On Constructivism[1]

New approaches, and the philosophy underlying Curriculum 2005 is based on a *socio-constructivist theory* of the nature of knowledge and the nature of learning:

- The learner is not viewed as a passive receiver of knowledge, an “empty vessel” into which the teacher must “pour” knowledge. Conceptual knowledge cannot be transferred ready-made and intact from one person (the teacher) to another person (the learner).
- Rather, the learner is viewed as an active participant who constructs his/her[2] own knowledge. The learner comes to the learning situation with his own existing knowledge; new ideas are understood and interpreted in the light of the learner’s existing knowledge, built up out of his previous experience. Learning from this perspective entails that the learner must re-organise and re-structure his present knowledge structures, and this can only be done by the learner himself.
- Learning is a social process. Learners learn from each other (and the teacher) through discussion, communication and sharing of ideas, by actively comparing different ideas, reflecting on their own thinking and trying to understand other people’s thinking by negotiating a shared meaning.

The constructivist perspective on learning mathematics is well captured in the following quotations:

> *At present, substantial parts of mathematics that is taught . . . are based on a conceptual model that children are "empty vessels", and that it is the teacher’s duty to fill those vessels with knowledge about how calculations are performed by standard methods, and to provide practice until the children can perform these methods accurately. . . . Recent work would suggest that another model of mathematics learning is in fact a better one; learners are conceptualized as active mathematical thinkers, who try to construct meaning and make sense for themselves of what they are doing, on the basis of their personal experience . . . and who are developing their ways of thinking as their experience broadens, always building on the knowledge which they have already constructed.*
>
> Hilary Shuard (1986)

> *The more explicit I am about the behaviour I wish my students to display, the more likely it is that they will display that behaviour without recourse to the understanding that behaviour is meant to indicate; that is, the more likely that they will take the form for the substance.*
>
> Guy Brousseau (1984)

> *All now agree that the mind can learn only what is related to other things learned before, and that we must start from the knowledge that the children really have and develop this as germs, otherwise we are . . . talking to the blind about colour. Alas for the teacher who does not learn more from his children than he can ever hope to teach them!*
>
> G. S. Hall (1907)

> *All students engage in a great deal of invention as they learn mathematics; they impose their own interpretation on what is presented to create a theory that makes sense to them. Students do not simply learn a subset of what they have been shown. Instead, they use new information to modify their prior beliefs. As a consequence, each student’s knowledge is uniquely personal.*
>
> National Research Council (1989)

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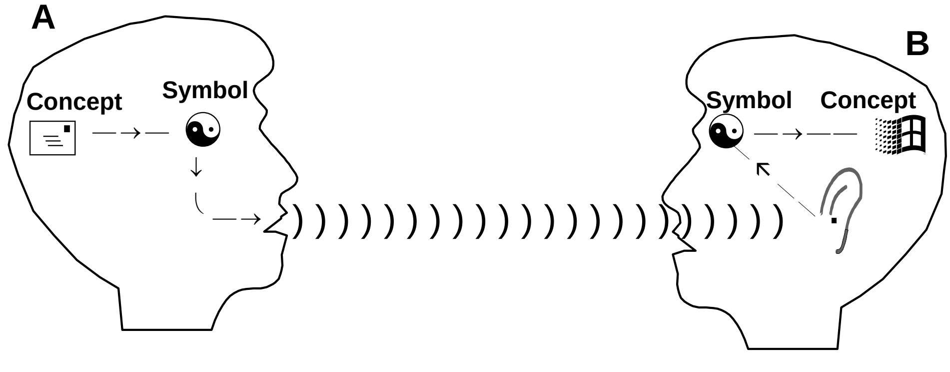
[1] Slightly adapted from Olivier, A. (1999). Constructivist learning theory. In Human, Olivier and Associates, **Advanced Numeracy Course. Facilitator's Guide**. Parow-East: Ebony Books CC.

[2] Instead of writing his/her throughout the text, we will write "his", but imply both male and female.

> *An imposition interaction pattern is characterized by the teacher maintaining tight controls over rules of procedure, over the kinds of acceptable contributions (usually of a very limited nature), over the amount of talk (teacher maximum), over meanings of terms and over the methods of solution. The mathematics teacher together with the textbook would represent the mathematical authority for the validity of solutions and the transmission of ideas and meaning from teacher to pupils would be emphasized.*
>
> *In a negotiation interaction pattern on the other hand, the rules of procedure are discussed and agreed on rather than imposed, the kinds of contributions from pupils will vary, there will be more equal amounts of teacher and pupil talk, and there will be discussion over meanings and over methods of solution. The mathematical context itself will offer the criteria for judging the acceptability and validity of solutions wherever possible, and in other cases the conventional criteria will be made explicit. In comparison with the transmission of ideas in the imposition pattern, here we would expect to find more of an emphasis on communication of ideas between teacher and pupils, and on establishing and developing shared meanings.*
>
> Alan Bishop (1987)

From the constructivist perspective, students must necessarily construct their own knowledge, irrespective of how they are taught. Even in the case of direct teaching ("telling"), students cannot absorb an idea exactly as it is taught, but must *interpret* it and give meaning to what the teacher says *in terms of their existing knowledge*. So they are *constructing* their knowledge (Cobb, Yackel & Wood, 1992).

Let us look a little closer at the process of communication. We think in terms of conceptual structures with a rich baggage of personal meanings and feelings. However when we talk to someone else, we must necessarily try to express those concepts in words, i.e. in symbols. So what B receives are not A's meanings and feelings, but A's *symbols* representing those meanings. B must now interpret these symbols in terms of his own concepts and meanings of these symbols. Because B has a different background and different life experiences than A, B's meanings of these symbols will invariably be *different* from A's meanings of the same symbols. This situation is depicted in the following diagram (Skemp,1979):

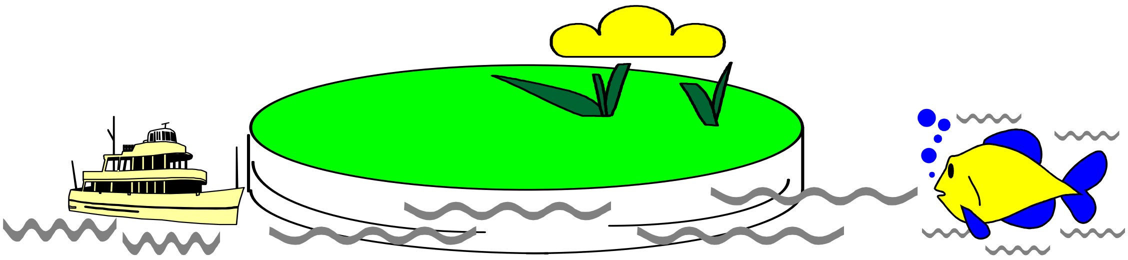


It should be clear that communication and in particular the transfer of knowledge is problematic. *One-way communication* and the transfer of knowledge (*telling*) can only be successful if A and B have nearly the same meanings for the symbols, i.e. if their *concepts* are nearly the same. If, however, they have vastly different concepts for the symbols, one of three things can happen:

- either there is a total breakdown of understanding, or
- B will *change* his *concepts* so that they are nearly the same as A's, and B will therefore understand what A is saying (Piaget's *accommodation*), or
- B will distort A's meanings to fit his own concepts behind the symbols (words), without changing his own concepts much (Piaget's *assimilation*), with the result that B will misunderstand A.

Let us illustrate this last point. A recent study addressed young children's "misconception" that the earth is flat. It is not surprising, it is based on their experience of their world – our immediate surroundings indeed look flat, not round. Most children probably at some stage believe that the earth is flat, and so did our forefathers before Diaz and Magellan. "No", explained the teacher, "the earth is round, because we can sail around it. Did you not learn in History that Magellan was the first person to sail around (circumnavigate) the earth in a boat?"

What the teacher meant when he said "round", was of course that the earth is a sphere (a "ball"). But what did the children "hear"? Most of them now agreed that the earth was round, and they would verbally say that the earth is *round*. So it seemed as if they understood. However, when they were later asked to draw the earth, it became apparent that their *meaning* for round was different from the teacher's: They still interpreted "round" in terms of their initial idea that the earth is *flat*. They therefore drew the earth as a *round, flat disc*!



This is a graphic illustration of how easily we can misinterpret new information, because *we must interpret it in terms of our own meanings*.

These problems with *one-way communication* are motivation for *two-way communication*, i.e. social interaction and discussion!

We should be quite clear about the different positions of the traditional behaviourist and the constructivist on the transfer of knowledge and the possibility of direct teaching (telling):

- The traditional behaviourist motto is: "There is nothing in the mind that was not first in the senses." It is assumed that a person can obtain direct knowledge of any reality, because, through the senses, we create an exact image (a replica or photocopy) of reality. Behaviourists therefore assume that knowledge can be transferred intact from one person to another. The learner is viewed as a passive recipient of knowledge, an “empty vessel” to be filled. The behaviourist teacher therefore tries to create a rich, concrete learning environment, because it is believed that *we understand what we see.*

- The constructivist perspective on learning, however, assumes that concepts are not taken directly from experience, that the learner does not passively absorb knowledge. Rather, the learner is an active participant in the construction of his own knowledge, because knowledge arises from the interaction of the learner’s existing ideas and new ideas, i.e. new ideas are interpreted and understood in the light of the learner’s own current knowledge. A person sees the world through the lens (“glasses”) of his existing knowledge and therefore each individual sees the world differently. The constructivist believes that *we see what we understand.* Conceptual knowledge can therefore not be transferred ready-made and intact from one person to another – each learner must necessarily construct his own conceptual knowledge.

  Ideas and thoughts cannot be communicated in the sense that meaning is packaged into words and "sent" to another who unpacks the meaning from the sentences. That is, as much as we would like to, we cannot put ideas in students' heads, they will and must construct their own meanings. Our attempts at communication do not result in conveying meaning but rather our expression ***evoke*** *meaning in another, different meanings for each person.*

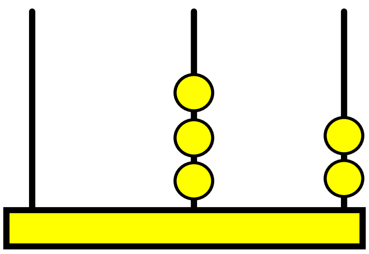
  Grayson Wheatley (1991)

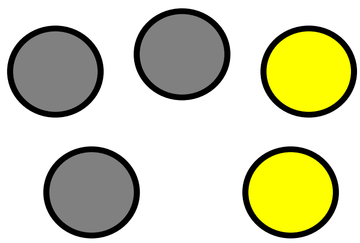
However, this does not mean that the teacher never “teaches” or tells. Following Piaget, we distinguish between three different types of knowledge, and between the sources of these different types of knowledge:

- *Social knowledge* is knowledge about the conventions of society, e.g. that Christmas is on 25 December, that the capital of France is Paris, that a tree is called “tree”, that in South Africa we drive on the left-hand side of the road. The terminology, notations and conventions used in mathematics may be regarded as social knowledge, e.g. that the number three is called “three” and that the symbol “3” is used for it, that there are 60 minutes in an hour, and that we multiply before we add, are all examples of social knowledge. Social knowledge originates outside the learner, it is *external knowledge*. Social knowledge is learned through *social transmission* – it can, and must be taught to learners.
- *Physical knowledge* is knowledge about the properties of physical objects. Such properties include, for example, that a glass jug is heavy, that the jug can hold water, but that it will overflow if you put too much water in it, that the jug will break if it falls, that the glass feels smooth and cold, that the jug is transparent (you can see through it), and the water feels wet. We learn these things through the experience of handling and manipulating physical objects. Physical knowledge is therefore learned through *empirical abstraction*, i.e. we abstract physical knowledge directly from the physical objects and from our actions on physical objects.
- *Conceptual (or logico-mathematical) knowledge* is knowledge about mathematical concepts and relationships. The structure of a mathematical idea cannot be abstracted directly from concrete objects or from our actions on concrete objects, but must be abstracted from relational systems that humans *impose* on objects, i.e. conceptual knowledge is constructed in the mind of the individual. Consider

this example: The idea of a line of symmetry is not really part of any object – it is an abstract idea created in the mind and then used as a "lens" to look at objects and so to "see" symmetry in objects. A person who has not yet constructed the idea of symmetry in his mind does not "see" symmetry in the world around him! Here is a numerical example: Knowledge about relationships between objects, e.g. relationships between a collection of 2 stones and a collection of 3 stones such as 3 − 2 = 1 or 2 + 3 = 5 or *3 is more than 2* are not part of the stones or part of our actions on the stones, but can exist only in the mind of a person. Conceptual knowledge originates in the mind of the individual person, through *reflective abstraction* (*thinking* about ideas and actions). It is *internal knowledge*.

One of the major problems with traditional behaviourist instruction is that it does not distinguish between these different types of knowledge. It treats conceptual knowledge as either social knowledge, and therefore tries to *tell* learners the knowledge, or it treats conceptual knowledge as physical knowledge and assumes that learners will understand transparent concrete embodiments of ideas. The problem with such an approach is that it assumes that learners see what we teachers see! Take as example the apparatus below often used to explain, or to model, place value concepts. The two representations each show 32. On the abacus, the *position* of the three beads on the middle wire makes them tens; in the case of the chips, we use different *colours* to convey that we can exchange 10 of the lighter coloured chips for one darker chip.

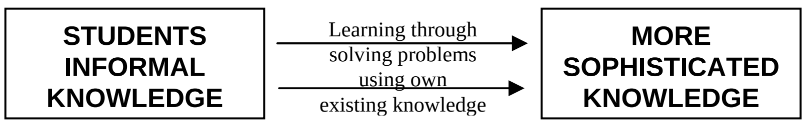




However, place value is *conceptual knowledge.* You only *see* 32 if you *already* understand the whole concept of place value. "Place value", as a structured conceptual scheme, acts as a lens through which you look at, interpret and see the situation – you *impose* meaning on the situation. However, learners who have not yet abstracted (constructed) the concept of place value mostly see 5, not 32! Let us again draw the essential distinction:

The empiricist believes *you understand what you see*.
The constructivist believes *you see what you understand*.

In modern approaches conceptual knowledge is not *taught*. Rather, learners' intuitive and informal numerical knowledge is taken as the point of departure and they are presented and challenged with sensible problems and activities that help them to build on their existing ideas, and re-organise and re-structure these ideas towards more sophisticated notions (Carpenter et al., 1999; Hiebert et al., 1996; Murray, Olivier & Human, 1993; Murray, Olivier & Human, 1998; Olivier, Murray & Human, 1990).

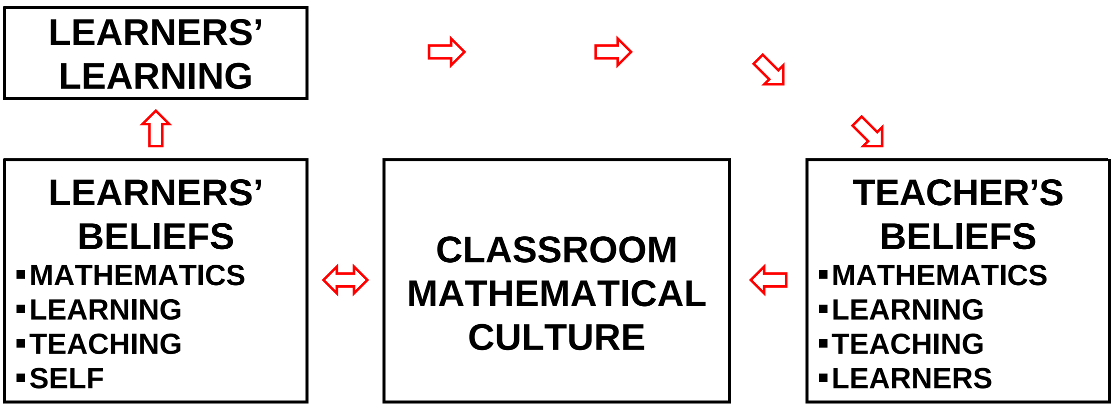


However, putting this into practice requires

- that an appropriate classroom *learning culture* be established between the teacher and the learners, including agreement about the aims and objectives and the norms of interaction in the classroom, and
- that realistic problem situations and tasks be utilised as a *vehicle* for learning.

The learning culture

Learners' learning is radically influenced by their beliefs about the nature of mathematics, about the nature of learning mathematics, about the nature of teaching and by their beliefs about their own abilities. These beliefs of learners are in turn influenced by the classroom mathematical culture, the day-to-day happenings in the classroom, which in turn are to a large extent determined by the beliefs of the teacher. These relationships are illustrated below:



It is therefore crucially important that the right kind of classroom mathematical culture be established and maintained as a *prerequisite* for learners to learn through problem solving and sharing of ideas.

The teacher and learners must negotiate a suitable *learning contract*, i.e. come to an agreement about the assumptions, expectations and obligations that will determine their respective roles in the classroom. This contract defines the classroom organisation and atmosphere or *culture*.

It must be established that the teacher will not teach (instruct!) number concepts and methods of computation. The teacher will not act as an authoritarian source of knowledge, but as an *organiser* of learning activities from which learners will learn through their own *activity*, and as a facilitator of the learning process by stimulating *reflection* on and *discussion* of learners' efforts.

Learners should not expect to be shown or told how to execute arithmetical tasks and solve problems, but should accept the given challenges to attempt to execute the tasks and solve the problems in their own way, taking responsibility for their own learning, realising that the teacher actually expects that they will make progress in solving the problem without his help (the teacher actually believes, and must be *seen* to believe, that the learners do have the ability to solve the problems). Learners should also accept that they are expected to demonstrate and explain their thinking, both verbally and in writing; and that they should listen carefully to and try to understand other learners' explanations, thereby learning from each other.

The teacher must listen to learners, accepting their explanations and justifications in a non-evaluative manner, with the purpose of identifying, understanding and interpreting the learners' present knowledge. This enables the teacher to provide appropriate further learning experiences that will facilitate the learners' development.

An essential part of the required learning contract concerns learners' *attitude* to learning. An orientation of understanding, of being "open" to try to really understand, and of *not* being dependent on the teacher to "show" them and "tell" them are fundamental to the success of the programme. *Intellectual independence* and *understanding* should be important objectives (intended outcomes) of the learning program; they are also *prerequisites* for true learning to take place. There is no place for any rote learning (memorising other people's methods without understanding). It serves no purpose and sabotages the objectives of understanding and independence:

> *I have observed, not only with other people but also with myself . . . that sources of insight can be clogged by automatisms. One finally masters an activity so perfectly that the question of how and why is not asked any more, cannot be asked any more, and is not even understood any more as a meaningful and relevant question.*
>
> Hans Freudenthal (1983)

The learning contract must therefore be based on *intrinsic* motivation (the *internal* satisfaction learners get from solving problems) and not *extrinsic* motivation (the *external* satisfaction of competition and reward, e.g. from getting praise from the teacher):

> *. . . a thinking subject has no occasion to feel the intellectual satisfaction of having solved a problem, if the solution did not result from his or her own management of concepts and operations but was supplied from outside. Here again the behaviorist notion of social approval as the prime reinforcement has helped to distort schooling practice. It is not that a teacher's approval and pat on the head have no effect on the student, but the effect is to strengthen the student's inclination to please the teacher rather than to build up <u>understanding</u> of the conceptual <u>area</u> in which the task was situated. Thus students are prevented from experiencing the rewarding elation that follows upon having found <u>one's own way and recognizing it as a good way</u>. If students are not oriented or led towards autonomous intellectual satisfaction, we have no right to blame them for their lack of proper <u>motivation</u>. The motivation to please superiors without understanding why they demand what they demand, may be required in an army – in an institution that purports to serve the propagation of knowledge, it is out of place.*
>
> Ernst von Glasersfeld (1991)

Another important facet of the required learning contract concerns the teacher's and the student's views about the nature of mathematics. Both teacher and students must believe that mathematics is not a finished formal body of knowledge, ("a bag of rules and prescriptions") to be learned and mastered, but that mathematics is a *human endeavour* (the solving of problems), and that each individual person has the ability to create the mathematical knowledge he needs, to "puzzle things out". Mathematics is also a *social activity* – individuals can "puzzle things out" and create the mathematical knowledge they need to solve problems by working together, by learning from each other. Learning mathematics is not a matter of finding out what some other people want you to do under certain circumstances, it is a matter of personally constructing the knowledge you need to solve problems. Learning mathematics is not to submit to an authoritative set of prescriptions and rules; it is to become *empowered* to solve problems.

Effective discussion (social interaction) among learners concerning problems, proposed solutions to problems and methods used, is crucially important:

- It forces learners to reflect on the work they have done, e.g. the way they solved a problem.

  > *From the constructivist point of view, there can be no doubt that reflective ability is a major source of knowledge on all levels of mathematics. That is the reason why . . . it (is) important that students be led to talk about their thoughts, to each other, to the teacher, or to both. To verbalize what one is doing ensures that one is examining it. And it is precisely during such examinations of mental operating that insufficiencies, contradictions, or irrelevancies are likely to be spotted. . . . leading students to discuss* <u>*their*</u> *view of a problem and their own tentative approaches, raises their self-confidence and provides opportunities for them to reflect and to devise new and perhaps more viable conceptual strategies.*
  >
  > Ernst von Glasersfeld (1991)

- It provides opportunities for learners to learn to communicate about mathematics and through mathematics.

- It provides opportunities for learning from other people without endangering the autonomy of individual learners, since social interaction with *peers* as equals does not suggest the obligation to adopt knowledge from one another, as is the case when interacting with an "authority".

- It provides an atmosphere in which learners are willing to think.

- Through classroom social interaction, the teacher and learners construct a consensual domain of taken-to-be-shared mathematical knowledge that both makes possible communication about mathematics and serves to constrain individual students' mathematical activity. In the course of their individual construction of knowledge, students actively participate in the classroom community's negotiation and institutionalisation of mathematical knowledge (Cobb et al., 1992).

The *quality* of social interaction among learners is a critical factor with respect to the success of problem-centred learning. *One of the most important tasks of the teacher is therefore to ensure that discussion among learners is at a high level*.

Learners have to understand their *social obligations* clearly, and the teacher should make inputs with respect to the following aspects where necessary.

- Learners should respect each other's needs for undisturbed thinking when trying to solve a problem, e.g. a learner who finishes before the others should not immediately announce his answer or try to force assistance on others.

- Where different solutions are proposed within a group, the group should not simply decide "by vote" and assume that the majority is right. All different proposals should be critically considered and real consensus should be sought.

Learners should not simply try to get their own answers accepted as right, but should test their own answers and those of other learners against the requirements of the problem with an open mind and "seek the truth".

- All learners in a group should get opportunities to submit their proposed solutions and methods.
- Learners should take pride in the quality of their explanations to others: methods should be described and justified clearly and systematically.
- Learners should respect each other's autonomy with respect to choice of methods. When a learner explains his method, it should not be with the attitude of *showing other learners how to do it*, but rather with the attitude "*This is how I did it and why, what do you think about it?"* Learners should also respect each other's need for understanding and should know that it is highly disadvantageous to use methods that are not well understood.

We summarise below the beliefs underlying the traditional classroom culture, and the opposing views underlying a child-centred curriculum and a problem-centred learning culture:

VIEW OF MATHEMATICS	
A finished body of facts, rules and skills	Dynamic continually expanding field
It can be discovered	A human social invention
VIEW OF TEACHING	
Instructor/Explainer	**Facilitator/organiser**
Break the maths into small logical pieces Explain, drill each piece in sequence	Present tasks and problems that lead to learners inventing mathematics
VIEW OF LEARNING	
Passive reception of knowledge	Active construction of knowledge
Learn by imitation, practice and repetition	Learn through social interaction and reflection
OBJECTIVES / OUTCOMES	
Submissiveness	Intellectual independence
Obedience	Understanding
Compliance	Problem solving and exploration
Following rules	Communication and reasoning
⇩	⇩
DISEMPOWERMENT	**EMPOWERMENT**

References

Bishop, A.J. (1987). *The social psychology of mathematics education*. **Educational Studies in Mathematics 19**, 1-13.

Brousseau, G. (1984). *The crucial role of the didactical contract in the understanding and construction of situations in teaching and learning mathematics*. In H.G. Steiner (Ed.) **Theory of Mathematics Education** (pp 110-119). Occasional paper 54. Bielefeld, IDM.

Carpenter, T.P., Fennema, E., Fuson, K., Hiebert, J., Human, P., Murray, H., Olivier, A. & Wearne, D. (1999). Learning basic number concepts and skills as problem solving. In E. Fennema & T. A. Romberg (Eds.), **Mathematics Classrooms that Promote Understanding** (pp. 45-62)**.** London: Lawrence Erlbaum.

Cobb, P., Yackel, E. & Wood, T. (1992). *A constructivist alternative to the representational view of mind in mathematics education*. **Journal for Research in Mathematics Education**, **23**(1), 2-33.

Freudenthal, H. (1983). **The Didactical Phenomenology of Mathematical Structures**. Dordrecht: Reidel.

Hall, G. S. (1907). **Aspects of Child life and Education**. Athenæum Press, Boston.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A. & Wearne, D. (1996). Problem-solving as a basis for curriculum development and instruction: The Case of Mathematics. **Educational Researcher**, 25(4), 12-21.

Murray, H., Olivier, A. & Human, P. (1993). *Learning through problem solving and social interaction*. In V. Reddy (Ed.), **Proceedings of the First Annual Meeting of the Southern African Association for Research in Mathematics and Science Education** (pp 246-255). Durban: Maskew Miller Longman.

Murray, H., Olivier, A. & Human, P. (1998). Learning through problem solving. In A. Olivier & K. Newstead (Eds.), **Proceedings of the Twenty-second International Conference for the Psychology of Mathematics Education: Vol. 1.** (pp. 169-185). Stellenbosch, South Africa.

National Research Council (1989). **Everybody Counts: A Report to the Nation on the Future of Mathematics and Education**. Wahington: National Academic Press.

Olivier, A.I., J.C. Murray and Human, P.G. (1990). *Building on young children's informal mathematical knowledge*. In G. Booker, P. Cobb & T.N. Mendicuti (Eds.), **Proceedings of the Fourteenth International Conference for the Psychology of Mathematics Education**, Vol. 3 (pp 297-304). Mexico City, Mexico.

Shuard, H. (1986). *Primary mathematics: towards 2000*. **The Mathematical Gazette 70**, 175-185

Skemp, R. R. (1979). **Intelligence, Learning, and Action**. Chichester: Wiley.

Von Glasersfeld, E. (Ed.) (1991). **Radical Constructivism in Mathematics Education**. Dordrecht: Reidel.

Wheatley, G.H. (1991). *Constructivist perspectives on science and mathematics learning*. **Science Education 75**(1), 9-21.