Effens aangepaste inleiding tot 'n module oor wiskundige probleemoplossing wat ek in 2001 vir UNISA geskryf het. Dit dien hier as gedeeltelike rasionaal vir die Kurrikulumstudie Wiskunde 174 module.

1. INTRODUCTION

Mathematics Education has failed our children!

We merely have to refer to the poor national matriculation results, the high failure rate in mathematics in all grades, the fact that too few learners take mathematics as subject in the FET phase (on HG), and too few study mathematical sciences at tertiary level.

Urgent and radical change in all aspects of education, and specifically Mathematics Education is necessary if the promise of a better future for our children in our new democracy is ever to become a reality.

What we need is

- a high quality mathematics curriculum (*excellence*), and
- *equity* for all, so that children do not drop out along the way, but the doors to higher levels of education are kept open to all children.

However, it is not easy, and not enough, to simply improve the mathematics curriculum. Oakes (1985) warns us that

> *the unquestioned assumptions that drive school practice and the basic features of schools may themselves lock schools into patterns that make it difficult to achieve either excellence or equality.*

Some of these "unquestioned assumptions" that makes it impossible to improve the teaching and learning of Mathematics includes outdated perspectives and beliefs about the nature of mathematics and therefore the mathematical diet we offer our children, how children learn mathematics, and the role of the teacher in teaching (teaching is not learning!).

We distinguish between

- *micro problems*: Problems internal to mathematics education, e.g. curriculum, teacher development, textbooks, the use of calculators, problem solving, etc., and
- *macro problems:* Problems affecting mathematics education because of external pressures from other sectors of society, economy, politics, culture, language, ...

Both macro and micro problems must be addressed and solved!

Curriculum 2005 is a good start! It is rightly a political statement formulating different outcomes, i.e. different products of schooling (this means delivering a different kind of *person!*). This view of different outcomes is again based on

- different views on children
- different views on the nature of mathematics
- different views on how children learn mathematics.

These fundamentally different assumptions require that teachers change radically in fundamental ways.

These beliefs underlying traditional teaching, and the new views underlying Curriculum 2005 are broadly summarised in the following table:

Traditional	**Curriculum 2005**
VIEW OF MATHEMATICS	
A finished body of facts, rules and skills	Dynamic continually expanding field
It can be discovered	A human social invention
VIEW OF TEACHING	
Instructor/Explainer	**Facilitator/organiser**
Break the maths into small logical pieces	Present tasks and problems that lead to
Explain, drill each piece in sequence	learners inventing mathematics
VIEW OF LEARNING	
Passive reception of knowledge	Active construction of knowledge
Learn by imitation, practice and repetition	Learn through social interaction and reflection
OBJECTIVES / OUTCOMES	
Submissiveness	Intellectual independence
Obedience	Understanding
Compliance	Problem solving and exploration
Following rules	Communication and reasoning
⇩	⇩
DISEMPOWERMENT	**EMPOWERMENT**

A teacher preparation course such as this that wants to contribute to the state of mathematics education in the country, should to our mind

- address macro and micro issues undermining and sabotaging an improvement in mathematics education
- unpack the underlying theoretical assumptions of Curriculum 2005
- offer teachers encouraging alternative approaches to the teaching of different topics
- help teachers in their new roles in the classroom.
- Etc.

Taylor and Vinjevold (1999: 159-161) point to a culture of rote learning, as evidenced by the recent President's Education Initiative research:

The most unequivocal finding about teachers is that a poor grasp on the part of teachers of the fundamental concepts in the knowledge area they are responsible for is a major problem in disadvantaged classrooms. . . . reform initiatives aimed at revitalising teacher education and classroom practices must not only create a new ideological orientation consonant with the goals of the new South Africa. They also need to get to grips with what is likely to be a far more intractable problem: the massive upgrading and scaffolding of teachers' conceptual knowledge and skills.

... the fundamental mechanism for its propagation [the vicious cycle of rote learning] is the lack of conceptual knowledge, reading skills and spirit of enquiry amongst teachers.

This is an echo of a previous report (ANC, 1994):

Science and mathematics … is characterised by a "cycle of mediocrity". … Under-qualified and poorly prepared teachers in turn produce weak and poorly prepared school students, and they cannot be expected to teach the subject with enthusiasm.

Surely we have made major progress in the years since 1994, especially in the improvement of macro problems. But it is this "cycle of mediocrity", which at the micro level is propagated by *rote learning*, that must be broken.

We believe the most pressing content, and therefore the focus of this course, is teachers' *mathematical content knowledge* (what to teach) and *pedagogical content knowledge* (how to teach it).

What mathematical content is worth teaching and learning? Beeby (1935: 10) draws our attention to the importance of revisiting our assumptions about the content of mathematics:

The black doubt that lurks in the bottom of every honest pedagogue's heart is not so much whether he is teaching correctly as whether what he is teaching is worth teaching at all. The real danger is not that we shall teach the right things inefficiently, but that we shall teach the wrong things more and more efficiently.

Schoenfeld describes traditional teaching:

All too often we focus on a narrow collection of well-defined tasks and train students to execute those tasks in a routine, if not algorithmic fashion. Then we test the students on tasks that are very close to the ones they have been taught. If they succeed on those problems, we and they congratulate each other on the fact that they have learned some powerful mathematical techniques. In fact, they may be able to use such techniques mechanically while lacking some rudimentary thinking skills. To allow them, and ourselves, to believe that they "understand" the mathematics is deceptive and fraudulent. (p. 30)

The point to make is that while the mastery of techniques may have been the major objective of mathematics teaching, the goals have changed! The South African Curriculum 2005 and the Revised Curriculum Statements set ambitious goals for Mathematics (Department of Education, 2001: 17):

The teaching and learning of Mathematics aims to develop in learners:
- *A critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations.*
- *The necessary confidence to deal with any mathematical situation without being hindered by the fear of mathematics.*
- *An appreciation for the beauty and elegance of Mathematics.*
- *A spirit of curiosity.*
- *A love for the Learning Area.*

According to the Revised Curriculum Statements (Department of Education, 2001: 16), the Mathematics Learning Area includes interrelated knowledge and skills:

Knowledge	**Skills**
Numbers, operations and relationships	Representation and interpretation
Patterns, functions and algebra	Estimation and calculation
Shape and space (geometry)	Reasoning and communication
Measurement	Problem-posing
Data handling	Problem-solving and investigation
	Describing and analysing

Table 1

In describing the content, the Revised Curriculum Statements (p. 18) use *verbs* like describing, representing, interpreting, analysing, synthesising, conjecturing, inferring, deducing, reflecting, generalising, predicting, refuting, explaining, specialising, defining, modelling, validating, justifying and to collect, summarise, display and critically analyse data to draw conclusions and make predictions.

Our changing society requires a changed perspective on the nature of mathematics and what is worth teaching and learning. This is eloquently described by the National Research Council (1989):

Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. It is important that students move beyond rules to be able to express things in the language of mathematics. This suggests changes both in curricular content and instructional style. It involves renewed effort to focus on:

- *Seeking solutions, not just memorizing procedures;*
- *Exploring patterns, not just memorizing formulas;*
- *Formulating conjectures, not just doing exercises.*

Students should have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be memorized. They should be encouraged to see that mathematics is really about patterns and not merely about numbers.

The National Council of Teachers of Mathematics (1989) advocates *mathematical power* for all and describes "mathematical power" as follows:

Mathematical power includes the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions. Students' flexibility, perseverance, interest, curiosity, and inventiveness also affect the realization of mathematical power.

For this course we adhere to the following view of Schoenfeld:

Mathematics is an inherently social activity, in which a community of trained practitioners (mathematical scientists) engages in the science of patterns – systematic attempts, based on observation, study, and experimentation, to determine the nature or principles of regularities in systems defined axiomatically or theoretically ("pure mathematics") or models of systems abstracted from real world objects ("applied mathematics"). The tools of mathematics are abstraction, symbolic representation, and symbolic manipulation. However, being trained in the use of these tools no more means that one thinks mathematically than knowing how to use shop tools makes one a craftsman. Learning to think mathematically means (a) developing a mathematical point of view -- valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure – mathematical sense-making.

This vision not only challenges teachers' assumptions about mathematics and mathematics teaching and learning, but also asks them to teach a mathematics that they may never have experienced themselves. Teachers are themselves victims of their own previous education and are likely to continue to teach the way they were taught unless a way is found to interrupt this self-perpetuating cycle. This course is an attempt to help students to break out of this cycle by experiencing *mathematical power* as described above.

Theory and research shows that we develop habits and skills of interpretation and meaning construction though a process of *socialization* or *enculturation* rather than through *instruction:*

… becoming a good mathematical problem solver -- becoming a good thinker in any domain -- may be as much a matter of acquiring the habits and dispositions of interpretation and sense-making as of acquiring any particular set of skills, strategies, or knowledge. If this is so, we

may do well to conceive of mathematics education less as an instructional process (in the traditional sense of teaching specific, well-defined skills or items of knowledge), than as a socialization process. In this conception, people develop points of view and behavior patterns associated with gender roles, ethnic and familial cultures, and other socially defined traits. When we describe the processes by which children are socialized into these patterns of thought, affect, and action, we describe long-term patterns of interaction and engagement in a social environment. (Resnick, 1989: 58)

This view of *enculturation* highlights the importance of *perspective* and *point of view* as core aspects of knowledge. The case can be made that a fundamental component of thinking mathematically is *having a mathematical point of view,* or having a mathematical *attitude of mind* -- seeing the world in ways like mathematicians do.

This course is about developing a mathematical attitude of mind, and the way to do it is to immerse participants in a typical *mathematical culture*. This course is about *mathematical thinking*, emphasising the “skills” mentioned in Table 1: Representation and interpretation; Reasoning and communication; Problem-posing; Problem-solving and investigation; Describing and analysing*.*

The medium through which the course is presented is through *problem solving*, i.e. non-routine problem solving – either through illustration of the process of problem solving, or through students' own engagement with problems.

One aspect of the course is about *our own mathematical activity* – we will reflect on the process and product of developing mathematics and we will reflect on our own experiences in doing mathematics (mathematical content knowledge). The other aspect of the course is, through our engagement with the content, to reflect on *learning obstacles* for our school learners, and how we can adapt our *teaching* to address their needs (pedagogical content knowledge).

We trust that you find the course worthwhile, empowering, and enjoyable! However, mathematics is not a spectator sport! You will only really *learn* if you *actively* engage the activities and *reflect* on what you are doing. The famous mathematician Paul Halmos says this about reading mathematics:

Don't just read it; fight it! Ask your own questions, look for your own examples, discover your own proofs. Is the hypothesis necessary? Is the converse true? What happens in the classical special case? What about the degenerate cases? Where does the proof use the hypothesis?