Mathematics - HG – Mar 2003 National Paper 2

INSTRUCTIONS

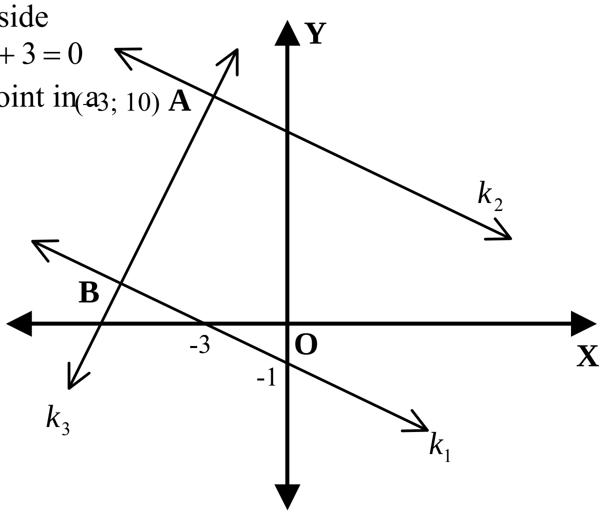
1. This paper consists of 10 pages with 10 questions.
2. A formula sheet is included on page 10 in the question paper. Detach it and use it to answer the questions in this question paper.
3. Answer ALL the questions.
4. All the necessary working details must be shown.
5. Clearly number all the answers correctly.
6. The diagrams are not drawn to scale.
7. A diagram sheet is included. Detach it and place it inside the ANSWER BOOK.
8. Non-programmable calculators may be used, unless the question states otherwise.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALYTICAL GEOMETRY

NOTE: - USE ANALYTICAL METHODS IN THIS SECTION.
- CONSTRUCTION AND MEASUREMENTS ARE NOT TO BE USED.

QUESTION 1

1.1 In the diagram alongside k_1 is the line $x + 3y + 3 = 0$ and A(–3; 10) is a point in a Cartesian plane.



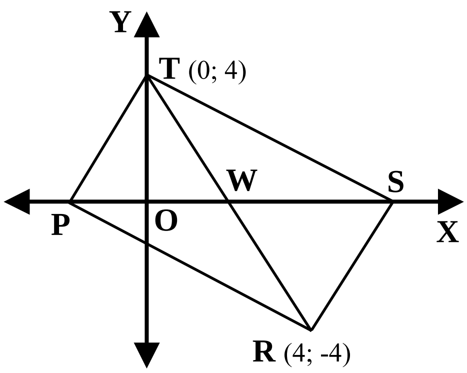
1.1.1 Determine the equation of line k_2 if $k_2 \,||\, k_1$ and k_2 passes through point A. (3)

1.1.2 Determine the equation of line k_3 if $k_3 \perp k_1$ and k_3 passes through point A. (4)

1.1.3 Calculate the distance AB between the lines k_1 and k_2. Leave the answer in surd form if necessary. (5)

1.1.4 If P(x; y) is a point on k_1 such that BP = AB, calculate the possible co-ordinates of P. (6)

1.2 In the diagram alongside, P, R(4; – 4), S and T(0; 4) are the vertices of a rectangle. P and S lie on the x-axis. The diagonals intersect at W.



1.2.1 Show that the co-ordinates of S are $\left(2 + 2\sqrt{5}\,;\ 0\right)$ (5)

1.2.2 Determine the gradient of TS rounded off to two decimal digits. (2)

1.2.3 Calculate $R\hat{T}S$ rounded off to two decimal digits. (4)

[29]

QUESTION 2

2.1 The point P(x; y) is twice as far from the point A (4; –2) as it is from the origin. Prove that the equation of the locus of P is $3x^2 + 3y^2 + 8x - 4y - 20 = 0$ (4)

2.2 2.2.1 Show that the equation of the tangent to the circle $x^2 + y^2 - 4x + 6y + 3 = 0$ at the point (5;–2) is $y = -3x + 13$ (8)

2.2.2 If T(x; y) is a point on the tangent in QUESTION 2.2.1, such that its distance from the centre of the circle is $\sqrt{20}$ units, determine the values of x and y (7)

2.3 In an attempt to find the condition that the line $y = mx + c$, $m \neq 0$, is a tangent to the graph of $y^2 + 4x = 0$, the following solution was given.
State the line in which an error appears and give the correct answer for that line.

2.3.1 The line meets the graph where $y^2 + 4\left(\dfrac{y-c}{m}\right) = 0$

2.3.2 That is where $my^2 + 4y - 4c = 0$

2.3.3 If the line is a tangent to the graph, this equation will have real unequal roots.

2.3.4 The required condition therefore is $mc = -1$ (3)

[22]

TRIGONOMETRY

QUESTION 3

3.1 If $\operatorname{cosec}\theta = \frac{1}{2k} + \frac{k}{2}$ $(0 < k < 1)$ and $90° \leq \theta \leq 270°$, determine, with the aid of a diagram, the value of $\operatorname{cosec}\theta + \cot\theta$ in terms of k (7)

3.2 Simplify to one trigonometric ratio of α: (9)

$$\frac{2\tan(180° - \alpha) - 2\cot(\alpha - 180°)}{\operatorname{cosec}(\alpha - 90°)\sin(360° - \alpha)}$$

3.3 Determine the values of $x \in [-90°; 90°]$ for which $\tan^2 x = \frac{\sin 600° \tan(-300°)}{\cos(-120°)}$ (6)

[22]

QUESTION 4

4.1 Solve for x in $\tan x = \sin 2x$ if $x \in [-180°; 180°]$. (11)

4.2 Use the set of axes provided on the diagram sheet to draw sketch graphs of $f(x) = \tan x$ and $g(x) = \sin 2x$ for $x \in [-180°; 180°]$. Indicate the intercepts with the axes as well as the co-ordinates of any turning points of the graphs. (6)

4.3 Use the graphs in QUESTION 4.2 as well as the answers to QUESTION 4.1 to determine the values of $x \in [-180°; 180°]$ for which $\tan x \geq \sin 2x$ (5)

[22]

QUESTION 5

5.1 Determine the general solution of x, rounded off to **TWO** decimal digits, if: (8)

$$3\sin x - 4\operatorname{cosec} x + 4 = 0$$

5.2 If $\sin(\theta - \alpha) = k\sin(\theta + \alpha)$, $k \neq 1$, determine $\tan\theta$ in terms of k and $\tan\alpha$ (6)

5.3

5.3.1 Prove the identity: $\frac{\tan 2A}{\tan A} = \frac{2\cos^2 A}{\cos 2A}$ (6)

5.3.2 For which values of $A \in [0°; 90°]$ is the identity not valid? (3)

[23]

QUESTION 6

C

6.1 6.1.1 In ΔABC, $90° < A < 180°$ (5)

Redraw this diagram in the answer book, or use the diagram on the diagram sheet to prove that

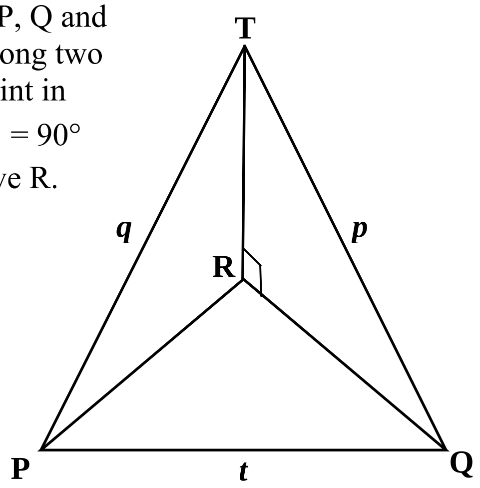
$a^2 = b^2 + c^2 - 2bc \cos A$

6.1.2 Hence write cos A in terms of a, b and c (1)

6.1.3 Deduce that (3)

$$1 - \cos A = \frac{(a-b+c)(a+b-c)}{2bc}$$

6.2 In the diagram alongside, P, Q and R represent three points along two walls of a room. R is a point in the corner such that $P\hat{R}Q = 90°$ T is a point vertically above R. QR = PR = 100 units.



6.2.1 Prove that $PQ = 100\sqrt{2}$ units. (2)

6.2.2 Use the result of QUESTION 6.1.3, or otherwise, to prove in ΔPTQ that:

$$\cos P\hat{T}Q = 1 - \frac{t^2}{2p^2}$$ (5)

6.2.3 Calculate the size of $P\hat{T}Q$, rounded off to the nearest integer, if $p = 2t$ (3)

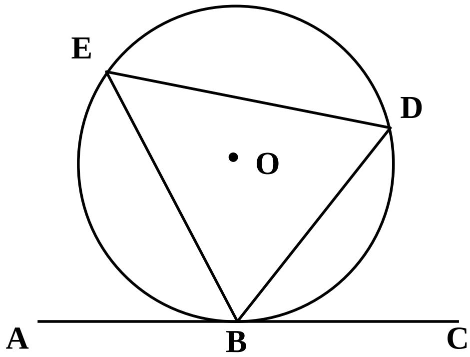
[19]

EUCLIDEAN GEOMETRY

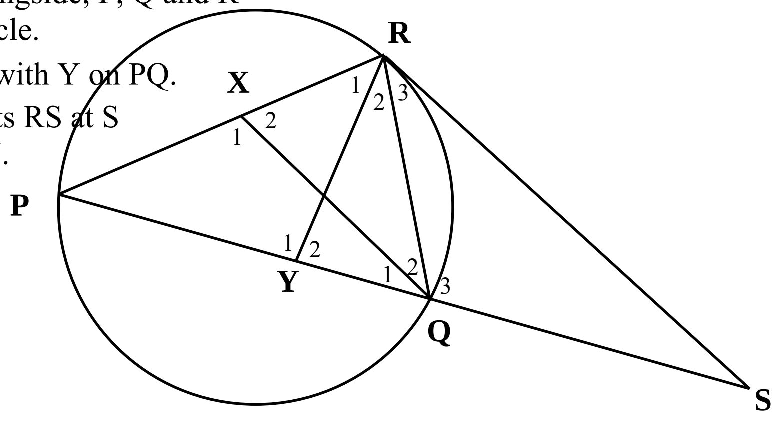
NOTE: - DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEET OR REDRAWN IN THE ANSWER BOOK.
- DETACH THE DIAGRAM SHEET FROM THE QUESTION PAPER AND PLACE IT INSIDE THE ANSWER BOOK.
- GIVE A REASON FOR EACH STATEMENT.

QUESTION 7

7.1 In the diagram alongside, DB is a chord of the circle such that $D\hat{B}C = D\hat{E}B$. ABC is a straight line. Redraw this diagram in the answer book or use the diagram on the diagram sheet, to prove the theorem which states that ABC is a tangent to the circle at B. (6)



7.2 In the diagram alongside, P, Q and R are points on a circle.
YR bisects $P\hat{R}Q$ with Y on PQ.
PQ produced meets RS at S such that SR = SY.
QX || SR



Prove that:

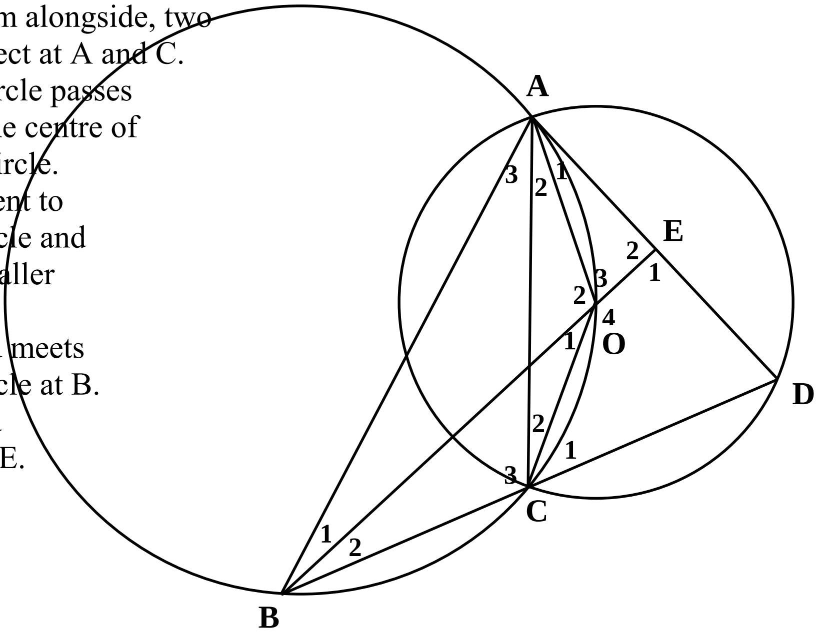
7.2.1 SR is a tangent to the circle (6)

7.2.2 QR is a tangent to the circle through Q, X and P (3)

[15]

QUESTION 8

In the diagram alongside, two circles intersect at A and C.
The larger circle passes through O, the centre of the smaller circle.
AD is a tangent to the larger circle and meets the smaller circle at D.
DC produced meets the larger circle at B.
BO produced meets AD at E.
Let $\hat{A}_1 = x$.



Prove that:

8.1 OA bisects $D\hat{A}C$ (5)

8.2 $\hat{D} = 90° - x$ (3)

8.3 AE = ED (5)

8.4 BA = BD (4)

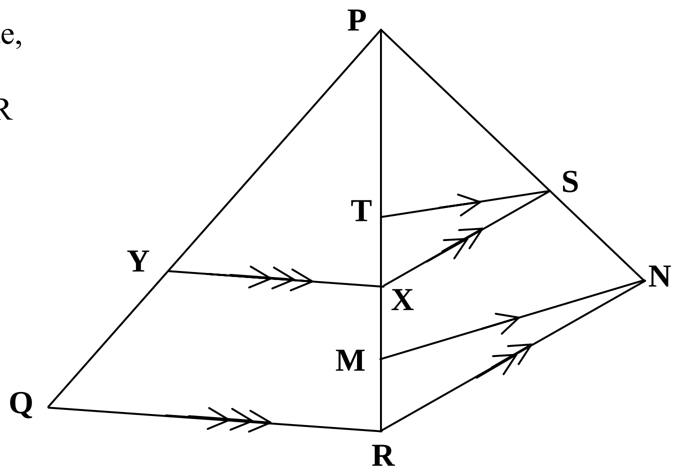
[17]

QUESTION 9

In the diagram alongside,
YX || QR and XS || RN
M is the midpoint of XR
TS || MN

PY = 4 units

PQ = 7 units



Write down the values of the following ratios, giving reasons:

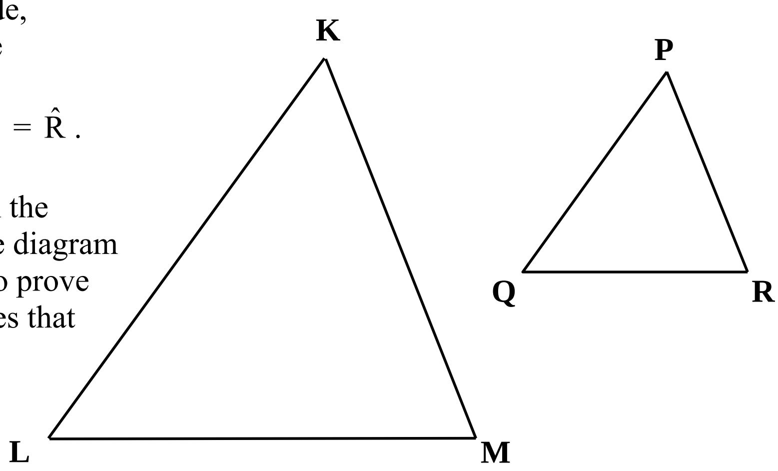
9.1 PS:SN (4)

9.2 MN:TS (3)

9.3 PX:XM (3)
[10]

QUESTION 10

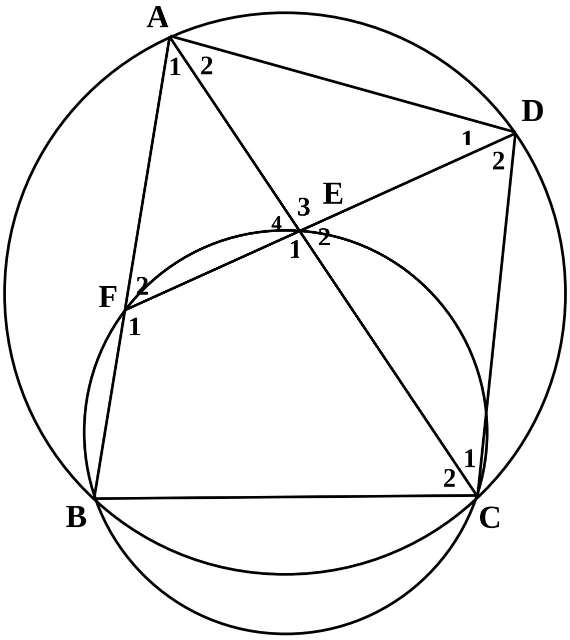
10.1 In the diagram alongside, ΔKLM and ΔPQR are two triangles such that $\hat{K} = \hat{P}$, $\hat{L} = \hat{Q}$ and $\hat{M} = \hat{R}$. (7)



Redraw this diagram in the answer book, or use the diagram on the diagram sheet, to prove the theorem which states that

$$\frac{KL}{PQ} = \frac{KM}{PR}$$

10.2 In the diagram alongside, two circles intersect at B and C. A is a point on the larger circle. AB and AC intersect the smaller circle at F and E respectively. D is a point on the larger circle.



Prove that:

10.2.1 $AD^2 = AC.AE$ (8)

10.2.2 $\Delta AFE \,|||\, \Delta ACB$ (4)

10.2.3 $AD^2 = AB.AF$ (2)

[21]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n - 1)d$ $\quad$ $S_n = \frac{n}{2}(a + l)$ $\quad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$T_n = a.r^{n-1}$ $\quad$ $S_n = \frac{a(1 - r^n)}{1 - r}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $\quad$ $S_\infty = \frac{a}{1 - r}$

$\mathbf{A} = \mathbf{P}\left(1 + \frac{\mathbf{r}}{100}\right)^{\mathbf{n}}$ $\quad$ $\mathbf{A} = \mathbf{P}\left(1 - \frac{\mathbf{r}}{100}\right)^{\mathbf{n}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta$$

$$\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$x^2 + y^2 = r^2$ $\quad$ $(x - p)^2 + (y - q)^2 = r^2$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta ABC = \tfrac{1}{2}ab.\sin C$$