DIE ROOS

Vooraf

Lees as agtergrond ook Fireworks Notes en Rose Notes.
Dit gaan hier nie soseer oor spesifieke *wiskundige inhoud* nie, maar oor *strategieë en houdings teenoor probleemoplossing*.

Kom ons kyk na leerlinge met Schoenfeld se bril van die faktore wat wiskundige probleemoplossing beïnvloed:

- "*Resources*": wiskundige kennis wat die individu ken wat hy/sy in die situasie kan gebruik.
- *Heuristieke*: probleemoplossingstrategieë (strategiese kennis) soos maak 'n skets, voer notasies in, ondersoek spesiale gevalle, maak 'n tabel, soek 'n patroon, . . .
- *Kontrole* of *meta-denke (denke oor denke):* die keuse en die monitering van kennis en heuristieke – weet hoe lank om te volhard met 'n poging en wanneer om te aborteer en iets anders te probeer.
- "*Belief systems*": perspektiewe op die aard van wiskunde, die leer van wiskunde en perspektiewe op eie vermoë.

Eerstens "beliefs" en houdings:
Glo hulle hulle kan die problem oplos – *probeer* hulle, of gee hulle sommer op en wag om gewys te word?
Hoe probeer hulle – probeer hulle iets onthou, of soek vir patrone? Wil hulle die koorde *tel*?
Wat is hul houding teenoor "waarheid" in Wiskunde – wat aanvaar hulle as geldig? Logika of outoriteit?

Heuristieke/probleemoplossingstrategieë:
Redeneer hulle *induktief* of *deduktief*?

Indien induktief – probeer hulle makliker spesiale gevalle? Is hulle sistematies, organiseer hulle die data, soek hulle vir 'n patroon? *Skryf* hulle – gebruik hulle skryf as 'n denkhulpmiddel?

Indien deduktief – is hul *logika* korrek? Wonder hulle of dit korrek is, d.w.s. is hulle *skepties* oor hul eie bewerings en hanteer dit as *hipoteses wat bewys moet word*, of aanvaar hulle dit onnadenkend? Hoe oortuig hul hulself dat dit korrek is? Hoe oortuig hulle ander dat dit korrek is?

Resources/kennis:
Soek hulle vir 'n *funksionele* formule, of gebruik hulle *rekursie*? Besef hulle dat die een nuttiger vir hierdie problem is as die ander?
As hulle korrek is vir 18 punte, kan hulle die resultaat veralgemeen na 50? Kan hulle veralgemeen na n? Wat beteken "n" vir hulle?

Het hul hul kennis in die groep gedeel? Was die groep meer suksesvol as individue? Het die groep mekaar gehelp? Wie het deelgeneem, wie was passief, het almal se idees ewe veel getel? Wie het geleer, wie het nie iets geleer nie? Wat het hulle geleer, wat het hulle nie geleer nie? *Hoe, en hoekom?*

Kontrole:
Hoe kontroleer hulle hul vordering?

Dit is nie belangrik as hulle nie by vraag 2 uitkom nie …

Agterna waarneming/refleksie (Jaylene Pheiffer – 2011)

The 5 boys from Paul Roos sat in 2 groups of 3 and 2. The problem was explained twice. They said that they understood and continued working on the problem. The Patel learner in the one group had experimented with this problem before. He was eager to get going and get an answer. The other boys were struggling with the concept *[Concept? Examples of concepts are "variable", "rectangle", ... So, with what concept were they struggling? You merely mean struggling to solve the problem? AIO].* Jeffery got his calculator from his bag without fully understanding what he was going to do. I did tell them that I am more interested in their calculations and that a number on its own does not carry any weight *[Yes! But I do not agree with the reason for it – this establishes the teacher as the authority who determines "what carries weight", "how many marks you will get". Rather, the reason is has everything to do with our view of what Mathematics is, what it means to "do mathematics". Mathematics is all about reasoning and logic ... so we want to establish a classroom atmosphere where the prime authority of whether something is right/wrong, true/false, is only reasoning and logic, not the teacher, or the majority, or ... AIO]*. Although they were split in groups they worked individually not sharing their ideas *[Why do you think that is? AIO?]*.

The other boys started scribbling but not really knowing what they were doing. Kreef had no understanding of the problem *[How do you know this, how did you find that out? AIO]*. He did not write down anything on his page. I explained to him again followed with some leading questions. How many points? How many cords at each point? *[Could you ask specific questions to find out if he did not understand <u>the problem</u>, or maybe understood the problem (question, context), but did not know how to start to <u>solve</u> it? AIO]*

Patel came up with a solution of 17+16+15+14+13+……….+1 *[He explained why: at the second point we have already counted the line connecting point 1, etc.]*. He added each number to get his total. I gave the rest of the class time to work with the problem, before Patel explained his answer.

Jeffery had an answer of 18 × 17. He knew 18 was the number of points and 17 was the number of cords *[? You mean the number of chords at each point. AIO]*. He failed to see that the cords were counted twice *[I had the impression that he learnt from Patel's explanation that he counted each line twice. And I checked at the end and he understood! AIO]*. All the boys looked at this problem of 18 points. No one considered that the same reasoning would hold for a smaller set of points. I moved on to show them how a smaller subset of numbers would work *[Why was it necessary for you to do this? AIO]*. Jeffery understood the /2 after Patel explained to him why we had to do that – to remove the duplicates.

I asked them to get a generic formula *[I am not sure about "generic" – I think rather "general" AIO]* for the calculation they have made. I suggested we use n as the number of points. They insisted on using x *[Why do you think this is? AIO]*. We used x after I explained to them that it could be anything as long as we remember what it represented, in our case the number of points. One boy said $x.y/2$. I asked them how they would express y in terms of x. They said $y = (x - 1)$. The number of cords is one less than the number of points. Thus they formulated the, x being the number of points. They could then easily do the problem with 25 point and 100 points on a circle.

However, Kreef did his calculation of 25 points as (25 x **17**)/2. This showed that he did not understand the concept of the cords. 17 still referred to the previous problem with the 18 points. I explained to him again what the formula $x(x-1)/2$ meant and what the x represented *[I am sure that Kreef did not use this formula at all!! Rather, he interpreted the structural result for 18 points, i.e. 18×17/2 ... He thought that because the number of points change from 18 to 25, only the 18 should change to 25!!! This really means that he does not understand the semantic references for the 18, 17 and 2, thus that he did not understand the crucial distinction between variable and constant, and all because he did not put in more energy into understanding the references for the objects! AIO]*.

I tried to get them to see that 17+16+15+14+13+……….+1 would come down to the same formula as $x(x-1)/2$, but we ran out of time *[I am very disappointed that it did not bother them, that they did not ask themselves why, how? AIO]*.

<u>Reflection:</u>

I talked too much and *[therefore! AIO]* did not give them enough time to work on the problem.
One learner already knew the answer, I should have asked him to explain last so that we could first understand where the others did not fully grasp the problem.
I should ask my questions to specific learners and not in general, because then the learner that knows the answer will always reply.
Speak to the whole class and not individuals *[sometimes ... it depends on ... AIO]*.
Follow up on problems, don't ignore wrong answers *[Wrong answers give us a great opportunity to learn about learners' thinking, and to analyse the mathematics! AIO]* .
More emphasis could have been places on mathematical terminology like the meaning and usage of variables and constants.

[Thanks Jaylene! Everybody please note: This is mostly a chronological description of what happened. However, the objective with this exercise is much more: <u>we want to learn to think like a mathematics teacher!</u> I am offering you lenses to look at what happened, and we must look at each little episode through these academic lenses – beliefs, heuristics, resources, control, etc. And we should not just say what children did, but describe it in academic terms, and theorise about the reasons for it …did they work inductively/deductively (everybody used deduction? But except for one, wrong deduction!), etc …
Also please note the fineprint on the lesanalise webpage: Die fasiliteerder behoort in die agterna refleksie minstens die volgende kategorieë te bespreek: die wiskundige inhoud (bv. die moeilikheidsgraad) ...; die leerders (bv. bestaande kennis, denkstrategieë, wankonsepte, ...); die leeromgewing/kultuur (bv. dissipline, leer in groepe) ...; die fasiliteerder (bv. intervensies, onsekerhede, ...) AIO]

PTO

Ouer … Agterna waarneming/refleksie (Lynn Jakins)

Note: in this problem there were 25 points on the circle.

Attitudes of the students

The students seemed willing to participate. They split themselves into voluntary friendship groups (not the ideal situation as a pre-established hierarchy is then in place, leaving no room to explore new developments by different students). In this case friendships grouping resulted in a racial divide, with the three coloured children in one group (Group 1)and the four white children in the other group (Group 2).

They received the worksheets eagerly but Group 2 seemed hesitant to write anything without first presenting their ideas to the class, possibly for fear of it being wrong. They also had an eraser with which they erased their "thinking" and enabled them to produce neat answers. They worked in pencil, giving me the feeling that they were less sure of their abilities and providing a "way out" so to speak.

Group 1, however, used pen and were not hesitant to write down answers. They seemed less worried about the neatness factor and more concerned about the correctness of their work. Both groups were very quiet in their explanations, leading me to think that group work was a new experience for them. At first some sat back and watched how the others were attempting the solution before becoming committed to the task. At the end though all of the students were participating.

Students' Strategies

In Group 1 they immediately assumed the answer was 25×25 and worked out the numerical answer. There seemed to be a general consensus in the group and no discussion was needed. In Group 2 one of the students had written 25×24 on his paper but when engaging in discussion with the group became "convinced" that his answer must be wrong. When I asked him why he was wrong, he replied that the 'rest of the group' has a different answer, therefore majority rules and he was wrong. So this resulted in both groups having the *wrong* 25×25 as their answer, leaving me with no choice but to present an example of a smaller case in order to bring about a *conflict* that would convince them that their thinking was wrong.

Once I had drawn a circle with 5 points and drawn in the connecting lines and counted the lines leaving from the first point, both groups realized that there were only $5 - 1$ lines leaving from each point and so changed their method to 25×24. We then counted the lines in the circle, coming to 10 lines while their new method gave $5 \times 4 = 20$ lines. Immediately a student from Group 2 said to divide it by 2 but was unable to explain the logic, his reason being that it 'gave the correct answer' *[So his saw an inductive pattern in one case. AIO]*. However, a student from Group 1 was able to explain that in counting each line at each point we were in fact counting it twice and should divide by 2 *[So his reasoning was deductive and therefore general and always true. AIO]*. They then came to the general formula of $n(n - 1) \div 2$, although most wrote it without the brackets as $n \times n - 1 \div 2$.

They did not make the doubling error from 25 to 50 as I had suspected they would and was thus quite pleased. However one of the students from Group 1 still asked "Is it right?" which indicates that he might not have fully understood the methods and so was unsure of his answer.

Since both groups had made the same mistake there was no chance for an inter-group explanation session (as I had hoped there might be) as both had followed the same incorrect method of analysis.

My Personal Experience

I was very nervous and unsure of how to begin, this being my first time ever of standing in front of a class. At first when hearing about how these classes would work I was worried that having classmates at the back of the class would hinder me. Funnily enough, once the lesson began I never even noticed them!

I was surprised at how quickly the students completed the activity, perhaps I should have given them more time to work on the problem before intervening. They seemed to be easily convinced of the correct answer, displaying enough knowledge to convince me that most of them at least understood where they had gone wrong and why.

I was afraid to offer too much help and in so doing 'lead' them along to a method which they would not have arrived at by themselves. But I was forced to do this once I saw that both groups had the same incorrect method and that the student from Group 2 was not going to explain his thought process (being convinced of its incorrectness). It disappointed me that this student was so easily convinced of being wrong, although even if he believed himself to be right, I still think he would have gone along with the group's method, either for popularity or fear of being the only one with a different and therefore wrong answer.

Altogether I found it a very positive experience and enjoyed watching how the students "thought" and attempted strategies. I was pleased that they were so willing to be part of the process and offered answers quite easily. I realize that I should have asked specific students to explain the processes instead of leaving it up to them to elect a spokesperson, as well as avoiding assuring them of their correctness of method. The reason for this is that they should have been convinced by the logic of their reasoning that their method was in fact correct and thus not need the "stamp of approval" of the teacher.

PTO

Ouer … Agterna waarneming/refleksie (Alettie van den Heever)

Note: in this problem there were 25 points on the circle.

Agtergrond

Die klas het bestaan uit 6 seuns wat in Paul Roos deur medium van Engels onderrig word. Drie van hulle se moedertaal is egter Xhosa en a.g.v my nalatigheid het hulle hulleself groepeer in twee groepe van drie Engels- en drie Xhosa-sprekers. Aanvanklik het almal Engels gepraat, maar later het die een groep oorgeslaan na Xhosa, sodat ek nie kon verstaan wat hulle sê nie. Ek kon uit hulle lyftaal aflei dat hulle steeds besig was met die probleem, maar ek dink hulle was aan die een kant bang dat ek hoor indien hulle iets verkeerds sou sê, en aan die ander kant was dit 'n natuurlike reaksie: wanneer 'n mens met 'n probleem gekonfronteer word, is dit immers net logies om eerstens van enige "language barriers" ontslae te raak. Hulle het ook nog tot aan die einde steeds weer en weer die probleem herlees, asof hulle bang was dat daar iets was wat hulle gemis het. Die hele situasie het net weer aan my bewys hoe belangrik moedertaalonderrig is en hoe verweefd wiskunde en taal is. Indien begrip van die taal wat as medium gebruik word, afwesig is, is begrip van die wiskunde baie onwaarskynlik. Dit sou interessant wees om te sien wat dieselfde leerders met dieselfde probleem (steeds in Engels) sou doen in 'n minder intimiderende situasie. Dit is moontlik dat hulle in 'n konteks waarin hulle heeltemal gemaklik is, anders sou reageer. Ek besef dus dat my gevolgtrekkings gegrond is op een observasie met baie beperkinge. *[Ek glo dat as ons sorg dat groepsamestellings heterogeen is om vir diversiteit te voorsien, baie van die probleme eintlik voordele kan word en almal by almal kan leer. AIO]*

Die oplos van die probleem

In die een groep (van nou af Groep X) het die "leier" dadelik besef dat daar iets gedoen moet word met 24, maar hulle was nie seker wat nie. Hulle het aanvanklik niks neergeskryf nie, maar net na die roos gekyk. In die ander groep (Groep E) was die eerste reaksie: 25 × 25. Hulle het onder andere die kleiner sirkels binne-in die sirkel getel. Die "leier" het ook die aantal koorde vanaf een punt getel en 24 gekry. Toe ek hom egter na die aantal vra, het hy gesê dat hy 24 gekry het, maar dat hy verkeerd moes getel het, want dit behoort 25 te wees. Die ander twee was nie gelukkig met 25 maal 25 nie en toe die twee groepe op die ou end hul bevindinge deel, het beide groepe gesê dat die antwoord op die eerste vraag 25 maal 24 is. Toe ek hulle vra na die rede vir hierdie antwoord, het hulle geantwoord dat elke punt met 24 ander punte verbind is en dat daar dus 24 koorde vanaf elke punt uitgaan.

Ek het toe op die bord vir hulle 'n *teenvoorbeeld* gewys, alhoewel ek dit nie so genoem het nie. Ons het 'n sirkel met drie punte se koorde getel. Hulle het saam met my die antwoord, 3, gekry en gesien dat hulle *metode* (25 × 24 vir 25 punte) die antwoord 6 (3 × 2) sou gee. Dit was egter duidelik dat hulle nie 'n verband getrek het tussen die *struktuur* van die roos met 25 punte en die roos met 3 punte nie. Toe ek hulle vra of die roos met 25 punte en die een met 3 dan nie iets met mekaar te doen het nie en of hulle steeds tevrede is met hulle antwoord, het die leier van Groep E baie definitief bevestig dat die antwoord 25 maal 24 moet wees. Die ander het nie so seker gelyk nie. Ek het toe voortgegaan om sirkels met 2, 4 en 5 punte te teken en ons het die koorde getel. Op hierdie punt het Groep E begin besef dat daar 'n verband moes wees – ek dink egter dit was meer die gevolg van my beklemtoning daarvan as wat dit die gevolg was van hulle herkenning van 'n struktuur.

Groep E het nie regtig soveel aandag aan die eenvoudiger voorbeelde op die bord gegee nie – hulle het steeds gekonsentreer op die roos met die 25 punte voor hulle. Ek het toe 'n tabel op die bord gemaak met die aantal punte en die aantal koorde as opskrifte. Ek dink met die maak van die tabel het Groep E finaal besef dat ek hulle nie net deurmekaar wil maak nie – ek dink hulle is al taamlik gewoond daaraan dat 'n tabel 'n sekere patroon impliseer. Toe hulle dit eers besef of gesnap het, het hulle vinnig met die korrekte formule vorendag gekom.

Toe ek hulle egter vra na die rede vir die formule, kon hulle slegs die patroon in die tabel as rede gee. Hulle het dus slegs *induktief* te werk gegaan. Daarmee in sigself is daar natuurlik nie fout te vind nie. Wat my egter bekommer, is my afleiding dat selfs die induktiewe benadering eintlik die gevolg was van wat ek dink 'n mens probleemoplossing-programmering sou kon noem. As hulle sekere dinge op die bord sien, sekere woorde hoor, dan weet hulle om x en y te doen of vir x en y te gaan soek. Dalk het ek hulle verkeerd opgesom, maar ek dink nie die vind van die formule was 'n werklike verstaan van die probleem en die struktuur van die probleem nie. Dis asof hulle eers die antwoord moet kry en dan die rede vir die antwoord kan gaan soek. *[Dit is eintlik heel natuurlik dat dit dikwels die verloop van wiskundige aktiwiteit is. Jy "ontdek" iets, en probeer dit eers agterna verklaar of bewys. AIO]*

Ek het toe weer die voorbeeld van 'n roos met 3 punte gevat. Ek het gesê dat ons moet kyk hoe ons die aantal koorde sou kry as ons hulle oorspronklike metode van 25 × 24 (nou 3 × 2) sou gebruik. Ek het letterlik elke koord getrek en toe weer gevra waarom die formule 'n "gedeeldeur 2" het. Die een seun in Groep E het toe gesê dat 'n mens elke koord twee maal tel en dat jy daarom die 3 × 2 deur 2 moet deel.

Groepwerk

In beide groepe was dit dadelik duidelik wie die "leier" was. Wat vir my die groepwerk doeltreffend gemaak het, was die feit dat dit juis die ander lede in die groep was wat dikwels die "stem van die rede" was wat die selfversekerdheid van die "leier" bevraagteken het. Op die manier het die groepwerk dus al die leerders gebaat.

Verwagtinge
Ek het nie geweet wat om te verwag nie. As ek eerlik moet wees, kan ek glad nie onthou op watter vlak die gemiddelde graad 9-leerder is nie. Ek het o.a. die klassieke foutiewe redenering van 25×25 en 24×25 verwag. Ek was daarom aan die een kant baie beïndruk daarmee dat hulle die formule gekry het.

Gevolgtrekkings
Ek het egter weer besef hoe moontlik dit is om probleme korrek op te los na aanleiding van 'n paar leidrade sonder om waarlik die struktuur van die probleem te verstaan en hoe maklik leerders daarmee kan wegkom as hulle "motiewe" nie bevraagteken word nie. Ek het in die praktyk gesien waarom een som eerder op tien maniere gedoen moet word as wat tien van dieselfde soort somme gedoen word. Leerders moet gelei word om vir hulleself te dink. Natuurlik is dit handig om te weet sekere metodes werk vir sekere probleme, maar dit help die leerder niks as hy nie self eers die probleem in sy geheel verstaan nie. Die belangrikheid van probleemoplossingstrategieë het weereens vir my duidelik geword. Ek was hartseer om te sien hoe die Xhosa-sprekers sukkel a.g.v. die taal. Hulle het my aanvanklik ge"fool", omdat hulle Engels op die oog af goed is. Dit was eers toe hulle na Xhosa oorgeslaan het, dat ek besef het dat die taal tog maar 'n kwessie moes wees. Ek dink dit het te make met die verskillende graad van taalvaardigheid wat 'n mens benodig vir kommunikasie in 'n taal en probleemoplossing in 'n taal.