DEPARTMENT OF EDUCATION
REPUBLIC OF SOUTH AFRICA

DEPARTEMENT VAN ONDERWYS
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION - 2005
SENIORSERTIFIKAAT-EKSAMEN - 2005

MATHEMATICS P2 : GEOMETRY
WISKUNDE V2 : MEETKUNDE

HIGHER GRADE
HOËR GRAAD

FEBRUARY/MARCH 2005
FEBRUARIE/MAART 2005

301-1/2

MATHEMATICS HG: Paper 2
Geometry, etc

301 1 2 HG

Marks: 200
Punte : 200

3 Hours
3 Ure

This question paper consists of 11 pages, 5 diagram sheets and 1 formula sheet.
Hierdie vraestel bestaan uit 11 bladsye, 5 diagramvelle en 1 formuleblad.

X05

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INSTRUKSIES

1. Hierdie vraestel bestaan uit **NEGE** vrae, 'n formuleblad en diagramvelle.

2. Gebruik die diagramvel om hierdie vraestel te beantwoord.

3. Maak die diagramvelle los van die vraestel en plaas dit binne jou **ANTWOORDEBOEK.**

4. Die diagramme is nie volgens skaal geteken nie.

5. Beantwoord **AL** die vrae.

6. Nommer **AL** die antwoorde korrek en duidelik.

7. **AL** die nodige bewerkings moet getoon word.

8. Nie-programmeerbare sakrekenaars mag gebruik word, tensy anders aangedui.

9. Die aantal desimale plekke waartoe afgerond moet word, sal aangedui word in die vraag waar nodig.

INSTRUCTIONS

1. This question paper consists of **NINE** questions, a formula sheet and diagram sheets.
2. Use the diagram sheet to answer this question paper.
3. Detach the diagram sheets from the question paper and place them inside your **ANSWER BOOK.**
4. The diagrams are not drawn to scale.
5. Answer **ALL** the questions.
6. Number **ALL** the answers correctly and clearly.
7. **ALL** the necessary calculations must be shown.
8. Non-programmable calculators may be used, unless otherwise stated.
9. The number of decimal digits to which answers must be rounded off will be stated in the question where necessary.

ANALITIESE MEETKUNDE

LET WEL:	– GEBRUIK ANALITIESE METODES IN HIERDIE AFDELING.
	– KONSTRUKSIE- EN METINGMETODES MAG NIE GEBRUIK WORD NIE.

VRAAG 1

In die diagram langsaan lê punte

A (–1 ; 0), C (2 ; – 2), E (2 ; 2) en B

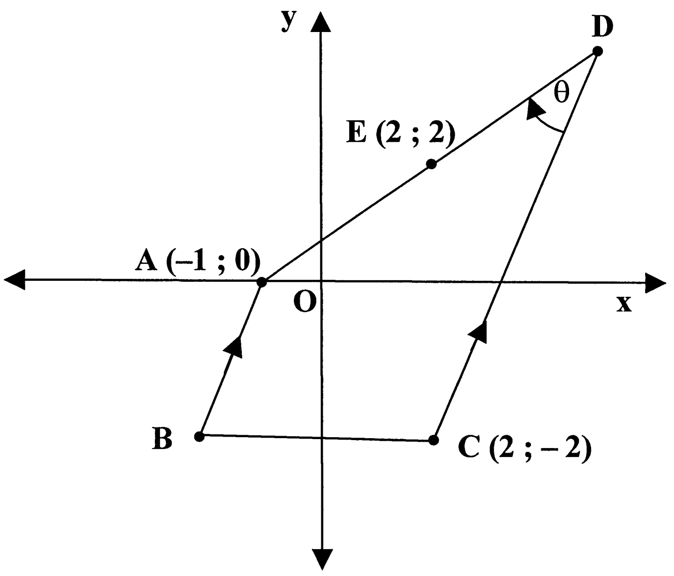
in 'n Cartesiese vlak.

E is die middelpunt van AD.

AB || DC

$A\hat{D}C = \theta$

3AB = DC



Bepaal:

1.1 Die koördinate van D (2)

1.2 Die waarde van θ, afgerond tot EEN desimale syfer (6)

1.3 Die koördinate van B (9)

1.4 Vervolgens, die koördinate van die ommiddelpunt (middelpunt van die omgeskrewe sirkel) H van Δ ABC (8)
[25]

ANALYTICAL GEOMETRY

NOTE :	– USE ANALYTICAL METHODS IN THIS SECTION.
	– CONSTRUCTION AND MEASUREMENT METHODS MAY NOT BE USED.

QUESTION 1

In the diagram alongside, points

A (–1 ; 0), C (2 ; – 2), E (2 ; 2) and B

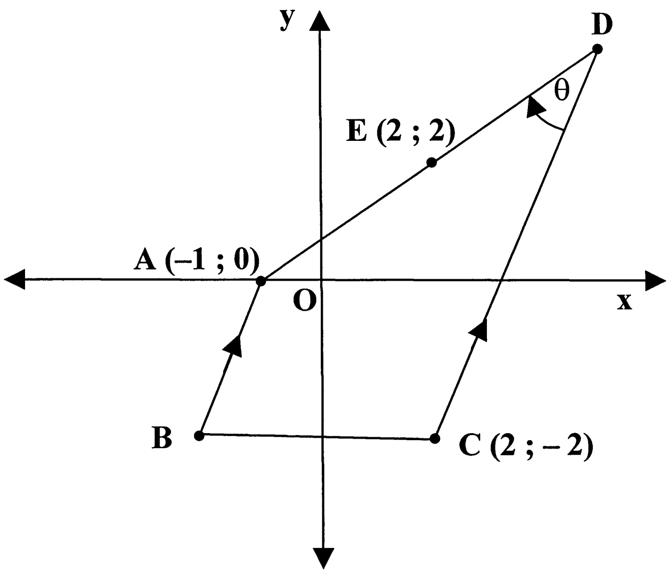
lie in a Cartesian plane.

E is the midpoint of AD.

AB || DC

$A\hat{D}C = \theta$

3AB = DC



Determine:

1.1 The coordinates of D (2)

1.2 The value of θ, rounded off to ONE decimal digit (6)

1.3 The coordinates of B (9)

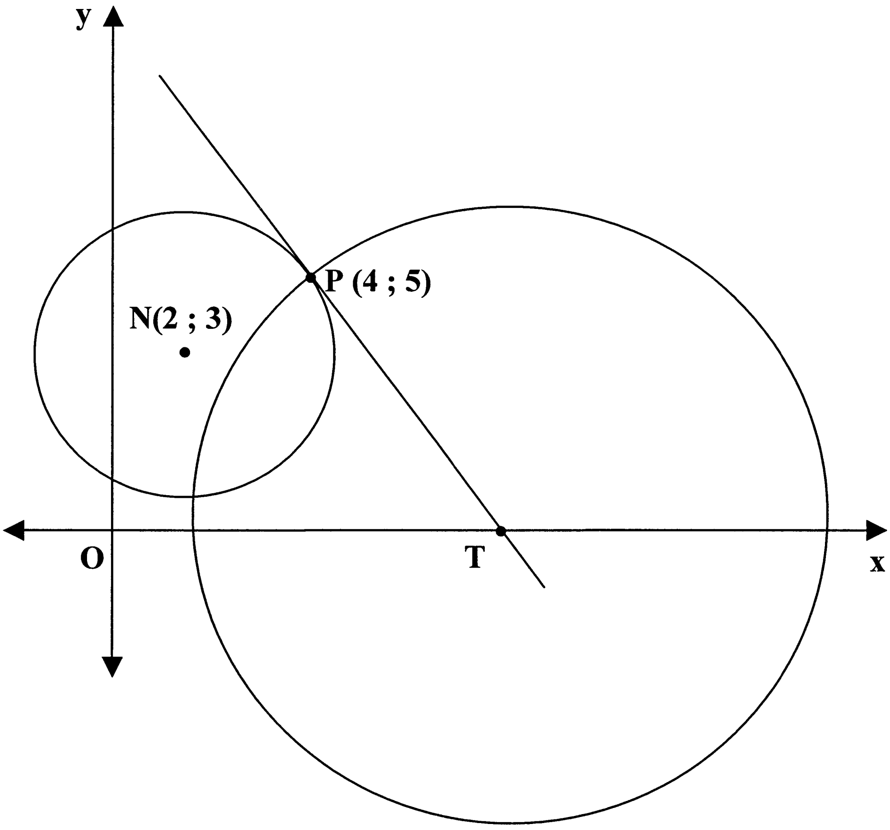
1.4 Hence, the coordinates of the circumcentre (centre of circumscribed circle) H of Δ ABC (8)

[25]

VRAAG 2

2.1 In die diagram hieronder word twee sirkels met middelpunte N (2 ; 3) en T gegee wat sny by punt P (4 ; 5).

Vanuit T, 'n punt op die x-as, word 'n raaklyn getrek aan sirkel N by P.



Bepaal:

2.1.1 Die vergelyking van sirkel N (4)

2.1.2 Die koördinate van T (5)

2.1.3 Die area van sirkel T, afgerond tot die naaste heelgetal (5)

2.2 Q (2 ; 4) en R (– 4 ; – 4) is twee punte in 'n Cartesiese vlak

2.2.1 Bepaal die vergelyking van die lokus van P(x ; y), as PQ loodreg op PR is. (5)

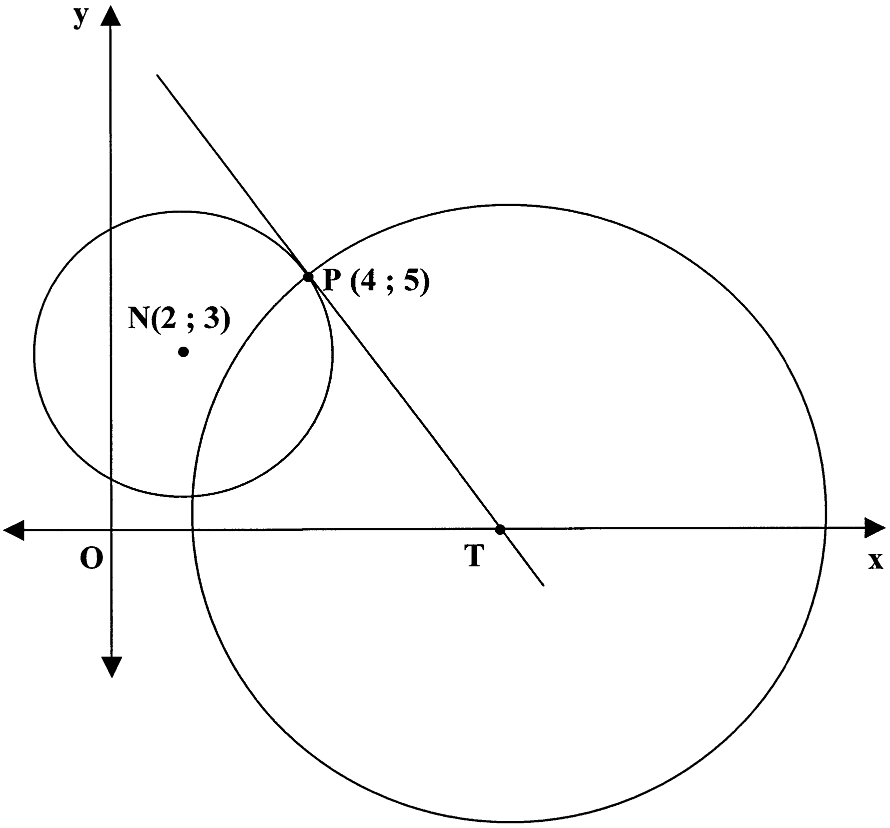
2.2.2 Bepaal vervolgens vir watter waardes van x die vergelyking van die lokus soos verkry in VRAAG 2.2.1 gedefinieer is. (3)

[22]

QUESTION 2

2.1 In the diagram below, two circles with centres N (2 ; 3) and T intersecting at point P (4 ; 5) are given.

From T, a point on the x-axis, a tangent is drawn to circle N at P.



Determine:

2.1.1 The equation of circle N (4)

2.1.2 The coordinates of T (5)

2.1.3 The area of circle T, rounded off to the nearest whole number (5)

2.2 Q (2 ; 4) and R (– 4 ; – 4) are two points in a Cartesian plane

2.2.1 Determine the equation of the locus of P(x ; y), if PQ is perpendicular to PR. (5)

2.2.2 Hence, determine for which values of x the equation of the locus obtained in QUESTION 2.2.1 is defined. (3)

[22]

TRIGONOMETRIE

VRAAG 3

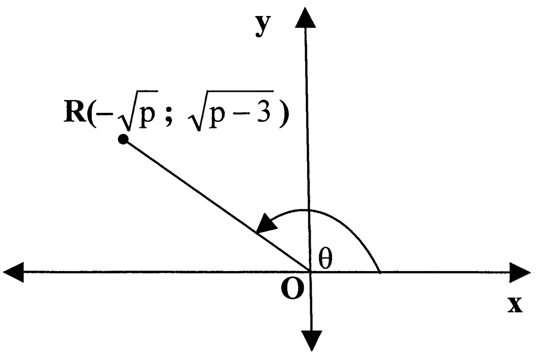
Beantwoord hierdie vraag sonder die gebruik van 'n sakrekenaar.

3.1 Vereenvoudig:

$$\frac{\cos^2 208°}{\tan 242°.\cos 28°}\ .\ \text{cosec}\ 928°\ .\ \cot(-120°)\ .\ \sec 30°$$ (10)

3.2 In die diagram langsaan is $R(-\sqrt{p}\ ; \sqrt{p-3})$ 'n punt in 'n Cartesiese vlak.

$\hat{ROX} = \theta$



3.2.1 Druk $\sin^2\theta$ uit in terme van p. (4)

3.2.2 Vervolgens, vir watter reële waardes van p is $\sin^2\theta$ gedefinieer? (2)

3.2.3 Bepaal $\sec\alpha$ in terme van p as α en θ supplementêr is. (3)

3.2.4 As $p = 6$, bewys dat

$$\cos(\theta - 30°) = \frac{-3\sqrt{2}+\sqrt{3}}{6}$$ (7)

[26]

TRIGONOMETRY

QUESTION 3

Answer this question without the use of a calculator.

3.1 Simplify:

$$\frac{\cos^2 208°}{\tan 242° . \cos 28°} . \operatorname{cosec} 928° . \cot(-120°) . \sec 30°$$

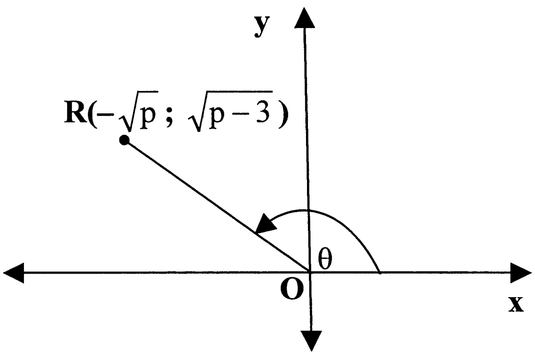
(10)

3.2 In the diagram alongside,

$R(-\sqrt{p}\ ; \sqrt{p-3})$ is a point in

a Cartesian plane.

$\hat{ROX} = \theta$



3.2.1 Express $\sin^2\theta$ in terms of p. (4)

3.2.2 Hence, for which real values of p is $\sin^2\theta$ defined? (2)

3.2.3 Determine $\sec\alpha$ in terms of p if α and θ are supplementary. (3)

3.2.4 If $p = 6$, prove that

$$\cos(\theta - 30°) = \frac{-3\sqrt{2}+\sqrt{3}}{6}$$

(7)

[26]

VRAAG 4

Gegee: $f(x) = \tan x$ en $g(x) = \sin 2x$

4.1 4.1.1 Bepaal die algemene oplossing van $f(x) = g(x)$ (9)

4.1.2 Bepaal vervolgens die waarde vir x as $f(x) = g(x)$ en $x \in [-180° ; 0°)$. (3)

4.2 Gebruik die assestelsel op die diagramvel om sketsgrafieke te teken van die krommes van f en g vir $x \in [-180° ; 90°]$. Dui ALLE draaipunte en snypunte duidelik aan en dui ook enige asimptote aan deur stippellyne te gebruik. (8)

4.3 Gebruik jou oplossings in VRAAG 4.1.2 verkry en die sketsgrafieke geteken in VRAAG 4.2 om te bepaal vir watter waarde(s) van x die volgende bewerings WAAR sal wees:

4.3.1 $\tan x - \sin 2x \geq 0$, waar $x \in [-180° ; 0°]$ (4)

4.3.2 Beide $f(x)$ en $g(x)$ neem toe soos x toeneem, vir $x \in [-90° ; -90°]$ (4)

[28]

VRAAG 5

5.1 Skryf 'n uitdrukking neer vir $\tan 2\alpha$ in terme van $\tan \alpha$. (1)

5.2 Gegee die identiteit: $\dfrac{1 + \cos 2A}{\cos 2A} = \dfrac{\tan 2A}{\tan A}$

5.2.1 Bewys die identiteit. (5)

5.2.2 Bepaal, deur as 'n algemene oplossing uit te druk, vir watter waardes van A die identiteit ongedefinieerd is. (5)

[11]

QUESTION 4

Given: $f(x) = \tan x$ and $g(x) = \sin 2x$

4.1 4.1.1 Determine the general solution for $f(x) = g(x)$ (9)

4.1.2 Hence, determine the values for x if $f(x) = g(x)$ and $x \in [-180° ; 0°)$. (3)

4.2 Use the system of axes provided on your diagram sheet to draw sketch graphs of the curves of f and g for $x \in [-180° ; 90°]$. Clearly show ALL turning points and points of intersection and also indicate any asymptotes using dotted lines. (8)

4.3 Use your solutions obtained in QUESTION 4.1.2 and the sketch graphs drawn in QUESTION 4.2 to determine for which value(s) of x the following statements will be TRUE:

4.3.1 $\tan x - \sin 2x \geq 0$, where $x \in [-180° ; 0°]$ (4)

4.3.2 Both $f(x)$ and $g(x)$ increase as x increases, for $x \in [-90° ; -90°]$ (4)

[28]

QUESTION 5

5.1 Write an expression for $\tan 2\alpha$ in terms of $\tan \alpha$. (1)

5.2 Given the identity: $\dfrac{1 + \cos 2A}{\cos 2A} = \dfrac{\tan 2A}{\tan A}$

5.2.1 Prove the identity. (5)

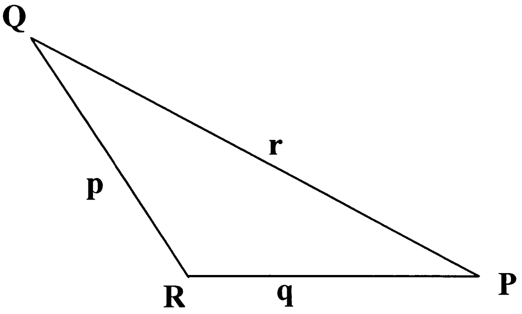
5.2.2 Determine, by expressing as a general solution, for which values of A the identity is undefined. (5)

[11]

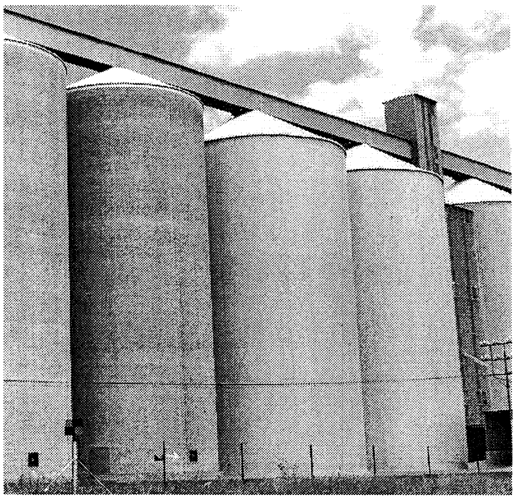
VRAAG 6

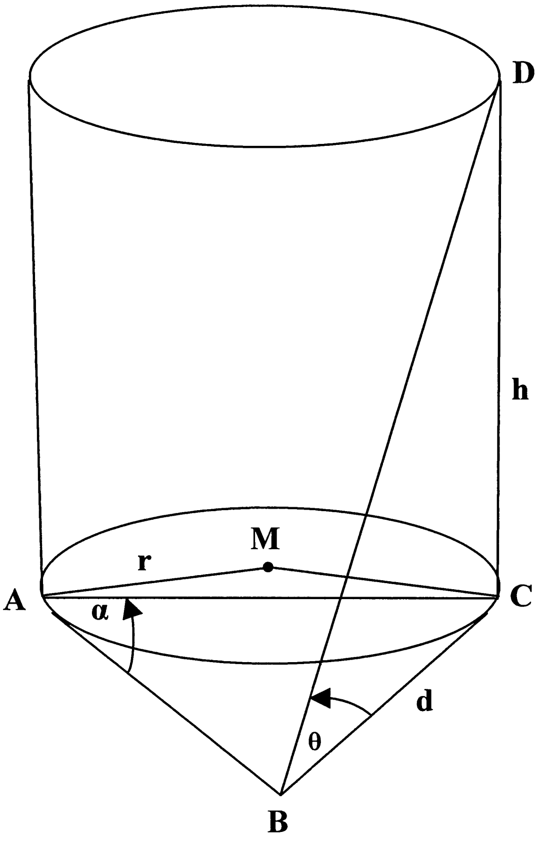
6.1 Gegee Δ PQR met $\hat{R}$ stomphoekig. Gebruik die diagram op die diagramvel of teken hierdie diagram oor in jou antwoordeboek, om te bewys dat

$r^2 = p^2 + q^2 - 2pq \cdot \cos R$ (6)



6.2





Die diagram langsaan stel een van die regte silindriese silo's hierbo voor. M is die middelpunt van die sirkelvormige basis met BA en BC raaklyne aan die basis by A en C. M, A, B en C lê op dieselfde horisontale vlak.
DC stel die vertikale hoogte van die silindriese deel van die silo voor.

$B\hat{A}C = \alpha$ and $D\hat{B}C = \theta$
$DC = h$
$BC = d$

6.2.1 Bewys dat $AC = d\sqrt{2 + 2\cos 2\alpha}$ (4)

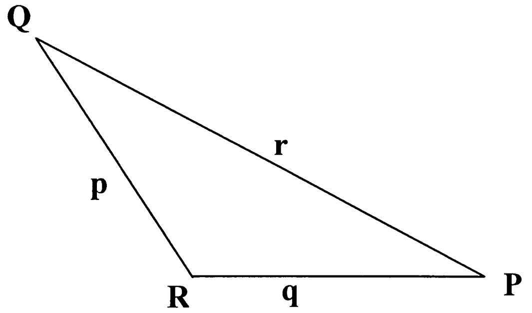
6.2.2 Bewys ook dat $AC = 2h \cdot \cos\alpha \cdot \cot\theta$ (5)

6.2.3 As $h = 36$ meter, $d = 10,3$ meter en $\alpha = 54°$, bereken die volume van die silindriese deel van die silo (rond af tot die naaste kubieke meter). (7)
[22]

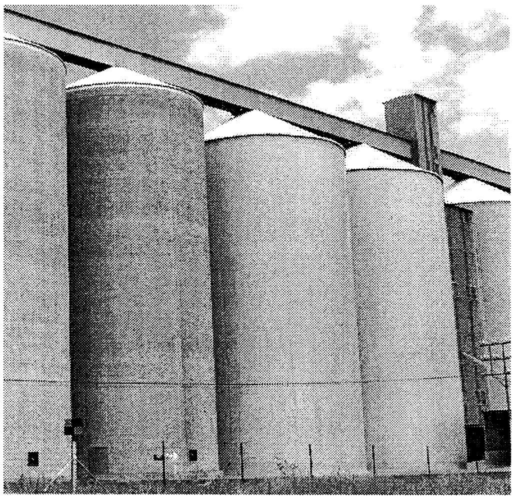
QUESTION 6

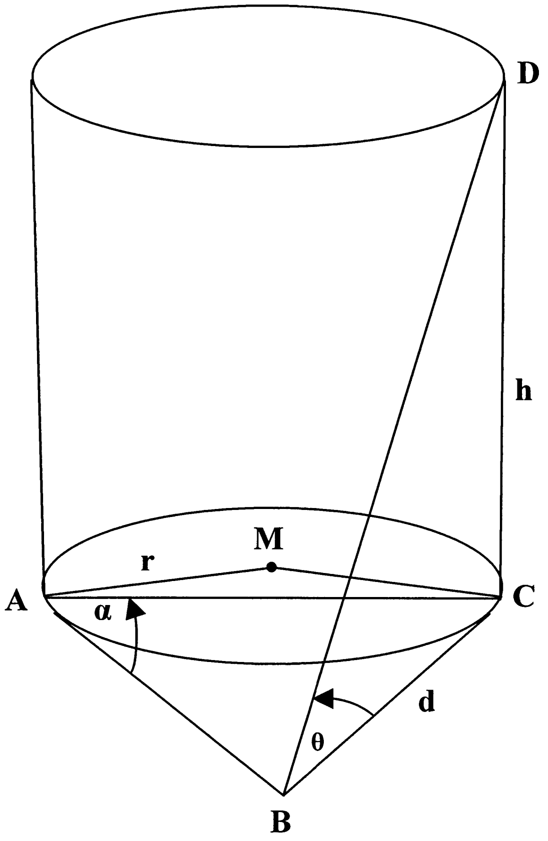
6.1 Given Δ PQR with $\hat{R}$ obtuse. Use the diagram on the diagram sheet or redraw the diagram in your answer book to prove that

$r^2 = p^2 + q^2 - 2pq \cdot \cos R$ (6)



6.2





The diagram alongside, represents one of the right cylindrical silos above. M is the centre of the circular base with BA and BC tangents to the base at A and C.
M, A, B and C lie on the same horizontal plane.
DC represents the vertical height of the cylindrical part of the silo.

$B\hat{A}C = \alpha$ and $D\hat{B}C = \theta$
DC = h
BC = d

6.2.1 Prove that $AC = d\sqrt{2 + 2\cos 2\alpha}$ (4)

6.2.2 Also prove that $AC = 2h \cdot \cos\alpha \cdot \cot\theta$ (5)

6.2.3 If h = 36 metres, d = 10,3 metres and α = 54°, calculate the volume of the cylindrical section of the silo (rounded off to the nearest cubic metre). (7)

[22]

EUKLIDIESE MEETKUNDE

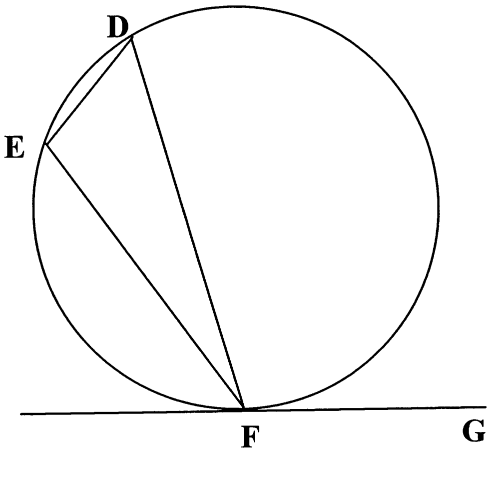
LET WEL:
- **DIAGRAMME VIR DIE BEWYS VAN TEORIE MAG OP DIE DIAGRAMVELLE GEBRUIK WORD, OF MAG IN JOU ANTWOORDEBOEK OORGETEKEN WORD.**
- **MAAK DIE DIAGRAMVELLE LOS VAN DIE VRAESTEL EN PLAAS DIT IN JOU ANTWOORDEBOEK.**
- **GEE 'N REDE VIR ELKE BEWERING.**

VRAAG 7

7.1 Stelling: 'Die hoek tussen die raaklyn aan 'n sirkel en die koord wat vanuit die raakpunt getrek word, is gelyk aan die hoek in die teenoorstaande sirkelsegment.'

7.1.1 Skryf die bewering neer van die **omgekeerde** van die bostaande stelling. (2)

7.1.2 In die diagram hieronder is FG 'n raaklyn aan sirkel DEF.



Gebruik die diagram op die diagramvel, of teken die diagram oor in jou antwoordeboek om die omgekeerde stelling, waarna in VRAAG 7.1.1 verwys word, te bewys. (5)

EUCLIDEAN GEOMETRY

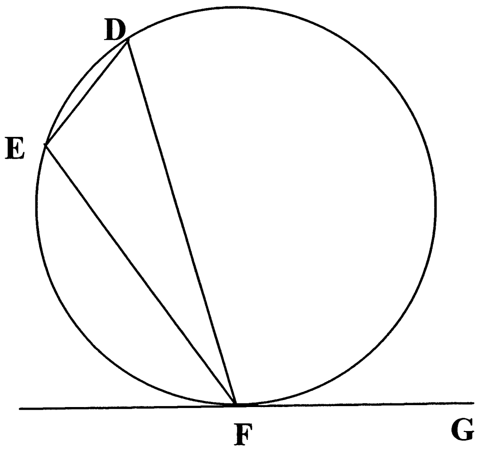
NOTE :
- **DIAGRAMS FOR PROVING THEORY MAY BE USED ON THE DIAGRAM SHEETS OR REDRAWN IN YOUR ANSWER BOOK.**
- **DETACH THE DIAGRAM SHEETS FROM THE QUESTION PAPER AND PLACE THEM IN YOUR ANSWER BOOK.**
- **GIVE A REASON FOR EACH STATEMENT.**

QUESTION 7

7.1 Theorem: 'The angle between the tangent to the circle and the chord drawn from the point of contact, is equal to the angle in the alternate segment.'

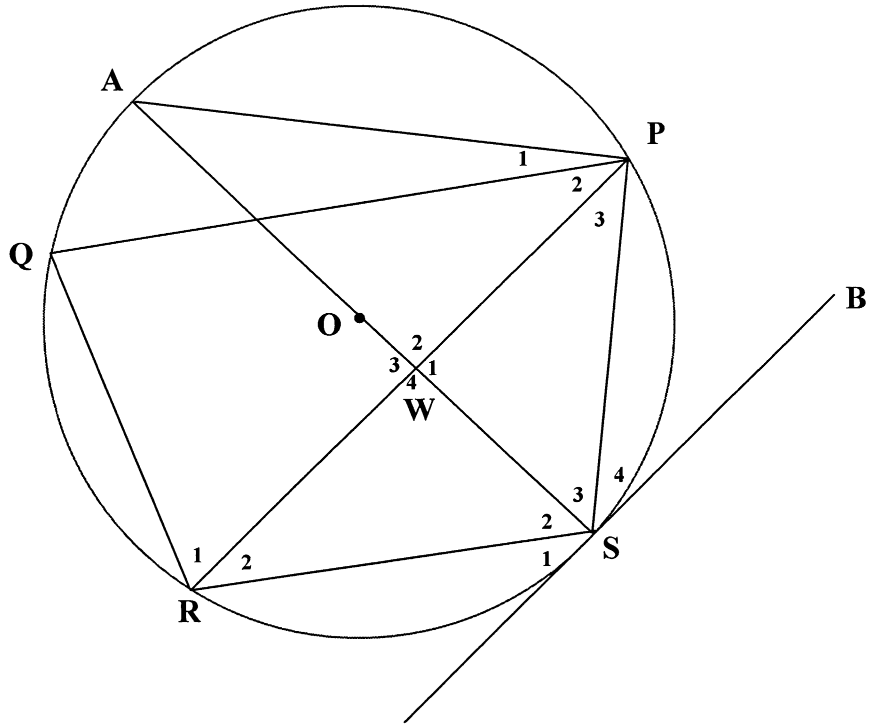
7.1.1 Write the statement of the **converse** of the above theorem. (2)

7.1.2 In the diagram below, FG is a tangent to circle DEF.



Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove the converse theorem referred to in QUESTION 7.1.1. (5)

7.2 In die diagram hieronder lê P, A, Q, R en S op die sirkel met middelpunt O.
SB raak die sirkel by S en RW = WP.
AOWS en RWP is reguitlyne.



Bewys dat:

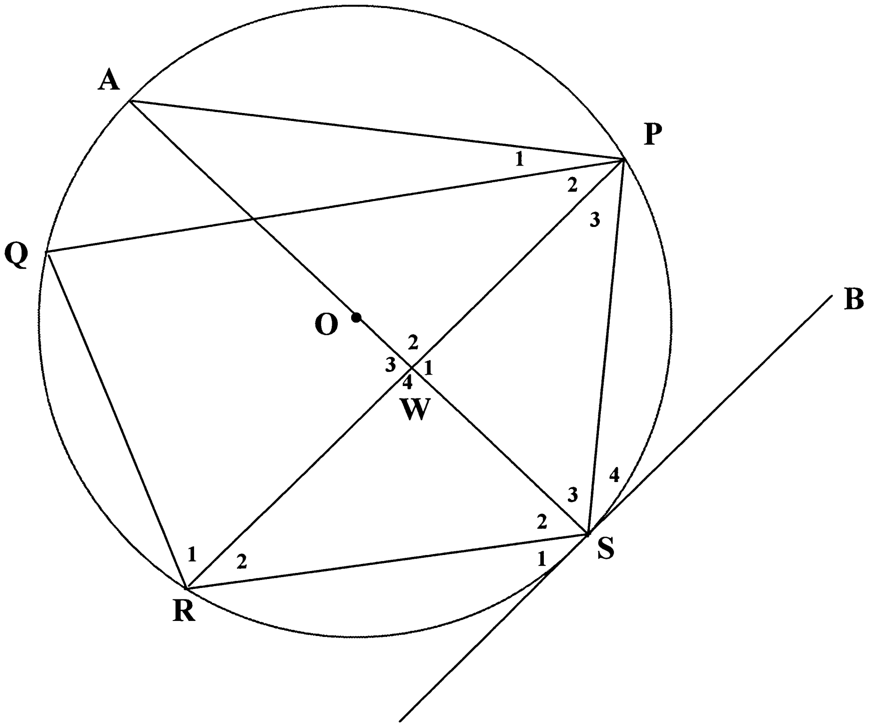
7.2.1 SB || RP (5)

7.2.2 $RS^2 = WS \cdot AS$ (8)

7.2.3 $AS = \dfrac{RW^2}{WS} + WS$ (4)

[24]

7.2 In the diagram below, P, A, Q, R and S lie on the circle with centre O.
SB touches the circle at S and RW = WP.
AOWS and RWP are straight lines.



Prove that:

7.2.1 SB || RP (5)

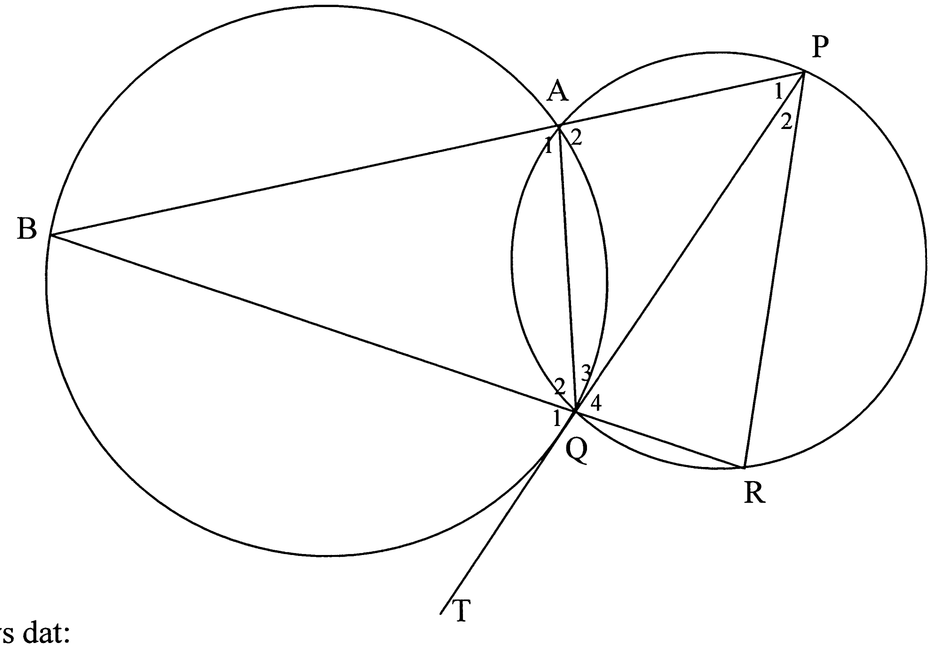
7.2.2 $RS^2 = WS \cdot AS$ (8)

7.2.3 $AS = \dfrac{RW^2}{WS} + WS$ (4)

[24]

VRAAG 8

8.1 In die diagram hieronder is PQT 'n raaklyn aan die groter sirkel ABQ by Q. BAP en BQR is reguitlyne met P en R op die kleiner sirkel.



Bewys dat:

8.1.1 PQ = PR (5)

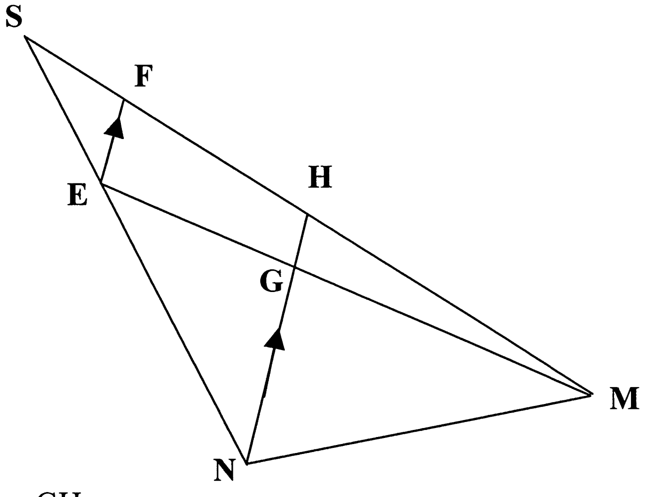
8.1.2 Die lengtes van PA, PR en PB (in hierdie volgorde) 'n meetkundige ry vorm (7)

8.2 In die diagram langsaan is E, F en H punte op Δ SNM.

EF || NH en EGM is 'n reguitlyn met G op HN en EM.

SH : SM = 3 : 8

2 SE = EN

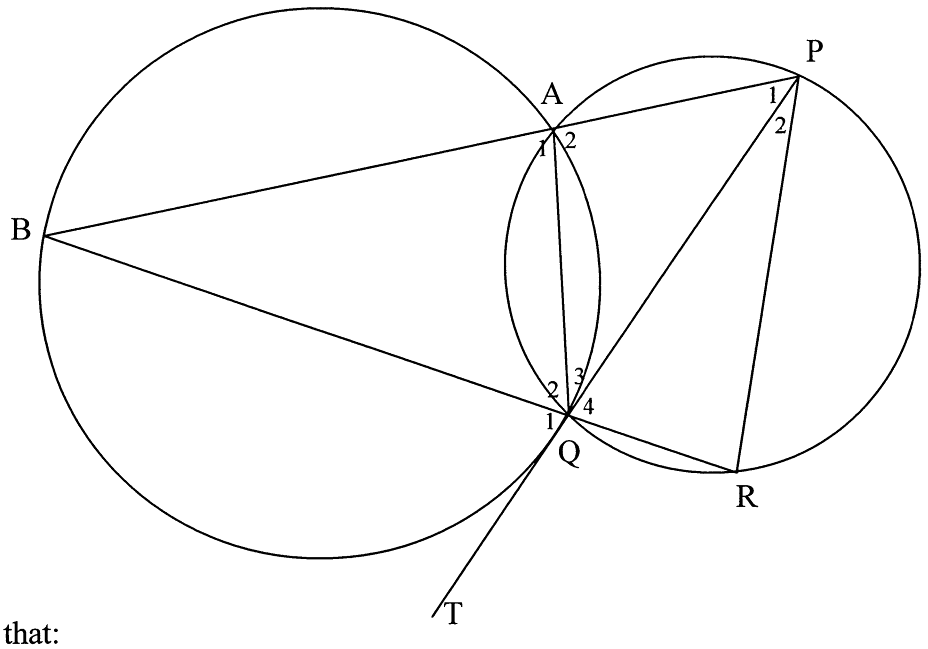


Bereken die numeriese waarde van $\frac{GH}{EF}$ (6)

[18]

QUESTION 8

8.1 In the diagram below, PQT is a tangent to the large circle ABQ at Q. BAP and BQR are straight lines with P and R on the smaller circle.



Prove that:

8.1.1 PQ = PR (5)

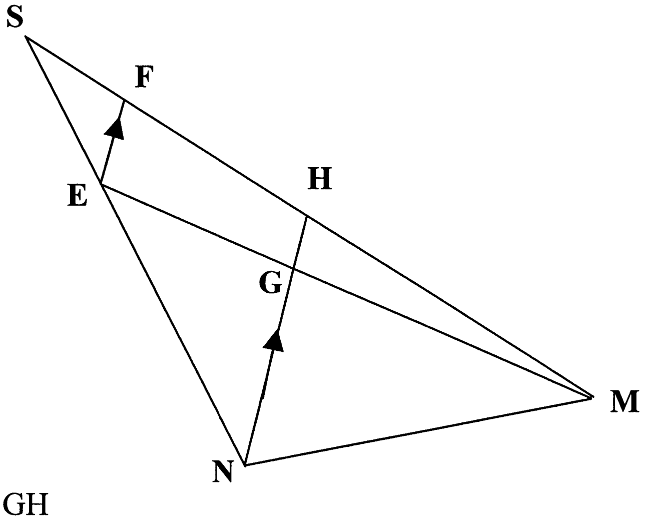
8.1.2 The lengths of PA, PR and PB (in this order) form a geometric sequence (7)

8.2 In the diagram alongside, E, F and H are points on Δ SNM.

EF || NH and EGM is a straight line with G on HN and EM.

SH : SM = 3 : 8

2 SE = EN



Calculate the numerical value of $\frac{GH}{EF}$ (6)

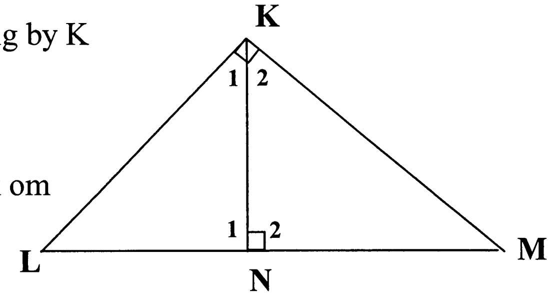
[18]

VRAAG 9

9.1 In die diagram langsaan is Δ KLM reghoekig by K en KN $\perp$ LM.

Gebruik die diagram op die diagramvel, of teken die diagram oor in jou antwoordeboek om die stelling te bewys wat beweer dat

Δ KLN ||| Δ MKN ||| Δ MLK (5)

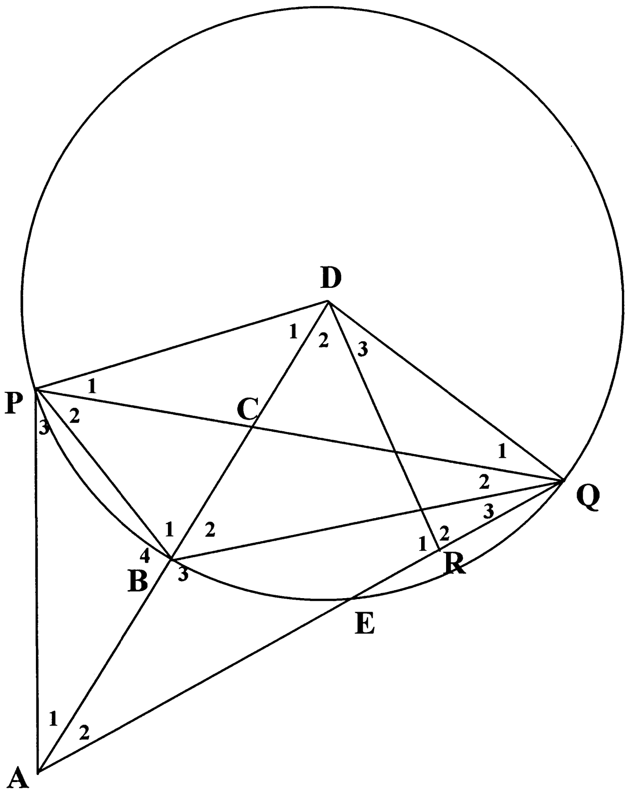


9.2 In die diagram langsaan is D die middelpunt van sirkel PBEQ.

DBA en PQ sny by C.

BQ is die halveerlyn van $P\hat{Q}A$ en

ER = RQ.



Bewys dat:

9.2.1 DPAQ 'n koordevierhoek is (4)

9.2.2 B die inmiddelpunt (middelpunt van die ingeskrewe sirkel) van Δ PQA is (4)

9.2.3 DQ is 'n raaklyn aan die sirkel ACQ (4)

9.2.4 $AQ^2 = AR \cdot AQ + DQ^2$ as AQ die middellyn van sirkel DPAQ is (7)
[24]

TOTAAL : 200

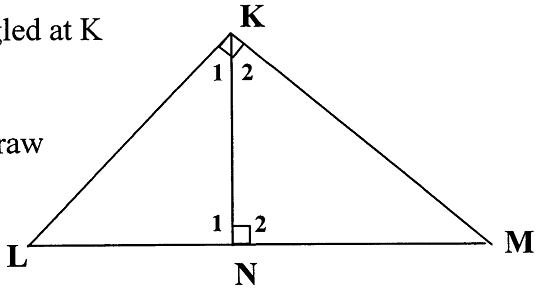
Kopiereg voorbehou

QUESTION 9

9.1 In the diagram alongside, Δ KLM is right angled at K and KN ⊥ LM.

Use the diagram on the diagram sheet, or redraw the diagram in your answer book to prove the theorem which states that

Δ KLN ||| Δ MKN ||| Δ MLK (5)

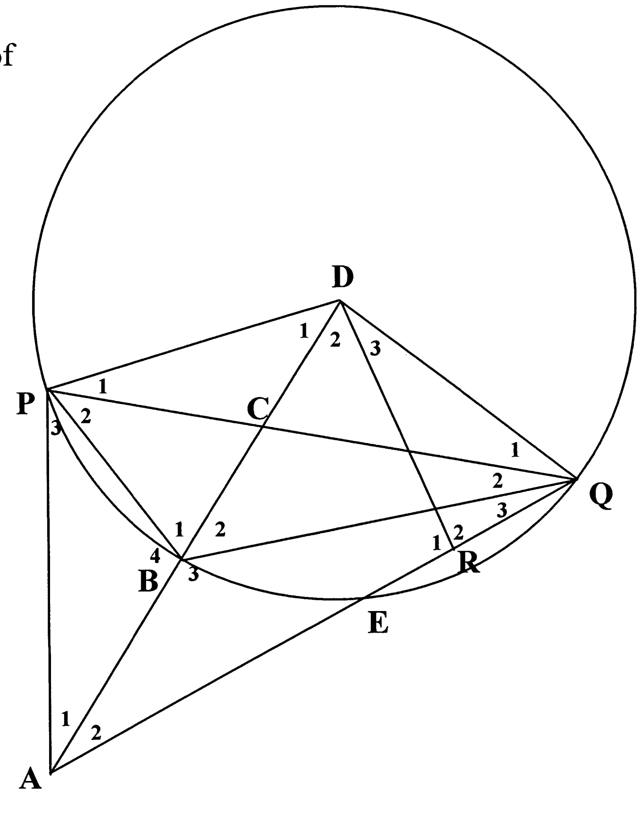


9.2 In the diagram alongside, D is the centre of circle PBEQ.

DBA and PQ intersect at C.

BQ is the bisector of $P\hat{Q}A$ and

ER = RQ.



Prove that:

9.2.1 DPAQ is a cyclic quadrilateral (4)

9.2.2 B is the incentre (centre of the inscribed circle) of Δ PQA (4)

9.2.3 DQ is a tangent to the circle ACQ (4)

9.2.4 $AQ^2 = AR \cdot AQ + DQ^2$ if AQ is the diameter of circle DPAQ (7)
[24]

TOTAL : 200

Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$T_n = a + (n-1)d$ $\qquad S_n = \frac{n}{2}(a + \ell)$ $\qquad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = a.r^{n-1}$; $\qquad S_n = \frac{a(1-r^n)}{1-r}$, $r \neq 1$ $\qquad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\qquad S_\infty = \frac{a}{1-r}$; $r \neq 1$

$A = P\left(1 + \frac{r}{100}\right)^n$ $\qquad A = P\left(1 - \frac{r}{100}\right)^n$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$y = mx + c$

$y - y_1 = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$x^2 + y^2 = r^2$ $\qquad (x-p)^2 + (y-q)^2 = r^2$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$\text{area } \Delta ABC = \frac{1}{2}ab.\sin C$

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DEPARTMENT OF EDUCATION

DEPARTEMENT VAN ONDERWYS

REPUBLIC OF SOUTH AFRICA
REPUBLIEK VAN SUID-AFRIKA

SENIOR CERTIFICATE EXAMINATION/SENIORSERTIFIKAAT-EKSAMEN
MATHEMATICS HG/WISKUNDE HG
PAPER II/VRAESTEL II
MARCH/MAART 2005

DIAGRAM SHEET/DIAGRAMVEL

INSTRUCTION

This diagram sheet must be handed in with your answer book. Please ensure that your details are complete.

INSTRUKSIE

Hierdie diagramvel moet saam met jou antwoordeboek ingelewer word. Maak asseblief seker dat jou besonderhede volledig ingevul is.

EXAMINATION NUMBER
EKSAMENNOMMER

CENTRE NUMBER
SENTRUMNOMMER

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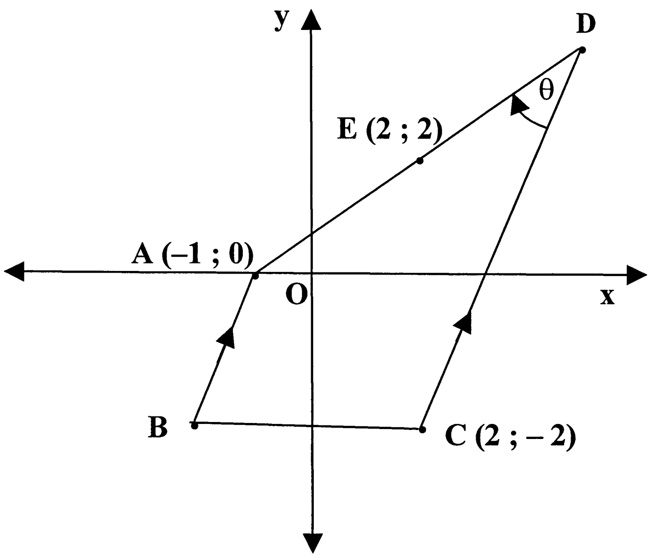
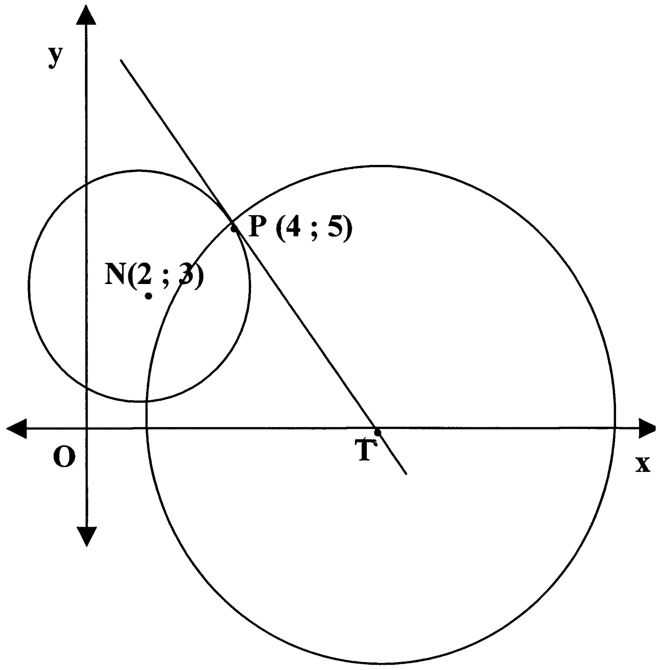
QUESTION 1/VRAAG 1



QUESTION 2/VRAAG 2



QUESTION 3.2/VRAAG 3.2

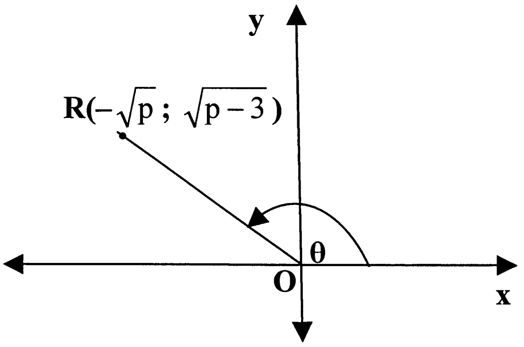
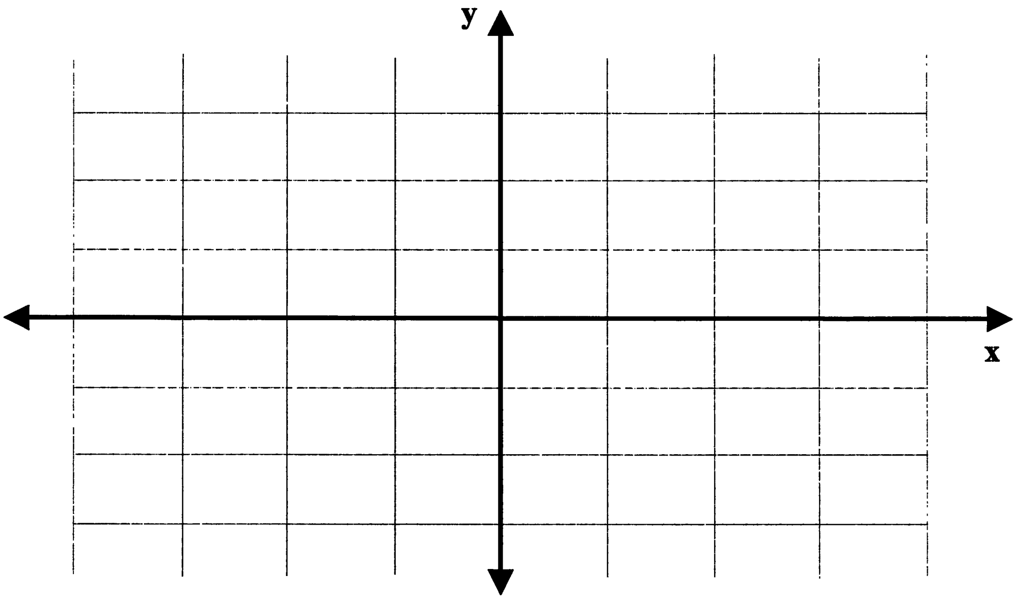


DIAGRAM SHEET/DIAGRAMVEL

QUESTION 4.2/VRAAG 4.2



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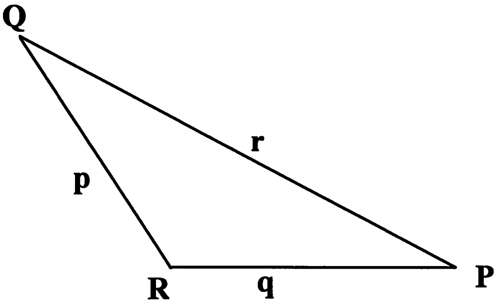
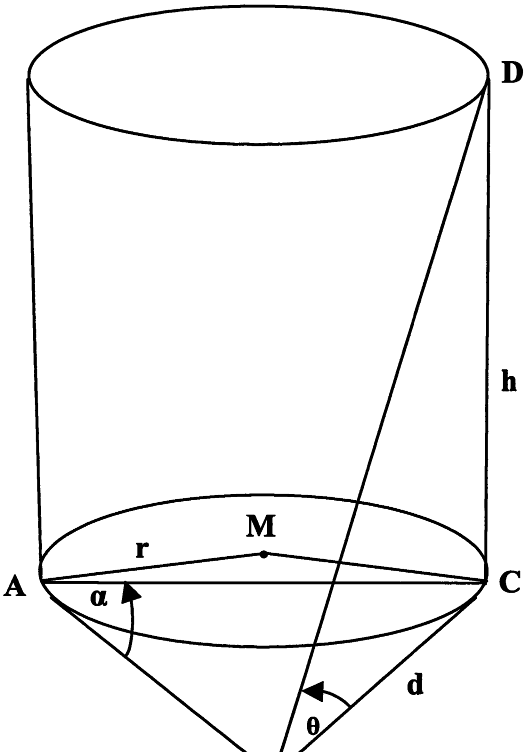


QUESTION 6. 2/VRAAG 6.2



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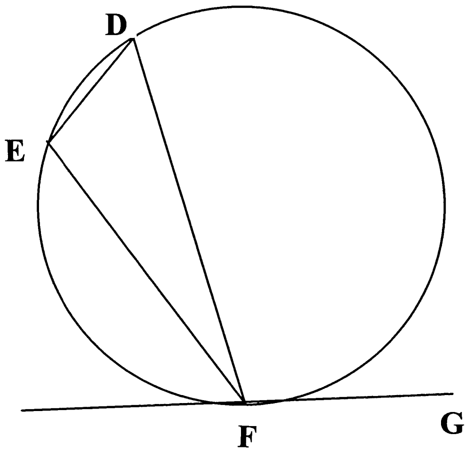
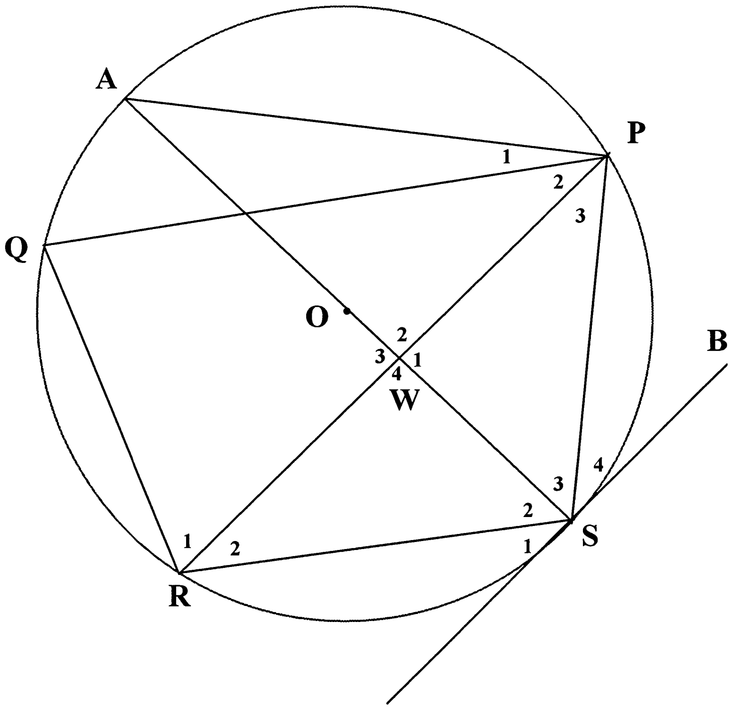
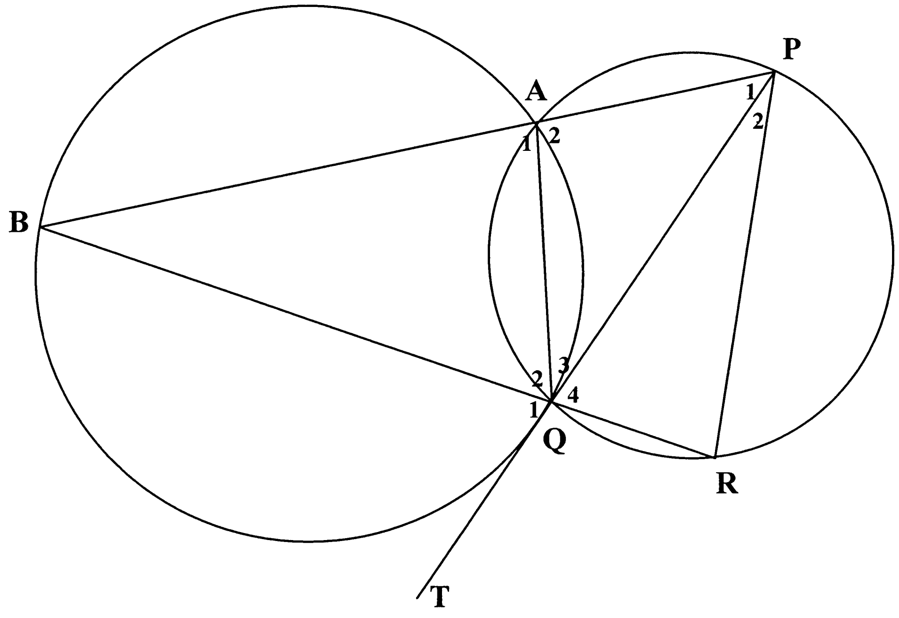


QUESTION 7.2/VRAAG 7.2



QUESTION 8.1/VRAAG 8.1



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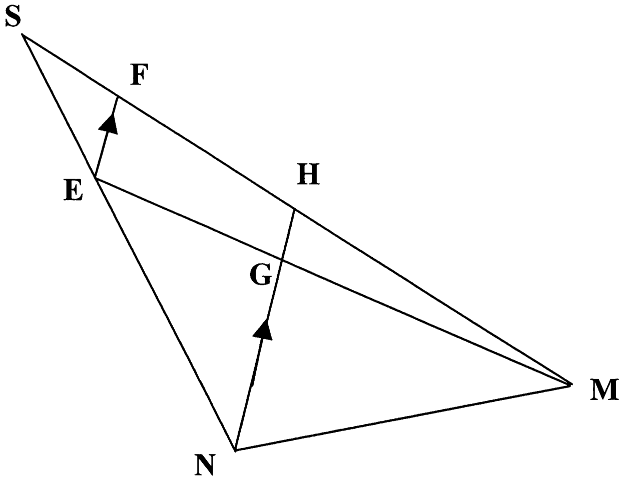
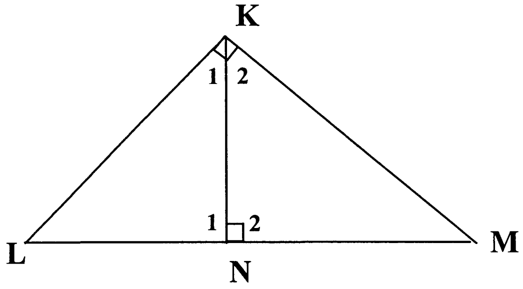
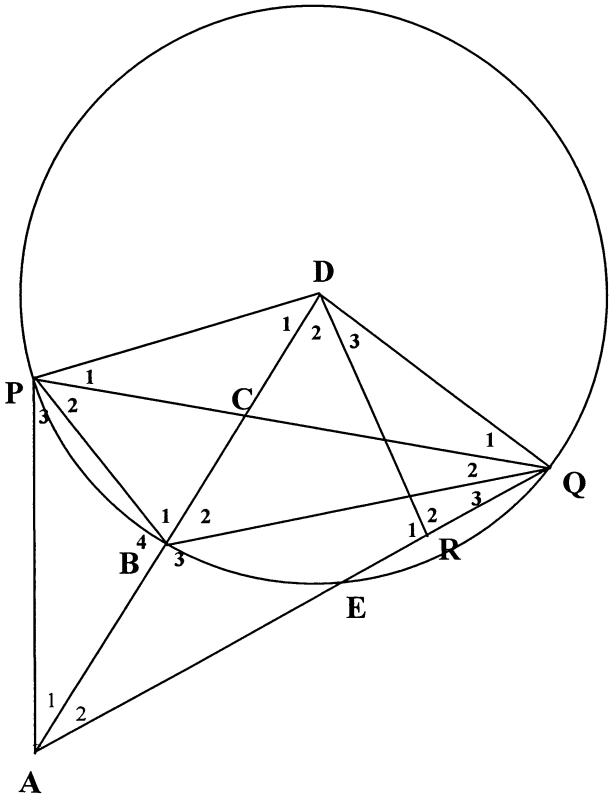


QUESTION 9.1/VRAAG 9.1



QUESTION 9.2/VRAAG 9.2



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