DEPARTMENT OF EDUCATION

NATIONAL CURRICULUM STATEMENT GRADES 10-12 (GENERAL)

MATHEMATICS

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(General)

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HOW TO USE THIS BOOK

This document is a policy document divided into four chapters. It is important for the reader to read and integrate information from the different sections in the document. The content of each chapter is described below.

■ Chapter 1 - Introducing the National Curriculum Statement

This chapter describes the principles and the design features of the National Curriculum Statement Grades 10 – 12 (General). It provides an introduction to the curriculum for the reader.

■ Chapter 2 - Introducing the Subject

This chapter describes the definition, purpose, scope, career links and Learning Outcomes of the subject. It provides an orientation to the Subject Statement.

■ Chapter 3 - Learning Outcomes, Assessment Standards, Content and Contexts

This chapter contains the Assessment Standards for each Learning Outcome, as well as content and contexts for the subject. The Assessment Standards are arranged to assist the reader to see the intended progression from Grade 10 to Grade12. The Assessment Standards are consequently laid out in double-page spreads. At the end of the chapter is the proposed content and contexts to teach, learn and attain Assessment Standards.

■ Chapter 4 – Assessment

This chapter deals with the generic approach to assessment being suggested by the National Curriculum Statement. At the end of the chapter is a table of subject-specific competence descriptions. Codes, scales and competence descriptions are provided for each grade. The competence descriptions are arranged to demonstrate progression from Grade 10 to Grade 12.

■ Symbols

The following symbols are used to identify Learning Outcomes, Assessment Standards, grades, codes, scales, competence description, and content and contexts.

= Learning Outcome		= Scale
= Assessment Standard		= Competence Description
= Grade		= Content and Contexts
= Code		

CONTENTS

ACRONYMS

2D/2-D	Two Dimensions (or two-dimensional)
3D/3-D	Three Dimensions (or three-dimensional)
AIDS	Acquired Immune Deficiency Syndrome
CASS	Continuous Assessment
DO	Developmental Outcome
FET	Further Education and Training
GET	General Education and Training
HIV	Human Immunodeficiency Virus
IKS	Indigenous Knowledge Systems
NCS	National Curriculum Statement
NQF	National Qualifications Framework
OBE	Outcomes-Based Education
SAQA	South African Qualifications Authority

CHAPTER 1

INTRODUCING THE NATIONAL CURRICULUM STATEMENT

The adoption of the Constitution of the Republic of South Africa (Act 108 of 1996) provided a basis for curriculum transformation and development in South Africa. The Preamble states that the aims of the Constitution are to:

- heal the divisions of the past and establish a society based on democratic values, social justice and fundamental human rights;
- improve the quality of life of all citizens and free the potential of each person;
- lay the foundations for a democratic and open society in which government is based on the will of the people and every citizen is equally protected by law; and
- build a united and democratic South Africa able to take its rightful place as a sovereign state in the family of nations.

The Constitution further states that 'everyone has the right … to further education which the State, through reasonable measures, must make progressively available and accessible'.

The National Curriculum Statement Grades 10 – 12 (General) lays a foundation for the achievement of these goals by stipulating Learning Outcomes and Assessment Standards, and by spelling out the key principles and values that underpin the curriculum.

PRINCIPLES

The National Curriculum Statement Grades 10 – 12 (General) is based on the following principles:

- social transformation;
- outcomes-based education;
- high knowledge and high skills;
- integration and applied competence;
- progression;
- articulation and portability;
- human rights, inclusivity, environmental and social justice;
- valuing indigenous knowledge systems; and
- credibility, quality and efficiency.

Social transformation

The Constitution of the Republic of South Africa forms the basis for social transformation in our post-apartheid society. The imperative to transform South African society by making use of various transformative tools stems from a need to address the legacy of apartheid in all areas of human activity and in education in particular. Social transformation in education is aimed at ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of our population. If social transformation is to be achieved, all South Africans have to be educationally affirmed through the recognition of their potential and the removal of artificial barriers to the attainment of qualifications.

Outcomes-based education

Outcomes-based education (OBE) forms the foundation for the curriculum in South Africa. It strives to enable all learners to reach their maximum learning potential by setting the Learning Outcomes to be achieved by the end of the education process. OBE encourages a learner-centred and activity-based approach to education. The National Curriculum Statement builds its Learning Outcomes for Grades 10 – 12 on the Critical and Developmental Outcomes that were inspired by the Constitution and developed through a democratic process.

The Critical Outcomes require learners to be able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively with others as members of a team, group, organisation and community;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

The Developmental Outcomes require learners to be able to:

- reflect on and explore a variety of strategies to learn more effectively;
- participate as responsible citizens in the life of local, national and global communities;
- be culturally and aesthetically sensitive across a range of social contexts;
- explore education and career opportunities; and
- develop entrepreneurial opportunities.

High knowledge and high skills

The National Curriculum Statement Grades 10 – 12 (General) aims to develop a high level of knowledge and skills in learners. It sets up high expectations of what all South African learners can achieve. Social justice requires the empowerment of those sections of the population previously disempowered by the lack of knowledge and skills. The National Curriculum Statement specifies the minimum standards of knowledge and skills to be achieved at each grade and sets high, achievable standards in all subjects.

Integration and applied competence

Integration is achieved within and across subjects and fields of learning. The integration of knowledge and skills across subjects and terrains of practice is crucial for achieving applied competence as defined in the National Qualifications Framework. Applied competence aims at integrating three discrete competences – namely, practical, foundational and reflective competences. In adopting integration and applied competence, the National Curriculum Statement Grades 10 – 12 (General) seeks to promote an integrated learning of theory, practice and reflection.

Progression

Progression refers to the process of developing more advanced and complex knowledge and skills. The Subject Statements show progression from one grade to another. Each Learning Outcome is followed by an explicit statement of what level of performance is expected for the outcome. Assessment Standards are arranged in a format that shows an increased level of expected performance per grade. The content and context of each grade will also show progression from simple to complex.

Articulation and portability

Articulation refers to the relationship between qualifications in different National Qualifications Framework levels or bands in ways that promote access from one qualification to another. This is especially important for qualifications falling within the same learning pathway. Given that the Further Education and Training band is nested between the General Education and Training and the Higher Education bands, it is vital that the Further Education and Training Certificate (General) articulates with the General Education and Training Certificate and with qualifications in similar learning pathways of Higher Education. In order to achieve this articulation, the development of each Subject Statement included a close scrutiny of the exit level expectations in the General Education and Training Learning Areas, and of the learning assumed to be in place at the entrance levels of cognate disciplines in Higher Education.

Portability refers to the extent to which parts of a qualification (subjects or unit standards) are transferable to another qualification in a different learning pathway of the same National Qualifications Framework band. For purposes of enhancing the portability of subjects obtained in Grades 10 – 12, various mechanisms have been explored, for example, regarding a subject as a 20-credit unit standard. Subjects contained in the National Curriculum Statement Grades 10 – 12 (General) compare with appropriate unit standards registered on the National Qualifications Framework.

Human rights, inclusivity, environmental and social justice

The National Curriculum Statement Grades 10 – 12 (General) seeks to promote human rights, inclusitivity, environmental and social justice. All newly-developed Subject Statements are infused with the principles and practices of social and environmental justice and human rights as defined in the Constitution of the Republic of South Africa. In particular, the National Curriculum Statement Grades 10 – 12 (General) is sensitive to issues of diversity such as poverty, inequality, race, gender, language, age, disability and other factors.

The National Curriculum Statement Grades 10 – 12 (General) adopts an inclusive approach by specifying minimum requirements for all learners. It acknowledges that all learners should be able to develop to their full potential provided they receive the necessary support. The intellectual, social, emotional, spiritual and physical needs of learners will be addressed through the design and development of appropriate Learning Programmes and through the use of appropriate assessment instruments.

Valuing indigenous knowledge systems

In the 1960s, the theory of multiple-intelligences forced educationists to recognise that there were many ways of processing information to make sense of the world, and that, if one were to define intelligence anew, one would have to take these different approaches into account. Up until then the Western world had only valued logical, mathematical and specific linguistic abilities, and rated people as 'intelligent' only if they were adept in these ways. Now people recognise the wide diversity of knowledge systems through which people make sense of and attach meaning to the world in which they live. Indigenous knowledge systems in the South African context refer to a body of knowledge embedded in African philosophical thinking and social practices that have evolved over thousands of years. The National Curriculum Statement Grades 10 – 12 (General) has infused indigenous knowledge systems into the Subject Statements. It acknowledges the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution. As many different perspectives as possible have been included to assist problem solving in all fields.

Credibility, quality and efficiency

The National Curriculum Statement Grades 10 – 12 (General) aims to achieve credibility through pursuing a transformational agenda and through providing an education that is comparable in quality, breadth and depth to those of other countries. Quality assurance is to be regulated by the requirements of the South African Qualifications Authority Act (Act 58 of 1995), the Education and Training Quality Assurance Regulations, and the General and Further Education and Training Quality Assurance Act (Act 58 of 2001).

THE KIND OF LEARNER THAT IS ENVISAGED

Of vital importance to our development as people are the values that give meaning to our personal spiritual and intellectual journeys. *The Manifesto on Values, Education and Democracy* (Department of Education, 2001:9-10) states the following about education and values:

Values and morality give meaning to our individual and social relationships. They are the common currencies that help make life more meaningful than might otherwise have been. An education system does not exist to simply serve a market, important as that may be for economic growth and material prosperity. Its primary purpose must be to enrich the individual and, by extension, the broader society.

The kind of learner that is envisaged is one who will be imbued with the values and act in the interests of a society based on respect for democracy, equality, human dignity and social justice as promoted in the Constitution.

The learner emerging from the Further Education and Training band must also demonstrate achievement of the Critical and Developmental Outcomes listed earlier in this document. Subjects in the Fundamental Learning Component collectively promote the achievement of the Critical and Developmental Outcomes, while specific subjects in the Core and Elective Components individually promote the achievement of particular Critical and Developmental Outcomes.

In addition to the above, learners emerging from the Further Education and Training band must:

- have access to, and succeed in, lifelong education and training of good quality;
- demonstrate an ability to think logically and analytically, as well as holistically and laterally; and
- be able to transfer skills from familiar to unfamiliar situations.

THE KIND OF TEACHER THAT IS ENVISAGED

All teachers and other educators are key contributors to the transformation of education in South Africa. The National Curriculum Statement Grades 10 – 12 (General) visualises teachers who are qualified, competent, dedicated and caring. They will be able to fulfil the various roles outlined in the Norms and Standards for Educators. These include being mediators of learning, interpreters and designers of Learning Programmes and materials, leaders, administrators and managers, scholars, researchers and lifelong learners, community members, citizens and pastors, assessors, and subject specialists.

STRUCTURE AND DESIGN FEATURES

Structure of the National Curriculum Statement

The National Curriculum Statement Grades 10 – 12 (General) consists of an Overview Document, the Qualifications and Assessment Policy Framework, and the Subject Statements.

The subjects in the National Curriculum Statement Grades 10 – 12 (General) are categorised into Learning Fields.

What is a Learning Field?

A Learning Field is a category that serves as a home for cognate subjects, and that facilitates the formulation of rules of combination for the Further Education and Training Certificate (General). The demarcations of the Learning Fields for Grades 10 – 12 took cognisance of articulation with the General Education and Training and Higher Education bands, as well as with classification schemes in other countries.

Although the development of the National Curriculum Statement Grades 10 – 12 (General) has taken the twelve National Qualifications Framework organising fields as its point of departure, it should be emphasised that those organising fields are not necessarily Learning Fields or 'knowledge' fields, but rather are linked to occupational categories.

The following subject groupings were demarcated into Learning Fields to help with learner subject combinations:

- Languages (Fundamentals);
- Arts and Culture;
- Business, Commerce, Management and Service Studies;
- Manufacturing, Engineering and Technology;
- Human and Social Sciences and Languages; and
- Physical, Mathematical, Computer, Life and Agricultural Sciences.

What is a subject?

Historically, a subject has been defined as a specific body of academic knowledge. This understanding of a subject laid emphasis on knowledge at the expense of skills, values and attitudes. Subjects were viewed by some as static and unchanging, with rigid boundaries. Very often, subjects mainly emphasised Western contributions to knowledge.

In an outcomes-based curriculum like the National Curriculum Statement Grades 10 – 12 (General), subject boundaries are blurred. Knowledge integrates theory, skills and values. Subjects are viewed as dynamic, always responding to new and diverse knowledge, including knowledge that traditionally has been excluded from the formal curriculum.

A subject in an outcomes-based curriculum is broadly defined by Learning Outcomes, and not only by its body of content. In the South African context, the Learning Outcomes should, by design, lead to the achievement of the Critical and Developmental Outcomes. Learning Outcomes are defined in broad terms and are flexible, making allowances for the inclusion of local inputs.

What is a Learning Outcome?

A Learning Outcome is a statement of an intended result of learning and teaching. It describes knowledge, skills and values that learners should acquire by the end of the Further Education and Training band.

What is an Assessment Standard?

Assessment Standards are criteria that collectively describe what a learner should know and be able to demonstrate at a specific grade. They embody the knowledge, skills and values required to achieve the Learning Outcomes. Assessment Standards within each Learning Outcome collectively show how conceptual progression occurs from grade to grade.

Contents of Subject Statements

Each Subject Statement consists of four chapters and a glossary:

- *Chapter 1, Introducing the National Curriculum Statement:* This generic chapter introduces the National Curriculum Statement Grades 10 – 12 (General).
- *Chapter 2, Introducing the Subject:* This chapter introduces the key features of the subject. It consists of a definition of the subject, its purpose, scope, educational and career links, and Learning Outcomes.
- *Chapter 3, Learning Outcomes, Assessment Standards, Content and Contexts:* This chapter contains Learning Outcomes with their associated Assessment Standards, as well as content and contexts for attaining the Assessment Standards.
- *Chapter 4, Assessment:* This chapter outlines principles for assessment and makes suggestions for recording and reporting on assessment. It also lists subject-specific competence descriptions.
- *Glossary:* Where appropriate, a list of selected general and subject-specific terms are briefly defined.

LEARNING PROGRAMME GUIDELINES

A Learning Programme specifies the scope of learning and assessment for the three grades in the Further Education and Training band. It is the plan that ensures that learners achieve the Learning Outcomes as prescribed by the Assessment Standards for a particular grade. The Learning Programme Guidelines assist teachers and other Learning Programme developers to plan and design quality learning, teaching and assessment programmes.

CHAPTER 2

MATHEMATICS

DEFINITION

The curriculum for Mathematics is based on the following view of the nature of the discipline.

Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself. It is a distinctly human activity practised by all cultures. Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical and symbolic relationships. Mathematics is based on observing patterns; with rigorous logical thinking, this leads to theories of abstract relations. Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives. Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.

PURPOSE

In an ever-changing society, it is essential that all learners passing through the Further Education and Training band acquire a functioning knowledge of the Mathematics that empowers them to make sense of society. A suitable range of mathematical process skills and knowledge enables an appreciation of the discipline itself. It also ensures access to an extended study of the mathematical sciences and a variety of career paths.

The study of Mathematics contributes to personal development through a deeper understanding and successful application of its knowledge and skills, while maintaining appropriate values and attitudes. Mathematics is a discipline in its own right and pursues the establishment of knowledge without necessarily requiring applications in real life. Competence in mathematical process skills such as investigating, generalising and proving is more important than the acquisition of content knowledge for its own sake.

Mathematical competence provides access to rewarding activity and contributes to personal, social, scientific and economic development. It is understandable, therefore, that a variety of stakeholders in society exert demands on school Mathematics. These stakeholders include parents, learners, educators, Mathematics educators, employers, professional mathematicians, tertiary institutions, and cultural and political organisations. Individual and collective engagement with Mathematics will provide valuable opportunities for the development of a variety of values, as well as personal and interpersonal skills.

Mathematics enables learners to:

- communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams;
- use mathematical process skills to identify, pose and solve problems creatively and critically;

- organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns;
- work collaboratively in teams and groups to enhance mathematical understanding;
- collect, analyse and organise quantitative data to evaluate and critique conclusions; and
- engage responsibly with quantitative arguments relating to local, national and global issues.

An important purpose of Mathematics in the Further Education and Training band is the establishment of proper connections between Mathematics as a discipline and the application of Mathematics in real-world contexts. Mathematical modelling provides learners with the means to analyse and describe their world mathematically, and so allows learners to deepen their understanding of Mathematics while adding to their mathematical tools for solving real-world problems. Mathematics can be used in a wide variety of physical, social and management sciences. An appreciation of the manner in which Mathematics has developed over time establishes its origins in culture and the needs of society.

SCOPE

Learners in the Further Education and Training band who are interested in the subject or who intend to follow a career path requiring Mathematics will, while ensuring that they are mathematically literate, work towards being able to:

- competently use mathematical process skills such as making conjectures, proving assertions and modelling situations;
- calculate confidently and competently with and without calculators, and use rational and irrational numbers with understanding;
- competently produce useful equivalents for algebraic expressions, and use such equivalents appropriately and with confidence;
- use Mathematics to critically investigate and monitor the financial aspects of personal and community life and political decisions;
- work with a wide range of patterns and transformations (translations, rotations, reflections) of functions and solve related problems;
- describe, represent and analyse shape and space in two and three dimensions using various approaches in geometry (synthetic, analytic transformation) and trigonometry in an interrelated or connected manner;
- collect and use data to establish basic statistical and probability models, solve related problems, and critically consider representations provided or conclusions reached;
- use and understand the principles of differential calculus to determine the rate of change of a range of simple, non-linear functions and to solve simple optimisation problems;
- solve problems involving sequences and series in real-life and mathematical situations;
- solve non-routine, unseen problems using mathematical principles and processes;
- investigate historical aspects of the development and use of Mathematics in various cultures; and
- use available technology (the minimum being a modern scientific calculator) in calculations and in the development of models.

Such mathematical skills and process abilities will, where possible, be embedded in contexts that relate to HIV/AIDS, human rights, indigenous knowledge systems, and political, economic, environmental and inclusivity issues.

EDUCATIONAL AND CAREER LINKS

Mathematics is an essential element in the curriculum of any learner who intends to pursue a career in the physical, mathematical, computer, life, earth, space and environmental sciences or in technology. Mathematics also has an important role in the economic, management and social sciences. It is an important tool for creating, exploring and expressing theoretical and applied aspects of the sciences. Mathematics is also important for the personal development of any learner.

The learning achieved in Mathematics in the General Education and Training band provides an essential base from which to proceed into the demands of Mathematics in the Further Education and Training band. The essentials of numeracy developed in the General Education and Training band are taken further, working in more symbolic ways. The General Education and Training engagement with space and shape becomes more formalised. The methods and uses of statistics and chance are dealt with in greater depth. How Mathematics can contribute to an understanding of financial issues is taken beyond dealing with budgets. The emphasis on contexts and integration within Mathematics and across the curriculum is maintained, while mathematical modelling becomes more prominent.

The subject Mathematics in the Further Education and Training band will provide a platform for linkages to Mathematics in Higher Education institutions. It will also provide for linkages to Mathematics of a complementary nature but specific to the needs of the individual, in appropriate Further Education and Training sites of learning. Learners proceeding to institutions of Higher Education should be mathematically literate, so that they are able to progress effectively in whatever discipline they decide to follow.

Mathematics is being used increasingly as a tool for solving problems related to modern society. The financial aspects of dealing with daily life are informed by mathematical considerations. Mathematical ways of thinking are often evident in the workplace. The Learning Outcomes and Assessment Standards in Mathematics are designed to allow all learners passing through this band to develop into citizens who are able to deal with the Mathematics that impinges on the society they live in and on their daily lives.

If a learner does not perceive Mathematics to be necessary for the career path or study direction chosen, the learner will be required to take Mathematical Literacy.

LEARNING OUTCOMES

Learning Outcome 1: Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

The range of numbers encountered in the Further Education and Training band will include irrational numbers as they occur in contextual problems. Learners will develop an understanding that not all numbers are real.

In this band learners will:

- expand the capacity to represent numbers in a variety of ways and move flexibly between representations;
- develop further the ability to estimate and judge the reasonableness of solutions and the ability to give solutions to an appropriate degree of accuracy, depending on the accuracy of measuring instruments and on the context;
- calculate confidently and competently, with and without a calculator, guarding against becoming over-dependent on the calculator;
- develop the concepts of simple and compound growth and decay;
- solve problems related to arithmetic, geometric and other sequences and series, including contextual problems related to hire-purchase, bond repayments and annuities; and
- explore real-life and purely mathematical number patterns and problems which develop the ability to generalise, justify and prove.

Learning Outcome 2: Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

A fundamental aspect of this outcome is that it provides learners with versatile and powerful tools for understanding their world while giving them access to the strength and beauty of mathematical structure. The language of algebra will be used as a tool to study the nature of the relationship between specific variables in a situation. The power of algebra is that it provides learners with models to describe and analyse such situations. It also provides them with the analytical tools to obtain additional, unknown information about the situation. Such information is often needed as a basis for reasoning about problem situations and as a basis for decision making.

Learners should:

- understand various types of patterns and functions;
- investigate the effect of changing parameters on the graphs of functions;
- use symbolic forms to represent and analyse mathematical situations and structures; and
- use mathematical models and analyse change in both real and abstract contexts.

The mathematical models of situations may be represented in different ways – in words, as a table of values, as a graph, or as a computational procedure (formula or expression). The information needed is mostly acquired in the following ways:

- finding values of the dependent variable (finding function values);
- finding values of the independent variable (solving equations);
- describing and using the behaviour of function values, periodicity;
- considering the increasing and decreasing nature of functions, rates of change, gradient, derivative, maxima and minima;
- finding a function rule (formula); and
- transforming to an equivalent expression ('manipulation' of algebraic expressions).

It is important that the Learning Programme provides for appropriate experiences of these problem types, and that it develops the underlying concepts and techniques to enable learners to experience the power of algebra as a tool to solve problems. The emphasis is on the objective of solving problems and not on the mastery of isolated skills (such as factorisation) for their own sake.

Learning Outcome 3: Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

The teaching and learning of space, shape and measurement in the Further Education and Training band must build on experiences from the General Education and Training band to make more formal and extended levels of knowledge accessible. Aspects that are important for the attainment of this Learning Outcome include location, visualisation and transformation. Learners' previous knowledge becomes deeper, they engage with new tools that can be used in a range of applications, and they become more proficient in processes leading to proof.

The study of space, shape and measurement enables learners to:

- explore relationships, make and test conjectures, and solve problems involving geometric figures and geometric solids;
- investigate geometric properties of 2-dimensional and 3-dimensional figures in order to establish, justify and prove conjectures;

- link algebraic and geometric concepts through analytic geometry;
- link the use of trigonometric relationships and geometric properties to solve problems;
- use construction and measurement or dynamic geometry software, for exploration and conjecture;
- analyse natural forms, cultural products and processes as representations of shape and space;
- investigate the contested nature of geometry throughout history and develop an awareness of other geometries;
- use synthetic, transformation or other geometric methods to establish geometric properties; and
- connect space, shape and measurement to other Learning Outcomes within Mathematics and where possible to other subjects.

Learning Outcome 4: Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

The focus of teaching and learning data handling in this band builds on what has been learned in the General Education and Training band. Learners will master further methods of organising, displaying and analysing data. Measures of central tendency and spread will be explored. A basic appreciation of the difference between data that is normally distributed about a mean and data that is skewed will be developed. Learners will become critically aware of the deliberate abuse in the way data can be represented to support a particular viewpoint. Learners will carry out practical research projects and statistical experiments. At least one project each year will involve the selection of a random sample of a specific population with a view to determining statistics that predict the corresponding parameters of the population.

The basic understanding of probability and chance gained at General Education and Training level will be deepened so that, for example, learners can compare the actual odds in winning popular games of chance with the odds offered by gaming houses. A basic understanding of the way the probability of everyday events can be calculated and used in prediction will be developed.

Wherever possible, contexts that are investigated will focus on human rights issues, inclusivity, current matters involving conflicting views, and environmental and health issues.

CHAPTER 3

LEARNING OUTCOMES, ASSESSMENT STANDARDS, CONTENT AND CONTEXTS

In the Learning Outcomes that follow, the bulleted items are the Assessment Standards. The alphabetical points that follow (e.g. a, b, c) introduce the sub-skills, knowledge and attitudes of which each Assessment Standard is constituted.

Grade 10

Learning Outcome 1

Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment Standards

We know this when the learner is able to:

10.1.1 Identify rational numbers and convert between terminating or recurring decimals and the form :

$$\frac{a}{b} \, ; \quad a, \, b, \, \epsilon \, \mathbb{Z}; \, b \neq 0.$$

10.1.2 (a) Simplify expressions using the laws of exponents for integral exponents.

(b) Establish between which two integers any simple surd lies.

(c) Round rational and irrational numbers to an appropriate degree of accuracy.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.1.1 Understand that not all numbers are real. (This requires the recognition but not the study of non-real numbers.)

11.1.2 (a) Simplify expressions using the laws of exponents for rational exponents.

(b) Add, subtract, multiply and divide simple surds

(*e.g.* $\sqrt{3}+\sqrt{12}=3\sqrt{3}$ *and* $\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$)

(c) Demonstrate an understanding of error margins.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.1.2 Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real-life problems (e.g. growth and decay see 12.1.4(a)).

Grade 10

Learning Outcome 1 Continued

Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment Standards

We know this when the learner is able to:

10.1.3 Investigate number patterns (including but not limited to those where there is a constant difference between consecutive terms in a number pattern, and the general term is therefore linear) and hence:
(a) make conjectures and generalisations;
(b) provide explanations and justifications and attempt to prove conjectures.

10.1.4 Use simple and compound growth formulae ($A = P(1+ni)$ and $A = P(1+i)^n$) to solve problems, including interest, hire-purchase, inflation, population growth and other real-life problems.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.1.3 Investigate number patterns (including but not limited to those where there is a constant second difference between consecutive terms in a number pattern, and the general term is therefore quadratic) and hence:
(a) make conjectures and generalisations;
(b) provide explanations and justifications and attempt to prove conjectures.

11.1.4 Use simple and compound decay formulae ($A = P(1 - ni)$ and $A = P(1 - i)^n$) to solve problems (including straight line depreciation and depreciation on a reducing balance) *(link to Learning Outcome 2)*.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.1.3 (a) Identify and solve problems involving number patterns, including but not limited to arithmetic and geometric sequences and series.
(b) Correctly interpret sigma notation.
(c) Prove and correctly select the formula for and calculate the sum of series, including:

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} a + (i-1)d = \frac{n}{2}[2a + (n-1)d]$$

$$\sum_{i=1}^{n} a.r^{i-1} = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$\sum_{i=1}^{\infty} a.r^{i-1} = \frac{a}{1 - r} \text{ for } -1 < r < 1$$

(d) Correctly interpret recursive formulae:
(e.g. $T_{n+1} = T_n + T_{n-1}$)

12.1.4 (a) Calculate the value of n in the formula $A = P(1 \pm i)^n$
(b) Apply knowledge of geometric series to solving annuity, bond repayment and sinking fund problems, with or without the use of the formulae:

$$F = \frac{x[(1+i)^n - 1]}{i} \text{ and}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

Grade 10

Learning Outcome 1
Continued

Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

Assessment Standards

We know this when the learner is able to:

10.1.5 Demonstrate an understanding of the implications of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).

10.1.6 Solve non-routine, unseen problems.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.1.5 Demonstrate an understanding of different periods of compounding growth and decay (including effective compounding growth and decay and including effective and nominal interest rates.

11.1.6 Solve non-routine, unseen problems.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.1.5 Critically analyse investment and loan options and make informed decisions as to the best option(s) (including pyramid and micro-lenders' schemes).

12.1.6 Solve non-routine, unseen problems.

Grade 10

Learning Outcome 2

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Assessment Standards

We know this when the learner is able to:

10.2.1 (a) Demonstrate the ability to work with various types of functions, including those listed in the following Assessment Standard.

(b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).

10.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence to generalise the effects of the parameters a and q on the graphs of functions including:

$$y = ax + q$$

$$y = ax^2 + q$$

$$y = \frac{a}{x} + q$$

$$y = ab^x + q; \ b > 0$$

$$y = a \sin(x) + q$$

$$y = a \cos(x) + q$$

$$y = a \tan(x) + q$$

Grade 11

Assessment Standards

We know this when the learner is able to:

11.2.1 (a) Demonstrate the ability to work with various types of functions including those listed in the following Assessment Standard.

      (b) Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).

11.2.2 Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures about the effect of the parameters k, p, a and q for functions including:

$y = \sin(kx)$

$y = \cos(kx)$

$y = \tan(kx);$

$y = \sin(x + p)$

$y = \cos(x + p)$

$y = \tan(x + p)$

$y = a\,(x + p)^2 + q$

$y = \dfrac{a}{x + p} + q$

$y = ab^{x + p} + q;\ b > 0$

Grade 12

Assessment Standards

We know this when the learner is able to:

12.2.1 (a) Demonstrate the ability to work with various types of functions and relations including the inverses listed in the following Assessment Standard.

      (b) Demonstrate knowledge of the formal definition of a function.

12.2.2 (a) Investigate and generate graphs of the inverse relations of functions, in particular the inverses of:

$y = ax + q$

$y = ax^2$

$y = a^x;\ a > 0$

      (b) Determine which inverses are functions and how the domain of the original function needs to be restricted so that the inverse is also a function.

Grade 10

Learning Outcome 2 Continued

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Assessment Standards

We know this when the learner is able to:

10.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed in 10.2.2 above:
   (a) domain and range;
   (b) intercepts with the axes;
   (c) turning points, minima and maxima;
   (d) asymptotes;
   (e) shape and symmetry;
   (f) periodicity and amplitude;
   (g) average gradient (average rate of change);
   (h) intervals on which the function increases/decreases;
   (i) the discrete or continuous nature of the graph.

10.2.4 Manipulate algebraic expressions:
   (a) multiplying a binomial by a trinomial;
   (b) factorising trinomials;
   (c) factorising by grouping in pairs;
   (d) simplifying algebraic fractions with monomial denominators.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of functions including those listed above:
   (a) domain and range;
   (b) intercepts with the axes;
   (c) turning points, minima and maxima;
   (d) asymptotes;
   (e) shape and symmetry;
   (f) periodicity and amplitude;
   (g) average gradient (average rate of change);
   (h) intervals on which the function increases/decreases;
   (i) the discrete or continuous nature of the graph.

11.2.4 Manipulate algebraic expressions:
   (a) by completing the square;
   (b) simplifying algebraic fractions with binomial denominators.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.2.3 Identify characteristics as listed below and hence use applicable characteristics to sketch graphs of the inverses of the functions listed above:
   (a) domain and range;
   (b) intercepts with the axes;
   (c) turning points, minima and maxima;
   (d) asymptotes;
   (e) shape and symmetry;
   (f) average gradient (average rate of change);
   (g) intervals on which the function increases/decreases.

12.2.4 Factorise third degree polynomials *(*including examples which require the factor theorem).

Grade 10

Learning Outcome 2 Continued

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Assessment Standards

We know this when the learner is able to:

10.2.5 Solve:
   (a) linear equations;
   (b) quadratic equations by factorisation;
   (c) exponential equations of the form
      $ka^{x+p} = m$
      (including examples solved by trial and error);
   (d) linear inequalities in one variable and illustrate the solution graphically;
   (e) linear equations in two variables simultaneously (numerically, algebraically and graphically).

10.2.6 Use mathematical models to investigate problems that arise in real-life contexts:
   (a) making conjectures, demonstrating and explaining their validity;
   (b) expressing and justifying mathematical generalisations of situations;
   (c) using the various representations to interpolate and extrapolate;
   (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and features.
   (Examples should include issues related to health, social, economic, cultural, political and environmental matters.)

Grade 11

Grade 12

Assessment Standards

Assessment Standards

We know this when the learner is able to:

11.2.5 Solve:
   (a) quadratic equations (by factorisation, by completing the square, and by using the quadratic formula) and quadratic inequalities in one variable and interpret the solution graphically;
   (b) equations in two unknowns, one of which is linear and one of which is quadratic, algebraically or graphically.

11.2.6 Use mathematical models to investigate problems that arise in real-life contexts:
   (a) making conjectures, demonstrating and explaining their validity;
   (b) expressing and justifying mathematical generalisations of situations;
   (c) using various representations to interpolate and extrapolate;
   (d) describing a situation by interpreting graphs, or drawing graphs from a description of a situation, with special focus on trends and pertinent features.
(Examples should include issues related to health, social, economic, cultural, political and environmental matters.)

Grade 10

Learning Outcome 2 Continued

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Assessment Standards

We know this when the learner is able to:

10.2.7 Investigate the average rate of change of a function between two values of the independent variable, demonstrating an intuitive understanding of average rate of change over different intervals (e.g. investigate water consumption by calculating the average rate of change over different time intervals and compare results with the graph of the relationship).

Grade 11

Assessment Standards

We know this when the learner is able to:

11.2.7 Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.2.7 (a) Investigate and use instantaneous rate of change of a variable when interpreting models of situations:

- demonstrating an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient of a function at a point;
- establishing the derivatives of the following functions from first principles:

$$f(x) = b$$

$$f(x) = x$$

$$f(x) = x^2$$

$$f(x) = x^3$$

$$f(x) = \frac{1}{x}$$

and then generalise to the derivative of $f(x) = x^n$

(b) Use the following rules of differentiation:

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

$$\frac{d}{dx}[k.f(x)] = k\frac{d}{dx}[f(x)]$$

(c) Determine the equations of tangents to graphs.
(d) Generate sketch graphs of cubic functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the *x*-axis.
(e) Solve practical problems involving optimisation and rates of change.

Grade 10

**Learning Outcome 2
Continued**

Assessment Standards

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.2.8 (a) Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by numerical search along the boundary of the feasible region.

(b) Solve a system of linear equations to find the co-ordinates of the vertices of the feasible region.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.2.8 Solve linear programming problems by optimising a function in two variables, subject to one or more linear constraints, by establishing optima by means of a search line and further comparing the gradients of the objective function and linear constraint boundary lines.

Grade 10

Learning Outcome 3

Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

Assessment Standards

We know this when the learner is able to:

10.3.1 Understand and determine the effect on the volume and surface area of right prisms and cylinders, of multiplying any dimension by a constant factor k.

10.3.2 (a) Through investigations, produce conjectures and generalisations related to triangles, quadrilaterals and other polygons, and attempt to validate, justify, explain or prove them, using any logical method (Euclidean, co-ordinate and/or transformation).
 (b) Disprove false conjectures by producing counter-examples.
 (c) Investigate alternative definitions of various polygons (including the isosceles, equilateral and right-angled triangle, the kite, parallelogram, rectangle, rhombus and square).

Grade 11

Assessment Standards

We know this when the learner is able to:

11.3.1 Use the formulae for surface area and volume of right pyramids, right cones, spheres and combinations of these geometric objects.

11.3.2 (a) Investigate necessary and sufficient conditions for polygons to be similar.
(b) Prove and use (accepting results established in earlier grades):
- that a line drawn parallel to one side of a triangle divides the other two sides proportionally (the Mid-point Theorem as a special case of this theorem);
- that equiangular triangles are similar;
- that triangles with sides in proportion are similar;
- the Pythagorean Theorem by similar triangles.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.3.2 (a) Accept the following as axioms:
- results established in earlier grades;
- a tangent is perpendicular to the radius, drawn at the point of contact with the circle, and then investigate and prove the theorems of the geometry of circles:
- the line drawn from the centre of a circle, perpendicular to a chord, bisects the chord and its converse;
- the perpendicular bisector of a chord passes through the centre of the circle;
- the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle;
- angles subtended by a chord at the circle on the same side of the chord are equal and its converse;
- the opposite angles of a cyclic quadrilateral are supplementary and its converse;
- two tangents drawn to a circle from the same point outside the circle are equal in length;
- the angles between a tangent and a chord, drawn to the point of contact of the chord, are equal to the angles which the chord subtends in the alternate chord segments and its converse.
(b) Use the theorems listed above to:
- make and prove or disprove conjectures;
- prove riders.

Grade 10

Learning Outcome 3 Continued

Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

Assessment Standards

We know this when the learner is able to:

10.3.3 Represent geometric figures on a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1 ; y_1)$ and $(x_2 ; y_2)$, a formula for calculating:
   (a) the distance between the two points;
   (b) the gradient of the line segment joining the points;
   (c) the co-ordinates of the mid-point of the line segment joining the points.

10.3.4 Investigate, generalise and apply the effect of the following transformations of the point $(x ; y)$:
   (a) a translation of p units horizontally and q units vertically;
   (b) a reflection in the x-axis, the y-axis or the line $y = x$.

10.3.5 Understand that the similarity of triangles is fundamental to the trigonometric functions $\sin \theta$, $\cos \theta$ and $\tan \theta$, and is able to define and use the functions.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.3.3 Use a Cartesian co-ordinate system to derive and apply:
    (a) the equation of a line through two given points;
    (b) the equation of a line through one point and parallel or perpendicular to a given line;
    (c) the inclination of a line.

11.3.4 Investigate, generalise and apply the effect on the co-ordinates of:
    (a) the point (x ; y) after rotation around the origin through an angle of 90° or 180° ;
    (b) the vertices $(x_1 ; y_1)$, $(x_2 ; y_2)$, …, $(x_n ; y_n)$ of a polygon after enlargement through the origin, by a constant factor k.

11.3.5 (a) Derive and use the values of the trigonometric functions (in surd form where applicable) of 30°, 45° and 60°.
    (b) Derive and use the following identities:
      • $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$
      • $\sin^2 \theta + \cos^2 \theta = 1$
    (c) Derive the reduction formulae for $\sin(90°\pm\theta)$, $\cos(90°\pm\theta)$, $\sin(180°\pm\theta)$, $\cos(180°\pm\theta)$, $\tan(180°\pm\theta)$, $\sin(360°\pm\theta)$, $\cos(360°\pm\theta)$, $\tan(360°\pm\theta)$, $\sin(-\theta)$, $\cos(-\theta)$ and $\tan(-\theta)$.
    (d) Determine the general solution of trigonometric equations.
    (e) Establish and apply the sine, cosine and area rules.

Grade 12

Assessment Standards

We know this when the learner is able to:

12.3.3 Use a two-dimensional Cartesian co-ordinate system to derive and apply:
    (a) the equation of a circle (any centre);
    (b) the equation of a tangent to a circle given a point on the circle.

12.3.4 (a) Use the compound angle identities to generalise the effect on the co-ordinates of the point (x ; *y*) after rotation about the origin through an angle θ.
    (b) Demonstrate the knowledge that rigid transformations (translations, reflections, rotations and glide reflections) preserve shape and size, and that enlargement preserves shape but not size.

12.3.5 Derive and use the following compound angle identities:

    (a) $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

    (b) $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$

    (c) $\sin 2\alpha = 2 \sin \alpha \cos a$

    (d) $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

$$= 2 \cos^2 \alpha - 1$$

$$= 1 - 2 \sin^2 \alpha$$

Grade 10

Learning Outcome 3 Continued

Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

Assessment Standards

We know this when the learner is able to:

10.3.6 Solve problems in two dimensions by using the trigonometric functions (sin θ, cos θ and tan θ) in right-angled triangles and by constructing and interpreting geometric and trigonometric models (examples to include scale drawings, maps and building plans).

10.3.7 Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through a project.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.3.6 Solve problems in two dimensions by using the sine, cosine and area rules; and by constructing and interpreting geometric and trigonometric models.

11.3.7 Demonstrate an appreciation of the contributions to the history of the development and use of geometry and trigonometry by various cultures through educative forms of assessment (e.g. an investigative project).

Grade 12

Assessment Standards

We know this when the learner is able to:

12.3.6 Solve problems in two and three dimensions by constructing and interpreting geometric and trigonometric models.

12.3.7 Demonstrate a basic understanding of the development and uses of geometry through history and some familiarity with other geometries (e.g. spherical geometry, taxi-cab geometry, and fractals).

Grade 10

Learning Outcome 4

Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

Assessment Standards

We know this when the learner is able to:

10.4.1 (a) Collect, organise and interpret univariate numerical data in order to determine:
- measures of central tendency (mean, median, mode) of grouped and ungrouped data, and know which is the most appropriate under given conditions;
- measures of dispersion: range, percentiles, quartiles, interquartile and semi-inter-quartile range.

(b) Represent data effectively, choosing appropriately from:
- bar and compound bar graphs;
- histograms (grouped data);
- frequency polygons;
- pie charts;
- line and broken line graphs.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.4.1 (a) Calculate and represent measures of central tendency and dispersion in univariate numerical data by:
- five number summary (maximum, minimum and quartiles);
- box and whisker diagrams;
- ogives;
- calculating the variance and standard deviation of sets of data manually (for small sets of data) and using available technology (for larger sets of data), and representing results graphically using histograms and frequency polygons.

(b) Represent bivariate numerical data as a scatter plot and suggest intuitively whether a linear, quadratic or exponential function would best fit the data (problems should include issues related to health, social, economic, cultural, political and environmental issues).

Grade 12

Assessment Standards

We know this when the learner is able to:

12.4.1 (a) Demonstrate the ability to draw a suitable sample from a population and understand the importance of sample size in predicting the mean and standard deviation of a population.

(b) Use available technology to calculate the regression function which best fits a given set of bivariate numerical data.

(c) Use available technology to calculate the correlation co-efficient of a set of bivariate numerical data to make relevant deductions.

Grade 10

Learning Outcome 4 Continued

Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

Assessment Standards

We know this when the learner is able to:

10.4.2 (a) Use probability models for comparing the relative frequency of an outcome with the probability of an outcome (understanding, for example, that it takes a very large number of trials before the relative frequency of throwing a head approaches the probability of throwing a head).

(b) Use Venn diagrams as an aid to solving probability problems, appreciating and correctly identifying:
- the sample space of a random experiment;
- an event of the random experiment as a subset of the sample space;
- the union and intersection of two or more subsets of the sample space;
- $P(S) = 1$ (where S is the sample space);
- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (where A and B are events within a sample space);
- disjoint (mutually exclusive) events, and is therefore able to calculate the probability of either of the events occurring by applying the addition rule for disjoint events: $P(A \text{ or } B) = P(A) + P(B)$;
- complementary events, and is therefore able to calculate the probability of an event not occurring: $P(\text{not } A) = 1 - P(A)$.

Grade 11

Assessment Standards

We know this when the learner is able to:

11.4.2 (a) Correctly identify dependent and independent events (e.g. from two-way contingency tables or Venn diagrams) and therefore appreciate when it is appropriate to calculate the probability of two independent events occurring by applying the product rule for independent events:
P(A and B) = P(A).P(B).

(b) Use tree and Venn diagrams to solve probability problems (where events are not necessarily independent).

Grade 12

Assessment Standards

We know this when the learner is able to:

12.4.2 Generalise the fundamental counting principle (successive choices from m_1 then m_2 then m_3 … options create $m_1.m_2.m_3$ … different combined options) and solve problems using the fundamental counting principle.

Grade 10

Learning Outcome 4 Continued

Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

Assessment Standards

We know this when the learner is able to:

10.4.3 (a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).

(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.

10.4.5 Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).

Grade 11

Assessment Standards

We know this when the learner is able to:

11.4.3 (a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).

(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.

11.4.4 Differentiate between symmetric and skewed data and make relevant deductions.

11.4.5 Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).

Grade 12

Assessment Standards

We know this when the learner is able to:

12.4.3 (a) Identify potential sources of bias, errors in measurement, and potential uses and misuses of statistics and charts and their effects (a critical analysis of misleading graphs and claims made by persons or groups trying to influence the public is implied here).

(b) Effectively communicate conclusions and predictions that can be made from the analysis of data.

12.4.4 Identify data which is normally distributed about a mean by investigating appropriate histograms and frequency polygons.

12.4.5 Use theory learned in this grade in an authentic integrated form of assessment (e.g. in an investigative project).

C CONTENT AND CONTEXTS FOR THE ATTAINMENT OF ASSESSMENT STANDARDS

In this section content and contexts are provided to support the attainment of the Assessment Standards. The content needs to be dealt with in such a way as to assist the learner to progress towards the achievement of the Learning Outcomes. Content must serve the Learning Outcomes but not be an end in itself. The contexts suggested will enable the content to be embedded in situations which are meaningful to the learner and so assist learning and teaching. The teacher should be aware of and use local contexts, not necessarily indicated here, which could be more suited to the experiences of the learner. Content and context, when aligned to the attainment of the Assessment Standards, provide a framework for the development of Learning Programmes. The Learning Programme Guidelines give more detail in this respect.

Grade 10

Learning Outcome 1

Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

The learner will use the following content in order to calculate and estimate accurately in solving standard problems, as well as those that are non-routine and unseen. The problems will be taken from mathematical and real-life contexts such as health and finance.

Proposed content

- Conversion of terminating and recurring decimals to the form:

  $\frac{a}{b}$; $a, b, \in \mathbb{Z}; b \neq 0.$

- The laws of exponents for integral exponents.

- Rational approximation of surds.

- Number patterns, including those where there is a constant difference between consecutive terms indicating that the general term is linear.

- Simple and compound growth formulae $A = P(1+ni)$ and $A = P(1+i)^n$; solving for any variable except in the compound growth formula.

Grade 11

Proposed content

- Recognition of non-real numbers.

- Use of the laws of exponents for rational exponents.

- Add, subtract, multiply and divide simple surds.

- Error margins.

- Number patterns, including those where there is a constant second difference between consecutive terms indicating that the general term is quadratic

Grade 12

Proposed content

- Definition of a logarithm and any laws needed to solve real-life problems (e.g. growth and decay).

- The calculation of n using the growth and decay formulae.

- Number patterns, including arithmetic and geometric sequences and series.

- Sigma notation.

- Proof and application of the formulae for the sum of series, including

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} a + (i - 1)d = \frac{n}{2}[2a + (n - 1)d]$$

$$\sum_{i=1}^{n} a.r^{i-1} = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$\sum_{i=1}^{\infty} a.r^{i-1} = \frac{a}{1 - r} \text{ for } -1 < r < 1$$

Grade 10

Learning Outcome 1
Continued

Number and Number Relationships

When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.

The learner will use the following content in order to calculate and estimate accurately in solving standard problems, as well as those that are non-routine and unseen. The problems will be taken from mathematical and real-life contexts such as health and finance.

Proposed content

■ Foreign exchange rates.

Grade 11

Proposed content

- Simple and compound decay formulae $A=P(1-ni)$ and $A=P(1-i)^n$. Calculation of all variables in $A=P(1-i)^n$ (for n by trial trial and error using a calculator).

- Different periods of compounding growth and decay.

Grade 12

Proposed content

- Recursive formulae
  (e.g. $T_{n+1} = T_n + T_{n-1}$)

- Annuities, bond repayments and sinking funds, with or without the use of the formulae:

$$F = \frac{x[(1+i)^n - 1]}{i} \text{ and}$$

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

- Loan options.

Grade 10

Learning Outcome 2

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

The approach to the content of this Learning Outcome should ensure that learning occurs through investigating the properties of functions and applying their characteristics to a variety of problems. Functions and algebra are integral to modelling and so to solving contextual problems. Problems which integrate content across Learning Outcomes and which are of a non-routine nature should also be used. Human rights, health and other issues which involve debates on attitudes and values should be involved in dealing with models of relevant contexts.

Proposed content

- Study of functions including

$$y = ax + q$$

$$y = ax^2 + q$$

$$y = \frac{a}{x} + q$$

$$y = ab^x + q; \; b > 0$$

$$y = a \sin(x) + q$$

$$y = a \cos(x) + q$$

$$y = a \tan(x) + q$$

- Conversion between numerical, graphical, verbal and symbolic representations.

- Investigation of the effects of the parameters a and q on the above functions.

- Sketch graphs of the above functions using the following characteristics:
  - domain and range;
  - intercepts with the axes;
  - turning points, minima and maxima;
  - asymptotes;
  - shape and symmetry;
  - periodicity and amplitude;
  - average gradient (average rate of change);
  - intervals on which the function increases/decreases;
  - the discrede or continuous nature of the graph;
  - the discrete or continuous nature of the graph.

Grade 11

Proposed content

- Study of functions including:

  $y = \sin(kx)$

  $y = \cos(kx)$

  $y = \tan(kx)$;

  $y = \sin(x + p)$

  $y = \cos(x + p)$

  $y = \tan(x + p)$

  $y = a\,(x + p)^2 + q$

  $y = \dfrac{a}{x + p} + q$

  $y = ab^{x + p} + q;\ b > 0$

- Conversion between numerical, graphical, verbal and symbolic representations.

- Investigation of the effects of the parameters *k, p, a*, and *q* on the above functions.

- Sketch graphs of the above functions using the following characteristics:
  - domain and range;
  - intercepts with the axes;
  - turning points, minima and maxima;
  - asymptotes;
  - shape and symmetry;
  - periodicity and amplitude;
  - average gradient (average rate of change);
  - intervals on which the function increases/decreases;
  - the discrete or continuous nature of the graph.

Grade 12

Proposed content

- Study of functions:
  - formal definition of a function;
  - the inverses of:

    $y = ax + q$

    $y = ax^2$

    $y = a^x;\ a > 0$

- Sketch graphs of the inverses of the functions above using the characteristics:
  - domain and range;
  - intercepts with the axes;
  - turning points, minima and maxima;
  - asymptotes;
  - shape and symmetry;
  - average gradient (average rate of change);
  - intervals on which the function increases/decreases.

Learning Outcome 2 Continued

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

The approach to the content of this Learning Outcome should ensure that learning occurs through investigating the properties of functions and applying their characteristics to a variety of problems. Functions and algebra are integral to modelling and so to solving contextual problems. Problems which integrate content across Learning Outcomes and which are of a non-routine nature should also be used. Human rights, health and other issues which involve debates on attitudes and values should be involved in dealing with models of relevant contexts.

Proposed content

- Algebraic manipulation:
  - multiplying a binomial by a trinomial;
  - factorising trinomials;
  - factorising by grouping in pairs;
  - simplifying algebraic fractions with monomial denominators.

- Solution of:
  - linear equations;
  - quadratic equations by factorisation;
  - exponential equations of the form $ka^{x+p} = m$ (including examples solved by trial and error);
  - linear inequalities in one variable and graphical illustration of the solution;
  - linear equations in two variables simultaneously (numerically, algebraically and graphically).
  - the discrete or continous nature of graph.

Grade 11

Proposed content

- Algebraic manipulation:
  - completing the square;
  - simplifying algebraic fractions with binomial denominators.

- Solution of:
  - quadratic equations (by factorisation, by completing the square, and by using the quadratic formula);
  - quadratic inequalities in one variable and graphical interpretation of the solution;
  - equations in two unknowns, one of which is linear and one which is quadratic, algebraically or graphically.

Grade 12

Proposed content

- Factorise third-degree polynomials (including examples which require the factor theorem).

- Differential calculus:
  - an intuitive understanding of the limit concept in the context of approximating the rate of change or gradient of a function at a point;
  - the derivatives of the following functions from first principles:

    $f(x) = b$

    $f(x) = x$

    $f(x) = x^2$

    $f(x) = x^3$

    $f(x) = \frac{1}{x}$

  - the derivative of $f(x) = x^n$ (proof not required);
  - the following rules of differentiation:

    $$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$$

    $$\frac{d}{dx}[k.f(x)] = k\frac{d}{dx}[f(x)]$$

  - the equations of tangents to graphs;
  - sketch graphs of cubic and other suitable polynomial functions using differentiation to determine the stationary points (maxima, minima and points of inflection) and the factor theorem and other techniques to determine the intercepts with the *x*-axis;
  - practical problems involving optimisation and rates of change.

Grade 10

Learning Outcome 2
Continued

Functions and Algebra

The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

The approach to the content of this Learning Outcome should ensure that learning occurs through investigating the properties of functions and applying their characteristics to a variety of problems. Functions and algebra are integral to modelling and so to solving contextual problems. Problems which integrate content across Learning Outcomes and which are of a non-routine nature should also be used. Human rights, health and other issues which involve debates on attitudes and values should be involved in dealing with models of relevant contexts.

Proposed content

- Average rate of change of a function between two values of the independent variable.

Grade 11

Proposed content

- Average gradient between two points on a curve and the gradient of a curve at a point.

- Linear programming:
  - optimising a function in two variables subject to one or more linear constraints, by numerical search along the boundary of the feasible region;
  - solving a system of linear equations to find the co-ordinates of the vertices of the feasible region.

Grade 12

Proposed content

- Linear programming:
  - optimisation of a function in two variables, subject to one or more linear constraints, by means of a search line and further comparing the gradients of the objective and constraint functions.

Learning Outcome 3

Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

An important aspect of this Learning Outcome is the use of the content indicated in the representation of contextual problems in two and three dimensions so as to arrive at solutions through the measurement and calculation of associated values. Powerful mathematical tools which enable the investigation of space are embedded in the content. The treatment of formal Euclidean geometry is staged through the grades so as to assist in the gradual development of proof skills and an understanding of local axiomatic systems. Opportunities for making connections across the geometries involved in this Learning Outcome as well as with the Mathematics of other Learning Outcomes should be sought in requiring the solution to standard as well as non-routine unseen problems.

Proposed content

- The effect on the volume and surface area of right prisms and cylinders, of multiplying one or more dimensions by a constant factor k.

- Investigation of polygons, using any logical method (Euclidean, co-ordinate and/or transformation).

- Alternative definitions of polygons.

- Co-ordinate geometry: for any two points $(x_1 ; y_1)$ and $(x_2 ; y_2)$, derive and use the formula for calculating:
  - the distance between the two points;
  - the gradient of the line segment joining the points;
  - the co-ordinates of the mid-point of the line segment joining the points.

Grade 11

Proposed content

■ Apply the formulae for the surface area and volume of right prisms, right cones, spheres and combinations of these shapes.

■ Euclidean geometry:
- necessary and sufficient conditions for polygons to be similar;
- the line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem);
- equiangular triangles are similar;
- triangles with sides in proportion are similar;
- Theorem of Pythagoras by similar triangles.

■ Co-ordinate geometry:
- the equation of a line through two points;
- the equation of a line through one point and parallel or perpendicular to a given line;
- the inclination of a given line.

Grade 12

Proposed content

■ Euclidean geometry: accepting as axioms all results established in earlier grades and the fact that the tangent to a circle is perpendicular to the radius, drawn to the point of contact, prove the following theorems:
- the line drawn from the centre of a circle perpendicular to a chord bisects the chord and its converse;
- the perpendicular bisector of a chord passes through the centre of the circle;
- the angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle;
- angles subtended by a chord at the circle on the same side of the chord are equal and its converse;
- the opposite angles of a cyclic quadrilateral are supplementary and its converse;
- two tangents drawn to a circle from the same point outside the circle are equal in length;
- the tangent-chord theorem and its converse.

■ Co-ordinate geometry:
- the equation of a circle (any centre);
- the equation of a tangent to a circle given a point on the circle.

Grade 10

Learning Outcome 3 Continued

Space, Shape and Measurement

The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification.

An important aspect of this Learning Outcome is the use of the content indicated in the representation of contextual problems in two and three dimensions so as to arrive at solutions through the measurement and calculation of associated values. Powerful mathematical tools which enable the investigation of space are embedded in the content. The treatment of formal Euclidean geometry is staged through the grades so as to assist in the gradual development of proof skills and an understanding of local axiomatic systems. Opportunities for making connections across the geometries involved in this Learning Outcome as well as with the Mathematics of other Learning Outcomes should be sought in requiring the solution to standard as well as non-routine unseen problems.

Proposed content

■ Transformation geometry:
  - translation of p units horizontally and q units vertically;
  - reflection in the x-axis, the y-axis and the line $y = x$.

■ Trigonometry:
  - introduction through the similarity of triangles and proportion;
  - scale drawing and the interpretation of maps and building plans;
  - definition and use of the definitions $\sin \theta$, $\cos \theta$ and $\tan \theta$;
  - the periodicity of trigonometric functions.

■ Research into the history of the development of geometry and trigonometry in various cultures.

Grade 11

Proposed content

- Transformation geometry:
  - rotation around the origin through an angle of 90˚ or 180˚;
  - the enlargement of a polygon, through the origin, by a factor of k.

- Trigonometry:
  - function values of the special angles 30˚, 45˚ and 60˚ (in surd form where applicable);
  - derivation and use of the identities

    $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2\theta = 1$;
  - derivation and use of reduction formulae for $\sin(90˚\pm\theta)$, $\cos(90˚\pm\theta)$, $\sin(180˚\pm\theta)$, $\cos(180˚\pm\theta)$, $\tan(180˚\pm\theta)$, $\sin(360˚\pm\theta)$, $\cos(360˚\pm\theta)$, $\tan(360˚\pm\theta)$, $\sin(-\theta)$, $\cos(-\theta)$ and $\tan(-\theta)$.;
  - the general solution of trigonometric equations;
  - proof and application to problems in two dimensions, of the sine, cosine and area rules.

- Research into the history of the development of geometry and trigonometry in various cultures.

Grade 12

Proposed content

- Transformation geometry:
  - the compound angle formula in generalising the effect on the co-ordinates of the point $(x \,;\, y)$ after rotation about the origin through an angle θ;
  - rigid transformations (translations, reflections, rotations and glide reflections) and enlargement.

- Trigonometry:
  - compound angle identities:

    $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

    $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$

    $\sin 2\alpha = 2 \sin \alpha \cos a$

    $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$

    $\qquad = 2 \cos^2 \alpha - 1$

    $\qquad = 1 - 2 \sin^2 \alpha$

  - Problems in two and three dimensions.

- Research into history and one or more other geometries such as:
  - spherical geometry;
  - taxi-cab geometry;
  - fractals.

Grade 10

Learning Outcome 4

Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

The content indicated below for this Learning Outcome really only becomes meaningful and alive when used to address issues of importance to the learner and to society. Activities should involve learners in the completion of the 'statistical cycle' of formulating questions, collecting appropriate data, analysing and representing this data, and so arriving at conclusions about the questions raised.

Proposed content

- Data handling (calculations):
    - measures of central tendency (mean, median, mode) of grouped and ungrouped data;
    - measures of dispersion: range, percentiles, quartiles, interquartile and semi-inter-quartile range;
    - errors in measurement;
    - sources of bias.

- Data handling (representation):
    - bar and compound bar graphs;
    - histograms (grouped data);
    - frequency polygons;
    - pie charts;
    - line and broken line graphs.

Grade 11

Proposed content

- Calculations and data representation:
  - measures of central tendency and dispersion in univariate numerical data by:
    * five number summary (maximum, minimum and quartiles),
    * box and whisker diagrams,
    * ogives,
    * calculating the variance and standard deviation sets of data manually (for small sets of data) and using available technology (for larger sets of data), and representing results graphically using histograms and frequency polygons;
  - scatter plot of bivariate data and intuitive choice of function of best fit supported by available technology;
  - symmetric and skewed data.

Grade 12

Proposed content

- Content from previous grades as used in statistical investigations.

- Sampling.

- An inituitive understanding of the least squares method for linear regression.

- Regression functions and correlation for bivariate data by the use of available technology.

- Identification of normal distributions of data.

- Probability problems using the fundamental counting principle.

Grade 10

Learning Outcome 4

Data Handling and Probability

The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.

The content indicated below for this Learning Outcome really only becomes meaningful and alive when used to address issues of importance to the learner and to society. Activities should involve learners in the completion of the 'statistical cycle' of formulating questions, collecting appropriate data, analysing and representing this data, and so arriving at conclusions about the questions raised.

Proposed content

■ Probability:
  • definition in terms of equally likely outcomes;
  • relative frequency after many trials approximating the probability;
  • Venn diagrams as an aid to solving probability problems:
    * the sample space of a random experiment (S),
    * an event of the random experiment as a subset of the sample space,
    * the union and intersection of two or more subsets of the sample space,
    * P(S) = 1 (where S is the sample space),
    * P(A or B) = P(A) + P(B) – P(A and B);
  • (where A and B are events within a sample space),
    * disjoint (mutually exclusive) events:
      P(A or B) = P(A) + P(B),
    * complementary events:
      P(not A) = 1 – P(A);
  • potential uses and misuses of statistics and charts.

Grade 11

Grade 12

Proposed content

- Probability:
  - dependent and independent events;
  - two-way contingency tables;
  - the product rule for independent events:
    $P(A \text{ and } B) = P(A).P(B)$;
  - Venn diagrams and other techniques to solve probability problems (where events are not necessarily independent).

Proposed content

Contexts: inclusivity, human rights and indigenous knowledge systems

Mathematics is often referred to as the 'queen of sciences and yet the servant of all' due to the evident power that it has in concisely formulating the theoretical aspects of the sciences and in providing tools for solving problems. This power extends beyond the natural sciences to the engineering, computing, actuarial, financial, economic, business, social and other sciences. Mathematics is, however, a human endeavour. Through the continuing inventiveness of the human mind, new aspects of Mathematics have been created and recreated through social interaction over the centuries of human existence.

The mastery of Mathematics depends to a large extent on mathematical processes such as investigating patterns, formulating conjectures, arguing for the generality of such conjectures, and formulating links across the domains of Mathematics to enable lateral thinking. Mathematics is a cognitive science. It requires understanding before competence in the Learning Outcomes can be achieved.

Mathematics is thus a key subject in providing access to a wide variety of learning. This curriculum focuses on the development of mathematical process skills, and in so doing endeavours to unlock the power of Mathematics. At the same time, the tools to enable this power to become effective are not neglected. Mathematics will become a 'pump' and not a 'filter' for the learner.

Mathematics has often been used as a filter to block access to further or additional learning, not only in Mathematics itself but also in areas and careers related or even unrelated to Mathematics. The past political history of our country is a prime example of how the deliberate lack of provision of quality learning for all in Mathematics was used to stunt the development of the majority of our people. Being literate in Mathematics is an essential requirement for the development of the responsible citizen, the contributing worker and the self-managing person. Being mathematically literate implies an awareness of the manner in which Mathematics is used to format society. It enables astuteness in the user of the products of Mathematics such as hire-purchase agreements and mathematical arguments in the media, hence the inclusion of Mathematical Literacy as a fundamental requirement in the Further Education and Training curriculum. The development of literacy in Mathematics, in the sense outlined here, is also a fundamental responsibility of the Mathematics teacher and other educators. The requirements of the Assessment Standards in Mathematics ensure this.

Many local and international studies have shown the existence of a set of attitudes, described as 'mathsphobia', in school-going learners and in the population at large. In implementing this curriculum, it is the responsibility of the teacher to endeavour to win learners to Mathematics. This will be ensured by complying with the Assessment Standards of the subject, not formalising into the abstract prematurely but first taking care to develop understanding and process skills. The teacher needs to be sensitive to the manner in which gendered attitudes towards Mathematics play themselves out in the classroom, particularly in co-educational schools. Stereotyping needs to be guarded against, as Mathematics is often seen to be a male preserve, leading to arrogance and domination by the boys in the class. The interests of all need to be taken into account in providing access to Mathematics.

Another aspect of providing access and affirmation for learners of Mathematics is to look at examples of Mathematics in the variety of cultures and societal practices in our country. Mathematics is embedded in many cultural artefacts which we experience in our daily lives: the murals of the Ndebele, the rhythm in the drums of the Venda, the beadwork of the Zulu and Vedic art, to name but a few. Architecture, games and music are rich fields to explore through the lens of Mathematics. Ethnomathematics provides a wealth of more recently developed materials, sensitive to the sacredness of culture, for use in the classroom. The flexibility allowed by the curriculum also promotes the incorporation of local practices as starting points for applications or investigations. Ethnomathematics also stresses that Mathematics originated in cultures other than the Greek, and that it continued to be developed in sophistication by many societies other than the European. Projects in the history of Mathematics can be used to explore this.

CHAPTER 4

ASSESSMENT

INTRODUCTION

Assessment is a critical element of the National Curriculum Statement Grades 10 – 12 (General). It is a process of collecting and interpreting evidence in order to determine the learner's progress in learning and to make a judgement about a learner's performance. Evidence can be collected at different times and places, and with the use of various methods, instruments, modes and media.

To ensure that assessment results can be accessed and used for various purposes at a future date, the results have to be recorded. There are various approaches to recording learners' performances. Some of these are explored in this chapter. Others are dealt with in a more subject-specific manner in the Learning Programme Guidelines.

Many stakeholders have an interest in how learners perform in Grades 10 – 12. These include the learners themselves, parents, guardians, sponsors, provincial departments of education, the Department of Education, the Ministry of Education, employers, and higher education and training institutions. In order to facilitate access to learners' overall performances and to inferences on learners' competences, assessment results have to be reported. There are many ways of reporting. The Learning Programme Guidelines and the Assessment Guidelines discuss ways of recording and reporting on school-based and external assessment as well as giving guidance on assessment issues specific to the subject.

WHY ASSESS

Before a teacher assesses learners, it is crucial that the purposes of the assessment be clear and unambiguous. Understanding the purposes of assessment ensures that an appropriate match exists between the purposes and the methods of assessment. This, in turn, will help to ensure that decisions and conclusions based on the assessment are fair and appropriate for the particular purpose or purposes.

There are many reasons why learners' performance is assessed. These include monitoring progress and providing feedback, diagnosing or remediating barriers to learning, selection, guidance, supporting learning, certification and promotion.

In this curriculum, learning and assessment are very closely linked. Assessment helps learners to gauge the value of their learning. It gives them information about their own progress and enables them to take control of and to make decisions about their learning. In this sense, assessment provides information about whether teaching and learning is succeeding in getting closer to the specified Learning Outcomes. When assessment indicates lack of progress, teaching and learning plans should be changed accordingly.

TYPES OF ASSESSMENT

This section discusses the following types of assessment:

- baseline assessment;
- diagnostic assessment;
- formative assessment; and
- summative assessment.

Baseline assessment

Baseline assessment is important at the start of a grade, but can occur at the beginning of any learning cycle. It is used to establish what learners already know and can do. It helps in the planning of activities and in Learning Programme development. The recording of baseline assessment is usually informal.

Diagnostic assessment

Any assessment can be used for diagnostic purposes – that is, to discover the cause or causes of a learning barrier. Diagnostic assessment assists in deciding on support strategies or identifying the need for professional help or remediation. It acts as a checkpoint to help redefine the Learning Programme goals, or to discover what learning has not taken place so as to put intervention strategies in place.

Formative assessment

Any form of assessment that is used to give feedback to the learner is fulfilling a formative purpose. Formative assessment is a crucial element of teaching and learning. It monitors and supports the learning process. All stakeholders use this type of assessment to acquire information on the progress of learners. Constructive feedback is a vital component of assessment for formative purposes.

Summative assessment

When assessment is used to record a judgement of the competence or performance of the learner, it serves a summative purpose. Summative assessment gives a picture of a learner's competence or progress at any specific moment. It can occur at the end of a single learning activity, a unit, cycle, term, semester or year of learning. Summative assessment should be planned and a variety of assessment instruments and strategies should be used to enable learners to demonstrate competence.

WHAT SHOULD ASSESSMENT BE AND DO?

Assessment should:

- be understood by the learner and by the broader public;
- be clearly focused;
- be integrated with teaching and learning;
- be based on the pre-set criteria of the Assessment Standards;
- allow for expanded opportunities for learners;
- be learner-paced and fair; and
- be flexible;
- use a variety of instruments;
- use a variety of methods.

HOW TO ASSESS

Teachers' assessment of learners' performances must have a great degree of reliability. This means that teachers' judgements of learners' competences should be generalisable across different times, assessment items and markers. The judgements made through assessment should also show a great degree of validity; that is, they should be made on the aspects of learning that were assessed.

Because each assessment cannot be totally valid or reliable by itself, decisions on learner progress must be based on more than one assessment. This is the principle behind continuous assessment (CASS). Continuous assessment is a strategy that bases decisions about learning on a range of different assessment activities and events that happen at different times throughout the learning process. It involves assessment activities that are spread throughout the year, using various kinds of assessment instruments and methods such as tests, examinations, projects and assignments. Oral, written and performance assessments are included. The different pieces of evidence that learners produce as part of the continuous assessment process can be included in a portfolio. Different subjects have different requirements for what should be included in the portfolio. The Learning Programme Guidelines discuss these requirements further.

Continuous assessment is both classroom-based and school-based, and focuses on the ongoing manner in which assessment is integrated into the process of teaching and learning. Teachers get to know their learners through their day-to-day teaching, questioning, observation, and through interacting with the learners and watching them interact with one another.

Continuous assessment should be applied both to sections of the curriculum that are best assessed through written tests and assignments and those that are best assessed through other methods, such as by performance, using practical or spoken evidence of learning.

METHODS OF ASSESSMENT

Self-assessment

All Learning Outcomes and Assessment Standards are transparent. Learners know what is expected of them. Learners can, therefore, play an important part, through self-assessment, in 'pre-assessing' work before the teacher does the final assessment. Reflection on one's own learning is a vital component of learning.

Peer assessment

Peer assessment, using a checklist or rubric, helps both the learners whose work is being assessed and the learners who are doing the assessment. The sharing of the criteria for assessment empowers learners to evaluate their own and others' performances.

Group assessment

The ability to work effectively in groups is one of the Critical Outcomes. Assessing group work involves looking for evidence that the group of learners co-operate, assist one another, divide work, and combine individual contributions into a single composite assessable product. Group assessment looks at process as well as product. It involves assessing social skills, time management, resource management and group dynamics, as well as the output of the group.

METHODS OF COLLECTING ASSESSMENT EVIDENCE

There are various methods of collecting evidence. Some of these are discussed below.

Observation-based assessment

Observation-based assessment methods tend to be less structured and allow the development of a record of different kinds of evidence for different learners at different times. This kind of assessment is often based on tasks that require learners to interact with one another in pursuit of a common solution or product. Observation has to be intentional and should be conducted with the help of an appropriate observation instrument.

Test-based assessment

Test-based assessment is more structured, and enables teachers to gather the same evidence for all learners in

the same way and at the same time. This kind of assessment creates evidence of learning that is verified by a specific score. If used correctly, tests and examinations are an important part of the curriculum because they give good evidence of what has been learned.

Task-based assessment

Task-based or performance assessment methods aim to show whether learners can apply the skills and knowledge they have learned in unfamiliar contexts or in contexts outside of the classroom. Performance assessment also covers the practical components of subjects by determining how learners put theory into practice. The criteria, standards or rules by which the task will be assessed are described in rubrics or task checklists, and help the teacher to use professional judgement to assess each learner's performance.

RECORDING AND REPORTING

Recording and reporting involves the capturing of data collected during assessment so that it can be logically analysed and published in an accurate and understandable way.

Methods of recording

There are different methods of recording. It is often difficult to separate methods of recording from methods of evaluating learners' performances.

The following are examples of different types of recording instruments:

- rating scales;
- task lists or checklists; and
- rubrics.

Each is discussed below.

Rating scales

Rating scales are any marking system where a symbol (such as A or B) or a mark (such as 5/10 or 50%) is defined in detail to link the coded score to a description of the competences that are required to achieve that score. The detail is more important than the coded score in the process of teaching and learning, as it gives learners a much clearer idea of what has been achieved and where and why their learning has fallen short of the target. Traditional marking tended to use rating scales without the descriptive details, making it difficult to have a sense of the learners' strengths and weaknesses in terms of intended outcomes. A six-point scale of achievement is used in the National Curriculum Statement Grades 10 – 12 (General).

Task lists or checklists

Task lists or checklists consist of discrete statements describing the expected performance in a particular task. When a particular statement (criterion) on the checklist can be observed as having been satisfied by a learner during a performance, the statement is ticked off. All the statements that have been ticked off on the list (as criteria that have been met) describe the learner's performance. These checklists are very useful in peer or group assessment activities.

Rubrics

Rubrics are a combination of rating codes and descriptions of standards. They consist of a hierarchy of standards with benchmarks that describe the range of acceptable performance in each code band. Rubrics require teachers to know exactly what is required by the outcome. Rubrics can be holistic, giving a global picture of the standard required, or analytic, giving a clear picture of the distinct features that make up the criteria, or can combine both. The Learning Programme Guidelines give examples of subject-specific rubrics.

To design a rubric, a teacher has to decide the following:

- Which outcomes are being targeted?
- Which Assessment Standards are targeted by the task?
- What kind of evidence should be collected?
- What are the different parts of the performance that will be assessed?
- What different assessment instruments best suit each part of the task (such as the process and the product)?
- What knowledge should be evident?
- What skills should be applied or actions taken?
- What opportunities for expressing personal opinions, values or attitudes arise in the task and which of these should be assessed and how?
- Should one rubric target all the Learning Outcomes and Assessment Standards of the task or does the task need several rubrics?
- How many rubrics are, in fact, needed for the task?

It is crucial that a teacher shares the rubric or rubrics for the task with the learners before they do the required task. The rubric clarifies what both the learning and the performance should focus on. It becomes a powerful tool for self-assessment.

Reporting performance and achievement

Reporting performance and achievement informs all those involved with or interested in the learner's progress. Once the evidence has been collected and interpreted, teachers need to record a learner's achievements. Sufficient summative assessments need to be made so that a report can make a statement about the standard achieved by the learner.

The National Curriculum Statement Grades 10 – 12 (General) adopts a six-point scale of achievement. The scale is shown in Table 4.1.

Table 4.1   Scale of achievement for the National Curriculum Statement Grades 10 – 12 (General)

Rating Code	Description of Competence	Marks (%)
6	Outstanding	80-100
5	Meritorious	60-79
4	Satisfactory	50-59
3	Adequate	40-49
2	Partial	30-39
1	Inadequate	0-29

SUBJECT COMPETENCE DESCRIPTIONS

To assist with benchmarking the achievement of Learning Outcomes in Grades 10 – 12, subject competences have been described to distinguish the grade expectations of what learners must know and be able to achieve. Six levels of competence have been described for each subject for each grade. These descriptions will assist teachers to assess learners and place them in the correct rating. The descriptions summarise the Learning Outcomes and the Assessment Standards, and give the distinguishing features that fix the achievement for a particular rating. The various achievement levels and their corresponding percentage bands are as shown in Table 4.1.

In line with the principles and practice of outcomes-based assessment, all assessment – both school-based and external – should primarily be criterion-referenced. Marks could be used in evaluating specific assessment tasks, but the tasks should be assessed against rubrics instead of simply ticking correct answers and awarding marks in terms of the number of ticks. The statements of competence for a subject describe the minimum skills, knowledge, attitudes and values that a learner should demonstrate for achievement on each level of the rating scale.

When teachers/assessors prepare an assessment task or question, they must ensure that the task or question addresses an aspect of a particular outcome. The relevant Assessment Standard or Standards must be used when creating the rubric for assessing the task or question. The descriptions clearly indicate the minimum level of attainment for each category on the rating scale.

The competence descriptions for this subject appear at the end of this chapter.

PROMOTION

Promotion at Grade 10 and Grade 11 level will be based on internal assessment only, but must be based on the same conditions as those for the Further Education and Training Certificate. The requirements, conditions, and rules of combination and condonation are spelled out in the *Qualifications and Assessment Policy Framework for the Grades 10 – 12 (General)*.

WHAT REPORT CARDS SHOULD LOOK LIKE

There are many ways to structure a report card, but the simpler the report card the better, provided that all important information is included. Report cards should include information about a learner's overall progress, including the following:

- the learning achievement against outcomes;
- the learner's strengths;
- the support needed or provided where relevant;
- constructive feedback commenting on the performance in relation to the learner's previous performance and the requirements of the subject; and
- the learner's developmental progress in learning how to learn.

In addition, report cards should include the following:

- name of school;
- name of learner;
- learner's grade;
- year and term;
- space for signature of parent or guardian;
- signature of teacher and of principal;
- date;
- dates of closing and re-opening of school;
- school stamp; and
- school attendance profile of learner.

ASSESSMENT OF LEARNERS WHO EXPERIENCE BARRIERS TO LEARNING

The assessment of learners who experience any barriers to learning will be conducted in accordance with the recommended alternative and/or adaptive methods as stipulated in the *Qualifications and Assessment Policy Framework for Grades 10 – 12 (General)* as it relates to learners who experience barriers to learning. *Refer to White Paper 6 on Special Needs Education: Building an Inclusive Education and Training System.*

COMPETENCE DESCRIPTIONS FOR MATHEMATICS

Grade 10

C Code   **S** Scale

Cd Competence Descriptions

6	**80%-100%** **Outstanding**

By the end of Grade 10 the learner with outstanding achievement can:

- produce clear, logical, geometric and algebraic solutions of simple (though not necessarily routine) problems;
- set up simple mathematical models and use them to draw appropriate real-life conclusions, linking across the four Learning Outcomes;
- make use of appropriate mathematical symbols and representations (graphs, sketches, tables, equations) to communicate ideas;
- provide logical arguments for the choice of definitions.

Grade 11

Cd Competence Descriptions

**By the end of Grade 11 the learner with
outstanding achievement can:**

- produce clear, logical, geometric, algebraic and
  trigonometric solutions and proofs to
  multi-step non-routine problems;
- justify conclusions as to the validity of
  self-formulated conjectures;
- set up mathematical models and draw
  appropriate conclusions in a manner that
  includes functions and techniques learned in
  this grade;
- use appropriate mathematical symbols and
  representations (graphs, sketches, tables,
  equations) to communicate ideas clearly and
  creatively, linking across Learning Outcomes.

Grade 12

Cd Competence Descriptions

**By the end of Grade 12 the learner with
outstanding achievement can:**

- think creatively and laterally on a broad range
  of complex mathematical concepts;
- communicate solutions effectively, thoroughly
  and concisely, making use of appropriate
  symbols, equations, graphs and diagrams;
- extend investigations, posing insightful
  questions;
- synthesise across different outcomes and make
  connections with other subjects;
- critically analyse and compare mathematical
  arguments and proofs;
- demonstrate an understanding of proof in local
  axiomatic systems.

Grade 10

C Code   **S** Scale

Cd Competence Descriptions

5

60%-79%
Meritorious

By the end of Grade 10 the learner with meritorious achievement can:

- make connections among basic mathematical concepts;
- investigate special cases methodically and hence derive generalisations and produce conjectures as well as supporting arguments for such generalisations and conjectures;
- check solutions and detect errors in calculations and in simple logical arguments or solutions to problems;
- provide logical arguments in support of solutions to problems.

Grade 11

Cd Competence Descriptions

By the end of Grade 11 the learner with meritorious achievement can:

■ make connections between important mathematical ideas from this and lower grades;

■ produce clear, logical solutions and proofs to routine and simple non-routine problems;

■ convert flexibly between equivalent representations;

■ set up and solve mathematical models for problems that depend on the direct use of expressions and formulae learned in this and lower grades;

■ detect errors in multi-step calculations and logical arguments and can generalise correctly on the basis of a structured investigation.

Grade 12

Cd Competence Descriptions

By the end of Grade 12 the learner with meritorious achievement can:

■ display sound reasoning in multi-step problems;

■ prove or disprove self-formulated conjectures;

■ set up and solve mathematical models which are dependent on the use of functions and statistical techniques learned in this and lower grades;

■ detect errors in multi-step calculations and logical arguments;

■ solve multi-step non-routine problems.

Grade 10

C Code   **S** Scale

Cd Competence Descriptions

4

50%-59%
Satisfactory

By the end of Grade 10 the learner with satisfactory achievement can:

- apply correctly the techniques, algorithms and formulae learned in this and lower grades;
- solve routine problems;
- make good estimates;
- simplify difficult expressions and complete difficult calculations correctly;
- set up mathematical models for straightforward situations that make direct use of the mathematics learned in this and lower grades;
- solve equations, draw and interpret graphs and diagrams;
- investigate problem situations methodically and accurately.

Grade 11

Cd Competence Descriptions

By the end of Grade 11 the learner with satisfactory achievement can:

- apply correctly the techniques, algorithms and formulae learned in this and lower grades;
- solve routine problems;
- simplify difficult expressions and complete difficult calculations correctly;
- solve equations;
- make good estimates;
- set up mathematical models for straightforward situations that make direct use of the mathematics learned in this and lower grades;
- draw and interpret graphs and diagrams;
- investigate problem situations systematically and thoroughly to arrive at conjectures.

Grade 12

Cd Competence Descriptions

By the end of Grade 12 the learner with satisfactory achievement can:

- correctly apply the techniques, algorithms and formulae learned in this and lower grades;
- solve multi-step routine problems with accuracy and purpose;
- simplify difficult expressions and complete difficult calculations correctly;
- solve equations;
- make good estimates;
- draw and interpret graphs and diagrams and investigate problem situations;
- make connections across important mathematical ideas and provide justifying arguments for inferences;
- detect errors in simple calculations and logical arguments.

Grade 10

C Code   **S** Scale

3   **40%-49%**
Adequate

Cd Competence Descriptions

By the end of Grade 10 the learner with adequate achievement can:

- demonstrate understanding of the techniques, algorithms and formulae learned in this and lower grades to arrive at correct solutions to simple routine problems related to everyday life;
- simplify basic expressions;
- complete numerical calculations correctly;
- solve simple equations.

Grade 11

Cd Competence Descriptions

By the end of Grade 11 the learner with adequate achievement can:

- demonstrate understanding of the techniques, algorithms and formulae learned in this and lower grades to arrive at correct solutions to simple routine problems;
- simplify basic expressions;
- complete numerical calculations correctly;
- solve simple equations;
- draw and interpret simple graphs and diagrams;
- do investigations in an unstructured, arbitrary manner.

Grade 12

Cd Competence Descriptions

By the end of Grade 12 the learner with adequate achievement can:

- demonstrate understanding of the techniques, algorithms and formulae learned in this and lower grades to arrive at correct solutions to simple routine problems;
- simplify basic expressions and complete numerical calculations correctly;
- solve simple equations;
- draw and interpret graphs and diagrams;
- do investigations in a structured manner to arrive at conjectures.

Grade 10

C Code   **S** Scale

Cd Competence Descriptions

2

30%-39%
Partial

By the end of Grade 10 the learner with partial achievement can:

- demonstrate understanding of the techniques, algorithms and formulae learned in this and lower grades to arrive at rote solutions to simple routine problems;
- partially simplify expressions, initiate calculations and solve simple equations;
- make attempts at drawing and interpreting graphs and diagrams;
- do investigations in an unstructured, arbitrary manner;
- communicate using memorised mathematical terminology.

Grade 11

Cd Competence Descriptions

By the end of Grade 11 the learner with partial achievement can:

- in a rote manner use the techniques, algorithms and formulae learned in this and lower grades;
- partially simplify expressions;
- partially complete numerical calculations;
- partially solve equations;
- do investigations in an unstructured, arbitrary manner.

Grade 12

Cd Competence Descriptions

By the end of Grade 12 the learner with partial achievement can:

- in a rote manner use the techniques, algorithms and formulae learned in this and lower grades;
- partially simplify expressions and partially complete numerical calculations;
- partially solve equations and simple problems;
- draw and interpret simple graphs and diagrams;
- do investigations in an unstructured, arbitrary manner.

Grade 10

C Code   **S** Scale

Cd Competence Descriptions

1   **0%-29%**
Inadequate

By the end of Grade 10 the learner with inadequate achievement can:

- in a rote manner use provided techniques, algorithms and formulae learned in this and lower grades;
- interpret simple diagrams and graphs.

Grade 11

Cd Competence Descriptions

By the end of Grade 11 the learner with inadequate achievement can:

- in a rote manner use provided techniques, algorithms and formulae learned from this and lower grades to complete numerical calculations correctly;
- interpret simple diagrams and graphs.

Grade 12

Cd Competence Descriptions

By the end of Grade 12 the learner with inadequate achievement can:

- in a rote manner use provided techniques, algorithms and formulae from this and lower grades to complete numerical calculations correctly;
- interpret simple diagrams and graphs;
- solve simple numerical problems.

GLOSSARY

affine transformation – a transformation (usually in a plane) which takes parallel lines to parallel lines (lengths and angles may change); includes **enlargement** and **reductions**, **shearing** and **stretching.**

amplitude – the maximum difference between the value of a periodic function and its mean.

annuity – a fixed sum payable each year or each month either to provide a pre-determined sum at the end of a number of years or months (sometimes referred to as a future value annuity) or a fixed amount paid each year or each month to repay (amortise) a loan (sometimes called a present value annuity).

Formula 1: $\quad F = \dfrac{x[(1 + i)^n - 1]}{i}$

F = future value of the annuity
x = fixed, regular payments
i = interest rate per compounding period, as a decimal fraction
n = number of compounding periods

Formula 2: $\quad P = \dfrac{[1-(1 + i)^{-n}]}{i}$

P $\quad=\quad$ present value of the annuity or the amount borrowed which is to be repaid by fixed repayments
x $\quad=\quad$ fixed repayments
i $\quad=\quad$ interest rate per compounding period as a decimal fraction
n $\quad=\quad$ number of compounding periods

association – a general term to describe the relationship between two variables. Two variables in bivariate data are *associated* or **dependent** if the pattern of frequencies of their bivariate values cannot be explained by only the frequencies of the univariate values. In contrast, two variables are *not associated* or **independent** if the frequencies of bivariate values can be determined simply from the frequencies of the values of each variable.

asymptote – a straight line to which a curve continuously draws nearer without ever touching it.

bar graph/diagram – a diagram that uses horizontal or vertical bars to represent the **frequency** of classes (or groups or labels) in data consisting of observations of a **categorical variable**. The height or length of each bar is proportional to the frequency of the corresponding class; the thickness of a bar has no meaning. The bars are not required to touch each other and may be separated. A bar graph is *not* a **histogram**.

bivariate data – two dimensions (of each object under observation) are recorded as a pair of variables (usually to investigate or describe an **association** or **correlation** or **relationship** between the variables). Numerical bivariate data are often presented visually as a **scatter plot** on a Cartesian plane where one variable (such as height) is read on the vertical axis and another variable (such as mass) is read on the horizontal axis.

box and whisker diagram – a diagram which graphically presents the **five-number summary** of numerical data values. The diagram shows the range and spread in quartiles around the median (a box indicates the range of the middle two quartiles and whiskers the range of the first and last quartiles).

categorical data – data that arises by observing the group or class to which an outcome or an object belongs; often recorded as *labels* which may be alphabetical or numerical (e.g. gender observed as male or female may be recorded as M or F, and also as 0 or 1).

categorical variable – a variable whose values indicate a category to which an observation or an object belongs; may be **nominal** (e.g. eye colour), **ordinal** (e.g. low, medium, high), or **cyclical** (e.g. Monday, Tuesday, … Sunday).

circumference – the (measure of the) perimeter of a circle.

compound event – an **event** consisting of **compound outcomes**, from a **compound experiment**. We say the compound event has *occurred* at one particular repetition of the experiment when *any one* of its compound outcomes is observed.

compound experiment – an **experiment** that consists of *more than one* simple component (e.g. throwing a die and also spinning a coin, or three successive spins of a coin). Each repetition of the compound experiment give rise to a single **compound outcome**, which is recorded as **multivariate data** (e.g. the outcome may be five and heads, written as (5: H); or heads then tails then heads, written as (H; T; H).

compound outcome – any **outcome** of a compound experiment; generally composed of a sequenced list of the outcome of each of the constituent simple experiments (e.g. throwing a head on spinning a coin and then a six and then a four on successive rolls of a die (H; 6; 4)).

conjecture – a tentative solution inferred from collected data.

constraints – limiting conditions (usually translated into linear inequalities) in a linear programming problem.

contingency table – the elements of a population or group may be classified according to qualitative (**categorical**) variables. A classification in a two-way table of the elements according to two such qualitative characteristics is called a contingency table. The rows of the table denote the categories of the first variable and the columns the categories of the second variable.

correlation – a term meaning **association** or **relationship**, often used for numerical data. A measure of correlation for numerical data is derived from a comparison of the **regression line** and the **bivariate data** in a **scatter plot.**

cumulative frequency – for data that has been ordered (from minimum to maximum values) the successive values can be assigned frequencies. The cumulative frequency for a value x is the *total count* of all the data values that are less than or equal in value to x.

data – items of information that have been observed and recorded; can be **categorical** (e.g. gender) or **numerical** (e.g. age). Data is often arranged in a list or table. Data can be **univariate**, **bivariate** or **multivariate**.

dependent variable – the element of the **range** of a function which depends on the corresponding value(s) of the **domain** (e.g. in $y = f(x) = \pi x^2$ the area of a circle (*y*) depends on the radius, *x*. *x* is the **independent variable** and *y* is the dependent variable).

derivative – the rate of change of a function with respect to the **independent variable.**

disjoint groups – two groups that have no common member (e.g. male, female); three or more groups of which no pair has any common member (e.g. planes, cars, trains).

effective interest – the annual rate which is equivalent to a **nominal rate** when compounding is effected more often than once a year (e.g. 12% p.a. compounded monthly is equivalent to 12,68% p.a.; the nominal interest rate (*i*) is 0,12 and the effective interest rate is 0,1268).

enlargement – a mapping that increases the distances between parallel lines by the same factor in all directions (in contrast, see **reduction).**

error margins – an understanding of the consequence of rounding or truncating in the course of calculations (the implication is not working with maximum accuracy as far as possible).

event – any subset of all the possible outcomes of an experiment. An event occurs at a particular experimental **trial** if *any one* of its constituent outcomes is the outcome observed for that trial.

exchange rate – the price of the currency of one country in terms of the currency of another.

experiment – a *statistical* experiment is a repeatable activity or process for which each repetition gives rise to exactly one outcome, drawn from the **sample space**; a *simple* experiment gives rise to **univariate data** on the outcome of each **trial** (e.g. the observed face of a die). The number of trials observed is the **sample size** n.

extrapolate – to estimate an unknown quantity by projecting from the basis of what is already known, but *outside* the limits of the known data (in contrast, see **interpolate**).

feasible region – the set of all points that satisfy the constraints of a linear programming problem.

five number summary – a summary of the observed values of a numerical variable, with **sample size** n, that consists of five values: the **minimum**, first quartile, **median**, third quartile, **maximum.**

fractals – curves or surfaces generated by some repeated process resulting in self-similarity.

frequency – a count of the number of times a particular outcome or event was observed in data with a **sample size** n.

frequency polygon – a polygon formed by joining the mid-points of the top of the columns of a **histogram.**

frequency table – a table reporting the groups into which data values were organised, as well as the **frequency** of each group.

glide reflection – a combination of a **translation** and a **reflection.**

grouped data – data arising from organising n observed values into a smaller number of disjoint groups of values, and then counting the frequency of each group; often presented as a **frequency table** or visually as a **histogram.**

histogram – a visual representation used for grouped data for a numerical variable. The histogram consists of *adjacent* rectangles, each standing on a class interval. The area of each rectangle is proportional to the frequency of observations falling in a class interval. Class intervals are often on the horizontal axis and all have the same width; the rectangles are vertical. Frequency density is read on the vertical axis. A **bar graph** is *not* a histogram.

independent events – the idea that two events do not connect with each other in any observable pattern, and hence that neither event can give any useful information about the other event. In contrast, if two events are *not* independent, they are said to **associated**. Numerical variables that are not independent are said to be **correlated**.

independent variable – as used in dealing with functions: the value that determines the value of the **dependent variable** (e.g. in $y = f(x) = \pi x^2$ the area of a circle (*y*) depends on the radius *(x); x* is the independent variable and *y* the dependent variable).

inflation rate – a quantitative measure of consumer goods price increases over time.

inflection (point of) – where a curve changes from being concave to convex or vice versa (e.g. the point (0˚ ; 0) is a point of inflection on the graph of *y = sin x);* horizontal points of inflection are called **stationary points.**

interpolate – to estimate an unknown quantity within the limits of what is already known (in contrast, see **extrapolate**).

isometric transformation – a mapping which preserves lengths and angles (in a plane); includes **translation**, **reflection**, **rotation** and **glide reflection.**

least squares line – a line in a **scatterplot** which is closest to the plotted points in that it will minimise the sum of squared vertical distances from the points to the line.

median – a value that splits the sample data of a numerical variable into two parts of equal size, one part consisting of all values less than the median and one part with all values greater than the median; most easily established if the data values are arranged in increasing or decreasing order.

multivariate data – two or more dimensions (of each object under observation) are recorded as an ordered string of variables (often to investigate or describe any **association** or **correlation** or **relationship** between the variables. The data is often arranged in a rectangular format of rows and columns, where a specific column is reserved for each variable and a row is allocated to each object that is observed (e.g. one row for the five variables of height, mass, age, eye colour and gender observed for each person, and five columns each with n entries). Categorical variables are summarised by **frequency** counts. Numerical multivariate data are often presented two variables at a time, with a **scatter plot** for each pair.

nominal interest rate – the quoted (annual) rate per annum. If interest is compounded more often than once a year, the **effective rate** is greater than the nominal rate (e.g. 12% p.a. compounded monthly is equivalent to 12,68% p.a.; the nominal interest rate (i) is 0,12 and the effective interest rate is 0,1268).

objective function – the function that one seeks to optimise in a linear programming problem.

ogive – a **cumulative frequency** curve with cumulative frequency plotted on the vertical axis against increasing values of the observed variate on the horizontal axis.

optimise – maximise or minimise (whichever is better in the particular context).

parameter – a constant whose value determines in part how other interrelated variables are expressed and through which they may then be regarded as being **dependent** upon one another.

percentiles – values of **ranked data** separated into one hundred groups of equal size, especially when **sample size** n is very large.

periodicity – the interval after which a periodic (or repeating) function takes the same values.

qualitative data – information or data arising from observations which are *not* numerical; qualitative data can be **categorical.**

quantitative data – data with values are numerical; can be discrete (counted) or continuous (measured).

quartiles – the sample quartiles are three values which split the ordered sample values into four groups of equal size; the second quartile is the **median.**

ranked data – numerical data, *ordered* from say minimum to maximum, and then allocated **ranks** 1 to n.

ranks – positions in an increasing or decreasing ordering, labelled as 1, 2, … n. The minimum (and maximum) are assigned rank either 1 or n. Ranks may be averaged where values are tied.

reduction – a mapping that reduces the distances between parallel lines by the same factor in all directions (in contrast, see **enlargement**).

reflection – a transformation which produces a mirror image of the same shape and size as the original, but reversed; sometimes referred to as a flip.

regression line – the straight line which minimises the sum of the squares of deviations of the dependent variable of individual data items from this line.

right circular cone – a cone that has its vertex directly above the centre of the circular base.

right cylinder – a solid that has one axis of symmetry through the centre of the circular base and a uniform, circular cross-section.

right prism – a prism that has lateral sides that are perpendicular to its base.

right pyramid – a pyramid that has its vertex directly above the centre of its base.

rigid transformation – any transformation which leaves the shape and size unchanged; includes **translation, reflection, rotation** and **glide reflection.**

rotation – a transformation under which a point or shape is rotated (turned) around a fixed point.

sample size – the number of **trials** or repetitions of an **experiment** or observation process; often the symbol for the sample size is n (e.g. n=30 implies thirty observations were made and recorded).

sample space – the collection of all the possible **outcomes** in a statistical **experiment**; may be *discrete* (consisting of categorised or counted values) or *continuous* (when a measurement is made on a scale that is continuous, e.g. mass, temperature, height).

scatter plot – a graph using a pair of x,y axes to represent **bivariate data**, each bivariate element being plotted at a position that represents its pair of values; often accompanied by a **least squares line** fitted to the data.

search line – the set of parallel lines used graphically to **optimise** the **objective function** $O = ax+by$, in a linear programming problem, subject to given constraints; the lines are generated by using various values for O with a and b fixed.

shearing – angular deformation of a plane figure without a change in area; a type of transformation in which parallel lines remain parallel, but distances and angles are not preserved.

sigma – the symbol Σ, denoting the sum.

sinking funds – an accounting term for cash set aside for a particular purpose and invested so that the correct

amount of money will be available when it is needed; this is an example of a **future value annuity.**

spherical geometry – the study of figures on a spherical surface.

stationary points – a point on a graph of $y=f(x)$ where the tangent is horizontal ($f'(x) = 0$).

stem and leaf diagrams – a way of organising and summarising numerical data values into a vertical array of stems and a horizontal array of leaves on each stem. This method conveys more than a **histogram** because it also stores the data and allows easy calculation of **median** and **quartiles**.

stretch – the shape of a graph or object undergoes a vertical or horizontal increase or decrease in scaling.

taxi-cab geometry – the study of routes in a rectangular grid including those which provide the shortest distance between two points.

transformation – the change of one figure (transformation geometry) or one expression (algebra) to another.

translation – a **transformation** that moves all points the same distance in a common direction.

trial – each repetition of a statistical **experiment**.

turning point – a maximum or minimum point on a curve where the y-value changes from increasing to decreasing or vice versa (and the tangent is horizontal).

univariate data – one dimensional data; any quantity or attribute whose value varies from one observation to another gives rise to univariate data, which may be qualitative or quantitative.

universal set – the set of all objects that are under discussion in a particular context.

Venn diagram – a diagrammatic method of representing the relationship between subsets of some **universal set.**

NATIONAL CURRICULUM STATEMENT GRADES 10-12 (GENERAL) POLICY

MATHEMATICS

DEPARTMENT OF EDUCATION

Working together to build an education and
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