircles and has the shape as shown in the figure below. The larger semicircles have radius R and the smaller semicircles have radius r. The values of R and r may vary but R + r = 200 mm. To minimise costs the company has stated that the area of the shape must be a minimum.



8.1.1 Show that the area of the figure is given by $A = 2\pi R^2 - 400\pi R + 40\,000\pi$ (4)

8.1.2 Use differential calculus to determine the values of R and r if the area, A, of the figure is a minimum. (4)

8.1.3 Consider your solution in QUESTION 8.1.2 and explain why the shape suggested by the company is not possible if you want to maintain a minimum area. (2)

8.2 The profit yielded on a taxi is dependent on the average speed at which it is being driven. The profit (P) in rands per hour is calculated from the formula $P = -\frac{3}{80}x^2 + 6x - 180$, where x is the average speed in kilometres per hour, and $x \geq 30$.

Determine:

8.2.1 The speeds at which no profit is yielded. (3)

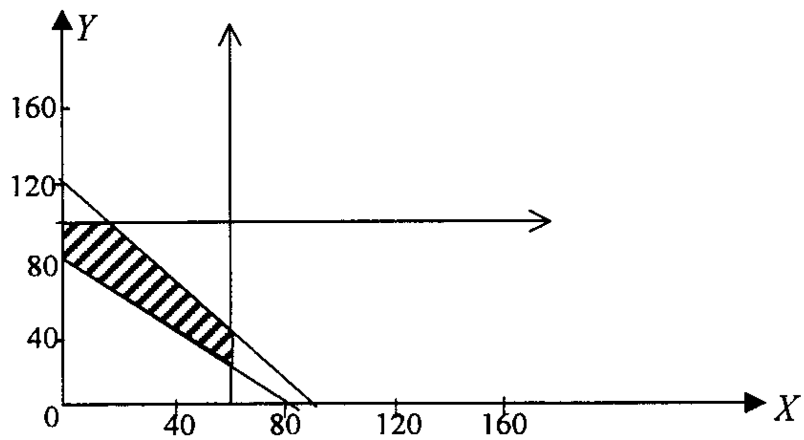
8.2.2 The most economical speed and the corresponding profit per hour. (5)

8.2.3 The speeds at which the owner will show a loss. (3)

[21]

QUESTION 9

In a certain week a radio manufacturer makes two types of portable radios, M(mains) and B(battery). Let x be the number of type M and y be the number of type B. In the sketch the shaded area represents the feasible region.



9.1 Write down the constraints to the linear programming problem, given:

9.1.1 At most 60 of type M and 100 of type B can be manufactured in a week. (2)

9.1.2 At least 80 radios in total must be produced in a week to cover costs. (1)

9.1.3 It takes $\frac{2}{3}$ hour to assemble a type M and $\frac{1}{2}$ hour to assemble a type B. The factory works a maximum of 60 hours per week. (2)

9.2 If the profit on a type M is R40 and on type B is R80, write down the equation in terms of x and y which will represent the profit (P). (1)

9.3 Draw the search line that represents the profit function on the diagram sheet provided. (1)

9.4 Use the graph to determine the pair $(x; y)$ in the feasible region where the profit is maximum. (3)

9.5 What is the maximum weekly profit? (2)

9.6 The manager is informed that the workers' union plans a strike for the following week, which will result in only 50 hours being worked. How many radios of each type should now be manufactured for maximum profit, and what will the maximum profit now be for the week? (4)

[16]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \qquad S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1) \qquad S_n = \frac{a(r^n - 1)}{r-1} \ (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \ (r \neq 1)$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2$$

$$(x-p)^2 + (y-q)^2 = r^2$$

In ΔABC: $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

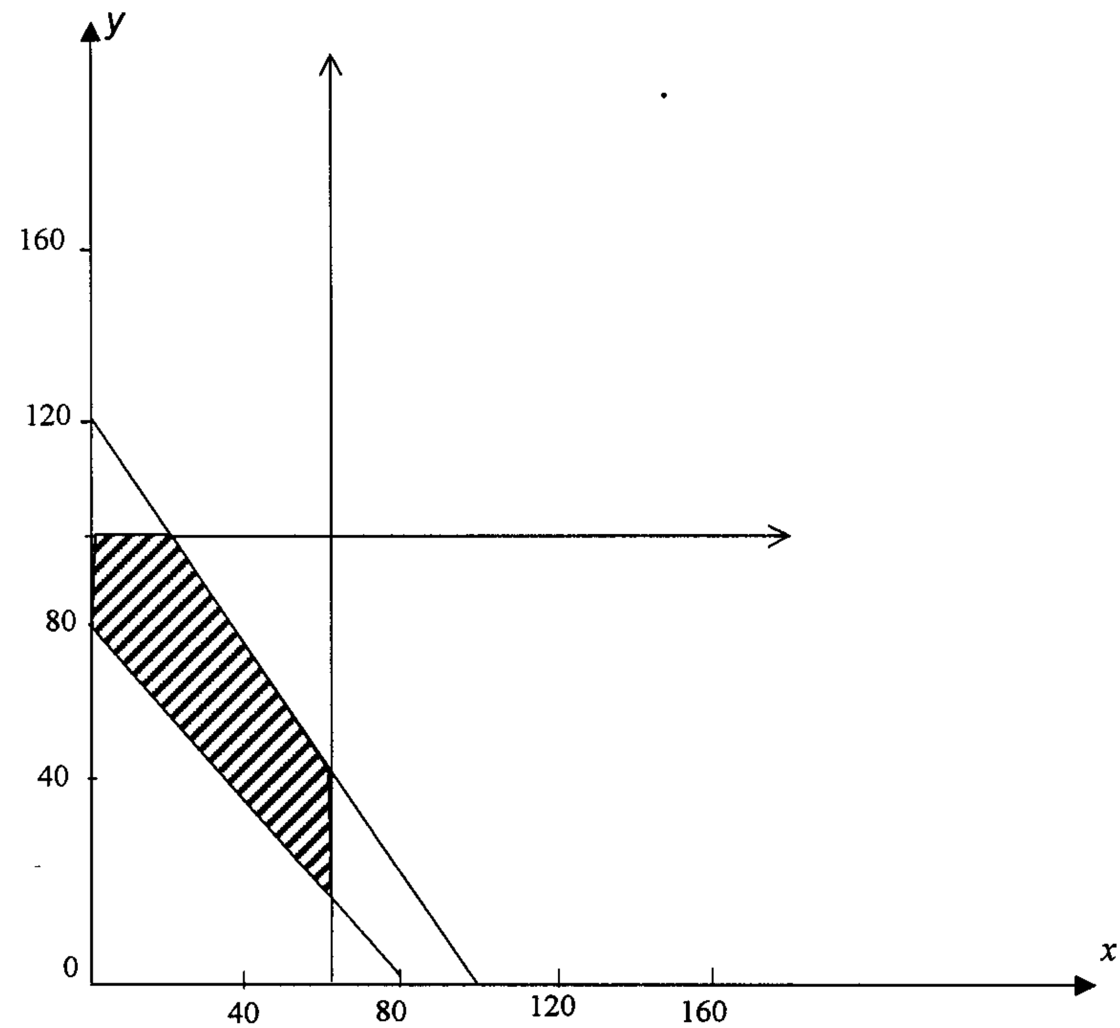
$$\text{area } \Delta ABC = \tfrac{1}{2}ab.\sin C$$

MATHEMATICS HG – PAPER 1
WISKUNDE HG – VRAESTEL 1

DIAGRAM SHEET **QUESTION 9/VRAAG 9** **DIAGRAMVEL**

NOTE: THIS SHEET MUST BE HANDED IN WITH THE ANSWER BOOK.
LET WEL: HIERDIE BLAD MOET SAAM MET DIE ANTWOORDEBOEK INGELEWER WORD.

CENTRE NUMBER SENTRUMNOMMER	
EXAMINATION NUMBER EKSAMENNOMMER	



Mathematics - HG - Nov 2002 National Paper 1 Memorandum
[Grade 12 Mathematics - HG]

Question 1 Final Version 8/11/2002

1 1 1 1

$$(2x+3)(3-x)=4$$
$$-2x^2+3x+9-4=0$$
$$2x^2-3x-5=0$$
$$(2x-5)(x+1)=0$$
$$x=\frac{5}{2} \text{ or } x=-1$$

(4)

✓ correct expansion of LHS
✓ std form (-1 if not = 0)
✓ both factors
✓ both answers, none rejected

1 2

$$x+2\sqrt{x}-8=0$$
$$(\sqrt{x}+4)(\sqrt{x}-2)=0$$
$$\sqrt{x}=-4 \text{ or } \sqrt{x}=2$$
N/A $x=4$
since $\sqrt{x}\geq 0$

(5)

✓ ✓ factorisation, M/A
[if $(\sqrt{x}-4)(\sqrt{x}+2)=0$ ✓]
✓ both answers
✓ ✓ rejecting one & accept the other

OR

$$(x-8)^2=(-2\sqrt{x})^2$$
$$x^2-16x+64=4x$$
$$x^2-20x+64=0$$
$$(x-16)(x-4)=0$$
$$x=16 \text{ or } x=4$$
N/A

(5)

✓ squaring both sides
✓ std form
✓ factors
✓ x = 4
✓ rejecting x = 16

OR

$$y^2+2y-8=0$$
$$(y+4)(y-2)=0$$
$$y=-4 \text{ or } y=2$$
$16+2\sqrt{16}-8=16\neq 0$ NA
$4+2\sqrt{4}-8=0$ $x=4$

(5)

✓ Let $y=\sqrt{x}$
✓ factorisation
✓ 2 y- values
✓ rejecting x = 16
✓ accepting x = 4
If $x^2+2x-64=0$ $\frac{0}{5}$

1 3

$$(x^2+1)(x-1)=0$$
$$x^2+1=0 \text{ or } x-1=0$$
$$x=1$$

(2)

✓ interpretation
✓ answer
If $x=\pm\sqrt{-1}$ or 1 – (2 marks)
If $x=\sqrt{-1}$ or 1 – (1 mark)
If $x=\pm\sqrt{-1}$ (0 marks)
Answer only x = 1 (2 marks)

1 4

$|4-x| \le 20$

$-20 \le 4 - x \le 20$

$-24 \le -x \le 16$

$x \le 24 \text{ and } x \ge -16$

Or $-16 \le x \le 24$

(4)

- ✔ removing | |
- ✔ transposing 4
- ✔ changing inequality Sign
- ✔ correct values

If or / , -1

$|4-x| \le 20$

$|4-x|^2 \le 20^2$

$16 - 8x + x^2 \le 400$

$x^2 - 8x - 384 \le 0$

$(x-24)(x+16) \le 0$

$-16 \le x \le 24$

(4)

- ✔ squaring both sides
- ✔ std form
- ✔ factors
- ✔ answer

$|4-x| \le 20$

$4 - x \le 20$	$-(4-x) \le 20$
$-x \le 16$	$x \le 24$
$x \ge -16$	$x \le 24$

$-16 \le x \le 24$

(4)

- ✔ definition
- ✔ accuracy
- ✔ two inequalities
- ✔ answer

1 2

$\frac{3-x}{x+7} \ge 0$

start with $\sqrt{\frac{3-x}{x+7}} \ge 0$ BD max $\frac{3}{5}$

no ≥ 0, but use a number line, x > -7 and x ≤ 3 $\frac{4}{5}$

- ✔ ✔ removing square & setting inequality

* = 0 1 mark

both 3 and –7 1 mark

– x > 0 & x + 7 > 0 [BD $\frac{3}{5}$]

		-7		3	
$3-x$	+		+	0	−
$x+7$	−	0	+		+
$\frac{3-x}{x+7}$	−	UD	+	0	−

$-7 < x \le 3,$

$-7 \le x \le 3$ $\left(\frac{4}{5} marks\right)$

(5)

- ✔ critical values (for both)
- ✔ ✔ each inequality

1 3

For non-real roots $\Delta < 0$

$b^2 - 4ac < 0$

$k^2 - 8k < 0$

$k(k-8) < 0$

$0 < k < 8$

[no mention of $\Delta < 0$ BD max $\frac{2}{5}$ marks]

(5)
[25]

✓ $\Delta < 0$
✓ $b^2 - 4ac$
✓ sustitution in Δ

✓ ✓ critical values & ienquality signs

Question 2

2 1 1 1

$xy = k$ or $y = \frac{k}{x}$ [but not $y = \frac{x}{k}$ $\frac{0}{3}$]

$(4)(2) = k$

$f(x) = \frac{8}{x}$ or $xy = 8$ or $y = \frac{8}{k}$

(3)

✓ formula
✓ substitution in formula
✓ equation

1 2

$x^2 + y^2 = r^2$

$r^2 = 16 + 4 = 20$

$g(x) = \sqrt{20 - x^2}$

(3)

✓ formula
✓ substitution
✓ equation

OR

$g(x) = \sqrt{r^2 - x^2}$

$2 = \sqrt{r^2 - 4^2}$

$4 = r^2 - 16$

$r^2 = 20$

$g(x) = \sqrt{20 - x^2}$

$If\ y = \sqrt{x^2 - r^2}$
$2 = \sqrt{16 - r^2}$
$r = \sqrt{12}$
$BD\ \frac{1}{3}\ marks$

(3)

✓ formula
✓ substitution

or $x^2 + y^2 = 20,\ y \geq 0$
✓ equation

2 2

$y = |x - p|$

$(4, 2)$ $2 = |4 - p|$

$4 - p = 2$ *or* $-4 + p = 2$

$p = 2$ *or* $p = 6$

From sketch $p = 2$

$2 = \|4 - p\|$
$4 - p = 2$
$p = 2\ \ BD\frac{2}{4}$
$\|4 - p\| = 2$
$\|4 - 2\| = 2$
$p = 2$ max $\frac{2}{4}$

(4) with $y = x - p$ and then substitutes

(4,2) $\frac{4}{4}$

✓ subsitution
✓ both equation
✓ both values of p
✓ selecting $p = 2$

OR

$$m = \frac{{}^{y}A - {}^{y}C}{{}^{x}A - {}^{x}C}$$

$C(p,0)$ & $m = 1$

$$\frac{4-p}{2-0} = 1$$

$4 - p = 2 \Rightarrow p = 2$

✔ form
✔ ✔ point C & value of *m*
✔ simplification

2 3

$y = -x + 2$ *and* $y = \sqrt{20 - x^2}$

$-x + 2 = \sqrt{20 - x^2}$

$(-x+2)^2 = (\sqrt{20 - x^2})^2$

$x^2 - 4x + 4 = 20 - x^2$

$2x^2 - 4x - 16 = 0$

$2(x-4)(x+2) = 0$

$x = 4$ *or* $x = -2$

$x = 4$ N/A for B $x = -2$

$B(-2,4)$ or $x = -2$ and $y = 4$

(6)

$x - 2 = \sqrt{2 - x^2}$ $\frac{5}{6}$
✔ equating the equations
✔ squaring both sides
or $|x-2|^2 = \left(\sqrt{20 - x^2}\right)^2$
✔ std form
✔ factors
✔ correct values of *x*
✔ answer (answer only $\frac{1}{6}$)

2 4

$-2 \le x \le 4$

$x_B \le x \le 4$ (accept) or $x_B \le x \le x_A$

(2)
[18]

✔ ✔ answer
$-2 < x < 4$ (1 mark)
$x \varepsilon [-2,4]$ (✔ ✔)

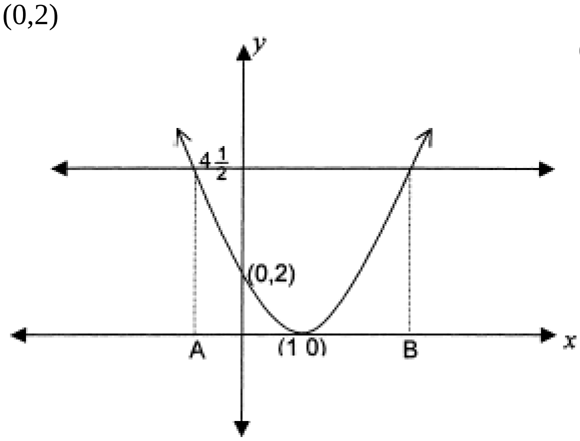
Question 3

3 1

y = 2(*x* - 1)²
Turning point (1,0)
x-intercept 2(*x* - 1)² = 0
x = 1

y-intercept *y* = 2(0 - 1)² = 2
(0,2)

graph not drawn, i e only calculations max $\frac{2}{4}$

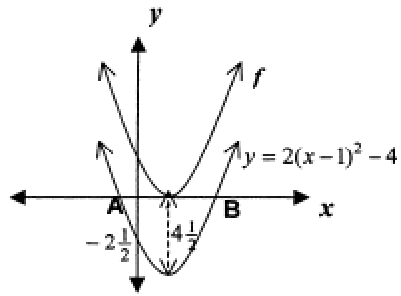


(4)

✔ *y*-intercept
✔ ✔ turning point
(1 mark: *x* value, 1 mark equal roots)
✔ shape

OR

3 1



3 2 2 1 $k \geq 0$ For correct answers non-graphical techniques $\frac{1}{2}$ (2) 2 is C/A marked form 3 1

✔ ✔ answer, $k > 0$ ✔

3 2 2 2 $k < -2$ / accept $k \leq -2$ (2) ✔ ✔ answer

3 3 off at A and B (2) ✔ ✔ on graph

[Calculation, i e not using graph max $\frac{1}{2}$]
Draw line $y = 4½$ only ✔
Calculate x values and show A and B on x-axis ✔ ✔

at points of intersection max $\frac{1}{2}$

[10]

Question 4

4 1

$$f(x) = 2x^3 + ax^2 + ax - 2$$
$$f(-\tfrac{1}{2}) = b$$
$$2(-\tfrac{1}{2})^3 + a(-\tfrac{1}{2})^2 + a(-\tfrac{1}{2}) - 2 = b$$
$$-\frac{1}{4} + \frac{1}{4}a - \frac{1}{2}a - 2 = b$$
$$-1 + a - 2a - 8 = 4b \qquad (\times \text{ by LCD} = 4)$$
$$a = -4b - 9$$

(5)

✔ $x = -\frac{1}{2}$ & knowing rem thm ✔
✔ substitution
✔ simplification
✔ answer

4 2

$$f(x) = (x-8)p(x) + k$$
$$f(2) = 0$$
$$(2-8)p(2) + k = 0$$
$$k = 6p(2)$$
$$\text{but } p(2) = 5$$
$$(2-8)(5) + k = 0$$
$$k = 6(5)$$
$$= 30$$

(5)
[10]

✔ knowing factor theorem
✔ substitution
✔ knowing remainder theorem
✔ substitution
✔ answer

$$\boxed{\begin{array}{l} \textit{If } p(2)=5 \\ \textit{then } p(x)=x+3 \\ f(x)=(x-8)(x+3)+k \\ f(2)=(-6)(5)+k=0 \\ k=30 \\ \max \tfrac{4}{3}\ \textit{marks} \end{array}}$$

Question 5

5 1

$$1+4\log_4 3\log_9 \frac{1}{2} = 1+4\left(\frac{\log 3}{\log 4}\frac{\log \frac{1}{2}}{\log 9}\right)$$

$$=1+4\,\frac{\log 3}{2\log 2}\,\frac{-\log 2}{2\log 3}$$

$$=1+4\left(-\frac{1}{4}\right)=1-1=0$$

(3)

✔ change of base (once only)

✔ simplification
write in terms of log 2 & log 3

✔ for -1

OR

$$1+4(\log_{2^2} 3)(\log_{3^2} 2^{-1})$$ ✔

$$=1+4(\tfrac{1}{2}\log_2 3)(-\tfrac{1}{2}\log_3 3)$$ ✔

$$=1+4(-\tfrac{1}{4})=1-1=0$$ ✔

OR

$$1+4\log_4 3\frac{\log_4 \frac{1}{2}}{\log_4 9}$$ ✔

$$=1+4\log_4 3\frac{\log_4 \frac{1}{2}}{2\log_4 3}$$

$$1+2\log_4 \tfrac{1}{2}$$ ✔

$$1+\log_4 (\tfrac{1}{2})^2 = 1-1=0$$

✔ If start with = 0 max $\frac{2}{3}$

5 2 2 1

$$2^{x+1}+7=2^{2-x}$$

$$2\ 2^x+7=\frac{4}{2^x}$$

$$2\ 2^{2x}+7\ 2^x-4=0$$

$$(2^x+4)(2\ 2^x-1)=0$$

$$2^x=-4 \quad \textit{or} \quad 2\ 2^x=1$$

$$\text{impossible} \quad 2^x=\tfrac{1}{2}=2^{-1}$$

$$x=-1$$

(6)

✔ exponential law $\frac{4}{2^x}$

✔ multiplying by LCD & std

✔ factorizing or $k=2^x$

✔ both equations or 2k = 1 only

✔ $2^x>0$

✔ x = value

$$\boxed{\begin{array}{l} 2k+7-\frac{4}{k}=0 \\ 2k^2+7k-4=0 \\ (2k-1)(k+4)=0 \\ k=\frac{1}{2} \text{ or } k=-4 \\ 2^x=2^{-1} \text{ or } 2^x=-4 \\ x=-1 \qquad\qquad N/A \end{array}}$$

2 2

$$x\log 5=\log\frac{3}{5}+x\log 3$$

(4)

$$\log 5^x=\log 3^x=\log\frac{3}{5}$$

✔ log law

$$\log\left(\frac{5}{3}\right)^x=\log\frac{3}{5}$$

✔ single log

$$\left(\frac{5}{3}\right)^x=\frac{3}{5}=\left(\frac{5}{3}\right)^{-1}$$

✔ common base
✔ answer

$$x=-1$$

OR

$$x\log 5=\log\frac{3}{5}+x\log 3$$

(4)

$$x\log 5=\log 3-\log 5+x\log 3$$

$$\log 5^{x+1}=\log 3^{x+1}$$

$$5^{x+1}=3^{x+1}$$

$$x+1=0$$

$$x=-1$$

✔ log law
✔ removing logs
✔ $5^0 = 3^0$
✔ answer

OR

$$x(\log 5-\log 3)=\log\frac{3}{5}$$

(4)

✔ factorisation

$$x\log\frac{5}{3}=\log\frac{3}{5}=-\log\frac{5}{3}$$

$$\boxed{\begin{array}{l} x=\dfrac{\log\frac{3}{5}}{\log\frac{5}{3}} \\ =-\log_{\frac{3}{5}}\frac{3}{5} \\ =-1 \end{array}}$$

$$(x+1)\log\frac{5}{3}=0$$

$$x+1=0$$

$$x=-1$$

✔ single log
✔ factorisation

✔ x-value

OR

$$0=\log\frac{3}{5}+x(\log 3-\log 5)$$

(4)

✔ factorisation

$$=\log\frac{3}{5}+x\log\frac{3}{5}$$

$$0=(x+1)\log\frac{3}{5}$$

$$x=-1$$

✔ single log
✔ factorisation
✔ x-value

OR

$x\log 5 = \log\frac{3}{5} + x\log 3$

$\log 5^x = \log\frac{3}{5}3^x$ ✓ ✓ ✓ ✓

$5^x = \frac{3}{5}3^x$

$\frac{5x}{3x} = \frac{3}{5} = \left(\frac{5}{3}\right)^{-1}$

$x = -1$

2 3 $\log(2x-3) \geq -\log(x-2)$

$\log(2x-3) + \log(x-2) \geq 0$

$\log(2x-3)(x-2) \geq \log 1$ ✓ single log

$2x^2 - 7x + 6 \geq 1$ ✓ removing of logs

if $2x^2 - 7x + 6 \geq 0$ BD $\frac{7}{10}$

if $2x^2 - 7x + 5 \leq 1$ BD $\frac{9}{10}$

$2x^2 - 7x + 5 \geq 0$ (10) ✓ std form

*A $(x-1)(2x-5) \geq 0$ ✓ factorization

$x \leq 1$ or $x \geq \frac{5}{2}$ ✓ ✓ each answer

By definition of logs ✓ use of definition

if ≥ is used –1

$2x-3>0$ and $x-2>0$ ✓ values of *x*

*B $x > \frac{3}{2}$ and $x > 2$

and $x \leq 1$ or $x \geq \frac{5}{2}$ ✓ ✓ solution

writes down final solution from *A to *B full marks

$x \geq \frac{5}{2}$

OR

$\log(2x-3) \geq \log\frac{1}{x-2}$ ✓

$2x - 3 \geq \frac{1}{x-2}$ ✓

$\frac{(2x-3)(x-2)-1}{(x-2)} \geq 0$ ✓ ✓

$\frac{2x^2 - 7x + 5}{x-2} \geq 0$ ✓ + last 4 marks

$\frac{(2x-5)(x-1)}{x-2} \geq 0$

$x \geq 2\frac{1}{2}$ or $1 \leq x \leq 2$ 6 marks

5 3 $f(x)=3^{-x}$

3 1

$y=3^{-x}$

$\log_3 y=-x$

For f^{-1} $x=3^{-y}$ $x \leftrightarrow y$

$-y=\log_3 x$ $\log_3 x=-y$

$y=-\log_3 x$ $y=-\log_3 x$

Or $y=\log_{\frac{1}{3}} x$ OR or $y=\log_{\frac{1}{3}} x$

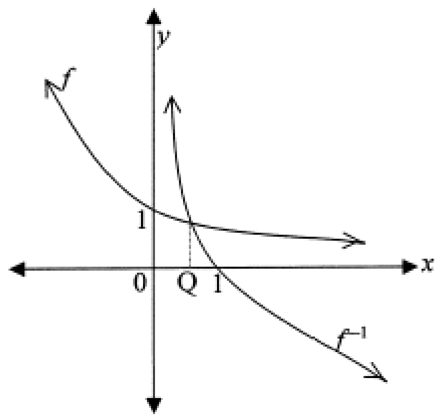
(2)

✓ writing in log form

$\boxed{\begin{array}{l} y=\left(\frac{1}{3}\right)^x \ (1\ mark) \\ f^{-1}(x)=\log_{\frac{1}{3}} x \ (1\ mark) \end{array}}$

✓ equation

3 2



(4)

✓ curve of f / shape
✓ y-intercept
✓ curve of f^{-1} / shape
✓ x-intercept

no or incorrect label: $\frac{3}{4}$
if f incorrect max 2 marks

3 3 Q on graph

(1)

✓ answer
no mark if no intersect

5 4

Let $x=\log_3 90$

$90=3^x$

$30=\frac{3^x}{3}=3^{x-1}$

$3^{3\ 096}=3^{x-1}$

$3,096=x-1$

$x=4,096$

$\boxed{\begin{array}{l} 30=3^{3\ 096} \\ 3\ 30=3\ 3^{3\ 906}\ (1\ mark) \\ 90=3^{4\ 096}\ (1\ mark) \\ \log_3\ 90=4,096\ (1\ mark) \end{array}}$

OR

✓ expressing in log from
✓ exponential form

(3)

✓ answer

$30 = 3^{3\,096}$

$\log_3 30 = 3{,}096$

$\log_3 90 = \log_3 3 + \log_3 30$

$= 1 + 3{,}096$

$= 4{,}096$

(3)

✔ log form
✔ log law

✔ answer

OR

$3^{3\,096} = 30$

$\log_3 30 = 3{,}096$

$\log_3 30 + \log_3 3 = 3{,}096 + \log_3 3$

$\log_3 90 = 3{,}096 + 1$

$= 4\ 096$

(3)

✔
✔

✔

[33]

Question 6

6 1

$S_n = \frac{n}{2}(7n + 15)$

1 1

$425 = \frac{n}{2}(7n + 15)$

$850 = 7n^2 + 15n$

$7n^2 + 15n - 850 = 0$

$(7n + 85)(n - 10) = 0$

$n = -\frac{85}{7} / -12{,}14$ or $n = 10$

N/A

(5)

✔ substitution $S_n = 425$

✔ standard form
✔ factors or
$n = \frac{-15 \pm \sqrt{24025}}{14}$
✔ accepting $n = 10$
✔ rejecting the other solution

correct answer only $\frac{3}{5}$

1 2

$T_6 = S_6 - S_5$

$= \frac{6}{2}(7 \times 6 + 15) - \frac{5}{2}(7 \times 5 + 15)$

$= 3(57) - 5(25)$

$= 171 - 125$

$= 46$

(4)

✔ ✔ interpretation
if $d = S_2 - S_1 = 18$ (max 1)
✔ substitution

✔ answer

OR

$T_1 = S_1 = \frac{1}{2}(7.1 + 15) = 11$

$T_2 = S_2 - S_1 = \frac{2}{2}(7.2 + 15) - 11 = 18$

$d = T_2 - T_1 = 7$

$T_6 = a + 5d$

$= 11 + 5.7$

$= 46$

(4)

✔ calculating term 1

✔ calculating 2nd term
✔ common difference

✔ answer

OR

$425 = 5(2a + 9d)$

$85 = 2a + 9d \quad (1) \quad 1\ mark$

$171 = 3(2a + 5d)$

$57 = 2a + 5d \quad (2) \quad 1\ mark$

$(1) - (2) \quad 28 = 4d$

$d = 7$

$a = 11 \quad 1\ mark\ for\ a\ and\ d$

$T_6 = 11 + 5.7 = 46 \quad 1\ mark$

6 2 with a = 400, r = 1,1 — ✔ values for a and r

2 1

$T_7 = ar^{n-1}$

or

$T_7 = 400\left(1 + \frac{10}{100}\right)^6$ (4) — ✔ formula ✔ substitution ✔ answer

$= 400(1,1)^6$

$= R708,62$

Accept T_7 even if AP Max $\frac{1}{4}$ marks

OR

, 440, ✔ 484, ✔ 532,40, 585,64, 644,20, 708,62 ✔ ✔ — marks

2 2

$S_n = \frac{a(r^n - 1)}{r - 1}$ (3) — ✔ formula

$= \frac{400\left[(1,1)^7 - 1\right]}{1,1 - 1}$ — ✔ substitution

$= R3\ 794,87$ — ✔ answer

Accept S_7 even if AP Max $\frac{1}{3}$ marks

OR

+ 440 + 484 + 532,40 + 585,04 + 644,20 = 3794,86

all the terms ✔ ✔ , answer ✔

$T_6 = 3086,24 \quad (\frac{2}{3})$

2 3

$T_n = ar^{n-1} > 1500$

$400(1,1)^{n-1} > 1500$

$(1,1)^{n-1} > 3,75$

$n-1 > \frac{\log 3,75}{\log 1,1}$

$n-1 > 13,9 \quad n > 14,9$

in the 15th month

OR

7 = 708,62, 779, 857, 943, 1037, 1141, 1255, 1380, T_{15} = 1518

In the 15th month

OR

$S_n > 1500$

$\frac{400(1,1)^n - 1}{1.1-1} > 1500$

$400[(1,1)^n - 1] > 150$

$(1,1)^n - 1 > 0,375$

$(1,1)^n > 1,375$

$n > 3,35$

$n = 4 \quad (\text{BD } \frac{4}{5})$

(5)

✔

✔ substitution

✔ log law

✔ simplification

✔ value of *n* & answer

if n = 14,9 max $\frac{3}{5}$

Full marks

6 3

a + 1, a – 1, 2a - 5

3 1

$\frac{a-1}{a+1} = \frac{2a-5}{a-1}$ or $\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$(2a-5)(a+1) = (a-1)^2$

*A $a^2 - a - 6 = 0$

$(a-3)(a+2) = 0$

*B $a = 3$ or $a = -2$

$a = 3 \quad 4,\ 2,\ 1,$

$a = -2 \quad -1,\ -3,\ -9,$

$a = 3 \quad$ series convergent

$$\boxed{\begin{array}{l} OR \\ r = \frac{a-1}{a+1} \\ r = \frac{1}{2} \text{ or } r = 3(N/A) \\ a = 3 \end{array}}$$

OR

only 4, 2, 1 r = ½ and series convergent

(7)

✔ equation

✔ std form

✔ factors

✔ both values of *a*

✔ sequence when *a* = 3

✔ sequence when *a* = -2

✔ value of *a*

*if goes directly from *A to *B full marks*

marks

a = 3

OR

r only a = 3 max $\frac{1}{7}$

3 2		$S_\infty = \dfrac{a}{1-r}$ $\quad r = \frac{1}{2}$ $= \dfrac{4}{1-\frac{1}{2}}$ $= 2(4)$ $= 8$	(4) [32]	✔ ✔ formula & value for *r* ✔ substitution ✔ answer If working with a = -2 or with both values of a max $\frac{2}{4}$

Question 7

7 1		$f(x) = 3x - x^2$ $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} \quad h \neq 0$ $f'(x) = \lim_{h\to 0} \dfrac{3(x+h) - (x+h)^2 - (3x - x^2)}{h}$ $= \lim_{h\to 0} \dfrac{3x + 3h - x^2 - 2xh - h^2 - 3x + x^2}{h}$ $= \lim_{h\to 0} \dfrac{3h - 2xh - h^2}{h}$ $= \lim_{h\to 0} \dfrac{h(3 - 2x - h)}{h}$ $= \lim_{h\to 0}(3 - 2x - h)$ $= 3 - 2x$	(6)	✔ definition/formula ✔ substitution ✔ simplification/expansion ✔ simplification ✔ factorization $\boxed{-1 \ \lim_{h\to 0} = \text{ or } \lim_{h\to 0} \text{ missing}}$ ✔ answer answer only no marks
7 2	2 1	$xy = 5$ $y = \dfrac{5}{x} = 5x^{-1}$ $\dfrac{dy}{dx} = -5x^{-2}$ or $-\dfrac{5}{x^2}$	(2)	✔ *y* subject with negative exponent ✔ derivative
	2 2	$y = \dfrac{1 - 2x + \sqrt{x}}{x^2}$ $= x^{-2} - 2x^{-1} + x^{-\frac{3}{2}}$ $\dfrac{dy}{dx} = -2x^{-3} + 2x^{-2} - \frac{3}{2}x^{-\frac{5}{2}}$	(4)	✔ simplification ✔ ✔ ✔ derivative of each term C/A if simplifying incorrectly, but 3rd mark for similar difficulty of 3rd term Notation -1

7 3

$f(x) = 2x^2 + x - 1$ (6)

$f'(x) = -3$ ✔ ✔ derivative & = -3

$4x + 1 = -3$

$4x = -4$

$x = -1$ ✔ value of *x*

$y = 2(-1)^2 + (-1) - 1$ ✔ value of *y*

$= 0$

$y - y_1 = m(x - x_1)$ ✔ substitution

$y = -3(x + 1)$ ✔ equation

$y = -3x - 3$

$$\boxed{\begin{array}{l} \text{If } f'(-3) = -11 \\ f(-3) = 14 \\ y - 14 = -11(x + 3) \\ y = -11x - 19 \\ \text{max } \frac{4}{6} \end{array}}$$

$$\boxed{\begin{array}{l} y = mx + c \\ y = -3x + c \\ 0 = -3(-1) + c \\ c = -3 \\ y = -3x - 3 \end{array}}$$

7 4

$f(x) = x^3 - x^2 - 5x - 3$

$y - \text{intercept } (0, -3)$

$f(-1) = 0 \quad x + 1$ is a factor of $f(x)$ ✔ factor or (x - 3)

$f(x) = (x + 1)(x^2 - 2x - 3)$ ✔ quadratic factor

for x-intercepts $f(x) = 0$ ✔ $y = 0$

i e $(x + 1)^2 (x - 3) = 0$ ✔ factorization

$x = -1$ or $x = 3$ ✔ both values

For turning points $f'(x) = 0$ ✔ definition = 0

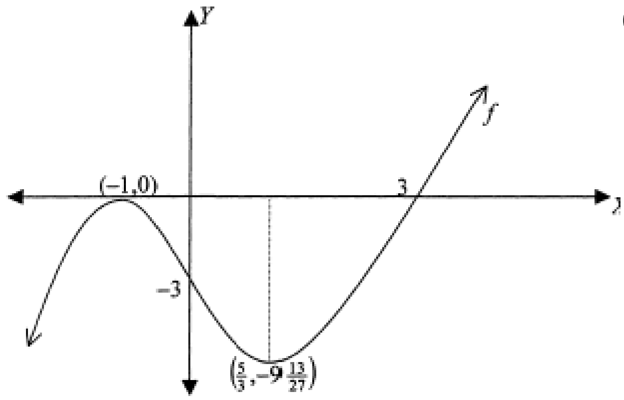
$3x^2 - 2x - 5 = 0$ ✔ derivative

$(3x - 5)(x + 1) = 0$ ✔ factorization

$x = \frac{5}{3}$ or $x = -1$ ✔ both values

$f(-1) = 0 \quad (-1, 0)$ ✔ *y*-value/TP

$f(\frac{5}{3}) = -9\frac{13}{27} \quad (\frac{5}{3}, -9\frac{13}{27})$ / $(\frac{5}{3}, -9{,}48)$ ✔ *y*-value/TP



(17)

✔ ✔ each turning point
✔ y-intercept on graph or in calculations
✔ x-intercept
✔ shape

[35]

Question 8

1 1

$A = \pi R^2 + \pi r^2$ (1)

$R + r = 200$ (2)

$r = 200 - R$

Subst (2) in (1)

$A = \pi R^2 + \pi(200 - R)^2$

$= \pi R^2 + \pi(40\ 000 - 400R + R^2)$

$= 2\pi R^2 - 400\pi R + 40\ 000\pi$

(4)

✔ ✔ equation on A (2 or 0)

✔ equation

✔ substitution

1 2

At minimum $\dfrac{dA}{dR} = 0$

i e $4\pi R - 400\pi = 0$

$R = \dfrac{400\pi}{4\pi}$

$= 100\ mm$

$r = 100\ mm$

(4)

✔ derivative = 0

✔ correct calculation of derivative

✔ value for R
✔ value for r

1 3 $R = r = 100$ one will not get the desired shape but a shape with <u>two equal circle</u> which <u>touch externally</u>.
Equal radius 1 mark
If a diagram is drawn, showing 2 touching cicles
(2 marks)

(2)

✔ ✔ valid explanantion

8 2 2 1

No profit $\Rightarrow P = 0$

$-\dfrac{3}{80}x^2 + 6x - 180 = 0$

$x^2 - 160x + 4800 = 0$

$(x - 40)(x - 120) = 0$

$x = 40\ km/h$ or $x = 120\ km/h$

(3)

✔ P = 0

✔ factorization
✔ both values of x

2 2

$P = -\frac{3}{80}x^2 + 6x - 180$

$\max P \; \frac{dP}{dx} = 0$

$-\frac{6}{80}x + 6 = 0$

$480 - 6x = 0$

$x = 80 \; km/h$

OR

$x = \frac{40+120}{2} = 80 \; km/h$

$P = -\frac{3}{80}(80)^2 + 6(80) - 180$

$P = -240 + 480 - 180$

$= R\;60,00$

OR

$x = -\frac{b}{2a}$

$x = -\frac{6}{2\left(\frac{-3}{80}\right)}$

$x = (-6)\left(-\frac{80}{6}\right)$

$x = 80$ km/h

$P = R\;60,00$

(5)

if units is left out in 8 2 2

✓ interpretation / = 0
✓ derivative

✓ value of x

✓ ✓ ✓

✓ substitution

✓ value for P

✓ formula

✓ substitution

✓ speed
✓ ✓ substitution & answer

8 2 2 3loss P < 0

$-\frac{3}{80}x^2 + 6x - 180 < 0$

$x^2 - 160x + 4800 > 0$

$(x-40)(x-120) > 0$

$30 \le x < 40$ km/h or $x > 120$ km/h

$30 \le x \le 40$ km/h or $x \ge 120$ km/h

OR

40 < x < 120 or

If 40 ≤ x ≤ 120

$\frac{1}{3}$

(3) ✓ setting up the inequality

[21] ✓ ✓ accept any of these 2 possible answers
ignore $x \ge 30$ (given)

Question 9

9 1 1 1 $x \le 60$
$y \le 100$

(2)

✓ ✓ each inequality
if x = M, y = B in inequalities –
if = sign left out –1 once

	C/A marks throughout the question		
1 2	$x + y \geq 80$	(1)	✓ inequality
1 3	$\frac{2}{3}x + \frac{1}{2}y \leq 60$ or $4x + 3y \leq 360$	(2)	✓ LHS ✓ RHS + inequality
9 2	$= 40x + 80y = P$	(1)	✓ equation
9 3	graph below	(1)	✓ either dotted line
9 4	(accept any x between 15 and 20)	(3)	profit if $x = 15$ and $y = 100$ ✓ ✓ ✓ or 0
	If m = -2 is used for search line give $\frac{3}{3}$ for (60,40) and $\frac{2}{2}$ for $P_{max} = 5600$		
9 5	$P_{max} = 40(15) + 80(100)$ $= R\ 8\ 600$	(2)	✓ substitution ✓ answer
9 6	$\frac{2x}{3} + \frac{y}{2} = 50$ $\Rightarrow 4x + 3y = 300$ max P now if x = 0 and y = 100 i e 0 type M, 100 type B $P_{max} = 40(0) + 80(100)$ $= R\ 8\ 000$	(4)	✓ ✓ values of x & y If m = -2 feasible region is lin segment Answer (60, 20) P_{max} = R4000 ✓ answer
		[16]: 200