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Tell Me With Whom You're Learning, and I'll Tell You How Much You've Learned: Mixed-Ability Versus Same-Ability Grouping in Mathematics

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In this article we report on 3 studies in which we investigated the effects of teaching mathematics in a mixed-ability setting on students' achievements and teachers' attitudes. The findings of the first 2 studies indicate that the achievements of students need not be compromised in a heterogeneous setting; on the contrary, the achievements of our average and less able students proved to be significantly higher when compared to their peers in the same-ability classes, whereas highly able students performed about the same. In the 3rd study we show that participating in the project workshops had a positive effect on teachers' attitudes toward teaching in mixed-ability mathematics classes.

Key words: Cooperative learning; Equity/diversity; Grouping for instruction; Longitudinal studies; Quasi-experimental design

The degree of influence of school grouping methods on the individual student's scholastic achievements is a central issue in educational research. One of the most widespread methods of grouping students in the same grade is ability grouping, either on a subject-by-subject basis (tracking) or for all subjects at once (streaming). Tracking and streaming are widely viewed as the best way to improve the scholastic achievements of all students.

Studies have shown that most teachers have a positive attitude toward ability grouping (Barker-Lunn, 1970; Chen & Addi, 1990; Chen & Goldring, 1994; Guttman, Gur, Kaniel, & Well, 1972; Husén & Boalt, 1967; McDermott, 1976; Oakes, 1985). Many of them justify ability grouping on the basis of the need to adapt class content, pace, and teaching methods to students functioning on different levels (Dar, 1985; Slavin, 1988, 1990; Sørensen & Hallinan, 1986). In the case of mathematics it is also justified by the "nature" of the subject. Mathematics is perceived as "graded," "linear," "structured," "serial," and "cumulative"—making it difficult to work with groups of students with different levels of knowledge and ability. And, indeed, the central issues for supporters of ability grouping relate to "ability to learn mathematics" and "the hierarchical nature of the subject" (Ruthven, 1987). They view students' abilities as the major explanation for differences in their achievements in mathematics (Lorenz, 1982).

Recent research, however, has cast doubt on whether placing students into ability groups is the correct method for dealing with the diversity of abilities. It has generally been shown that the scholastic achievements of students assigned to higher tracks are better than those of students who are judged to have similar abilities but who have been placed in lower tracks. Researchers conducting studies of this sort have concluded that the placement of students in ability groups in and of itself increases the gap between students beyond what would be expected on the basis of the initial differences between them (Alexander, Cook, & McDill, 1978; Gamoran & Berends, 1987; Gamoran & Mare, 1989; Kerckhoff, 1986; Oakes, 1982; Slavin, 1990; Sørensen & Hallinan, 1986).

The discouraging results of tracking studies, on the one hand, and evidence of the promising potential of cooperative learning, on the other (Crain & Mahard, 1983; Crain, Mahard, & Narot, 1982; Davidson & Kroll, 1991; Goldring & Eddi, 1989; Wortman & Bryant, 1985; Willie, 1990), have prompted attempts to cope with student diversity within the mathematics classroom. Most of those using these approaches argue that low-ability settings lead to low-quality teaching. Low-quality teaching is characterized by teachers' low expectations; a low-status, nonacademic curriculum; valuable class time spent on managing students' behavior; and most class time devoted to paperwork, drill, and practice. Moreover, the nature and quality of the oral interaction is fundamentally different in low-track and high-track settings (Gamoran, 1993). This last crucial aspect—the role and quality of discussions—is highly emphasized in theoretical approaches that describe learning as an individual process nourished by interpersonal interaction (Bandura, 1982; Carver & Scheier, 1982; Voigt, 1994; Wood & Yackel, 1990). For these theorists the study group is not a mere administrative division but is a crucial component of the learning environment. They suggest that two hypothetically identical students may end up with different mathematical knowledge if they are assigned to two study groups with significantly different participants and styles of interaction. Ability grouping is an obvious case of creating unequal learning groups within the same school. It is thus unsurprising that it has been criticized on this ground and that alternative solutions for dealing with student diversity, such as mixed-ability settings, have been investigated.

Thus, past research has shown that ability grouping results in an increase in the gap between high- and low-ability students beyond that expected on the basis of initial differences between them. What has not been shown is, first, whether this growth in inequality is avoided in mixed-ability settings and, second, whether this gap in achievement (because of tracking) occurs because tracking helps students in the higher ability groups, harms students in the lower ability groups, or because of some combination of the two. In Part I of this article we report on two studies (Study 1 and Study 2) that were designed to address these two questions. In Part II we report on a third study that was designed to examine the effects on teachers' attitudes of teaching in mixed-ability classes. These three studies took place within the framework of a large, ongoing project, Project TAP, in which mathematics is taught in mixed-ability settings in Israeli junior high schools. In the following section we briefly describe this project.

The TAP Project

The junior high schools participating in the TAP project are comprehensive schools. Each of these schools draws its students from at least two elementary schools that are located in neighborhoods of differing socioeconomic levels. The heterogeneity of each of the classes participating in TAP reflected the heterogeneity of the population of its school.

The major principle of the TAP (*T*ogether and *AP*art) project is to keep a class together as one learning unit while responding to the different needs of the students. This principle does not necessarily mean bringing all the students to the same level of achievement. Instead, it means enabling them to progress to the fullest extent of their abilities through a combination of the following: (a) meaningful instructional activities for cooperative learning by all students throughout the school year in heterogeneous settings whether the whole class or smaller groups—activities henceforth called *shared topics*—and (b) differential instructional activities for cooperative learning by different students according to their abilities and prior achievements in homogeneous settings—henceforth called *differential topics*.

Thus, in each class the teaching was conducted within four major settings: (a) students working in a whole-class setting; (b) students working in small mixed-ability groups; (c) students working in small homogeneous groups; and (d) students working in large homogeneous groups. In the first and last settings teachers played an active role, whereas in the others they were in a supportive role only. Each of these settings was designed to respond to different needs for interaction among the students and between the teacher and the students.

During whole-class discussions the teachers could develop conceptions about what mathematics is; create an appropriate learning atmosphere; and foster essential norms such as listening to classmates, legitimizing errors as part of the learning process, and allowing expression of ideas and tolerance of ambiguity (Davis, 1989; Gooya & Schroeder, 1994). The whole-class discussions established a basis for collaborative dialogues that are known to be a major feature of productive small-group interactions. These discussions also allowed the weaker students to participate, albeit many times passively via "legitimate peripheral participation" (Lave & Wenger, 1991) and "cognitive apprenticeship" (Brown, Collins, & Duguid, 1989), in a challenging intellectual atmosphere.

Justifications for small-group interaction within mathematics classrooms have been presented in many recent papers (Brown et al., 1989; Cobb, 1994; Good, Mulryan, & McCaslin, 1992; Schoenfeld, 1989; Shimizu, 1993; Yackel, Cobb, & Wood, 1991). Cobb (1994) has pointed out that productive small-group interactions involve multivocal interactions, which at first glance seem to require homogeneous grouping. Further, according to Cobb, "Homogeneous grouping

... clashes with a variety of other agendas that many teachers rightly consider important, including those that pertain to issues of equity and diversity" (p. 207). Brown et al. (1989) emphasized the cognitive value of collaborative learning via cognitive apprenticeship in heterogeneous groups. We thus chose for our project the strategy of alternately using small homogeneous groups and small heterogeneous groups so that each child was simultaneously a member of two groups (the composition of the groups changed from time to time, depending on the topics, activities, and students' past achievements). The work of the heterogeneous groups focused on the shared topics that met all the requirements of the official curriculum. The homogeneous groups, however, usually dealt with completely different mathematical topics, prepared in accordance with the groups' needs, and sometimes the groups were presented with alternative approaches to the same mathematical topic. In the homogeneous setting opportunities for multivocal interactions were created naturally.

Whenever a teacher felt that a large, specific homogeneous group of students would benefit from the teacher's direct intervention, that setting was used—for example, to better prepare weaker students to be integrated into a planned heterogeneous group activity. Silver, Smith, and Nelson (1995) described activities of this sort as "preteaching." Large homogeneous groups were also used to investigate enrichment topics.

To involve the project teachers in developing the appropriate strategies, tools, and instruments needed to teach effectively in their heterogeneous mathematics classes, we held weekly workshops in which all teachers participated. For example, activities were prepared for different ability levels or to encourage interactions in heterogeneous groups. An equally important aspect of our workshop meetings was discussion and sharing of problems that had arisen in the teachers' classes that week.

The present studies do not specifically examine any particular aspects of the TAP project. The purpose of the project description is to give the reader some flavor and familiarity with the project because the schools involved in these studies were all project schools.

In the next section we report on two studies in which we examined how learning mathematics in mixed-ability settings affected students' achievements.

PART I: STUDENTS' ACHIEVEMENTS

Rationale for Our Research Questions and Study Design

In Study 1 and Study 2 we compared achievements of students studying in mixed-ability versus same-ability systems to determine (a) which format leads to greater achievements on the part of the students and (b) specifically, which system leads to greater achievements for the better students, the weaker students, and the intermediate ones, when parallel levels for each of these systems are compared.

The appropriate design for examining these questions is an experiment involving the random assignment of classes of students to either heterogeneous or homogeneous classes. In the heterogeneous classes the students are hypothetically assigned to ability-group levels, whereas in the homogeneous classes the students actually study according to the assigned ability levels. However, the difficulty in performing random experiments in the educational system and the methodological problems associated with post hoc comparisons between schools with and without ability grouping (for a review see Slavin, 1990) led to a less ambitious design, one that compared ability-group levels within schools. In the latter type of study, one investigates whether the gap between better and weaker students after placement in ability groups for a certain period of time differs from the gap expected on the basis of initial differences (Kerckhoff, 1986; Oakes, 1982). In this type of study the main methodological problem is to separate the two effects that might influence final achievements: the effect of belonging to groups at different levels and the effect of the initial differences between the students placed in these groups (Cahan, Linchevski, & Ygra, 1992).

The methodological problem can be overcome if students are divided into group levels by establishing agreed-upon cutoff points based on a measure of ability, previous achievements in the subject matter at hand (henceforth called the pretest), or both. Using information on the cutoff points and each student's group level and pretest score, one can see the variance among the student scores on a common achievement test some time later (henceforth called the posttest) as the sum of two effects: (a) the effect of the initial differences among the students and (b) the effect of the group levels in which students worked. These two effects can be disentangled by means of a regression discontinuity design (Cook & Campbell, 1979). The effect of initial differences is estimated by the regression line of the posttest on the pretest within each group level, whereas the effect of the group level is estimated by the discontinuity between the regression lines of consecutive group levels. This research design is known as the quasi-experimental regression discontinuity design. Figure 1 shows an example of all possible combinations of the two effects.

Students close to a given cutoff point on either side can be seen as identical from the viewpoint of the selection criterion. Figure 1a is a hypothetical exam-

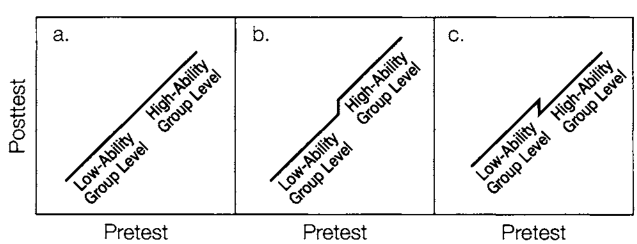

Figure 1. An example of the various combinations of the values of the two effects: a. Lack of grouping effect. b. Favoring high-ability group. c. Favoring low-ability group.

ple of "no grouping effect": After a period of treatment no gap has been created at the cutoff points; that is, after a period of treatment students on either side of and close to a cutoff point had similar scores. Thus the variance among the students' scores is due only to the initial differences among the students. Figures 1b and 1c are hypothetical examples in which gaps have been created at the cutoff points. Thus the variance among the students' scores is due to the initial differences among the students and to a grouping effect. In Figure 1b students in the higher group level gained more than similar students in the lower group level, whereas in Figure 1c the opposite occurred.

A design of this sort was used successfully by Abadzi (1984, 1985) in the United States for investigating the effect of streaming in elementary schools and in Israel by Cahan and Linchevski (1996) for investigating the effect of tracking in mathematics in junior high schools. In the latter investigation the findings clearly indicated that the differences among the scholastic achievements of the students at the different group levels at the end of the first and the third years of junior high school were greater than would be predicted by the data at the time of placement. Moreover, in most of the schools, after 3 years, the effect of the group level was greater than the effect of the initial differences among the students.

Study 1

Because widening the gap between stronger and weaker students might occur in heterogeneous settings as well as homogeneous settings, the results reported above have no clear bearing on the comparative benefits of homogeneous and heterogeneous educational settings (Cahan et al., 1992; Linchevski, Cahan, & Dantziger, 1994). Study 1 was designed to answer the following question: Is the gap between better and weaker students learning together in mixed-ability settings for a certain period of time different from the gap that would be expected on the basis of initial differences between the two groups? We used the regression discontinuity design to investigate this question. In the study we also compared the results with those reported in the ability-grouping study of Cahan and Linchevski (1996). This comparison was possible because identical research designs were used in the two studies.

Our conjecture was that in the schools that participated in the TAP project, no gaps between students would be created beyond the one expected on the basis of initial differences among them. This outcome was expected because the project was designed using the theoretical considerations and previous research results reported in this article (e.g., Brown et al., 1989; Cobb, 1994; Gamoran, 1993; Lave & Wenger, 1991; Schoenfeld, 1989).

Design

In Study 1, the unit of analysis was a school. This choice was made for several reasons. First, the fact that the effects were estimated separately for each of the schools investigated actually constitutes independent replications of the study.

Second, we could compare this study with the earlier ability-grouping study by Cahan and Linchevski (1996) because the same unit of analysis was used in the two studies. Last but not least, the choice of a school as a unit of analysis is an improvement on earlier studies that compared the gap between heterogeneous and homogeneous settings. Other earlier studies calculated effects in a pooled sample of schools, hence obtaining results that might have hidden the variability in mathematics achievement by aggregation of schools to the pool level.

Because teaching actually took place in heterogeneous settings in this study, the application of the regression discontinuity design required a hypothetical division of the students into the various group levels *as if* these students were actually going to study in separate ability groups. At the beginning of the seventh grade, each school therefore assigned each student his or her ability-group level according to the school's previous tracking policy, although in effect these students studied mathematics in heterogeneous settings. This assigning procedure was done without the knowledge of the students' mathematics teachers. The hypothetical group level and the placement scores (the pretest) served as the independent variables, whereas the achievement test scores in mathematics after 1 year and after 2 years—at the end of the seventh grade and at the end of the eighth grade (the posttests)—served as the dependent variables. On the basis of these variables, the effects of grouping and initial differences on achievement were calculated separately for each school. For a detailed description of these calculations see the appendix.

Sample. All 1730 seventh-grade students in the 12 Israeli junior high schools that participated in the TAP project were tested (posttest) at the end of the seventh grade. We have complete data for 1629 of these students.

In 4 of the 12 schools, the students (389 students) were tested at the end of the eighth grade as well.

Tests. Achievements in mathematics were measured by tests constructed according to the topics covered in the schools, as detailed in the national mathematics curriculum (first- and second-year algebra, problem solving, and geometry). The contents of the seventh- and eighth-grade tests were confirmed with the schools involved, and the tests were validated by experts and by the General Inspector for Mathematics Teaching in the Israeli Ministry of Education. The size of the research population and the nature of the study determined to a large extent the type of test that could be administered to the students. We are aware that these tests were traditional in form. However, the questions were not multiple-choice but instead were open-ended, allowing for more flexible types of questions to be asked. To control for between-school differences in mean achievement levels and test-score variance, the scores on pretests and posttests were standardized separately for each school, with a mean of 0 and standard deviation of 1.

Results

Results at the end of the seventh grade. The measures of effects of the hypothetical ability groupings on the students' achievements in mathematics at the end of the seventh grade are presented separately for each school (see Table 1).

Table 1
The Results of the Regression at the End of the 7th Grade

		Hypothetical-grouping effect		Initial-differences effect
School	No. of groups	(α_H)[a]	p value for F-test	(α_P)[a]
1	4	-0.36	.37	2.75
2	3	-0.42	.18	3.17
3	3	-0.26	.54	2.30
4	3	-0.60 *	.04	2.75
5	4	-0.12	.64	2.71
6	4	-0.87 *	.03	3.45
7	3	0.10	.75	2.78
8	4	0.09	.75	3.15
9	3	-0.34	.42	3.11
10	3	-0.40	.20	3.57
11	3	0.26	.59	2.20
12	3	0.10	.71	2.90

[a] For details, see the the appendix.
* $p < .05$.

A "negative" grouping-effect measure (e.g., -0.36, see Table 1) means that a gap that was not expected on the basis of initial differences was created at a cut-off point. This negative grouping effect means that students close to a cutoff point gained more on the average if they were in a higher hypothetical group than if they were in the next lower group (see Figure 1b). A "positive" grouping effect means that students close to a cutoff point gained more being hypothetically part of a lower ability group than of the next higher group (see Figure 1c). Zero effect measure means grouping had no effect—neither increasing nor decreasing the gap (see Figure 1a). As can be seen in Table 1, the effects were not uniform and in 10 of the 12 schools were nonsignificant at the p = .05 level. In 8 of the 12 schools the effect was negative, whereas in the others it was positive. The size of the effect measure was different in different schools, ranging from +0.26 *SD* to –0.87 *SD*, with a median of –0.31 *SD*.

Figures 2a and 2b show two examples of the regression discontinuity of the posttest on the pretest. School 4 is a case in which the effect was in the direction of increasing the variance. The difference between the better and the weaker students was 3.35 standard deviation units (2.75 - [–0.60] = 3.35; see appendix) at the end of the year.

In School 11, the difference between the two groups at the end of the year was less than would be expected on the basis of the initial differences—a gap of 1.94 standard deviation units (2.20 - [+0.26] = 1.94; see appendix).

A comparison of the hypothetical-grouping effect with the initial-differences effect (see Table 1) in each of the schools shows that the former was very small

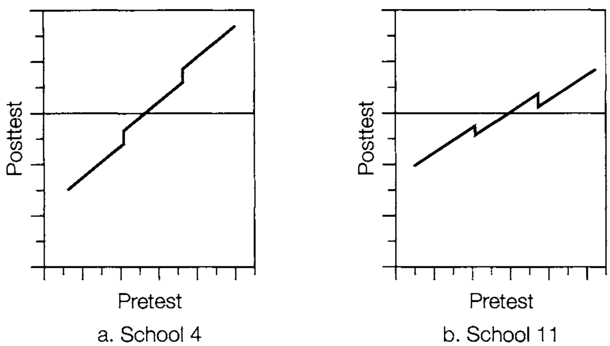

Figure 2. Representative cases of discontinuity in posttest and pretest grades.

in all schools, relative to the initial-differences effect. Because the hypothetical-grouping effect was nonsignificant in 10 out of the 12 schools and the trend was not uniform—that is, the effect was positive in some of the schools and negative in others—the conclusion is that in 10 of the schools there was no effect and in 2 schools there was a negative effect.

Results at the end of the eighth grade. Four of the 12 schools that participated in the study maintained heterogeneous classes in the eighth grade as well. Table 2 shows the hypothetical-grouping effects and the initial-differences effects at the end of 2 years of such studying in these four schools.

Table 2
The Results of the Regression at the End of the 8th Grade

School	No. of groups	Hypothetical-grouping effect $(\alpha_H)^a$	Hypothetical-grouping effect: p values for F-test	Ability effect $(\alpha_P)^a$
4	3	−0.52*	.04	2.70
6	4	−0.36	.40	3.00
7	3	−0.24	.49	2.80
11	3	0.10	.70	2.70

[a] For details, see the the appendix.
$*p < .05$.

In three of the four schools there was a negative treatment effect—that is, the variance at the end of the 2 years was greater than would be expected on the basis of the placement data, whereas the opposite effect was found in the remaining school. In Schools 4 and 6 the effect at the end of the seventh grade had been negative, and it continued to be negative at the end of the eighth grade, but its absolute value at the end of the second year was smaller than it had been at the end of the first. In School 7 the effect changed from positive at the end of the seventh grade to negative at the end of the eighth grade, whereas in School 11 it remained positive, although its absolute value decreased. In three of the four schools the effect was not significant. Only in School 4 was the effect significant; the effect had been significant there at the end of the seventh grade as well. Generally speaking, inasmuch as the effects were not significant (other than in School 4), the conclusion is that in heterogeneous classroom instruction the differences in achievement are explained mainly by the initial differences.

Comparison between the present study and the ability-group study

The main difference between the results of this study and those of the same-ability-group study previously described (Cahan & Linchevski, 1996) is the near absence of any treatment effect in this mixed-ability-group study and the presence of a unidirectional treatment effect in all the schools in the previous same-ability-group study.

In the same-ability-group study the variance among the students at the end of the seventh grade was greater than would be expected according to the placement data in each of the schools. Moreover, the grouping effect increased over the years in all the schools. The grade of a student in a higher ability group was always higher than the grade he or she would have received if he or she had hypothetically been placed, with the same initial data, in the next lower ability group. Thus it seems that in a tracking system the achievements of students close to the cutoff points are largely dependent on their being arbitrarily assigned to a lower or higher group level. In the present study, in which the students attended heterogeneous classes, there was no significant effect in 10 of the schools at the end of the seventh grade. In the other two schools (Schools 4 and 6) there was a significant effect, and it was in the same direction as in the same-ability study. At the eighth-grade level, three of the cases showed no significant effect, whereas in the fourth, there was a significant difference but with a smaller absolute value than that found at the end of the seventh grade. One important difference between the two studies, however, is that in the same-ability-grouping study the students were tested at the end of 1 year and 3 years of grouping, whereas in the mixed-ability study they were tested at the end of 1 year and 2 years.

Study 2: Another Perspective

In Study 2 we compared the mathematical achievements (actual grades) of students placed in same-ability classes with those of students placed in mixed-ability classes to investigate our second research question: Which of the two systems—placement in heterogeneous classes as described earlier in the project description or placement in same-ability classes—leads to greater student achievement? Moreover, which system leads to greater achievements for the better students, the intermediate students, and the weaker students, when parallel levels in each of these systems are compared?

Our conjecture was that the achievements of lower and intermediate level students who learned in heterogeneous settings would be higher than those of students in parallel levels who learned in homogeneous settings. This outcome was expected because the students in the TAP classes had the advantage of participating in a rich learning environment that included legitimate peripheral participation (Lave & Wenger, 1991) and cognitive apprenticeship (Brown et al., 1989) via cooperative learning.

With respect to the highest level students, no differences were expected for two reasons: (a) In Gamoran's (1993) analysis, the gap found in the ability-grouping system emanates more from the weaker students' loss than from the stronger students' gain because of the qualitative differences in the students' learning environments; (b) the learning environments in our heterogeneous classes, experienced by all students and in particular the strongest ones, incorporate most of the positive factors encountered by the highest level ability groups in the tracking systems.

Design

In a junior high school not associated with TAP the mathematics faculty considered the possibility of joining the project because they had been quite unhappy with their instruction in the lower tracks. However, they considered same-ability grouping the only fair, effective way to deal with student heterogeneity. There was a conflict between this belief, on the one hand, and the project's stated benefits of learning in heterogeneous classes, on the other hand. After a long process of deliberation that included meetings, discussions, and reading some of the relevant research literature, the mathematics teachers decided to participate in a "real experiment" for which they obtained the parents' agreement. Thus, this school was selected for this study.

For the study we used a random experimental design, with the class as the unit of analysis. At the beginning of the seventh grade all the students were new to the school and were randomly assigned to four mixed-ability homeroom classes. (All content areas other than mathematics were taught, as in the past, in these homeroom settings.) Thereafter, using the same procedure the school had always used for tracking in mathematics, all students were assigned to one of three ability-group levels for mathematics. Two of the large mixed-ability homeroom classes were tracked into three smaller separate, same-ability mathematics classes according to the assignment procedure, and the other two homeroom classes studied mathematics in their original mixed-ability homeroom classes. Thus, there were five mathematics classes in all: two mixed-ability classes and three same-ability classes. In the two mixed-ability classes, the same tracking procedures were used to assign students to hypothetical ability groups. This hypothetical assignment was disclosed neither to the teachers nor to the students. Teachers were randomly assigned to the different mathematics classes, and teachers of both kinds of groups were involved in weekly workshops: the mixed-ability-group teachers' workshop was led by a TAP counselor, and the same-ability-group teachers' workshop was led by the school mathematics coordinator. Whereas the project workshop concentrated on discussions and activities appropriate for the heterogeneous classes, the same-ability workshop concentrated on discussions and activities appropriate for the same-ability levels.

Students remained in the groups for 2 years. At the end of the eighth grade, achievement tests were administered to all students. Two alternatives had been discussed: One was giving all students the same test regardless of their placement; the other alternative was writing three different tests for the three different ability-group levels, with students in the true same-ability classes and students of the equivalent hypothetical level in the mixed-ability classes being given the same test. This alternative was discussed to reduce anxiety of students who had learned in the lower same-ability classes and had been accustomed to tests specially prepared for their levels. In the end it was decided that both forms of testing would be used. The questions for these tests were proposed by the teachers and the mathematics coordinator, and the final versions were written by a representative of the Ministry of Education.

Results

Table 3 displays the average scores for each group level on the final achievement tests for the differential tests and for the common test. The data have been presented for same-ability groups and for mixed-ability groups. *T*-tests were used to compare, for each level, the achievements of the students in the mixed-ability and same-ability classes. The average scores of high-level students in the same-ability classes were higher (but not significantly) than those of the students in the mixed-ability classes on both versions. The average scores of intermediate- and low-level students in the same-ability grouping system were significantly lower than those in the mixed-ability system on both versions. Moreover, it seemed that most low-level students in the same-ability classes were unable to answer the questions on the common test inasmuch as they handed in almost empty test

Table 3
Achievements (Means in Percentages) in Mathematics at the End of 8th Grade

	Same-ability groups			Mixed-ability groups		
Tests	High	Intermediate	Low	High	Intermediate	Low
Differential test						
Mean	85	64	55	82	80*	78*
SD	7.8	5.6	6.2	7.5	4.3	5.1
n	33	27	14	35	26	15
Common test						
Mean	88	41	—[a]	85	65*	54
SD	8.1	5.1	—	6.9	6.1	3.9
n	33	27	14	35	26	15

[a] Because many of these students did not complete the test, we could not do a *t*-test, but because the mean would have been exceedingly low, we assume that it would be significantly different from 54, the score of the low-ability students in the mixed-ability group.
* $p < .05$ (significant *t*-test value between the same-ability mean and the mixed-ability mean).

papers, whereas the equivalent students in the heterogeneous classes scored an average of 54%. The students in the mixed-ability classes who had been hypothetically assigned to the intermediate and low tracks found the differential tests, written for the actual lower tracks, relatively easy; they were accustomed to much higher demands and expectations. Thus our hypotheses concerning all levels were confirmed.

An analysis of the test papers showed that the better students in the mixed-ability classes lost points for formal presentation and notation, such as the symbolic notation of truth set or the formal presentation of geometric proofs, whereas high-level students in the same-ability classes were quite competent in this area. The teachers of the mixed-ability classes confirmed that some symbolic notations and formal presentations, which traditionally constitute a major problem for average students, were given less attention in their classes.

PART II: TEACHERS' ATTITUDES

Another purpose of our research project was to examine the attitudes of the TAP teachers toward teaching in heterogeneous classes and the effects of workshops on these attitudes. Study 3 was designed for this purpose.

Past research (Chen & Addi, 1990; Chen & Goldring, 1994; Oakes, 1985) has shown that teachers have conflicting attitudes about ability grouping. Ideologically they favor diversity; practically they support ability grouping. Dar (1985) showed that teachers who work in heterogeneous settings within a school that actively supports and maintains such classes have a more positive attitude about the effectiveness of heterogeneity than their colleagues who teach in schools with a policy of tracking students into homogeneous classes. Thus, direct experience of teaching in a heterogeneous setting within a supportive framework might have a positive effect on teachers' perspectives about nontracking of students. Dar (1985) emphasized, however, that even these teachers still maintained a negative attitude toward teaching mathematics and English in heterogeneous classes. Schools with teachers who have succeeded in teaching effectively in heterogeneous classrooms were reported to have involved the teachers in the ideological development and implementation of a commitment to education in diverse classrooms (Wheelock, 1992). Professional training of the teachers is another factor crucial to successful implementation if tracking is to be eliminated (Gamoran, 1992). Thus, in planning the project-workshop guidelines we took into consideration the aspects mentioned above. All the project teachers participated in weekly workshops in which discussions evolved concerning the project's rationale and ideological basis. All project workshops were led by TAP project counselors who had participated in a special course developed for them. Thus all project teachers received approximately the same inservice training. The project teachers developed an awareness of the different needs of the students and were involved in constructing instruments, tools, and appropriate strategies for cooperative and differential teaching in their heterogeneous mathematics classes.

Preparation for differential teaching included gaining familiarity and proficiency with differential classroom strategies and class organization, preparing relevant activities, preparing alternative assessment tools, and the like. An equally important purpose of our workshop meetings was the opportunity given for discussing and sharing problems that had arisen in the teachers' classes that week. Frustrations, successes, and failures were all exposed and possible solutions were sought by the group; thus the teachers received essential support and practical solutions for their needs.

Study 3

In Study 3 we examined the attitudes of all project teachers who taught mathematics in heterogeneous settings while participating in the project, investigating the following research question: What are the attitudes of the teachers participating in the TAP project toward teaching mathematics in heterogeneous classes, and how do the project workshops affect these attitudes?

Design

The target population for this study was the group of all the project teachers, 58 teachers from demographically diverse regions. All answered a written questionnaire. Shortly thereafter individual oral interviews were conducted with 5 of these teachers. We defined (teacher) seniority as the number of school years the teacher had participated in the TAP project workshops. For example, if a teacher participated in both a seventh- and eighth-grade workshop during one school year, he or she accumulated two workshop-years. The teachers' seniority in the project varied from one to five workshop-years. Our conjecture was that positive attitudes toward teaching mathematics in heterogeneous settings would be directly related to the teachers' number of workshop-years. This outcome was expected because the project workshop guidelines had incorporated the suggestions emanating from the relevant research reported in this section.

The survey questionnaire contained 51 items. All items were constructed as statements, to be evaluated according to a 5-point Likert-type scale. Scores lower than 3 represented support for heterogeneity in terms of attitudes, workshop contribution, and absence of instructional difficulties. The questionnaire comprised four parts: The first part was subdivided into four topic factors and the second part into two topic factors; the third and fourth parts had only one topic factor each, for a total of eight topic factors.

1. *Teachers' attitudes toward children's learning in heterogeneous classes* included 14 items. This part of the questionnaire included four factors: (a) Factor 1: Affective Impact of Heterogeneity (e.g., "Heterogeneous classes rid weak pupils of feelings of inferiority"); (b) Factor 2: Cognitive Effects of Heterogeneity (e.g., "Studying in a heterogeneous class challenges the low-ability students"); (c) Factor 3: Equality of Educational Opportunity Selection (e.g.,

"Learning in ability groups increases the gap between the high- and low-ability students"); and (d) Factor 4: Reliability and Validity of Educational Selection (e.g., "It is possible to place students accurately into ability groups").

2. *Instructional difficulties in heterogeneous classes* included 12 items divided topically into two factors: (a) Factor 5: Solutions by Training meant instructional difficulties that could be resolved by appropriate training (e.g., "I lack knowledge of mathematical instructional methods needed to teach mathematics in a heterogeneous class") and (b) Factor 6: External Constraints meant instructional difficulties such as class size, inadequate teaching or learning materials, and so on (e.g., "It is impossible to teach mathematics in a heterogeneous class with 40 pupils").

3. *The importance of various items in differential teaching in heterogeneous classes* included 8 items related to Factor 7: Item Importance in Differential Teaching (e.g., "Pupils assisting each other in learning is important to me in differential teaching").

4. *The workshop's contribution to the teachers* included 17 items related to Factor 8: Workshop Contribution (e.g., "The workshop allows me to raise and solve instructional problems that come up in class").

Relevant items relating to teachers' attitudes toward children's learning in heterogeneous classes were adapted from an attitude inventory (Dar, 1985). The items relating to instructional difficulties in heterogeneous classes were in part extracted from a survey conducted by Chen, Kfir, and Addi (1990) and in part derived from interviews with teachers in heterogeneous mathematics classes. Items for Factors 7 and 8 were based on interviews with project counselors and former project teachers. The questionnaire was pilot tested among those teachers who had participated in the project but had since left the schools for various reasons. Internal reliability of the questionnaire was measured using Cronbach's coefficient alpha. Reliability coefficients ranged between 0.73 and 0.88. This result confirmed the validity of the distinction for the eight factors.

For statistical reasons emanating from the sample size, the teachers were grouped into three seniority groups: (a) Seniority 1 consisted of teachers with experience of one workshop-year, (b) Seniority 2 & 3 consisted of teachers with two or three workshop-years, and (c) Seniority 4+ consisted of teachers with four to seven workshop-years' experience.

Results

The measures of attitudes toward teaching in heterogeneous classes are shown in Figure 3. The scores showed, generally, that the attitudes of project teachers with more seniority were more positive toward student learning in a heterogeneous class than the attitudes of new project teachers.

Using analysis of variance, we compared the three seniority groups. The biggest difference found was with respect to cognitive effects on the students when learning in a heterogeneous class ($p = .004$, cognitive effects of heterogeneity, Factor 2, Figure 3). Whereas the novice project teachers had reservations ($\overline{x} = 3.16$), the

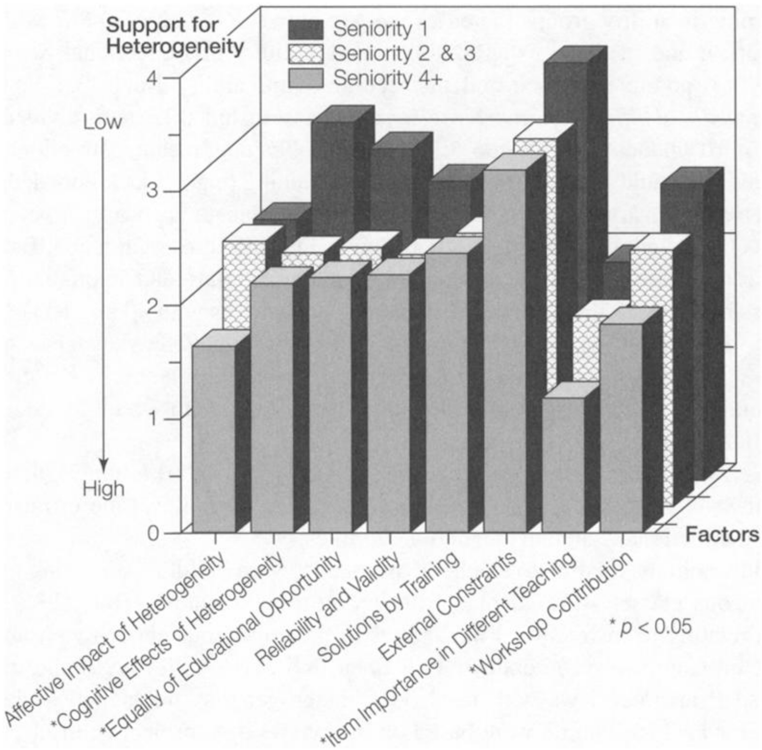

Figure 3. Support for heterogeneity according to teacher seniority groups (by questionnaire factors).

teachers with four or more years' seniority were more confident of positive effects of heterogenization ($\bar{x}$ = 2.16). These findings contradict Dar's (1985) results, which showed that even those teachers who were currently teaching in heterogeneous classes and had reservations favored ability grouping in mathematics (and in English). It might thus be assumed that participation in the workshop training program positively affected the teachers' attitudes.

Using the fact that scores lower than 3 represent support for heterogeneity, we can say that all three groups of project teachers agreed that heterogeneous grouping enhanced the weaker students' self-images and motivation (Factor 1) and all groups disagreed that the assignment of all students to a certain ability group was either reliable or valid (Factor 4). Significant differences (p = .03) appeared among the groups of teachers in their appraisals of the effect of equal educational opportunity on all students. Whereas the novice project teachers were but slightly positively inclined, the teachers with seniorities of 2 and 3 or 4+ were far more positive of the effects (Factor 3). All groups of project teachers agreed that they were capable of teaching mathematics in heterogeneous classes (Factor 5).

However, large classes, the lack of necessary equipment, and the curricular demands dictated by higher grade levels were difficulties with which all project teachers were unhappy (Factor 6). The findings presented in Figure 3 show that not all the problems were solved through the workshops. Teacher training-related problems were resolved; difficulties stemming from external constraints were not.

All items included as items important in differential teaching were found to be very important to all project teachers with significant differences among them ($p = .002$, Factor 7); the teachers with greater project seniority found these items of greater consequence to their lessons. Seemingly, experience in differential teaching and, perhaps, positive pupil feedback raise the teachers' consciousness of the usefulness of these items. A similar trend was found for items associated with the workshop contribution ($p = .034$, Factor 8); the project teachers of greater seniority felt that the workshop had better equipped them for the task of teaching in a heterogeneous class more so than did the novices. Thus, the more experienced project teachers seem to echo Gamoran's (1992) suggestion that professional training is essential for the successful implementation of teaching in heterogeneous (mathematics) classes.

The question arises whether these teachers feel capable of coping with the task of teaching mathematics in diverse classrooms without the continuous support of regular workshop meetings. To find out we examined the teachers' responses in the oral interviews. All the teachers felt the need for regular meetings in which lessons would be discussed and planned and common problems resolved, in effect emulating the format of the project workshop.

The picture that emerges is that continuous intercollegial support seems to be crucial for the success of implementing a program that requires fundamental changes in instructional methods.

CONCLUSIONS AND DISCUSSION

In this article we report on three studies of teaching mathematics in mixed-ability and same-ability settings and the effects of the settings on students' achievements and on teachers' attitudes. These studies took place within the framework of the TAP project. The reported results, to a great extent, support our conjectures.

We first examined the ways in which teaching in mixed-ability mathematics settings affects students' achievements. In Study 1 we investigated whether studying in mixed-ability classes would prevent formation of a gap (usually found when students are grouped by ability) between high- and low-ability students greater than that expected on the basis of the initial differences between them. In Study 2 we compared the effects of mixed-ability and same-ability grouping on the mathematics performance of students classified as having high ability, intermediate ability, and low ability. In Study 3 we examined how teaching in mixed-ability classes affects teachers' attitudes.

The results of Study 1 showed that after 1 year, in 10 of the 12 schools investigated, there was no significant change in the achievement differences among students of different ability levels as a result of using mixed-ability grouping. The 2 remaining schools showed a statistically significant increase in this gap after 1 year. By the end of 2 years the effects in both of these schools had been decreased; in only 1 school was the effect still significant. Thus we may conclude that within the TAP schools the added gap that is created in a tracking system was nearly nonexistent.

The results of Study 2 showed that placement of students in mixed-ability mathematics classes was not detrimental to their achievements when compared to achievements of students of similar ability levels who had learned in separate same-ability classes. On the contrary, the average and weaker students' achievements showed significant gains, whereas the loss in achievements of the stronger students was negligible. Demonstrating this result was one of the project developers' main goals.

By integrating the results of Studies 1 and 2, we might conclude that an increase in the gap, due to learning in the tracking system, emanates mainly from the loss in the weaker students' achievements instead of from the stronger students' gains. In Study 1 we have shown, generally, that in mixed-ability classes the gap did not increase nor were achievements significantly impaired. In Study 2 the comparison between the achievements of the mixed-ability students and their same-ability counterparts indicates that the achievements of the average and lower ability groups in the mixed-ability classes were higher. We may, then, conclude that in our case all levels progressed reasonably well. As such, we may infer that the increase in the gap due to learning in the same-ability classes emanates mainly from the loss for the students in the lower ability levels instead of from gain for the stronger ones. Better understanding of cognitive differences among students requires further investigation. By using alternative assessment tools that necessarily require a smaller research population, researchers can take a closer look at the students' thinking processes. Such alternative analyses may explain differences among students of different levels in the two systems.

In Study 3 we investigated the TAP-project teachers' attitudes. It must be remembered that the schools involved in the TAP project were not randomly sampled. Most principals were interested in the program; perhaps they felt increasingly skeptical toward ability grouping because of difficulties encountered in the lower ability levels or because they really believed that this program offered a chance to achieve equity. It does not necessarily follow that all participating teachers wanted to participate in this project. On the contrary, most of the teachers did not initially believe it was possible to successfully implement a mixed-ability mathematics program.

Study 3 results suggest that TAP-project participation had a positive effect on teachers' attitudes toward teaching in mixed-ability mathematics classes. Those teachers of higher project seniority consistently felt more positive than project newcomers about teaching in mixed-ability mathematics classes. They believed that they were capable of conducting mathematics classes in a manner that would not be detrimental to students at any ability level. They also felt confident that they had acquired tools to challenge all levels of students in a heterogeneous class. Our findings, in contrast with Dar's (1985) results, indicate that mathematics teachers can develop positive attitudes toward teaching in mixed-ability classes.

Thus we learn from the results of Study 3 that it is possible to teach mathematics in a heterogeneous setting to the satisfaction of the teachers involved. All the teachers felt that their success was to some extent dependent on continual support of a workshop type of framework, supporting Gamoran's (1992) suggestions for successfully implementing innovative programs.

Our studies indicate that it is possible for students of all ability levels to learn mathematics effectively in a heterogeneous class, to the satisfaction of the teacher.

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APPENDIX

Calculation of Effects

The effects of grouping and the initial differences on achievement in each school were calculated separately for each posttest. The overall grouping effect α_H was defined as equal to $\sum_{j=1}^{m-1} H_j$, in which m indicates the number of ability groups in the school and H_j is the effect of ability group j (see Figure 4). Similarly, the overall effect α_P of the initial differences between the students was defined as equal to $\sum_{j=1}^{m} P_j$, in which P_j is the effect of initial differences for ability group j (see Figure 4). The overall difference D (in SD units) between the strongest and the weakest students for each posttest is $D = \alpha_P - \alpha_H$.

The following two examples are taken from Table 1:

1. In School 1, $\alpha_P = 2.75$, $\alpha_H = -0.36$; therefore $D = 2.75 - (-0.36) = 3.11$. The latter result means that the actual difference between the strongest and the weakest students (3.11) was greater than the expected one (2.75). The added gap was 0.36.

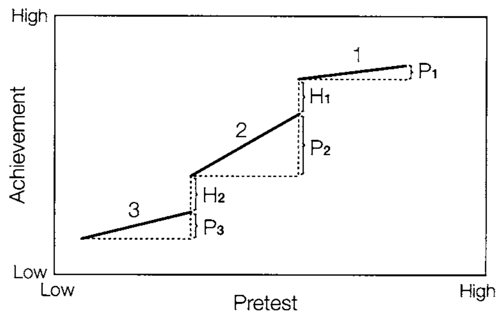

Figure 4. A hypothetical example of initial differences ($P_1 + P_2 + P_3$) and grouping effect ($H_2 + H_1$).

2. In School 11, $\alpha_P = 2.20$, $\alpha_H = 0.26$; therefore $D = 2.20 - 0.26 = 1.94$. The latter result means that the actual difference between the strongest and the weakest students (1.94) was smaller than the expected one (2.20). The reduced gap was 0.26.

The actual calculation of effects used the multiple regression equation of posttest scores on pretest scores and ability-group levels. In this design the regression coefficients of ability level and pretest equal the mean, across ability groups, of the effects of ability level and initial differences, respectively. Thus, the overall effect of grouping in each school is $(m - 1)\ \beta_H$, in which β_H is the regression coefficient of ability level. The overall effect of initial differences is $(X_{max} - X_{min})\ \beta_P$, in which β_P is the regression coefficient of the pretest score and $(X_{max} - X_{min})$ is the range of the pretest scores within the school (Cook & Campbell, 1979).

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