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Interference of Instrumental Instruction in Subsequent Relational Learning

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To balance their professional obligation to teach for understanding against administrators' push for higher standardized test scores, mathematics teachers sometimes adopt a 2-track strategy: teach part of the time for meaning (relational learning) and part of the time for recall and procedural-skill development (instrumental learning). We explore a possible negative effect of this dual approach when relational learning is preceded by instrumental learning. A group of students who received only relational instruction outperformed a group of students who received instrumental instruction prior to relational instruction. Interview data show aspects of cognitive, metacognitive, and attitudinal interference that may have been caused by the juxtaposition of instructional modes.

Key Words: Children's strategies; Cognitive theory; Intermediate grades; Manipulatives; Metacognition; Reform in mathematics education; Teaching (role, style, methods); Teaching effectiveness

Debates on how to characterize learning and understanding of school mathematics reflect different and perhaps irreconcilable interests of various constituencies within the education enterprise. Those in the administrative infrastructure, reflecting the responsibilities of political and public oversight of education, demand unambiguous documentation of learning (Tyack, 1980). This demand creates pressure to regard knowledge as comprised of discrete elements that can be individually assessed and evaluated (Madaus, 1988). In contrast, according to some theories of learning, "the degree of understanding is determined by the number and strength of the connections [among representations]" (Hiebert & Carpenter, 1992, p. 67).

The brunt of this conflict is borne by the classroom teacher. On the one hand, the organizations in which teachers' professional identities are vested regularly call for teachers to provide instruction that leads to meaningful learning (America 2000,

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1991; Cockcroft, 1982; Cohen & Spillane, 1992; Collins, 1988; Howson &

Wilson, 1986; Mathematical Association of America, 1991; Mathematical Sciences Education Board, 1991; National Council of Teachers of Mathematics, 1989). Instruction should involve students in reflecting, explaining, reasoning, connecting, and communicating. Students should develop relational understanding--"understand[ing] both what to do and why" (Skemp, 1987, p. 9). This message is a consistent one that mathematics teachers receive in journals, at conferences, and during in-service programs.

On the other hand, administrators, parents, and political agents often press for instruction that is based on the belief that students must learn skills first and foremost. Curriculum guidelines list individual topics and skills. Structured time lines and specific texts are recommended or required to guarantee that all curriculum topics are "covered." Teachers are encouraged to drill students and have them practice throughout the academic year--particularly during the weeks immediately preceding standardized assessments. Such instruction leads to what Skemp referred to as instrumental learning, "learning rules without reasons" (1987, p. 9).

Eisenhart et al. (1993) found that classroom teachers trying to address all these needs are under great pressure. Caught between professional and administrative demands, they frequently adopt a two-track strategy: They spend some time on drill and practice to provide for skills and facts and some time on integrating understandings (Borko et al., 1992). Because relational instruction usually is assumed to take more time than instrumental instruction to implement (Hiebert & Carpenter, 1992; Skemp, 1987), time constraints are often cited as a principal reason for teachers' relying mostly on instrumental instruction.

Because alternating instructional modes is an obvious resolution of the dilemma, exploring effects of the practice on students' learning is worthwhile. Indeed, several mathematics education researchers have reported finding interference effects when initial instrumental learning is followed by relational learning (Kieran, 1984; Mack, 1990; Wearne & Hiebert, 1988). Of course, the theoretical constructs in each study are somewhat different from Skemp's rather nontechnical distinction; however, the basic insights in these studies seem to share a family resemblance.

Kieran (1984) investigated equation equivalence with seventh-grade students who were not yet initiated to algebra. The students were separated into two groups: those who had been taught transposing (instrumental instruction) as a way to solve arithmetic sentences (e.g., when asked the value for the variable in $5 + a = 12$, they responded $12 – 5 = 7$) and those who had no prior instruction in transposing. Both groups received instruction that provided meaning for the terms *variable* and *equality*. They were encouraged to use trial and error as a means to balance an equation. By the end of the experiment, the students with no previous transposing skills were much better able to apply the procedure of performing the same operation on both sides of an equation. Students in the transposing group resisted the new ideas for equation solving and tended to err by overextending their transpositions. Kieran suggested that interference in relational understanding had been created by the prior learning of transposition (instrumental skills) without an understanding of equality.

In their study of decimal concepts, Wearne and Hiebert (1988) conjectured that "students who have already routinized syntactic rules without establishing connections between symbols and referents will be less likely to engage in the semantic processes than students who are encountering decimal symbols for the first time" (p. 374). They observed that students who had received syntactic instruction prior to a semantic presentation on working with decimals scored significantly lower than students having no prior instruction. Students who chose semantic approaches to problem solving scored significantly better than students using nonsemantic methods.

Mack (1990), in her research on rational number concepts, reached similar conclusions about interference. She conducted 6 weeks of individual instruction for sixth-grade students on fraction concepts and symbolism. The initial lessons were designed to build on the students' informal knowledge of fractions by avoiding fraction symbols and procedures. But Mack found that students' knowledge of rote procedures frequently interfered with their attempts to build on their informal knowledge. The students who had previously acquired rote procedural knowledge tended to focus on symbolic manipulations and did not seem to consider the validity of their responses. The influence of the rote procedural knowledge could be overcome, she found, but only with a great deal of time and effort.

Collectively these studies suggest an educationally relevant generalization concerning the negative influence of instrumental instruction on subsequent efforts to teach mathematics for meaning. But their collective effect is weakened by the relatively incidental nature of the observations. Wearne and Hiebert (1988) and Mack (1990) did not seek the interference effects; the effects were attributed only post hoc to the data at hand. Kieran (1984) did set out to investigate sequence effects in instruction, but she had no experimental control of the instrumental curriculum. Our study is an extension of the research into interference effects through systematic experimental control. We set out to determine if relational instruction alone could be more effective than relational instruction that is preceded by instrumental instruction. Note that for the purposes of experimental control, we necessarily created exaggerated and unifocal (instrumental or relational) learning environments instead of replicating more typical classroom learning experiences. This change allowed us to capture in the laboratory the phenomena of interest, which we then reinterpreted relative to more normal classroom situations.

INTERFERENCE OF PRIOR LEARNING

In the literature two sorts of interference of prior learning on subsequent learning have been identified. The more important of these, *cognitive interference*, occurs when previous understandings in a domain are so powerful as to spontaneously obtrude into subsequent learning. Cognitive interference is often associated with Piaget's (1977) genetic epistemology. For instance, Karmiloff-Smith and Inhelder (1975, as cited in Confrey, 1990) described how children who have developed the theory that "the geometric center of a[n unevenly weighted] bar ought to be the

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balance point persist in their efforts to balance at that point despite contrary evidence *and despite their own success when blindfolded*" (p. 9, emphasis added). Similarly, science educators have documented a large number of cases in which Aristotelian models of physics overwhelm students' development of Newtonian models (Wandersee, Mintzes, & Novak, 1994).

The second type of interference we call *attitudinal interference*. In this case the student's previously acquired opinions and attitudes serve to block full engagement in a situation that might otherwise be productive for learning. For instance, a student's low self-efficacy may decrease that student's willingness to engage fully in instructional activities (Lester, Garofalo, & Kroll, 1989). Or school-based views of mathematics problems as never requiring more than 5 minutes to solve may mitigate against students' persisting with more substantial mathematical problems (Schoenfeld, 1994). Schoenfeld (1987) referred to this type of interference as *metacognitive*, but we reserve that term for a third category.

Metacognitive interference in some sense is intermediate between the cognitive and attitudinal varieties. As with cognitive interference, previous content obtrudes on the new. But unlike in cognitive interference, the previous competence is not firmly entrenched. Rather, for metacognitive interference, the student's initial competence is effortful, not automatic. Indeed, maintaining this competence requires substantial rehearsal or other mental effort. Instruction in new methods in this domain can threaten the student's existing competence by drawing away the mental resources needed to maintain it. Therefore, the student ignores new instruction not because some salient model interposes itself (as in cognitive interference) but because (possibly unconscious) strategic control mechanisms reject the disruptive influence of the new explanations. This analysis is consistent with psychological models of limited cognitive resources (e.g., Kahneman, 1973) as well as with studies that document the degradation of performance when attentional resources are divided between two tasks that both demand cognitive resources (e.g., Hirst, Spelke, Reaves, Caharack, & Neisser, 1980). Note that these forms of interference are not mutually exclusive. For instance, negative attitudes toward relational mathematics instruction may express a student's discomfort with the excessive cognitive demands required to simultaneously maintain instrumental competencies.

METHOD

Design

Our objective in this study was to compare the effects of students' receiving instrumental instruction prior to relational instruction (the I-R treatment) with the effects of relational instruction only (the R-O treatment). In designing the treatments, we decided to provide a longer period of time (5 days) for the instrumental instruction than for the relational instruction (3 days). This decision was based on two considerations. First, we felt that this ratio reflects the situation in many classrooms in which relational methods are employed as "extra" activities within a classroom culture generally dominated by instrumental instruction. Second, using a

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shorter relational treatment provided the possibility of highlighting the effectiveness of relational teaching, challenging the usual assumption that meaningful instruction requires more time for addressing the same content (Hiebert & Carpenter, 1992; Skemp, 1987).

The design included a written pretest for all students, 5 days of instrumental instruction given to half the students (I-R group), an intermediate test covering the instrumental instruction for the I-R group, a 3-day relational treatment given to all students (I-R and R-O groups combined), a posttest following the relational instruction, and a retention test 2 weeks later. The pretest was used as a covariate to control for previous exposure to the content of instruction. The intermediate test was administered to establish the effectiveness of the instrumental instruction. The posttest and the retention test measured the students' instrumental and relational knowledge in the domain of instruction. California Achievement Test (CAT) scores were used as a second covariate to control for the effects of students' prior mathematical achievement.

In addition to using these quantitative measures, we selected six students from each group for three interviews each: before any instruction, following the intermediate test, and following the posttest. The interview data supplemented the quantitative data to provide us with a better understanding of any interference effects that might be revealed in the test scores. The description of the interview sample, coding methods, and so on are explained in the qualitative analysis.

To detect bias and assess validity, we asked a mathematics teacher from another school to observe one instructional unit each day as well as some of the interviews. No instructional biases were reported.

While controlling for the effects of the two covariables, pretest and CAT score, we conducted an analysis of covariance to test whether or not the two treatments differed in effectiveness. Data for students who missed 2 or more days of the 8 days of instruction and for students who had extremely low CAT scores (at or below the 5th percentile) were not considered in the analysis. Because the students in the I-R group took an intermediate test following the instrumental instruction (in addition to the pretest and the posttest), the effects of this intermediate test score and CAT score on the posttest score also were examined using multiple regression analyses. A regression model that included a term for the intermediate test-CAT interaction was used, with the predictor variables centered (that is, expressed as deviations around their means) to reduce the multicollinearity that is possible in an interaction regression model. All significances were tested at the .05 level.

Participants

Six intact regularly scheduled fifth-grade mathematics classes were used for this study; three classes were taught by each of 2 fifth-grade mathematics teachers in a middle-class, semirural school. All these classes were grouped heterogeneously for mathematics instruction. To control for the class variable, we separated each class into two treatment groups by using random stratification by gender and

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achievement level. In selecting six students from each treatment group to be interviewed, we sought stratified random samples that included three students of high ability (as measured by the CAT score) and three students of low ability, as well as three girls and three boys.

Instructional Content?

The mathematical content chosen for this study was area and perimeter of squares, rectangles, triangles, and parallelograms. This content was selected because it was believed to be conceptually accessible to these students and because formulas for calculations had not yet been introduced in the curriculum. (Instruction in formulas for area and perimeter was scheduled for the end of the fifth-grade curriculum.) The formulas taught instrumentally included those for perimeter and area of squares ($P = 4s$, $A = s^2$), rectangles ($P = 2[l + w]$, $A = LW$), triangles ($P = a + b + c$, $A = 1/2\ bh$), and parallelograms ($P = 2[b + w]$, $A = bh$). In the relational instruction, we tried to use students' intuitions about size and distance to develop measurement strategies for area and perimeter in the context of these geometric shapes.

Instrumental instruction. Much effort was put into the five lessons designed to facilitate the I-R students' memorization and routine application of the formulas. A typical day's lesson for the instrumental instruction started with a review of formulas previously learned. Then a shape was drawn; the dimensions were labeled with appropriate variables; and the formula for finding the desired measure was written on the overhead. The students were asked to write the new formula 10 times. The instructor then explained the indicated operations in the formula and connected the variables in the formula to the corresponding features of the shape. Values for the variables were assigned, and the teacher demonstrated the use of the formula for finding the desired measure. The students worked three problems with the instructor and then five additional problems in cooperative groups. The period ended with a quick review of the formulas presented that day. At no time were the formulas justified in terms of the characteristics of the geometric figures to which they applied. During these 5 days, the R-O group remained with their regular teacher and reviewed material unrelated to the content of this study.

Relational instruction. The first author taught each intact class (consisting of both the I-R and R-O students) in three 1-hour lessons over a 3-day period. We designed the instruction to encourage students to construct relationships. Area and perimeter for each shape were presented together to help students contrast and compare these constructs. The shapes were discussed in the following order: squares, rectangles, parallelograms, and triangles. Connections were developed through concrete materials, questioning, student communication, and problem solving.

We designed the relational instruction to assist students in constructing their own ways to calculate measures of area and perimeter on the basis of their understandings of these two concepts. During the 3 days of relational instruction, we progressed from using personal and concrete objects (e.g., hands, tiles, and geoboards) to using

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less concrete models (e.g., pictures of a geometric shape onto which the students could draw square units). For example, in the initial activity, we asked students to find the areas and perimeters of their table tops using their hands as units of measure. The students then discussed the reasons different solution measures were found. We asked, "If basketball star Magic Johnson was measuring the area and perimeter, would the solutions be larger or smaller numbers? Why?" Materials were used in the following sequence: students' hands, 1-inch square tiles (to cover their textbooks), geoboards (for constructing and counting the area and perimeter units), grid paper (so that larger numbers could be used), and, finally, unlined paper (for students to either draw square units or apply some constructed algorithm). The teacher initially presented each problem to the whole class to ensure that all students understood what was required. Then the students worked in cooperative groups of four to solve the problem. Finally, the class as a whole discussed the solutions and strategies used by the groups.

The teacher never specified strategies for efficient or effective calculation. Initially most students used counting strategies to find the area or perimeter of the given figure. Because small numbers were used in the initial problems, such counting strategies could be used effectively. But as larger numbers were used in the problems, students had greater difficulty counting the area and perimeter units. We chose such numbers to encourage students to develop more sophisticated methods for calculation.

Students were always allowed time to explain how they had arrived at their solutions. Students who arrived at generalizations, such as the multiplication strategy for area or the doubling strategy for perimeter, shared them with their peers. Gradually most students used increasingly sophisticated methods, though a few did not progress beyond using counting strategies. In all cases, the strategies students used evolved out of their intuitive understandings of area and perimeter and were sensible to them. Our instructional methods placed the responsibility for finding efficient solution methods with the students. We provided the sequenced problems, diverse materials, and peer discussions to facilitate students' constructions of their own generalizations. No pencil-and-paper calculations were taught during this relational unit.

In addition to carefully attending to these pedagogical practices, we designed the instructional sequence following a careful analysis of the conceptual difficulties posed by specific geometric figures. For instance, triangles were addressed only on part of the last day because area calculations for triangles are complex. The connections from quadrilaterals to triangles were carefully orchestrated. First, students were asked to construct a 2-by-2 square on their geoboards and to explain how they could calculate the area. (We encouraged students to count the square units at this point.) They then constructed a diagonal modeled by the instructor on the overhead. They were asked questions such as "What was our original shape? What two shapes do we now have? What was the area of the square? Does anyone know the area of each triangle? How many times as large as the triangle is the square? How do you know that?" For this initial problem, students could use a

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counting strategy of adding together the halves and wholes of the geoboard squares comprising each triangle or a computational strategy of dividing the area of the square by 2. Subsequent problems involved rectangles and, finally, parallelograms for which the counting strategy was intractable. This task sequence tended to push students from the more concrete counting strategy to a more abstract computational strategy that shows the relationship between areas of triangles and areas of quadrilaterals.

Instruments

In this study we used four tests: pretest, intermediate test, posttest, and retention test. The intermediate test, administered after the instrumental instruction was completed, was designed to evaluate the students' abilities to calculate the areas and perimeters of squares, rectangles, triangles, and parallelograms. This test had only eight items, one for the area and one for the perimeter of each shape. The items were presented in the same format used during the instrumental instruction: We gave students a drawing of the shape and only the measures necessary for determining its area or perimeter. Students were not required to use formulas to get full credit for an item.

The pretest, posttest, and retention test were nearly identical. These instruments consisted of 37 open-ended items; however, three items on the pretest were deleted from the analysis because they proved to be poorly constructed. These tests incorporated a variety of kinds of test items to evaluate a student's ability to calculate the areas and perimeters of shapes (squares, rectangles, triangles, parallelograms, and irregular shapes).

Twenty-four of the items on the tests could be solved using one of the formulas presented in the instrumental instructional unit. In 8 of these items, shapes were presented in a diagram with only necessary measures provided (e.g., "Find the area of the following square" with the length of one side given). In 4 of the 24 items presented with diagrams, the necessary measures were embedded in a word-problem context (e.g., "This room [shown in the diagram] is 12 feet by 10 feet. How many square feet of carpet are needed for the room?"). Five of the 24 items were presented as word problems with only necessary measures given but without accompanying diagrams. For these problems the students were required to draw the diagram (e.g., "Draw a rectangle below. Label the length 8 feet and the width 5 feet. Find the area.") Finally, 7 of the 24 problems were presented, with no word problem contexts, in diagrams in which more measures than necessary were included (e.g., "Find the area of the following square" for which the measure of each side was given).

The other 13 items could *not* be solved by straightforward use of the formulas presented in the instructional unit: Seven of these items gave the perimeters or areas (and possibly one dimension) and required the student to derive a dimension (e.g., find the measure of each side of a square, given the perimeter). The remaining items involved complex shapes constructed by adjoining two or more simple figures (triangles, squares, and rectangles), for example, an L-shaped figure formed by adjoining two rectangles. Four of these items involved determination of the areas of the complex figures. Two of these items involved determination of the perimeters of the complex figures. ?

In scoring the pretest, posttest, and retention test, we assigned each correct response a value of 1 point. The maximum score was 34. The criterion for scoring the responses was whether the student displayed an understanding of area or perimeter, as required. For example, for one item, students were asked to draw a *triangle* having a perimeter of 10 feet. If a student drew a *rectangle* with a 10-foot perimeter, the answer was awarded 1 point because an understanding of perimeter seemed evident; the incorrectness of the shape was considered a slip. All tests were scored by the first author and by at least two additional educators. The reliability coefficients using Cronbach's alpha on the pretest, intermediate test, posttest, and retention test were .699, .754, .873, and .840, respectively.

QUANTITATIVE RESULTS

The central issue in this study was the comparison of the effect of instrumental instruction followed by relational instruction (I-R treatment) with relational-only instruction (R-O treatment). The analysis of covariance revealed that the covariates, pretest (F = 9.3, p = .0029) and CAT (F = 28.29, p = .0001), were both significant. The posttest mean score for students receiving only relational instruction was 16.42 in comparison with a mean score of 14.36 for students who were given instrumental instruction prior to relational instruction. The analysis of covariance did not determine these differences to be significant at the α= .05 level (F = 3.65, p = .059). The trends in the retention-test data were similar (p = .087). (See Table 1.)

Table 1
Descriptive Statistics by Treatment, on Pretest, Intermediate Test, Posttest, and Retention Test

	I-R group			R-O group		
Test	*n*	*M*	*SD*	*n*	*M*	*SD*
Pretest (max score = 34)	54	9.98	3.42	54	10.74	3.34
Intermediate test (max score = 8)	50	5.36	2.27	--	--	--
Posttest (max score = 34)	53	14.36	6.30	52	16.42	5.68
Retention test (max score = 34)	49	14.31	5.59	51	16.49	5.72
CAT (percentile)	54	59.52	21.42	54	59.19	20.47

Note. Dashes indicate that R-O students did not take the intermediate test because they did not receive the instrumental instruction.

The Minitab program was used to assess the statistical power of the analysis. Although the Minitab options for power calculations do not include covariate analyses, the square root of the mean square error that was input into the power

calculation already had the effects of the covariate removed; it should, therefore, be a valid estimate of the population standard deviation needed in calculating the power score. The power score computed in this fashion for the posttest was .58, considerably less than the .80 score that normally would be considered good power for an alpha level of .05. This low power indicates that had the means and standard deviations found in this study been achieved with a larger sample size, the accompanying ***p*** value would have been less. Because the ***p*** value only slightly exceeded the indicated alpha level of .05, the strong likelihood is that the results would have been adjudged to be significant.

The results of the multiple regression analysis conducted on data from the I-R treatment group indicated that for this group, the CAT score was a good predictor of posttest scores ($t = 5.070$, $p = .0001$). About 47% of the variance in posttest scores was explained by this model $R^2 = .4738$). But a scatterplot (Figure 1) shows that there appears to be a mild 3-way interaction among the CAT, posttest, and intermediate scores ($p = .079$). For strong students (as identified by the CAT), the intermediate score seems to be a predictor of the posttest score whereas for weak students, success in memorization as measured by the intermediate test tends to accompany decreased posttest performance. (As shown in the scatterplot, the change in effect seems to occur around the 60th percentile.) Thus stronger students seemed able to overcome the negative effects of instrumental instruction more easily than weaker students.

QUALITATIVE ANALYSIS AND RESULTS

Qualitative Analysis

To gain further insight into effects of sequencing instructional modes, the first author conducted three interviews with 12 stratified, randomly selected students (6 from each group including 3 boys and 3 girls of varying mathematical achievement). The first set of interviews, conducted prior to any treatment, probed students' attitudes toward schooling, learning, and mathematics. The second set was conducted after the instrumental teaching unit to check for students' understandings and skills in calculating area and perimeter. (Students from both groups were interviewed at this time even though only half had received any treatment.) The third (final) set was conducted after the relational instruction in order to assess attitudes toward the instructional methods and to probe students' concepts, skills, and confusions pertaining to area and perimeter. The final interview questions addressed eight topics:

1. (I-R group only) Feelings about the relational instruction compared with the instrumental instruction (e.g., Which did you like better? Which was easier? In which did you learn more?)

2. Feelings about the relational instruction (e.g., What did you like most this week? How did using the manipulatives help you learn?)

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3. Understanding of area (e.g., What is area? How would you find the area? Why do we learn area? Who uses area in their jobs?)

4. Understanding of perimeter (e.g., What is perimeter? How would you find perimeter? Why do we learn perimeter?)

5. Application of area and perimeter (e.g., To know how much carpet to buy for this room, do you need to know the area or the perimeter?)

6. Application of calculation methods (e.g., Draw a shape and explain how you could get the area of the shape. What is the perimeter of your shape?)

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7. Questions regarding how the student solved a specific item on the posttest

8. Interpretation of formulas: Students were given the formula for the perimeter of a rectangle and the area of a triangle and were asked why using the formulas could give the solutions.

All interviews were audiotaped, and the final interviews were videotaped, transcribed, coded, and analyzed. Only the final interviews are analyzed here.

We used the following coding system for the final interview. Each transcript was labeled to conceal the name, treatment, gender, and achievement level of the student. Each of the eight topics was assigned a color, and sections of the transcripts were coded accordingly. We analyzed, separately, data on the eight topics. We read and summarized each student's responses on the specific topic. After data for all topics had been summarized in this fashion, the transcripts were separated according to the treatment group of the student. We intensively studied the summaries for emerging patterns.

Qualitative Results

In the final-interview data we found further evidence of an interference of instrumental instruction on subsequent relational understanding, and these data helped us identify cognitive, attitudinal, and metacognitive characteristics of the interference.

Cognitive interference. In defining, describing, and applying area and perimeter concepts, interviewees from the two treatment groups frequently displayed both subtle and not-so-subtle differences in understanding. To describe the difference between area and perimeter, all the I-R students used the term *inside* in defining area, and three of the six used the word *outside* for perimeter. For instance, to define area, one student said, "The part that's inside an object," and for perimeter, "[Perimeter] is the outside." In contrast, five of the six R-O students interviewed used *whole* in defining area ("the whole room," "the whole thing," "the inside and the middle"), and four of them used the word *around* for perimeter. The interviewees in the two groups understood these terms in relation to familiar concepts and attributes, but these concepts and attributes were different for the two groups.

This difference in conceptual grounding was apparent in the responses to some of the tasks posed in the interviews and seems to partly explain the differential success rates of the two groups. Students were asked questions like "What if I wanted to know how much carpet to buy for this room? Would I need to know the area or the perimeter?" All but one student responded correctly that area was needed. But for a similar question regarding the amount of paint or wallpaper needed for the walls, five of the six I-R interviewees said either that they did not know or that one needs to know the perimeter of the room to determine the amount of paint or wallpaper required for the walls. A typical explanation was "Walls don't have area because they go around." In contrast, all six of the R-O interviewees recognized that area measurements are needed to determine the measure of the wall's surface.

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Differences between students in the two groups also emerged in their understandings of dimensions and dimensionality in discussion of real-world applications of area and perimeter. The I-R students mistakenly claimed that area is needed to measure "the lengths of boards," "how much liquid," "the thickness of concrete," and the "height of a pole." R-O students made no such errors in their applications of area to life situations. The concepts of area and perimeter appear to have been less clearly differentiated for the I-R students than for the R-O students.

These differences contributed to students' varying appreciation of the possible applications of this mathematical content to their out-of-school lives. In discussing who needs to understand area and perimeter and why they need to understand it, the R-O interviewees gave a greater number of concrete applications (for carpet, painting, wallpaper) than did the I-R students (for tests, later study, college).

Attitudinal interference. The experience of relational instruction was new for most of the students. They all agreed that their regular mathematics instruction was very different from the instruction in this study: "We don't use all that stuff [manipulatives]." Several students in the I-R group said that their regular mathematics class was similar to the formula instruction (instrumental) because their teacher also explained many problems on the overhead.

Perhaps because of the novelty, all of those interviewed said that they enjoyed the relational unit. Their favorite manipulative was the geoboard. They liked "playing" with the boards and bands. But students in the I-R group varied in their preferences for the two forms of instruction. Half of them stated that the formula instruction was easier and more enjoyable, and all but one thought that they had learned more from the formula instruction. The one student preferring the second treatment complained, "They [the formulas] got me confused. It's complicated to remember all the stuff about which formula goes with which problem." Perhaps the similarity of the instrumental instruction to their regular classroom practices influenced students' greater receptivity to the instrumental component.

Metacognitive interference. The interviewees' explanations of how to calculate area and perimeter reflected very different approaches to the subject matter. The I-R students consistently used operations (usually incorrectly or with incorrect reasons) in their approaches to calculating area and perimeter. For instance, when asked how one can find area, an I-R student said, "Sometimes you multiply; sometimes you add." For the area of a 2-by-2 shape, the student recommended multiplication, "2 times 2"; but the student gave a relationally incoherent reason: "Because there's four sides." One I-R student demonstrated understanding at the start of her explanation of perimeter but seemed to then apply operations at random by stating

> It's like when you have a rectangle or something, and if you have four on one side and three on one side, ... you walk around it and you count how [much] distance.... And then sometimes they will multiply it and get the answer, or they will add it and then they will add the two and then they will multiply the other ones with it.

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The R-O interviewees' general approaches tended to be grounded in the concrete methods used in the relational instruction ("use hands," "use books," "use tiles" to cover the surface). The solution methods they used during the interviews involved quantitative reasoning about diagrams constructed or provided during the interviews.

The I-R interviewees' ungrounded use of formulas carried over into their explanations of how formulas work. When asked to explain why the perimeter of a rectangle and the area of a triangle are given by the formulas $P = 2(l + w)$ and $A = 1/2\ bh$, respectively, all I-R students either said that they did not know or gave incorrect explanations. For instance, no one in this group could correctly explain the role of the 2 in the perimeter formula. Some erroneous explanations were "the length is 2 and the width is 2," "it has 2 numbers," "because it has 2 different sides," and it makes it "easier to remember." In explaining the triangle area formula, one of the I-R students simply said that he did not understand, and four students gave confusing explanations (e.g., "I think one measure is one half the other"; "you take the number and you add them together"; "you take one half away"; and "one half [is there] because the sides are not always going to be the same."). One displayed the alchemist's initiative in explaining "there's three sides on a triangle [therefore] ... there's three numbers."

Interestingly, the R-O interviewees were able to make more sense of the formulas, despite their lack of instructional exposure to them. In some cases R-O students reported that they simply could not see any connections. However, five of the six students formulated partial or complete explanations for formulas for at least one of the figures, and no explanation given was incorrect. "For $P = 2(l + w)$, one needs to add the length and width and then double it" was the gist of statements by several R-O students. In various ways they explained that the sides were repeated twice. The students who were able to connect the area of a triangle to its formula explained that a triangle is half of a rectangle.

That I-R interviewees relied so heavily on formulas but understood them so poorly provides conditions under which metacognitive interference might be expected to arise. Maintaining applicative skills through memorization requires concerted mental rehearsal and other attentional resources. To the extent that students are committed to maintaining these applicative skills, engaging with subsequent relational instruction might be experienced as a distraction. Engaging less fully with the relational instruction would lessen this distraction (Hirst et al., 1980).

DISCUSSION

This study, in conjunction with past research (Kieran, 1984; Mack, 1990; Wearne & Hiebert, 1988), indicates a danger in the compromise of teaching for rote skill development part of the time and for conceptual understanding part of the time: Initial rote learning of a concept can create interference to later meaningful learning. In this study students who were exposed to instrumental instruction prior to relational instruction achieved no more, and most probably less, conceptual under-

standing than students exposed only to the relational unit. Contrary to the common-sense expectation regarding time-on-task, more instruction does not necessarily translate into more learning.

Three possible mechanisms of interference were outlined in the introduction: *cognitive interference*, in which deeply rooted prior understandings of the content domain obtrude into attempts to construct new understandings; *attitudinal interference*, in which students' attitudes and beliefs about themselves or about the domain in question serve to prevent their full engagement in learning activities; and *metacognitive interference*, in which maintenance of prior instrumental understanding requires rehearsal or other mental effort and new learning is rejected (perhaps unconsciously) as disruptive to the existing competencies.

The interview data indicate that the interference effects of this study are partly cognitive in nature. For instance, an inability to find the surface area of a wall because walls "go around, thus one can only find the perimeter" was a consistent misconception among interviewees who had received instrumental instruction prior to relational instruction (the I-R group) but was not present at all for those interviewees who received relational instruction only (the R-O group). The I-R students seemed to have only partially understood concepts of perimeter (e.g., perimeter means "goes around"), and that partial understanding effectively blocked out notions of area (as "filling" or "covering") that were highlighted in the relational instruction and became operative for the R-O students.

Students in the two treatment groups emerged with very different approaches to solving problems involving area and perimeter. The interviewees who had received both instructional treatments referred to formulas, operations, and fixed procedures as the means for solving problems. Students who had received only the relational instruction used conceptual and flexible methods of constructing solutions from the units of measurement with which they had had concrete experiences.

To assess the relative contributions of metacognitive and attitudinal processes to these differences in approach is difficult. Although all the students said that they enjoyed the relational instruction, about half of the I-R group felt that they had learned more from the instrumental instruction. The greater similarity of the instrumental instruction to their regular classroom instruction (as reported in the final interview) may have contributed to a belief that learning rules and procedures is appropriate and hence deserving of one's full effort and attention. As well, the subsequent relational instruction, though enjoyable, may have constituted a threat (consciously perceived or not) to the practice and rehearsal needed to maintain the prior (instrumental) competencies. Thus metacognitive and attitudinal processes may have mutually supported each other in the students' greater attention to instrumental methods at the expense of relational understanding.

The rote-instruction unit was almost twice as long as the relational-instruction unit. This situation replicates the circumstance we believe characterizes many mathematics classes: Most instructional time is spent on routine exercises to consolidate rote or procedural knowledge; much less emphasis is given to students' intuitive and sense-making capabilities. Thus the interference effects suggested here in the exper-

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imental microcosm may reflect the learning experiences of many students in many classrooms.

These results may be most relevant for those teachers who do make an effort to respond to the recommendations of the professional organizations by incorporating relational learning activities but maintain a classroom culture dominated by instrumental learning. The hard truth is that real reform in mathematics education calls for a reorientation of classroom norms and practices (Cobb, Boufi, McClain, & Whitenack, 1997; Lampert, 1990). Relational units appended to the existing classroom regimen may effectively be blocked from achieving the relational understanding sought. We hope that the relative efficiency and effectiveness of relational instruction that is not preceded by instrumental instruction, demonstrated in this study, inspires other teachers to undertake fundamental classroom reform.

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[1]The terms *relational* and *instrumental* were coined by the late Richard R. Skemp, to whom we dedicate this report for his contributions to the topic and his encouragement of this study.

The study reported here was conducted by the first author as part of her dissertation (Simoneaux, 1992) at Louisiana State University under the supervision of the second author. A brief report (Simoneaux & Kirshner, 1994) of the dissertation was presented at PME-NA XVI.

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