Why is 1, 2, 3.. to the power 0 equal to 1?

The answer is: *Because we want it to be 1!*
But why do we want it to be 1??
First, note that 2^0 is purely a mathematical generalisation, building *mathematical structure*, i.e. we do not find 2^0 in practical situations[1]!

What I mean with building structure in this context is that as we extend or *generalise our number range* from
the Natural numbers, i.e. 1, 2, 3, 4, …
to the Whole numbers 0, 1, 2, 3, …
and later to the Integers ... $^-3$, $^-2$, $^-1$, 0, 1, 2, 3, 4, …
we want our *known structures to remain valid*, and this means that we must give new meanings to several operations. Below I mention three structures …

For Natural number exponents, we know
2^n by definition means $2 \times 2 \times 2 \times 2$ …. *n* times
So $2^3 = 2 \times 2 \times 2 = 8$, but 2^0 and 2^{-1} do not make sense in this definition, and basically we cannot use this known definition, because the definition is only for Natural number exponents, and 0 and $^-1$ are not Natural numbers …

Mathematics problem solving is all about *using what we know to deduce what we do not know …*

We know:	$2^4 = 16$
	$2^3 = 8$
	$2^2 = 4$
	$2^1 = 2$
What if …?	$2^0 =$ **?**

The inquisitive mathematician asks: *what if* we extend the pattern downwards?
So we *invent* 2^0 … *If* the pattern is extended downward, *then* $2^0 = 1$ and also $2^{-1} = \frac{1}{2}$.
So we simply *define $2^0 = 1$, because we want the pattern (structure) to continue!*

Similarly for other structures:
We know that $2^m \times 2^n = 2^{m+n}$ for *m* and *n* Natural numbers.
We extend mathematics by always asking *what if … ?*, or *what if not …?*
So, *what if m* = 0? Strictly mathematically we may not substitute *m* = 0 into the above equation, because *m* is not a Natural number, but informally, just suppose …
Then we would have $2^0 \times 2^n = 2^{0+n} = 2^n$, i.e. $2^0 \times 2^n = 2^n$ and the only way this can be true, is if $2^0 = 1$, then $1 \times 2^n = 2^n$. *That is why we want $2^0 = 1$, so we define $2^0 = 1$!*

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[1] E.g. if in a practical problem we get $\frac{2^5}{2^5}$, we merely say the answer is 1, and if we get $\frac{2^3}{2^5}$ we merely say it is $\frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{4}$.

Likewise, we know that $\dfrac{2^m}{2^n} = 2^{m-n}$ for *m* and *n* Natural numbers and *m* > *n*.

So we wonder, *what if m* = *n*? Then $\dfrac{2^m}{2^m} = 2^{m-m} = 2^0$, i.e. $\dfrac{2^m}{2^m} = 2^0$, but we know $\dfrac{2^m}{2^m} = 1$, so *if our system is to be consistent, we will have to have* $2^0 = 1$.

The same argument leads to the *definition* of $2^{-n} = \dfrac{1}{2^n},\ n \in N$.

Note again: these are not *proofs* – it is invalid logic to substitute *m* = 0 or m= -1 in the exponential laws, because they are only defined and proven for *m* ∈ N! These are *conventions* in the form of *definitions!* The definitions are *man-made*, i.e. *invented*, they were not *discovered* on Treasure Island! Pupils must understand the nature of conventions as social knowledge, i.e. they must understand the reasons behind the definitions. If we say "it just is so", we convey a misconception of the nature of mathematics as arbitrary rules!

If we worked *only with Natural number exponents*, we must have these *three* structures:

$$\frac{a^m}{a^n} = a^{m-n} \text{ if } m > n,\ m,n \in N$$

$$\frac{a^m}{a^n} = \frac{1}{a^{n-m}} \text{ if } m < n,\ m,n \in N$$

$$\frac{a^m}{a^n} = 1 \text{ if } m = n,\ m,n \in N$$

The power of generalisation in Mathematics is that once we have *given meaning* (note I say *given* meaning, i.e. *invented* meaning, not *discovered* meaning!) to a^0 and to $a^{-n},\ n \in N$, then we need *only one* structure, namely

$$\frac{a^m}{a^n} = a^{m-n},\ m,n \in Z$$

because we now have meanings for answers like a^0 and a^{-1}.

Note:

- We are still not exact enough! What is the meaning of 0^0, 0^2, 0^{-2}??
- We said nothing about meanings for $a^{\frac{1}{2}}$ and $a^{\frac{2}{5}}$!!?? This connects with the problem of the meaning of "square root" and $\sqrt{a}$.