DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2004
SENIORSERTIFIKAAT-EKSAMEN - 2004

MATHEMATICS P2 : GEOMETRY
WISKUNDE V2 : MEETKUNDE

HIGHER GRADE
HOËR GRAAD

OCTOBER/NOVEMBER 2004
OKTOBER/NOVEMBER 2004

301-1/2

Marks: 200
Punte : 200

3 Hours
3 Ure

This question paper consists of 12 pages, 1 formula sheet and 5 diagram sheets.
Hierdie vraestel bestaan uit 12 bladsye, 1 formuleblad en 5 diagramvelle.

MATHEMATICS HG: Paper 2
Geometry

301 1 2   HG

INSTRUKSIES

1. Hierdie vraestel bestaan uit **9** vrae, 'n formuleblad en diagramvelle.

2. Gebruik die formuleblad om hierdie vraestel te beantwoord.

3. Maak die diagramvelle los van die vraestel en plaas dit in jou **ANTWOORDEBOEK.**

4. Die diagramme is nie volgens skaal geteken nie.

5. Beantwoord **AL** die vrae.

6. Nommer **AL** die antwoorde korrek en duidelik.

7. **AL** die nodige bewerkings moet aangedui word.

8. Nie-programmeerbare sakrekenaars mag gebruik word, tensy anders vermeld.

9. Waar nodig, sal die aantal desimale syfers waartoe antwoorde afgerond moet word in die vraag gemeld word.

INSTRUCTIONS

1.   This question paper consists of **9** questions, a formula sheet and diagram sheets.

2.   Use the formula sheet to answer this question paper.

3.   Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK.**

4.   The diagrams are not drawn to scale.

5.   Answer **ALL** the questions.

6.   Number **ALL** the answers correctly and clearly.

7.   **ALL** the necessary calculations must be shown.

8.   Non-programmable calculators may be used, unless otherwise stated.

9.   The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALITIESE MEETKUNDE

LET WEL: - GEBRUIK ANALITIESE METODES IN HIERDIE AFDELING.
- KONSTRUKSIE- EN METINGMETODES MAG NIE GEBRUIK WORD NIE.

VRAAG 1

In die diagram langsaan,

A (– 4 ; 5) , C (–1 ; – 4) en B (4 ; 1)

is die hoekpunte van 'n driehoek in

'n Cartesiese vlak.

CE ⊥ AB met E op AB.

E is die middelpunt

van reguitlyn CD.
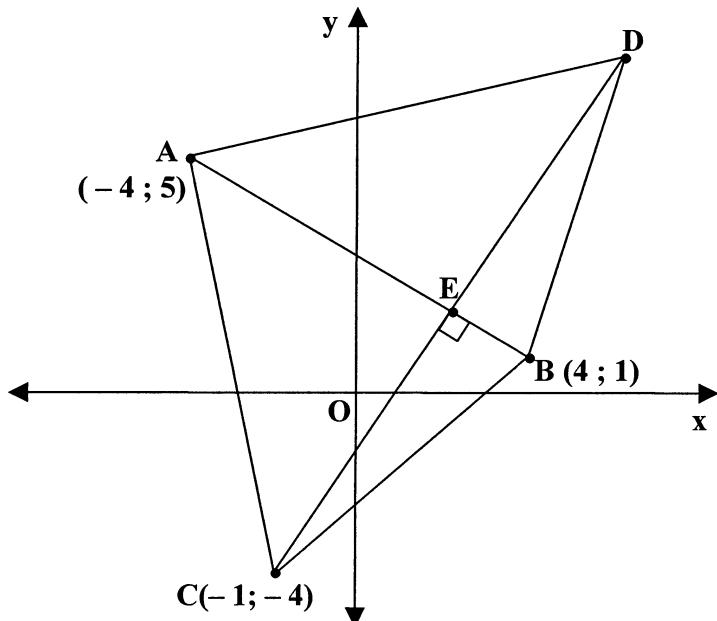
1.1   Bepaal:

1.1.1   Die vergelyking van CD   (6)

1.1.2   Die koördinate van E   (6)

1.1.3   Die vergelyking van die reguitlyn ewewydig aan AC en wat deur D gaan   (6)

1.2   Bepaal, deur alle bewerkings te toon, of die x-afsnit van reguitlyn CD ook op die hoogtelyn vanaf A tot BC lê.   (6)

[24]

ANALYTICAL GEOMETRY

NOTE:	-	USE ANALYTICAL METHODS IN THIS SECTION.
	-	CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.

QUESTION 1

In the diagram alongside,

A $(-4 ; 5)$, C $(-1 ; -4)$ and B $(4 ; 1)$

are the vertices of a triangle in a

Cartesian plane.

CE $\perp$ AB with E on AB.

E is the midpoint of

straight line CD.
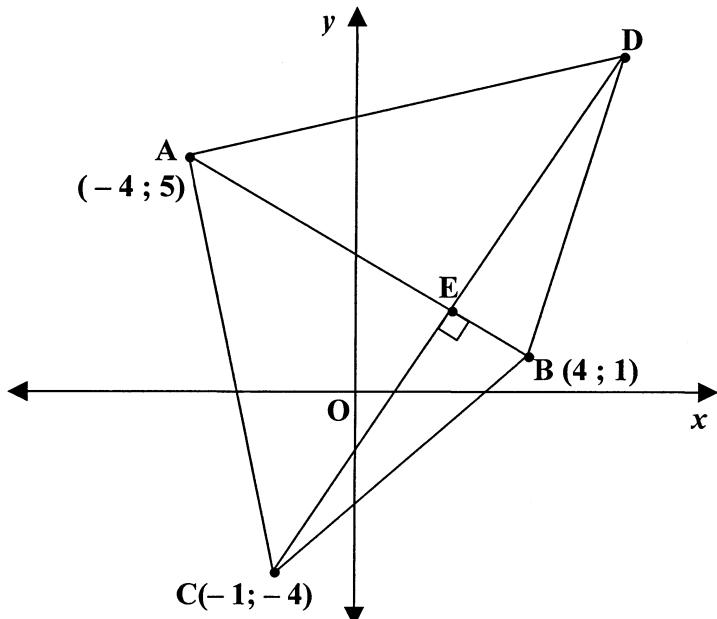
1.1   Determine:

    1.1.1   The equation of CD   (6)

    1.1.2   The coordinates of E   (6)

    1.1.3   The equation of the straight line parallel to AC and passing through D   (6)

1.2   Determine, showing all calculations, whether the x-intercept of straight line CD also lies on the altitude from A to BC.   (6)

[24]

VRAAG 2

2.1   In die diagram langsaan, sny die sirkel

met middelpunt C en met vergelyking

$x^2 - 6x + y^2 - 4y = 12$

die y-as by A.

BA is 'n raaklyn

aan die sirkel.

B lê op die x-as.

$B\hat{A}Y = \alpha$
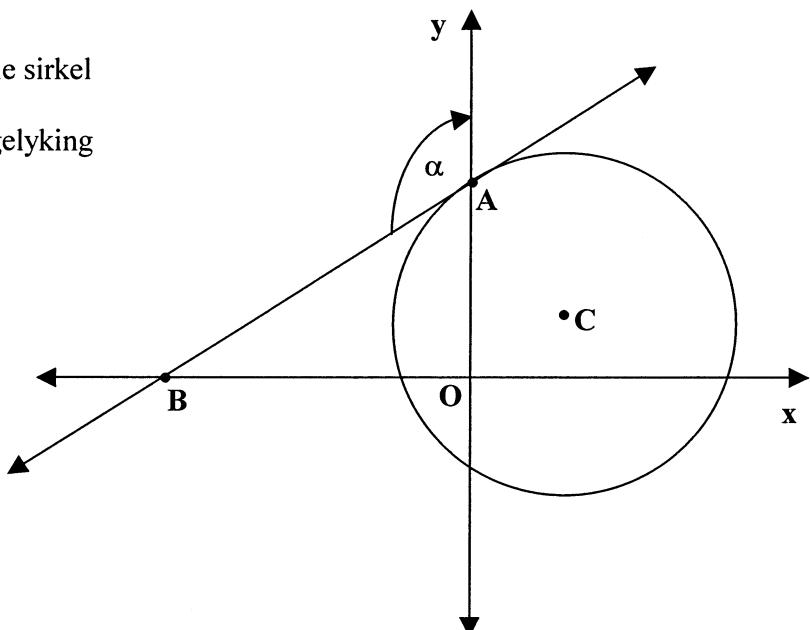
Bepaal:

2.1.1   Die koördinate van C   (5)

2.1.2   Die vergelyking van BA   (8)

2.1.3   Die grootte van α, afgerond tot EEN desimale syfer   (3)

2.2   P $(x ; y)$ en A $(3 ; 1)$ is twee punte in 'n Cartesiese vlak.

2.2.1   Bepaal die vergelyking van die lokus van P indien P ewe ver is van die reguitlyn $y = -2$ en van punt A.   (6)

2.2.2   Wat is die vorm van die lokus verkry in VRAAG 2.2.1?   (1)

2.2.3   Vervolgens, skryf die minimum waarde van y van die vergelyking verkry in VRAAG 2.2.1.   (2)

[25]

QUESTION 2

2.1　In the diagram alongside, the circle with

centre C and with equation

$x^2 - 6x + y^2 - 4y = 12$

cuts the y-axis at A.

BA is a tangent to the circle.

B lies on the x-axis.

$B\hat{A}Y = \alpha$
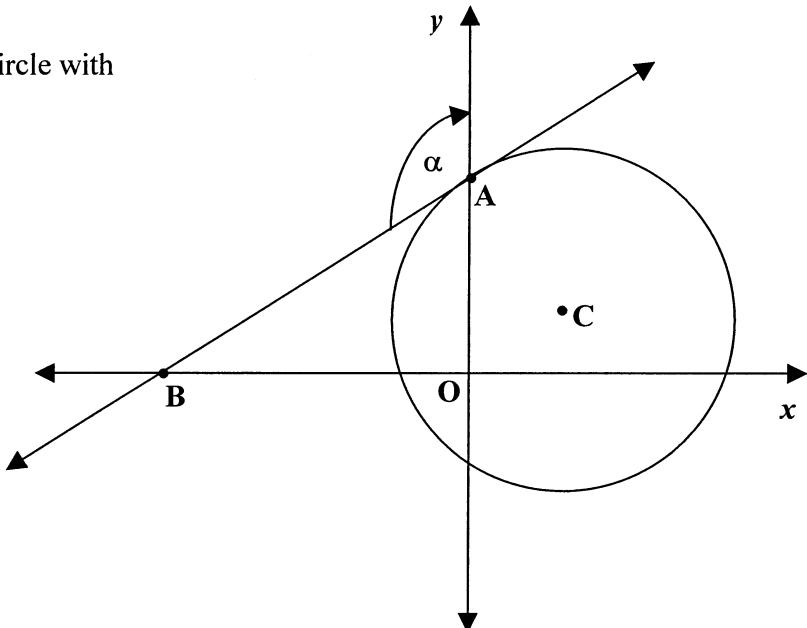
Determine:

2.1.1　The coordinates of C　(5)

2.1.2　The equation of BA　(8)

2.1.3　The size of α, rounded off to ONE decimal digit　(3)

2.2　P (x ; y) and A (3 ; 1) are two points in a Cartesian plane.

2.2.1　Determine the equation of the locus of P, if P is equidistant from the straight line $y = -2$ and from point A.　(6)

2.2.2　What is the shape of the locus obtained in QUESTION 2.2.1 ?　(1)

2.2.3　Hence, write the minimum value of y of the equation obtained in QUESTION 2.2.1.　(2)

[25]

TRIGONOMETRIE

VRAAG 3

3.1 In die diagram langsaan

$\stackrel{\wedge}{\text{POM}} = 90°$,

$\stackrel{\wedge}{\text{MOX}} = \theta$,

$\text{M}(6;a)$, $\text{P}(b;4)$

en $\sqrt{5} \cos \theta - 2 = 0$
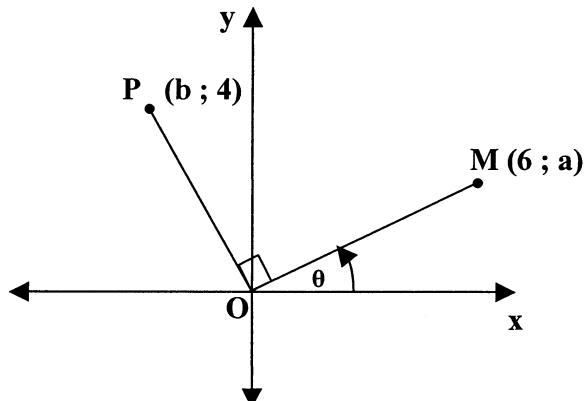
Bepaal, **sonder die gebruik van 'n sakrekenaar**, die numeriese waarde van:

3.1.1 a (3)

3.1.2 b (4)

3.2 Vereenvoudig die volgende **sonder die gebruik van 'n sakrekenaar:**

$$\frac{\tan(-420°) \,.\, \cos 156°}{\sin 492° \,.\, \sec 294°}$$ (9)

3.3 As α en θ komplementêre hoeke is en $\cot \alpha = k$, en $k \neq 0$, vereenvoudig

$$\frac{\cos ec^2(180° + \alpha) \,.\, \cos(\theta - 720°)}{\cos(\theta - 90°)}$$

in terme van k. (7)

[23]

TRIGONOMETRY

QUESTION 3

3.1 In the diagram alongside,

$$P\hat{O}M = 90°,$$

$$M\hat{O}X = \theta,$$

$$M(6;a), \quad P(b;4)$$

and $\sqrt{5} \cos\theta - 2 = 0$
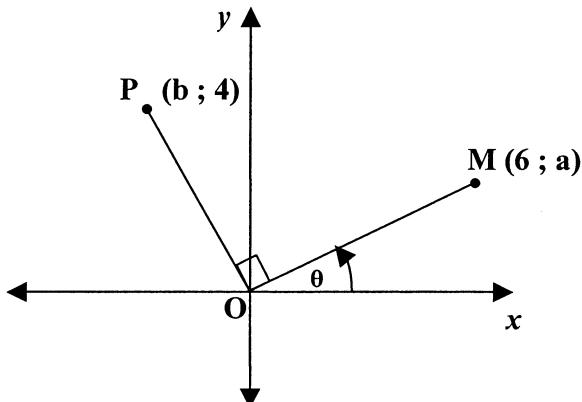
Determine, **without the use of a calculator**, the numerical value of:

3.1.1 a (3)

3.1.2 b (4)

3.2 Simplify the following **without the use of a calculator**:

$$\frac{\tan(-420°) \cdot \cos 156°}{\sin 492° \cdot \sec 294°}$$ (9)

3.3 If α and θ are complementary angles and $\cot\alpha = k$, and $k \neq 0$, simplify

$$\frac{\operatorname{cosec}^2(180° + \alpha) \cdot \cos(\theta - 720°)}{\cos(\theta - 90°)}$$

in terms of k. (7)

[23]

VRAAG 4

Gegee: $h(x) = \cos(x + 30°)$ en $g(x) = -2\sin x$

4.1   Bepaal die algemene oplossing, **sonder die gebruik van 'n sakrekenaar,** as

$$h(x) = g(x) \tag{7}$$

4.2   Gebruik die assestelsel wat voorsien is op die diagramvel om sketsgrafieke van
die kromme h en g vir $x \in [-120°\ ;\ 180°]$ te teken.
Toon die koördinate van alle draaipunte en afsnitte met die asse.   (8)

4.3   Gebruik die oplossing verkry in VRAAG 4.1 en die grafieke geteken in VRAAG 4.2
om te bepaal vir watter waardes van x :

     4.3.1 $2\sin x + \cos x . \cos 30° \geq \sin x . \sin 30°$ en $x \in [-120°\ ;\ 180°]$ (5)

     4.3.2   sal beide $h(x)$ en $g(x)$ toeneem soos x toeneem vir die interval
        $[-120°\ ;\ 0°]$ (3)

4.4   As die kromme h afgeskuif word met $\dfrac{\sqrt{3}}{2}$ eenhede, bepaal die gevolglike y-afsnit.   (1)
                                                                      [24]

QUESTION 4

Given: h (x) = cos $(x + 30°)$ and   g (x) = $-2 \sin x$

4.1   Determine the general solution, **without the use of a calculator,** if

$$h (x) = g (x)$$ (7)

4.2   Use the set of axes provided on the diagram sheet to draw sketch graphs
         of the curves h and g   for $x \in [-120° ; 180°]$.
         Show the coordinates of all turning points and intercepts with the axes.   (8)

4.3   Use the solution obtained in QUESTION 4.1 and the graphs drawn in
         QUESTION 4.2 to determine for which values of x :

         4.3.1 $2 \sin x + \cos x . \cos 30° \geq \sin x . \sin 30°$ and $x \in [-120° ; 180°]$ (5)

         4.3.2   will both h (x) and g (x) increase as x increases for
                    the interval $[-120° ; 0°]$ (3)

4.4   If the curve h is shifted $\dfrac{\sqrt{3}}{2}$ units down, determine the resulting y – intercept.   (1)
                                                                                                                   [24]

VRAAG 5

5.1   5.1.1   Skryf 'n uitdrukking vir cos (A – B) in terme van die sinusse en kosinusse van A en B.   (1)

       5.1.2   Vervolgens, lei af 'n uitdrukking vir sin (A – B) in terme van die sinusse en kosinusse van A en B.   (3)

5.2   Gegee: sec θ = $\sqrt{10}$ waar $0° < \theta < 90°$.

   5.2.1   Bewys dat $\sqrt{10}$ sin (A – θ) = sin A – 3 cos A.   (5)

       5.2.2   Bepaal vervolgens of andersins, die oplossing van

                    6 cos A + 3 = 2 sin A

                    vir A ∈ [– 180° ; 180°], afgerond tot EEN desimale syfer.   (9)
                                                                              [18]

VRAAG 6

6.1   In die diagram langsaan, $\hat{P}$ > 90°
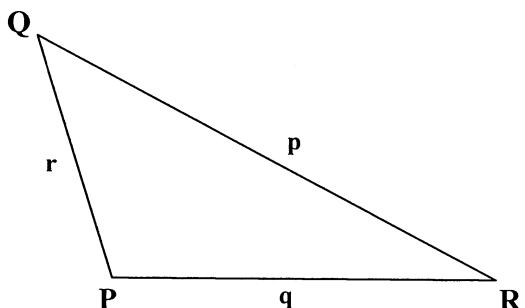
       6.1.1   Gebruik die diagram op die diagramvel of teken die diagram oor in jou antwoordeboek om te bewys dat $p^2 = q^2 + r^2 - 2qr.\cos P$ (6)

       6.1.2   Bewys vervolgens dat

                    $$tan\ P = \frac{4\ (area \triangle PQR)}{q^2 + r^2 - p^2}$$ (3)

QUESTION 5

5.1   5.1.1   Write an expression for cos (A − B) in terms of the sines and cosines of A and B.   (1)

        5.1.2   Hence, derive an expression for sin (A − B) in terms of the sines and cosines of A and B.   (3)

5.2   Given :   sec θ $= \sqrt{10}$ where $0° < θ < 90°$.

        5.2.1   Prove that $\sqrt{10}$ sin (A − θ) = sin A − 3 cos A.   (5)

   5.2.2   Hence, or otherwise, determine the solution of

                          6 cos A + 3 = 2 sin A

                    for A ∈ [− 180° ; 180°], rounded off to ONE decimal digit.   (9)

                                                                             [18]

QUESTION 6

6.1   In the diagram alongside, $\hat{P} > 90°$
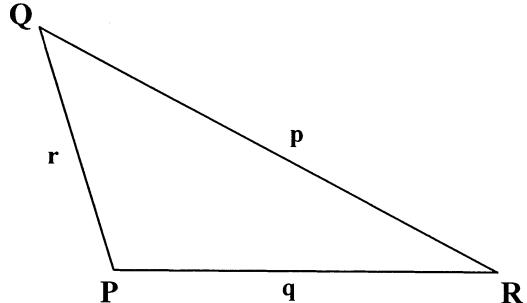
        6.1.1   Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove that $p^2 = q^2 + r^2 - 2qr.\,cosP$ (6)

        6.1.2   Hence, prove that

                    $$tan\,P = \frac{4\,(area\,\triangle PQR)}{q^2 + r^2 - p^2}$$ (3)

6.2 Bouregulasies vereis die voorsiening van weerligbeskerming vir strooidakstrukture.

In die diagram hieronder stel AD 'n vertikale weerligmas voor wat op

dieselfde horisontale vlak as 'n huis gekonstrueer is.

B en C is twee punte op die grond wat in lyn is met die voorkant MN van die

huis. B en C lê op die sirkel van weerligbeskerming

wat deur die weerligmas voorsien word.

$\overset{\wedge}{D B A} = \alpha$ is die hoogtehoek

vanaf B na die toppunt

van die mas by A.

$\overset{\wedge}{D C B} = \theta$
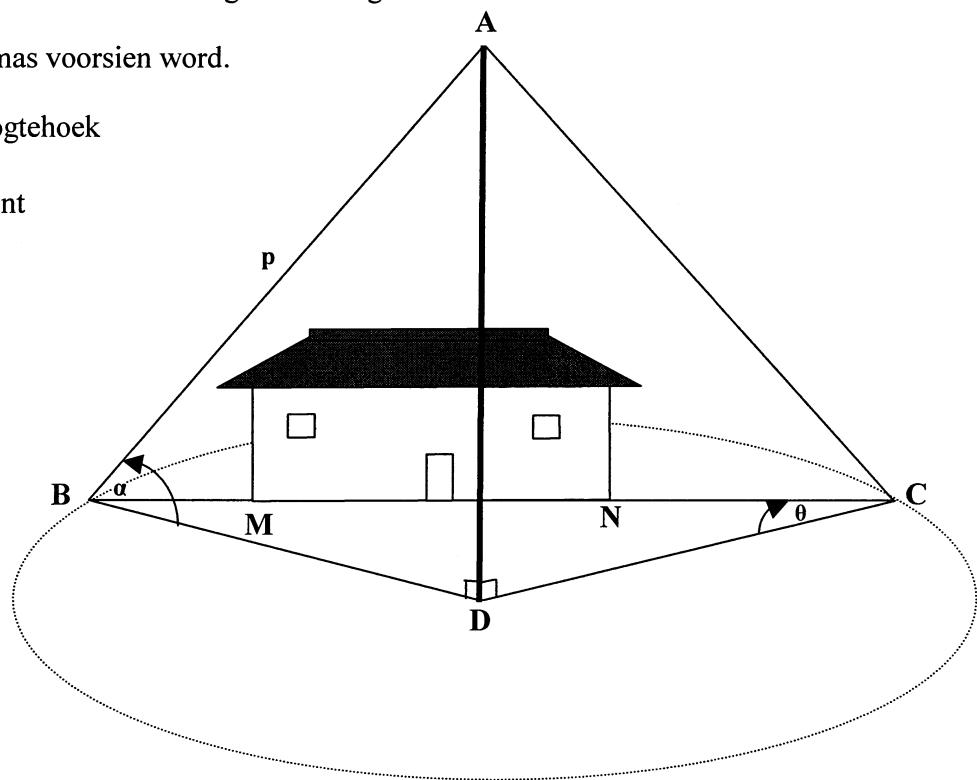
6.2.1 Druk BD, die radius van beskerming, uit in terme van p en α (2)

6.2.2 Toon vervolgens dat $BC = 2 p. \cos \alpha . \cos \theta$ (4)

6.2.3 As $BC = 29,5$ meter, $p = 21,2$ meter en $\alpha = 45°$, bereken die grootte van θ, afgerond tot EEN desimale syfer. (3)

6.2.4 Bereken, tot die naaste meter, die kortste afstand vanaf D na die huis. (3)

[21]

6.2   Building regulations require the provision of lightning protection for thatched roof

        structures. In the diagram below, AD represents a vertical lightning mast constructed

        on the same horizontal plane as a house.

        B and C are two points on the ground which are in line with the front side MN of the

        house. B and C lie on the circle of lightning

        protection provided by the lightning mast.

        $D\hat{B}A = \alpha$ is the angle of elevation

        from B to the top of the mast at A.

        $D\hat{C}B = \theta$
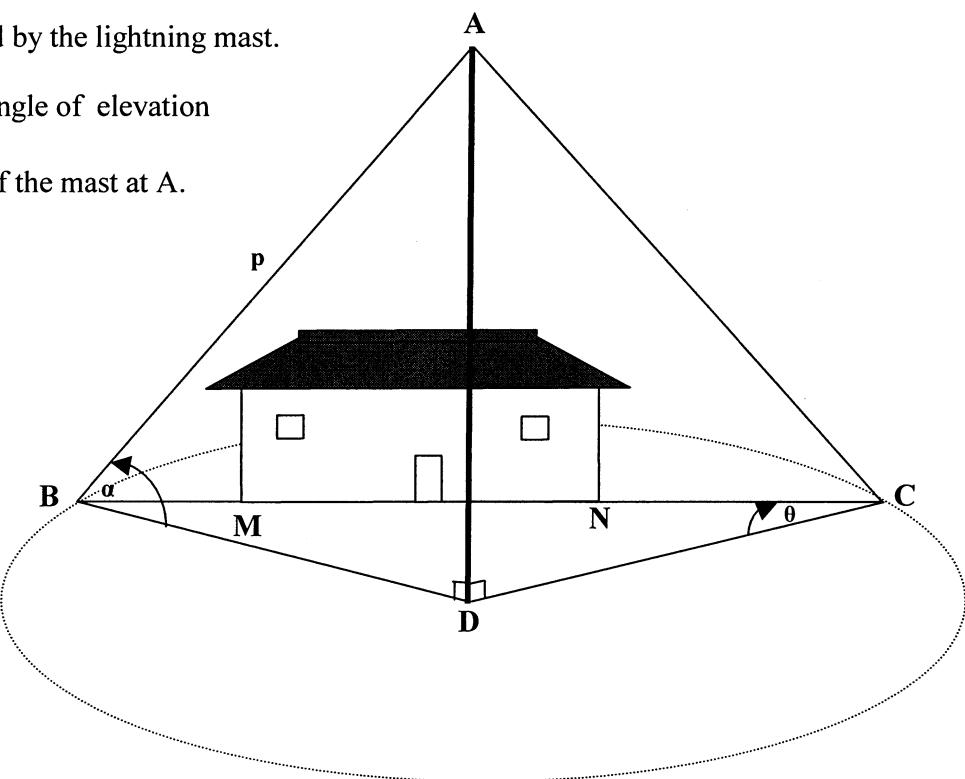
        6.2.1   Express BD, the radius of protection, in terms of p and α (2)

        6.2.2   Hence, show that $BC = 2p \cdot \cos \alpha \cdot \cos \theta$ (4)

        6.2.3   If BC = 29,5 metres, p = 21,2 metres and $\alpha = 45°$,
                  calculate the size of θ, rounded off to ONE decimal digit.   (3)

        6.2.4   Calculate, to the nearest metre, the shortest distance from D to the house.   (3)
                                                                                                   [21]

EUKLIDIESE MEETKUNDE

LET WEL : – **DIAGRAMME VIR DIE BEWYS VAN TEORIE MAG OP DIE DIAGRAMVELLE GEBRUIK WORD, OF IN JOU ANTWOORDEBOEK OORGETEKEN WORD.** – **MAAK DIE DIAGRAMVELLE LOS VAN DIE VRAESTEL EN PLAAS DIT IN JOU ANTWOORDEBOEK.** – **GEE 'n REDE VIR ELKE BEWERING, TENSY ANDERS VERMELD.**

VRAAG 7

7.1   In die diagram langsaan is OM die radius van

die groter sirkel en ook die middellyn van die

kleiner sirkel. Koord EM van die groter

sirkel sny die kleiner sirkel by N.

As EM = $(2x^2 - 2)$ eenhede en

ON = $2x$ eenhede,

druk, met opgaaf van redes, die lengte van die

radius van die groter sirkel uit in terme van x. (6)
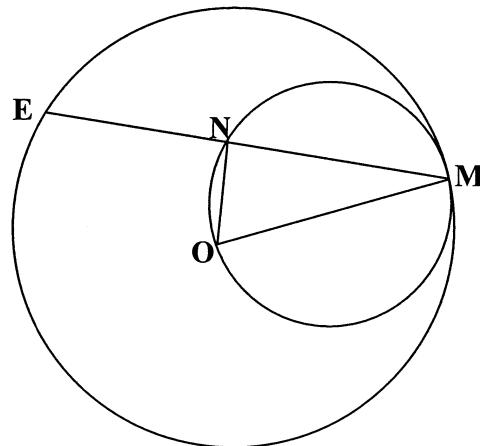
7.2   In die diagram langsaan is ABCD 'n vierhoek

sodat $\hat{B} + \hat{D} = 180°$.

Gebruik die diagram op die diagramvel of

teken die diagram oor in jou antwoordeboek

om die stelling te bewys wat beweer

dat ABCD 'n koordevierhoek is. (6)
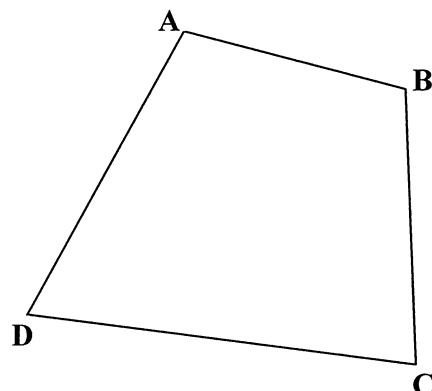
EUCLIDEAN GEOMETRY

NOTE:	-	**DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
	-	**DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
	-	**GIVE A REASON FOR EACH STATEMENT, UNLESS OTHERWISE STATED.**

QUESTION 7

7.1 In the diagram alongside, OM is the radius of

the larger circle and also the diameter of the

smaller circle. Chord EM of the larger

circle cuts the smaller circle at N.

If EM = $(2x^2 - 2)$ units and

ON = $2x$ units,

express, giving reasons, the length of the

radius of the larger circle in terms of x. (6)
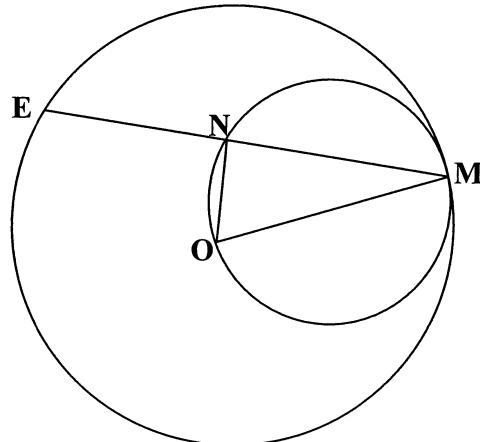
7.2 In the diagram alongside, ABCD is a quadrilateral

such that $\hat{B} + \hat{D} = 180°$.

Use the diagram on the diagram sheet or

redraw the diagram in your answer book,

to prove the theorem which states

that ABCD is a cyclic quadrilateral. (6)
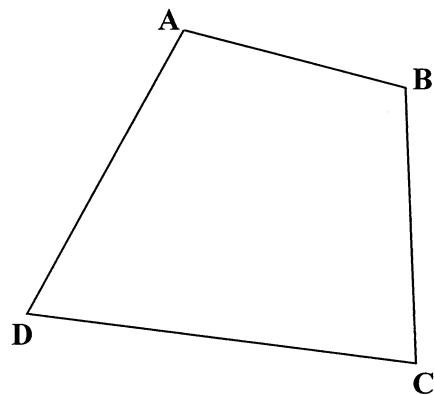
7.3   In die diagram langsaan is PQ 'n raaklyn by Q.

PRS is 'n snylyn van sirkel RSTWQ.

RW sny SQ by K en sny QT by L.

PS ∥ QT

RS = TW
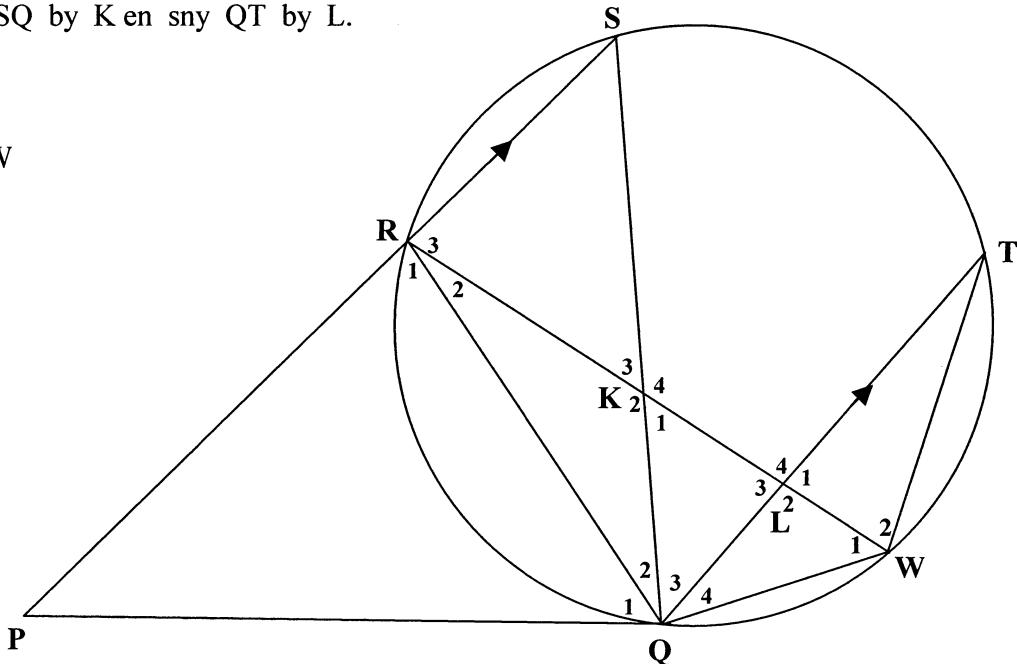
Bewys dat:

7.3.1   KQ 'n raaklyn aan sirkel LQW is   (5)

7.3.2 $\hat{R}_1 = \hat{L}_3$ (4)

7.3.3   PRKQ 'n koordevierhoek is   (4)

7.3.4   RSLQ   nie 'n koordevierhoek is nie   (2)

                                                                               [27]

7.3   In the diagram alongside, PQ is a tangent at Q.

PRS is a secant of circle RSTWQ.

RW cuts SQ at K and cuts QT at L.

PS ∥ QT

RS = TW
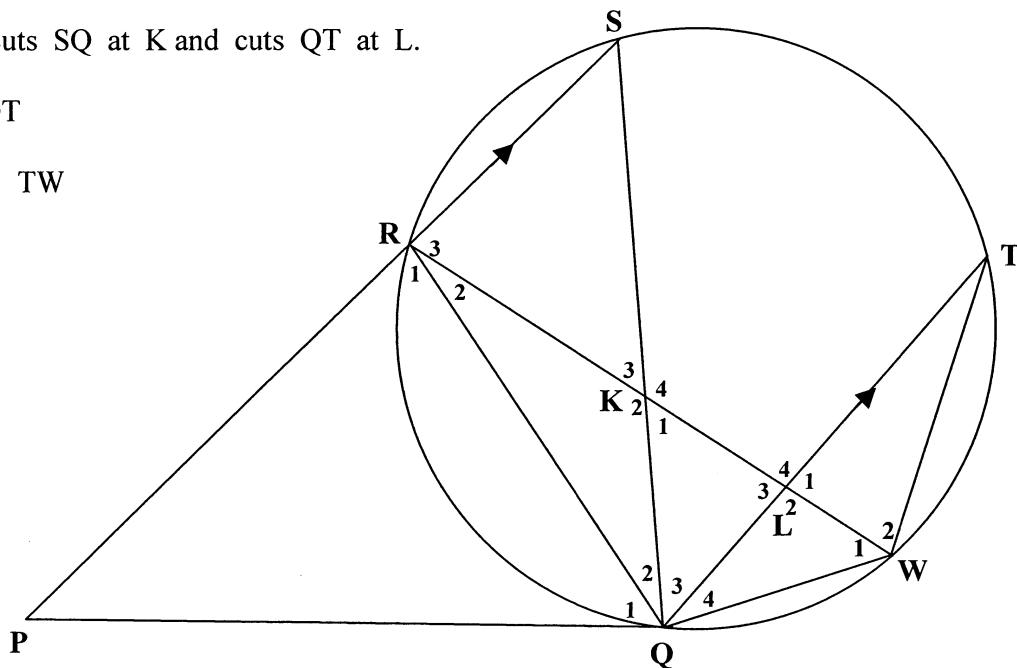
Prove that:

7.3.1	KQ is a tangent to circle LQW	(5)
7.3.2	$\hat{R}_1 = \hat{L}_3$	(4)
7.3.3	PRKQ is cyclic quadrilateral	(4)
7.3.4	RSLQ is a not a cyclic quadrilateral	(2)

[27]

VRAAG 8

In die diagram langsaan is

PS 'n swaartelyn van $\triangle$ PRQ.

T is die middelpunt van PS

en MTW $\parallel$ PQ.
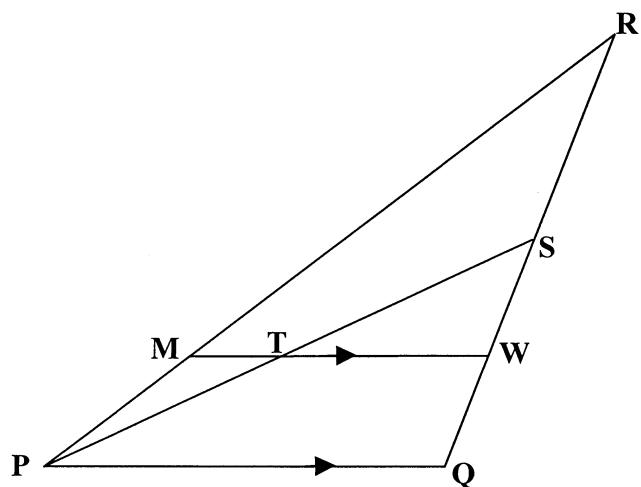
Bereken die numeriese waarde van die volgende:

8.1 $\quad \dfrac{RM}{RP}$ (6)

8.2 $\quad \dfrac{\text{area } \triangle RPS}{\text{area } \triangle RMW}$ (4)

[10]

VRAAG 9

9.1 In die diagram langsaan is

$\hat{X} = \hat{L}, \quad \hat{Y} = \hat{O} \quad$ en $\quad \hat{Z} = \hat{V}$

Gebruik die diagram op die diagramvel, of herteken die diagram oor in jou antwoordeboek om die stelling te bewys wat beweer dat

$$\frac{XY}{LO} = \frac{XZ}{LV}$$

(7)
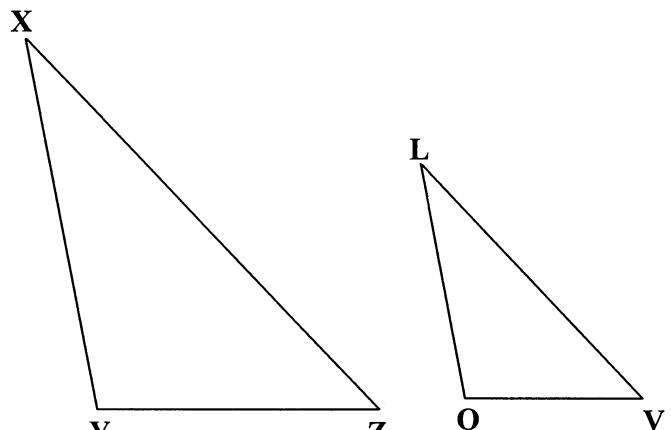
QUESTION 8

In the diagram alongside,

PS is a median of $\triangle$ PRQ.

T is the midpoint of PS

and   MTW $\parallel$ PQ.
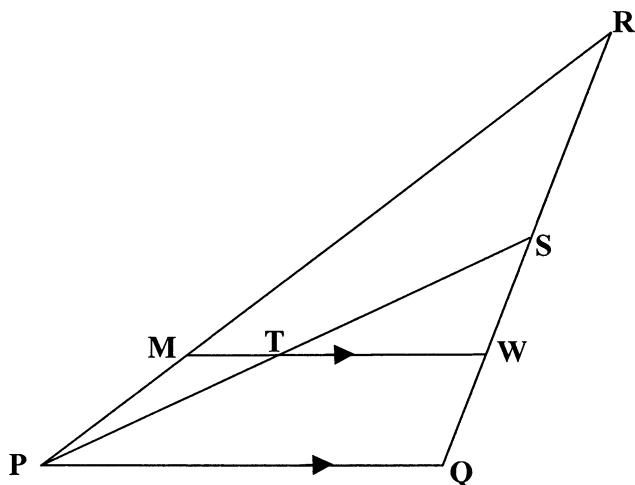
Calculate the numerical value of the following:

8.1 $\dfrac{RM}{RP}$ (6)

8.2 $\dfrac{\text{area } \triangle \text{ RPS}}{\text{area } \triangle \text{ RMW}}$ (4)

[10]

QUESTION 9

9.1   In the diagram alongside,

$\hat{X} = \hat{L}$, $\hat{Y} = \hat{O}$ and $\hat{Z} = \hat{V}$

Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove the theorem which states that

$\dfrac{XY}{LO} = \dfrac{XZ}{LV}$ (7)
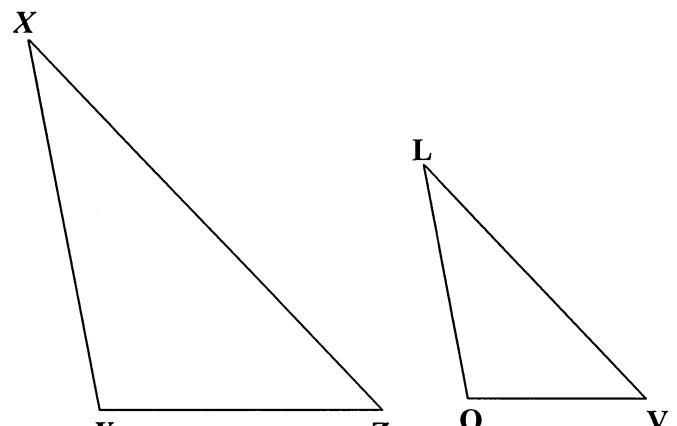
9.2 In die diagram hieronder is BC 'n gemene koord van sirkels BCG en ABC.
AD is 'n raaklyn aan sirkel ABC by A.
GHKN en GEBA is reguitlyne.
$AD \perp AC$ en $FE \perp GA$
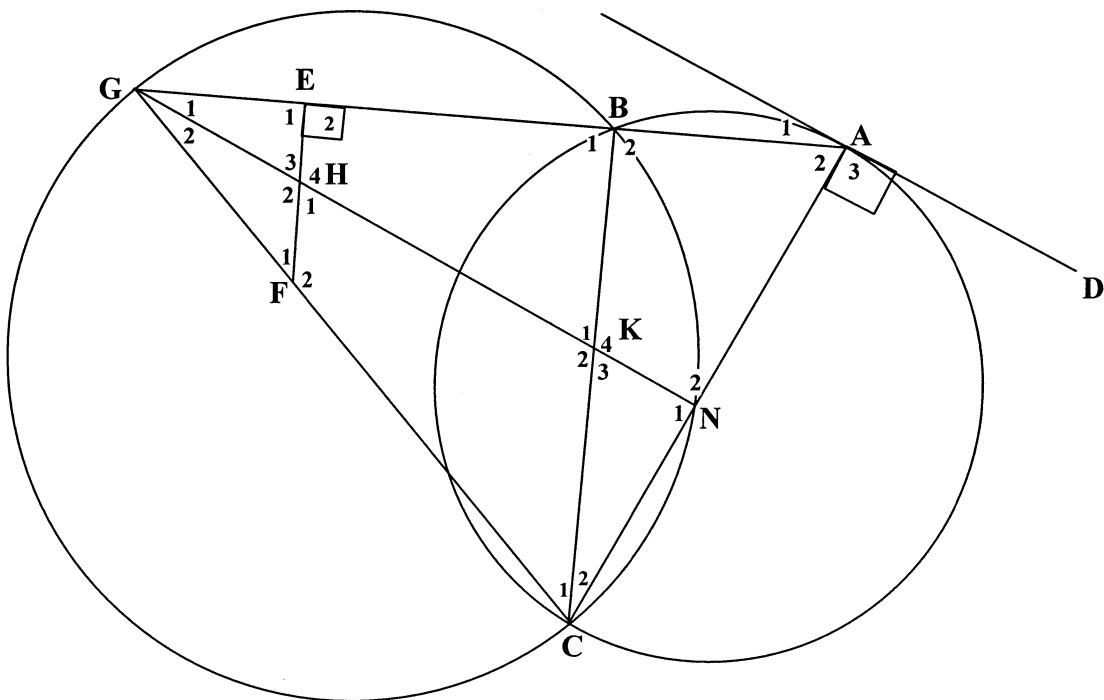
Bewys dat:

9.2.1 K die hoogtepunt van $\triangle AGC$ is. (6)

9.2.2 $BC . KG = AC . BG$ (5)

9.2.3 $\dfrac{BC^2}{BG^2} = \dfrac{EF . AC}{EG . KG}$ (6)

9.2.4 $\dfrac{GC^2}{BG^2} = \dfrac{EF . AC}{EG . KG} + 1$

(4)

[28]

TOTAAL: 200

9.2   In the diagram below, BC is a common chord of circles BCG and ABC.
AD is a tangent to circle ABC at A.
GHKN and GEBA are straight lines.
AD $\perp$ AC   and   FE $\perp$ GA
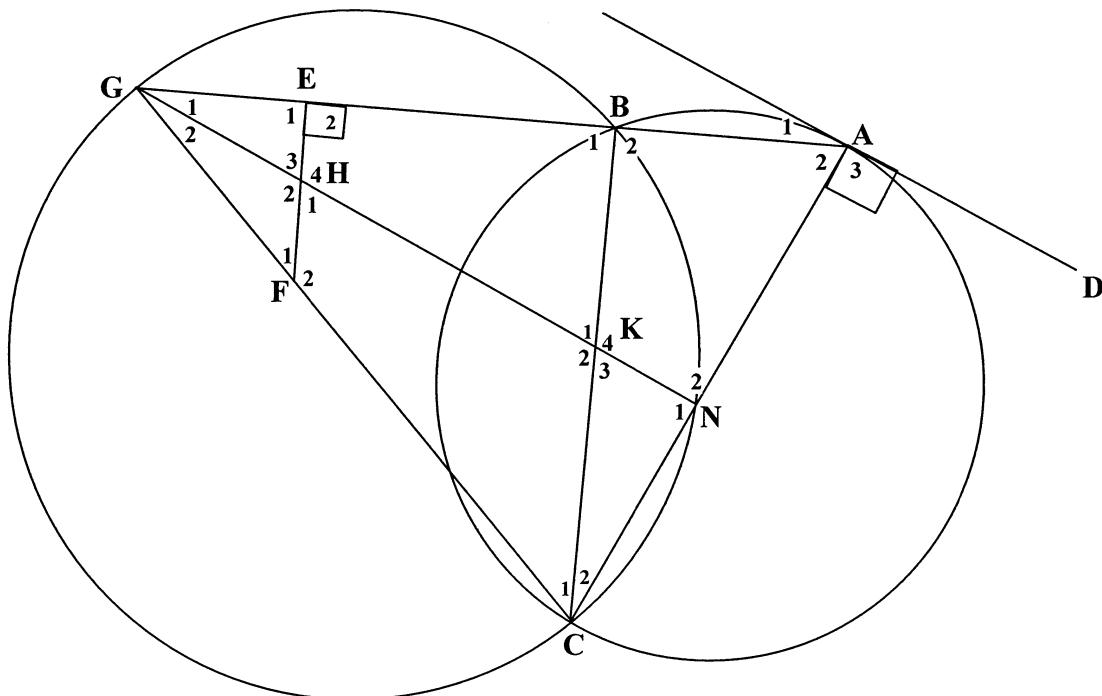
Prove that:

9.2.1   K is the orthocentre of $\triangle$ AGC   (6)

9.2.2   BC . KG = AC . BG   (5)

9.2.3 $\dfrac{BC^2}{BG^2}$ $=$ $\dfrac{EF . AC}{EG . KG}$ (6)

9.2.4 $\dfrac{GC^2}{BG^2}$ $=$ $\dfrac{EF . AC}{EG . KG}$ $+$ 1   (4)

[28]

TOTAL: 200

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}(a + l) \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = a.r^{n-1} \quad S_n = \frac{a(1 - r^n)}{1 - r} \; ; \; r \neq 1 \quad S_n = \frac{a(r^n - 1)}{r - 1} ; \; r \neq 1 \quad S_\infty = \frac{a}{1 - r} \; ; \; r \neq 1$$

$$A = P\left(1 + \frac{r}{100}\right)^n \qquad A = P\left(1 - \frac{r}{100}\right)^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$x^2 + y^2 = r^2 \qquad (x - p)^2 + (y - q)^2 = r^2$$

In $\triangle ABC$:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area \triangle ABC = \frac{1}{2}ab.\sin C$$

| DEPARTMENT OF EDUCATION |
| DEPARTEMENT VAN ONDERWYS |

REPUBLIC OF SOUTH AFRICA
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN
MATHEMATICS HG/WISKUNDE HG
PAPER II/VRAESTEL II

DIAGRAM SHEET / DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Please ensure that your details are complete.

INSTRUKSIE

Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak asseblief seker dat jou besonderhede volledig is.

EXAMINATION NUMBER
EKSAMENNOMMER

CENTRE NUMBER
SENTRUMNOMMER

SENIOR CERTIFICATE EXAMINATION – 2004 - (DIAGRAM SHEET)
SENIORSERTIFIKAAT-EKSAMEN – 2004 - (DIAGRAMVEL)

EXAMINATION NUMBER
EKSAMENNOMMER

QUESTION 2/VRAAG 2
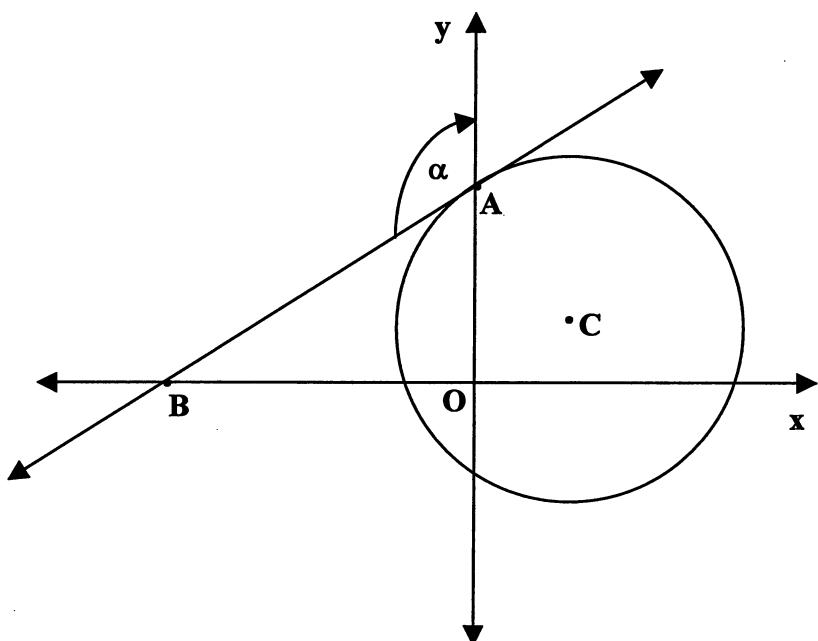
QUESTION 3.1/VRAAG 3.1
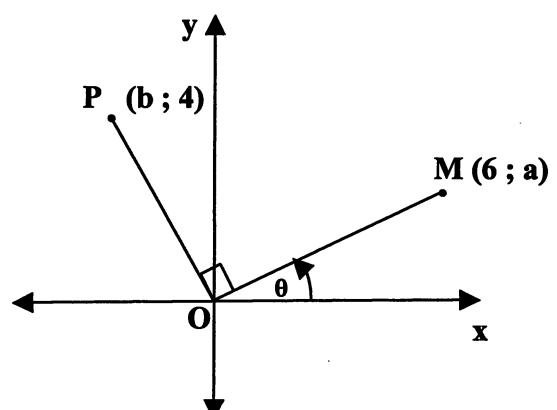
QUESTION 4.2/VRAAG 4.2
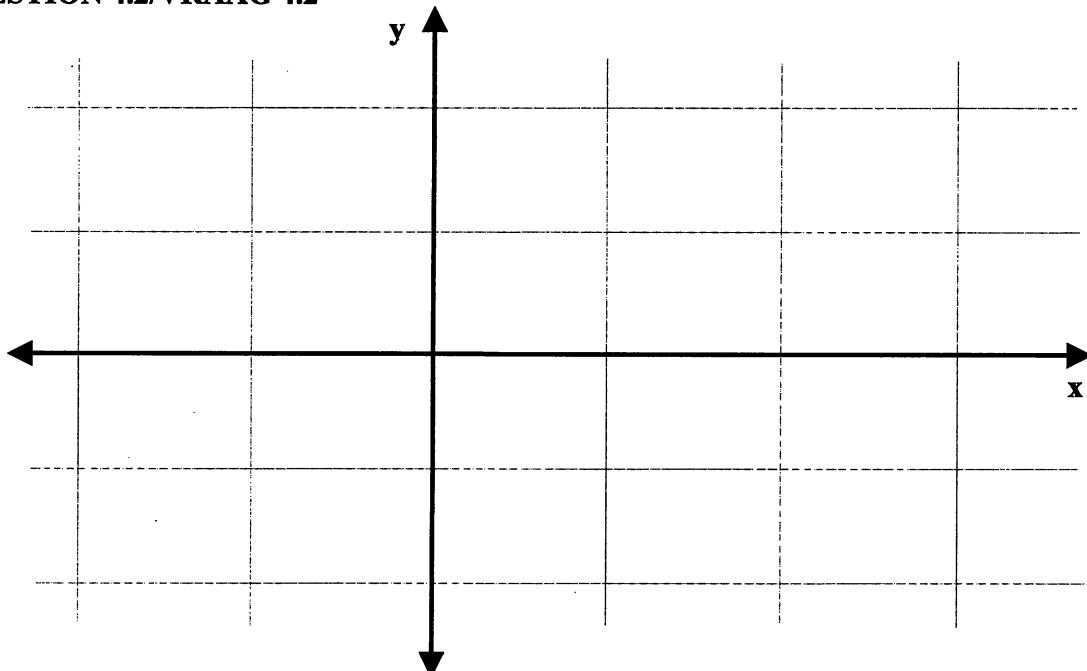
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QUESTION 6.1/VRAAG 6.1
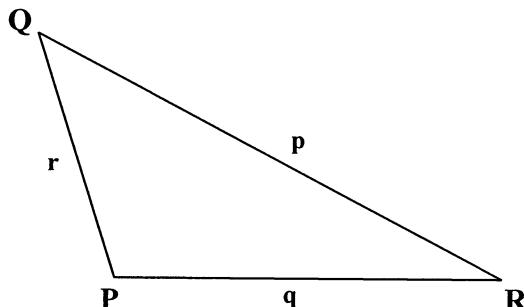
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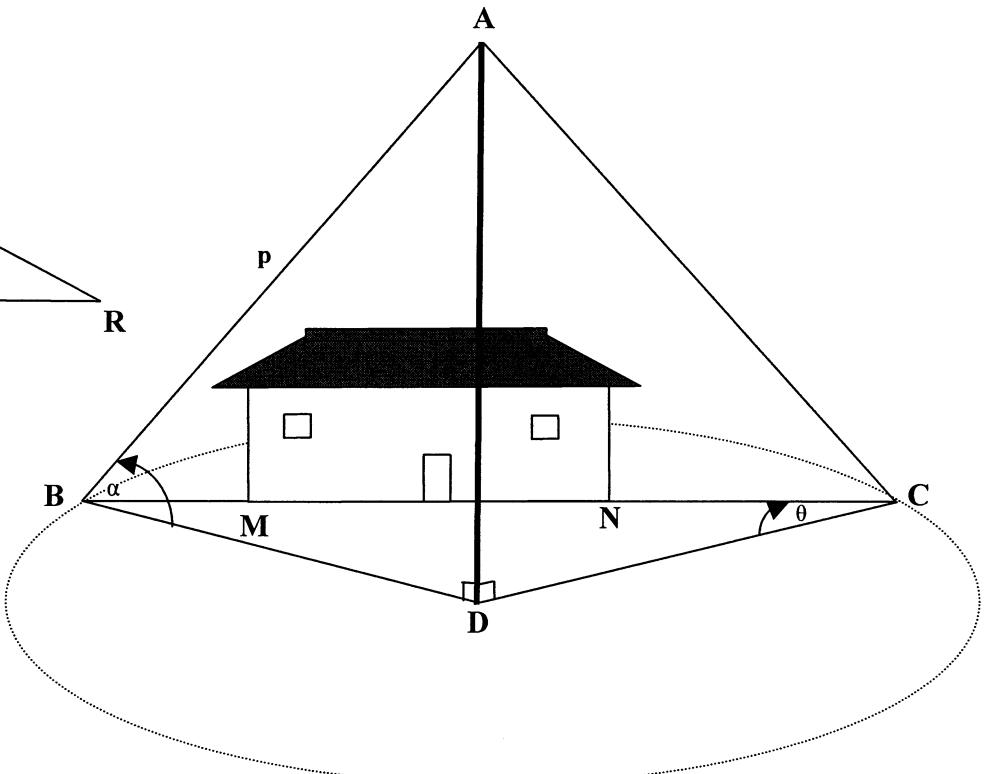
QUESTION 7.1/VRAAG 7.1

QUESTION 7.2/VRAAG 7.2

SENIOR CERTIFICATE EXAMINATION – 2004 - (DIAGRAM SHEET)
SENIORSERTIFIKAAT-EKSAMEN – 2004 - (DIAGRAMVEL)
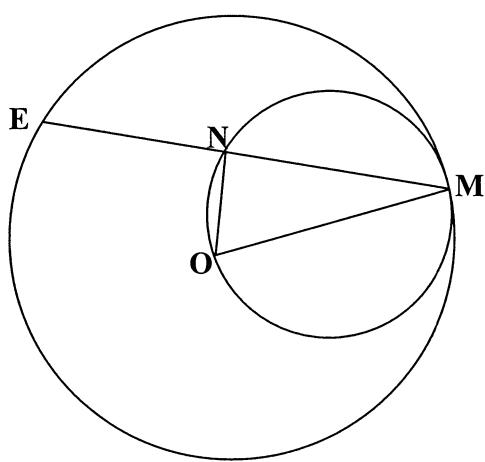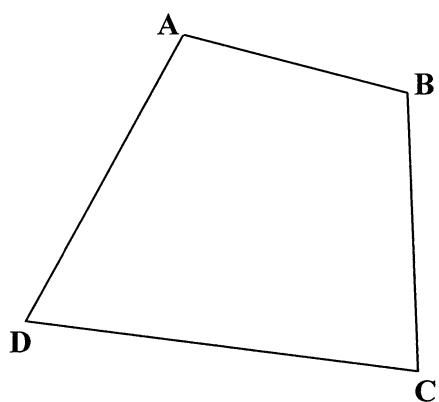
EXAMINATION NUMBER
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QUESTION 7.3/VRAAG 7.3
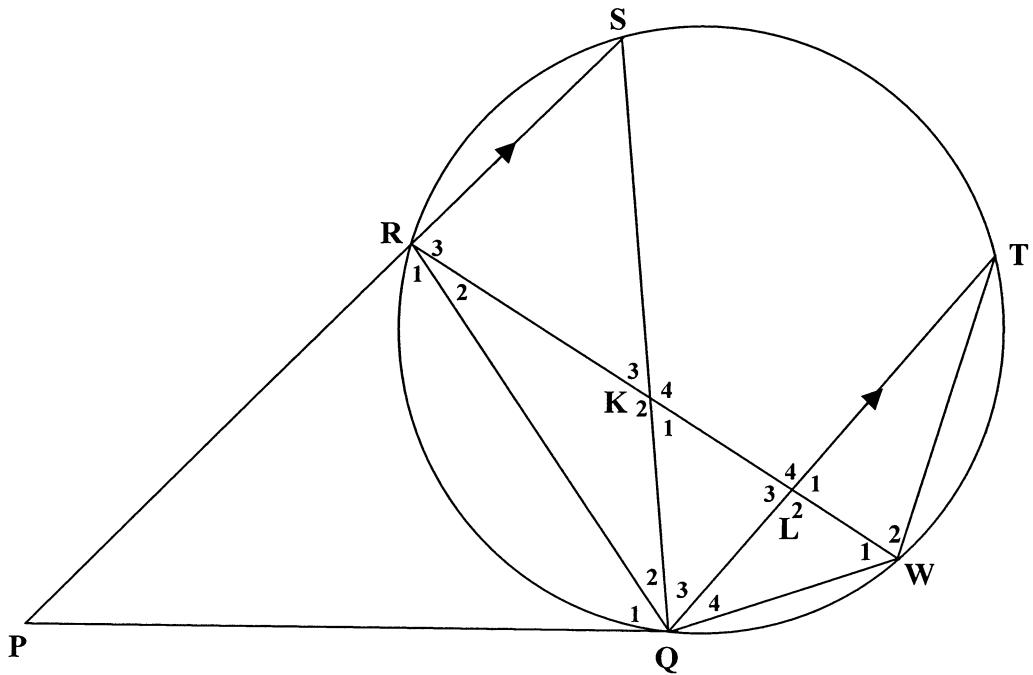
QUESTION 8/VRAAG 8
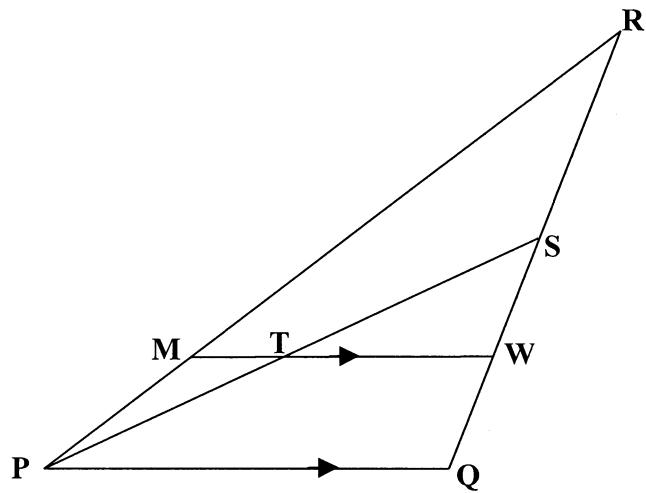
EXAMINATION NUMBER
EKSAMENNOMMER

QUESTION 9.1/VRAAG 9.1

QUESTION 9.2/VRAAG 9.2