HOD: WISKUNDE-DIDAKTIEK 174

HUISWERK 2 – SAKREKENAARAKTIWITEITE

Inhandigingsdatum:

1. Die Wiskunde
Doen die aktiwiteite!
Gebruik die grafiese sakrekenaar kwistig!
Wys/bespreek jou werk.

2. Die Didaktiek
Die aktiwiteite is ontwerp om gebruik te word in 'n probleem-gesentreerde benadering en 'n oop ondersoekende klaskamer wiskundekultuur, dit is, die onderwyser wys/leer/demonstreer nie vir kinders metodes hoe om die probleme te doen nie. Kinders pak die probleme aan *en leer in die proses!*

Ons onderskei tussen:

- *Onderrig vir probleemoplossing:* Die onderwyser onderrig *eers* vooraf en apart die "tools" – die nodige wiskundige konsepte en metodes in die abstrak, en dan word dit *agterna* toegepas.
- *Onderrig via (deur) probleemoplossing:* Die onderrig *begin* met relevante probleme en die wiskundige konsepte en metodes word ontwikkel *terwyl* leerlinge die probleme oplos.
- *Onderrig omtrent probleemoplossing:* Die onderwyser illustreer in die algemeen probleemoplossingsmetodes.

Wat is jou opinie omtrent die moontlikhede van hierdie aktiwiteite vir 'n *onderrig-via-probleemoplossing* benadering?

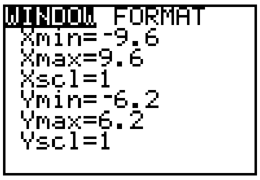
Graph Art 1

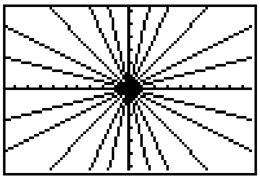
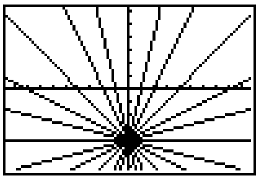
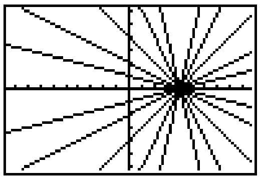
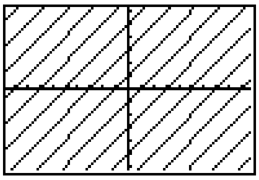
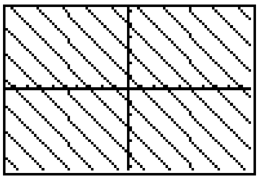
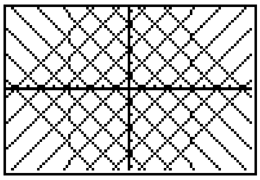
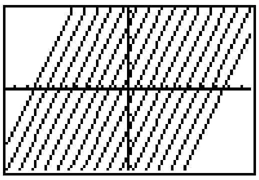
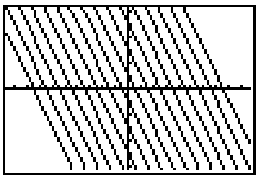
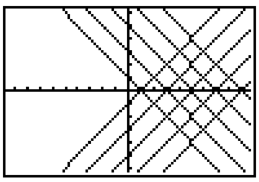
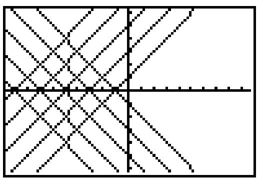
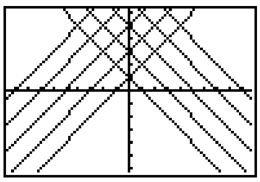
Make each of the designs below in your calculator window.

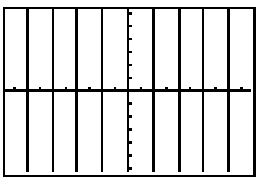
In each case *describe* the pattern in *words* and explain to others exactly how you designed the pattern (write down the functions you used).

Note:
To begin, set the **WINDOW** on your TI-82 graphing calculator as shown.

Also clear all functions in **Y=**





Graph Art 2

Make each of the designs below in your calculator window.

In each case *describe* the pattern in *words* and explain to others exactly how you designed the pattern (write down the functions you used).

Note:
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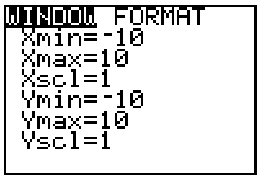
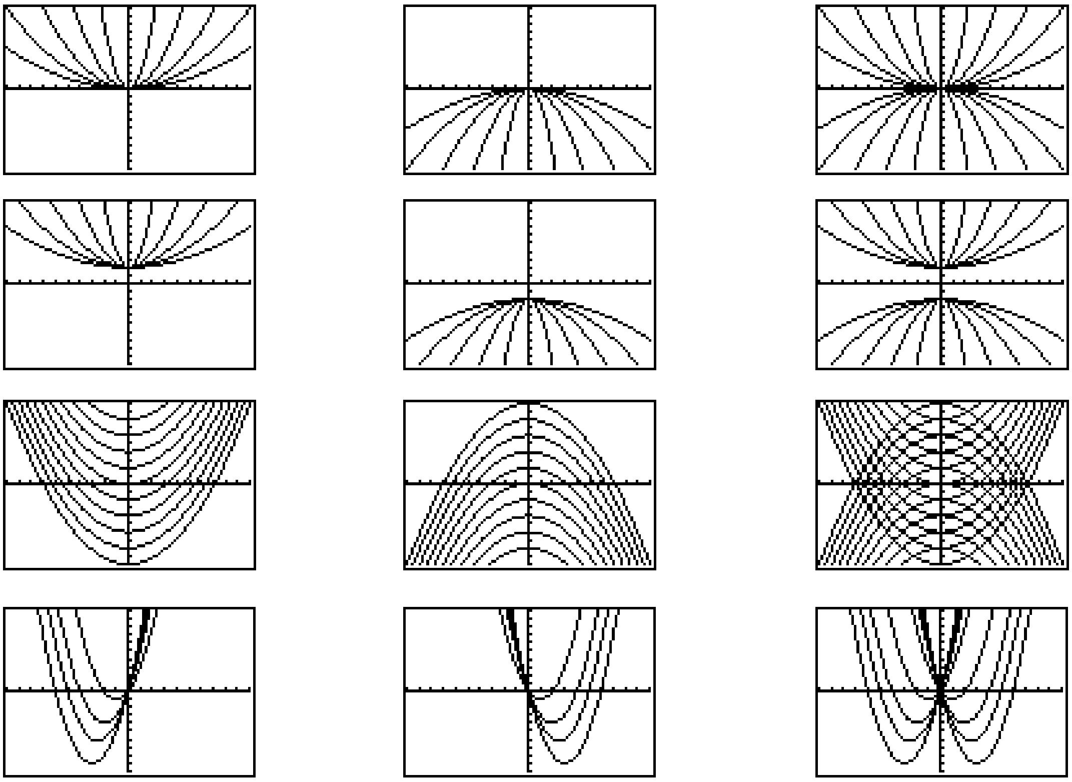
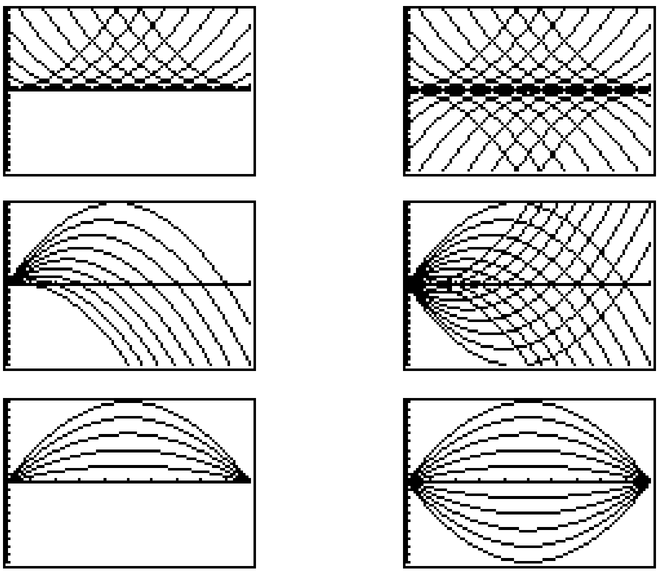
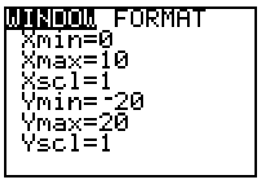


Also clear all functions in **Y=**





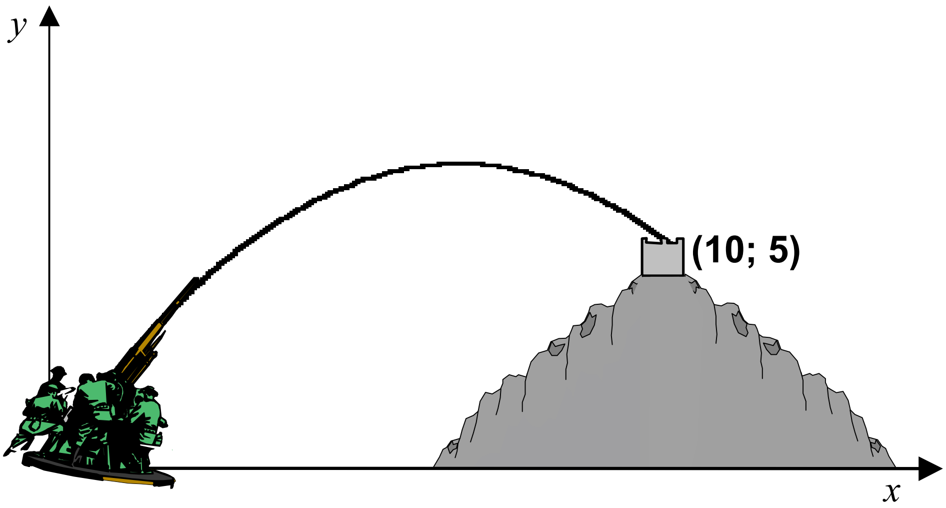
Use this WINDOW for the last six:



Canon

Imagine in the sketch below that you shoot a missile from a canon located at the origin (0; 0) and you want the stone to hit the enemy in their castle on the hill at the point (10; 5).

Assume that the trajectory (path) of the stone is described by a parabola.



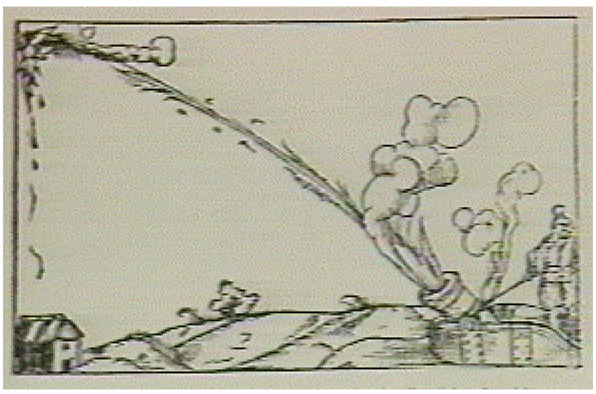
1. Find a formula for the parabola path that will hit the target.
   Prove that it will hit the target!

2. *Is there more than one formula that will hit the target?*
   Find another formula that will hit the target.
   Prove that it will hit the target!

3. A *mathematician* will want to find *all* the formulae that will hit the target!

Note: You may also want to look at this Excel canon equivalent!

Historical note:
For centuries man believed that that an object shot from a cannon, for example, followed a straight line until it "lost its impetus", at which point it fell abruptly to the ground, as illustrated in this drawing. Later, by more careful observation, it was realised that projectiles actually follow some sort of a *curved path*, but what sort of curve? No one knew until Galileo Galilei proved in 1638 that the curve has an exact mathematical shape called the parabola, with formula $y = ax^2 + bx + c$ where x is the horizontal distance and y is the corresponding height of the projectile at any given moment and a, b and c are situational constants (parameters).



Entrepreneur

An entrepreneur starts a new business.

He produces gadgets at a cost of $Y_1 = 4x + 200$ rands and then sells it at $Y_2 = 5x$ rands, where x is the number of gadgets.

Note: He assumes that he can sell all the gadgets he produces.

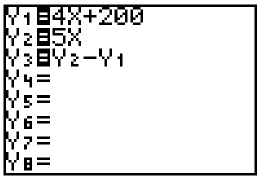
Profit = Selling price – Production cost

1. Calculate his profit for different numbers of gadgets.
2. Draw graphs of the *cost-function* Y_1 and the *selling price-function* Y_2 on the same system of axes.
3. How many gadgets must he produce and sell to make a profit? Can you show this on the graphs?
4. Of course he can increase his profit by making and selling more gadgets. But there are limits to the market and to his production capacity.
   Show how he can increase his (unit) profit by reducing his production costs and/or increasing his selling price. Illustrate it graphically . . .

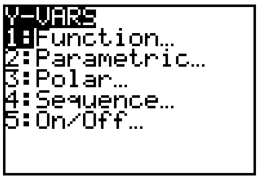
Challenge:

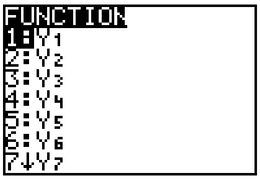
Draw the graph for the *profit-function* by entering $Y_3 = Y_2 – Y_1$.

Can you deduce the same information as above from this graph?



Note: to enter Y_2 and Y_1 directly into the equation for Y_3 you must press 2nd Y-VARS , choose FUNCTION and then select the appropriate function:

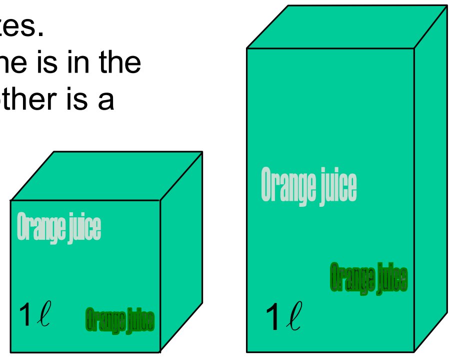




Can you write down a formula (in simplest form) for Y_3 using x?
How can you *check* that you are right?

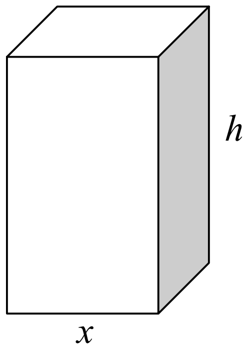
DESIGN A CONTAINER

Food containers come in many different shapes and sizes.
For example, here are two cardboard containers – the one is in the shape of a **cube** of 10 cm by 10 cm by 10 cm, and the other is a **rectangular box (cuboid)** of 10 cm by 5 cm by 20 cm.



Do you agree that both containers hold the same amount of orange juice? How much?

Which container requires the most material to make it? How much?



Is there a *best shape* that is the most economical for the manufacturer, i.e. that uses the minimum material for a given volume?

Suppose a manufacturer wants to make a cuboid, with a square base, that will hold 1 000 mℓ. What is the best shape?

Investigate the relationship between the height h and the side length x of the square base of the cuboid with volume 1 000 mℓ, and how different values of x and h influences the amount of material needed to build the shape.

Is this a special case for 1 000 mℓ, or is it also the case for other volumes?

Is there any other shape of container that needs *less* material to build for a given volume?

Making fences

1. (a) Thabo wants to build a run for his chickens with 20 m of fencing wire. He decides to make a rectangular run, and that the area of the run should be as large as possible.

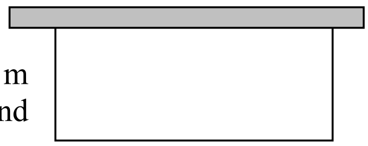
How long and how wide should he make the run? Complete a table like this for different lengths and widths to help you decide:

Length (m)	**Width (m)**	**Area (m^2)**
6	4	24
7		
8		

(b) What must the length be if he had

(1) 12 m (2) 28 m (3) 40 m (4) x m of wire?

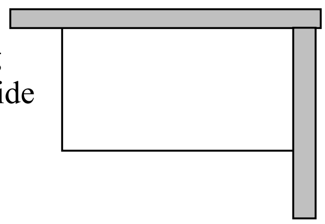
2. (a) If Thabo decides to build the chicken run against a wall of the house so that he uses the 20 m fencing wire for only three sides, how long and how wide should he make the run?



(b) What must the length be if he had

(1) 12 m (2) 28 m (3) 40 m (4) x m of wire?

3. (a) If Thabo decides to build the chicken run against a corner of the house so that he uses the 20 m fencing wire for only two sides, how long and how wide should he make the run?



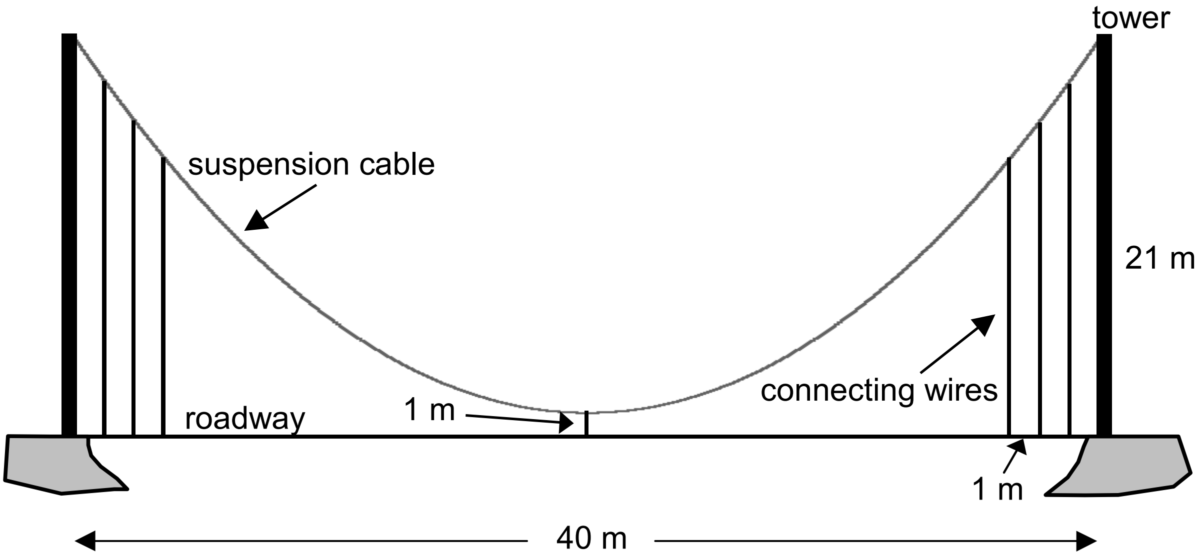
(b) What must the length be if he had

(1) 12 m (2) 28 m (3) 40 m (4) x m of wire?

Building bridges

The famous Golden Gate hanging bridge in San Francisco, USA

Engineers must build a hanging bridge over a river. The roadway of the bridge will be hung from a suspension cable with connecting wires, 1 m apart. At its lowest point, the connecting wire is 1 m long. The suspension cable is supported by two towers at the ends, which are each 21 m high and are 40 m apart.



It is essential that the lengths of the connecting wires should be *exact*, otherwise the bridge is unsafe!
The lengths cannot be found by *practical measurement while* building the bridge!
This is how engineers do it: They imagine a system of axes, find a *formula* for the shape of the suspension cable, then they use this formula as a *model* to *calculate* the lengths of all the connecting wires *beforehand*.

Make a table giving the length of each of the connecting wires.
Describe patterns in the table.

Can you, after calculating a few values, use the *table* as a *model* to easily calculate the other lengths?

Note: the *choice* of the system of axes influences the *form* of the formula and the *complexity* of the calculations! Choose at least *four* different positions for the axes, give the corresponding formula for the shape of the suspension cable and discuss the advantages/disadvantages of the four systems.