Malati

Mathematics learning and teaching initiative

Statistics

Probability 2

Grade 7

Teacher document

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Introduction to Probability Module 2

This module for Grade 7 develops on learners' use of systematic counting and basic understanding of chance established in Grades 4, 5 and 6 in Module 1. We recommend, therefore that teachers assess learners' understandings in these areas before proceeding with the activities in this Module. If necessary, the teacher can revisit the systematic counting activities and the "Likelihood Scale 1" in the Module 1.

We recommend the following diagnostic activities:

- *Systematic counting:* The teacher can use activities similar to those in Module 1, but change the context or numbers.
- *The notion of chance:* The teacher can use the diagnostic activity ("Diagnostic Activity 1") in Module 1 or a similar activity.

In this Module, methods for systematic representation are developed through the introduction of the grid ("*Throwing Dice*"). The likelihood scale is developed with the introduction of more descriptive terms, namely, "very likely" and "very unlikely" in the "*Likelihood Scale 2*". This activity also provides learners with an opportunity to revisit the ideas of the likelihood studied in the earlier grades. This understanding of the notion of chance is then developed into more formal probability using systematic representation as a cognitive tool to assist learners in calculating probabilities.

The Likelihood Scale 2

Mandisa's teacher gave the class this activity:

Impossible	Unlikely	50% chance	Likely	Certain

Sipho has a box containing coloured balls of the same size. Each time he closes his eyes and removes one ball from the bag.

Choose words from the scale above to describe the likelihood of each of these events:

1. There are three balls in the bag, a red ball, a blue ball and a green ball. Sipho removes a red ball.
2. There are six balls in the box, a black ball, a white ball, a red ball, a blue ball, a green ball and a yellow ball. Vusi removes a red ball .

Mandisa is confused. The members of her group say that both these events are "unlikely", but Mandisa says there is a difference between these events. In question 1 the red ball is one of three balls, but in question 2 the red ball is one of six balls. She wants to use different words to describe their likelihood because the one event is **more** likely than the other.

Do you agree with Mandisa? Which event is more likely?

Mandisa's teacher says this scale will help her:

Impossible	Very unlikely	Unlikely	50% chance	Likely	Very likely	Certain

Use this scale to describe the likelihood of the events in numbers 1 and 2.

In each row of this table, an **event** is described. Put a tick in the column which best describes the **likelihood** of each event.

Event	Impossible	Very unlikely	Unlikely	50% chance	Likely	Very likely	Certain
1. Paul throws a dice and a ‘4’ lands on top.							
2. Paul throws a dice and an even number lands on top.							
3. Paul throws a dice and a ‘1’ or ‘2’ lands on top.							
4. Jono drives his car from Cape Town to Johannesburg in two hours.							
5. Nomsa has ten balls in a box, 8 black balls and 1 white ball and 1 pink ball. She takes out a black ball when she puts her hand in the box without looking.							
6. 100 tickets are sold in a raffle. John takes two tickets and wins the raffle.							
7. There are 50 children in a class, 22 girls and 28 boys. The teacher writes each child’s name on a piece of paper and places it in a box. She closes her eyes and draws takes one name from the box. The paper she draws has a boy’s name on it.							
8. Nomsa has 4 balls in a box, 1 red, 1 blue, 1 green and 1 yellow. She takes out a green ball when she puts her hand in the box without looking.							

Teacher Notes: The Likelihood Scale 2

In the activity "The Likelihood Scale 1" learners were introduced to the mathematical use of the words "impossible", "unlikely", "50% chance", "likely" and "certain". The words "likely" and "unlikely" are, however, used to describe a range of events and do not give an indication of the relative likelihoods of each of these events. The words "very likely" and "very unlikely" are thus introduced. Learners are required to use the new words to describe the events given in the table and are provided with an opportunity to revisit the use of the words "impossible", '50% chance" and "certain".

The following responses should be expected:

1. *Since there are six possible ways the dice can land and each outcome is equally likely, it is "very unlikely" that the dice will land on a '4'.*
2. *Three out of the six numbers on the dice are even, so there is "50%chance" of the dice landing on an even number.*
3. *This is two out of the six possible ways the dice can land, so it is "unlikely" to get a '1' or a '2'.*
4. *It is "impossible' for Jono to cover this distance in two hours.*
5. *Since eight out of the ten balls are black, it is "very likely" that Nomsa will remove a black ball.*
6. *John bought only two out of the hundred raffle tickets, so it is "very unlikely" that he will win the raffle.*
7. *It is "likely" that a boy's name will be drawn.*
8. *There are four balls of different colours in the box – since the chance is 25% it is difficult to classify this as 'unlikely" or "very unlikely". If learners are concerned about the preciseness of the descriptive words, the teacher can ask them to place the event in position on the scale rather than describing the likelihood.*

Question 8 has been included in this activity as it shows the inadequacy of the descriptive terms in distinguishing the relative likelihoods of different events. Learners might also note that although the likelihoods in questions 1 and 6 can be described as "very unlikely", there is clearly a difference in likelihood of each of these events. It is for this reason that we introduce the notion of using rational numbers to describe probability more precisely. Some learners might do this automatically, for example, use percentages to refer to the probabilities. The use of rational numbers, however, can provide interference in the learning of probability and should be delayed until the teacher feels the learners have an understanding of rational numbers. As suggested in question 8, the teacher can, as an alternative to introducing the use of numbers, require that learners show the likelihoods on the scale, without having to name them precisely.

The President is holding a party for children at his house in Cape Town. He has invited sixteen pupils from Mountain Senior Primary.

1. Mr Shembe has to decide which pupils will go to the party. **He says that every pupil should have the same chance of being chosen.**

   He takes this list of pupils and says that five pupils from each grade can go.

GRADE	NO. OF PUPILS
Grade 4	48
Grade 5	52
Grade 6	43
Grade 7	40

   If you were a pupil at Mountain Senior Primary, would you be happy with the method Mr Shembe has used to choose pupils? Explain.

   If you do not agree, suggest a method that is fair.

2. Mr Jacobs, the Grade 5 teacher, has been told to choose 4 pupils from his class. He says his method for choosing pupils is fair: he places the 28 girls names in one box and the 24 boys names in another box and draws 2 names out of each box.

   Do you think this is a fair method? Explain.

   If you do not agree suggest a method which is fair.

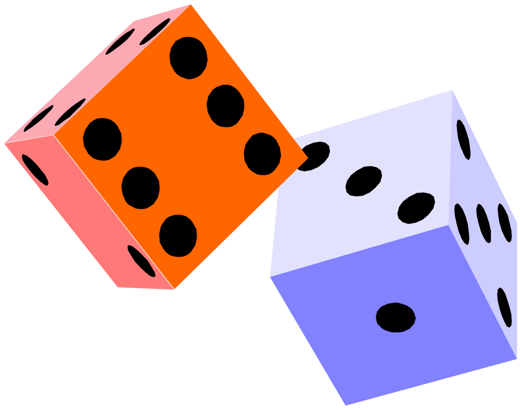
Teacher Notes: Party

Pupils are required to consider whether the decisions made have been fair, that is, whether all pupils have the same chance of being selected.

*In question 1 pupils should note that pupils in the smaller classes have a greater chance of being chosen. It would be better to place **all** the names in a box and select sixteen names.*

In question 2 Mr Jacobs' method favours the boys. It could be suggested that all the names be place in one box, or that the number drawn should be proportional to the total number in each box.

Throwing Dice



You throw two dice, a red dice and a blue dice. One possible result is that the red dice lands on ‘1’ and the blue dice on ‘4’.

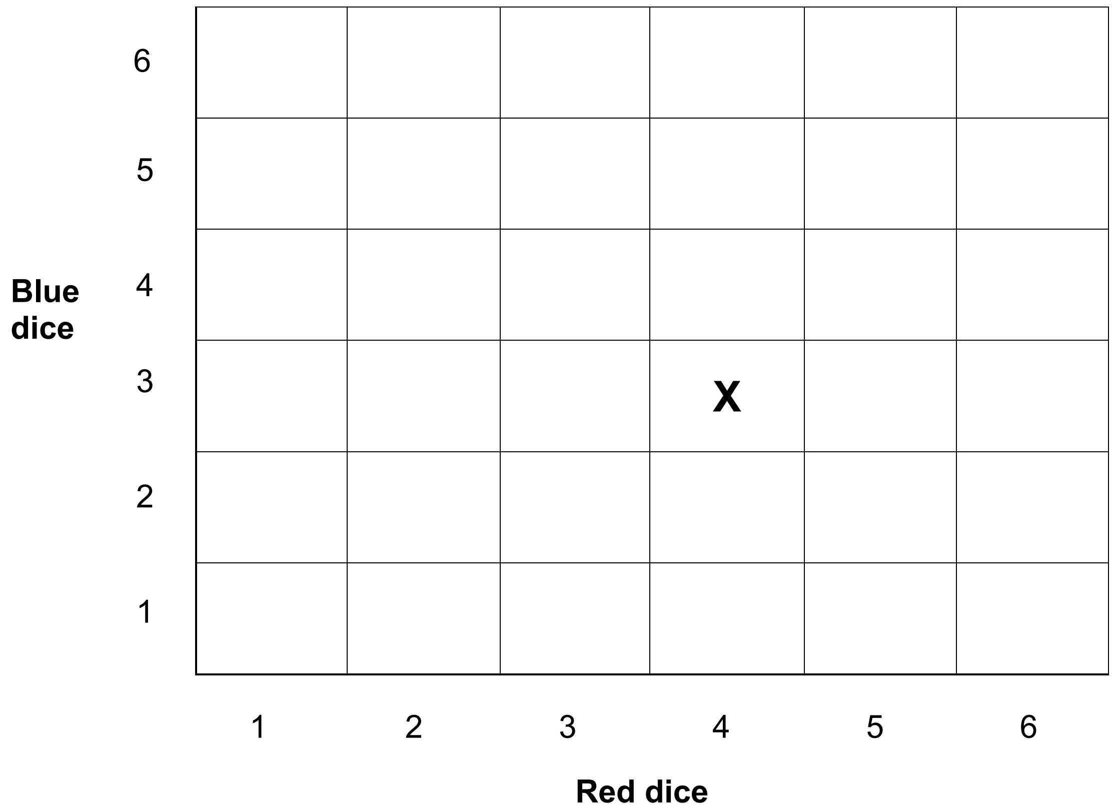
1. Use a tree diagram to show all the possible results. How many different results could you get?
2. We can also use a grid to represent the possible results of throwing two dice. On the grid below, colour in all the blocks which show that the blue dice has landed on a ‘3’.

Blue dice

6						
5						
4						
3						
2						
1						
	1	**2**	**3**	**4**	**5**	**6**

Red dice

3. The combination of a ‘4’ on the red dice and a ‘3’ on the blue dice is shown on the grid below:



(a) Show <u>another</u> point on the grid where a four and a three are thrown.

(b) In this case the sum of the numbers on the two dice is 7. What other combinations on the two dice will produce a sum of 7? Use **X** to show each combination on the grid. What do you notice?

(c) Is it possible for the sum of the two dices to be 1? Or 12? Or 13? Explain.

4. Here's another grid. Indicate with X on the grid all the blocks where the two dice produce the same numbers. What do you notice?

Blue dice							
	6						
	5						
	4						
	3						
	2						
	1						
		1	2	3	4	5	6
		Red dice					

5. Now use this grid to show all the possible outcomes (combinations) when two dice are thrown. How many different outcomes are there? Does this agree with your answer in question 1?

Blue dice							
	6						
	5						
	4						
	3						
	2						
	1						
		1	2	3	4	5	6
		Red dice					

6. Compare the tree diagram with the grid. What do you think are the advantages and disadvantages of each?

7. If you were throwing three dice, how many possible results would there be? Would you prefer to use a tree diagram or a grid to show all the possible results?

Teacher Notes: Throwing Dice

This activity provides more practice in using the tree diagram. Pupils should realise, and indeed by this stage suggest, that it is not necessary to complete all the branches of the tree diagram to count all the possible results.

Questions 2 to 5 give pupils more practice in using grids. They are given the opportunity to find various combinations of numbers on the grids and to see the pattern when they show given outcomes (Questions 3 and 4).
It should be clear to pupils that total scores (sums from the two dices) cannot be less than 2 or more than 12.

Question 6 highlights and compares the different ways of systematic counting. It is important that pupils reflect on this by discussing.

This activity also highlights (Question 7) to pupils that a tree diagram can be quite tedious in the case of more than two events, but that it is not possible to use a two-dimensional grid in this case.

Playing a Game with Coins

1. You have two coins, a 50c coin and a R1 coin. Show all the possible outcomes (results) of tossing these two coins.

2. Imagine you are playing a game with your friend. You each have a coin, and you toss your coins together. You repeat this again and again. The rules of the game are as follows:

• Player A scores a point if both players throw the *same* (in other words both throw heads or both throw tails). • Player B scores a point if his coin is *different* to Player A's once they've both tossed their coins. • The winner is the player with the most points.

(a) Which player would you want to be if you wanted to win this game? Explain.

(b) What is your chance of winning if you are Player A?
And if you are Player B? Discuss!

Teacher notes: Playing a Game with Coins

IT IS ESSENTIAL THAT THIS ACTIVITY IS COMPLETED BEFORE PUPILS ARE ALLOWED TO PLAY THE GAME*. Otherwise the mathematical value of the activity is limited. Experiments like this game are used to <u>check</u> the prediction.*

Pupils should be encouraged to read the problem carefully and explain it to each other before commencing.

The teacher can produce a coin if pupils are unfamiliar with 'heads' and 'tails'. The name allocated to each side should also be negotiated by the pupils themselves. If pupils cannot see that each of the four possible outcomes (HH, TT, HT, TH) is equally likely, they may also toss two coins a few times. These coins should preferably be different, to challenge the common belief that HT and TH are the same outcome. A tree diagram or grid can also illustrate the four different outcomes clearly. If pupils do not list all four of these outcomes they should be asked "How do you know that you have all the possible outcomes?"

Player A and Player B each have an equal chance of winning, namely 2 out of 4 or $\frac{2}{4}$ *or* $\frac{1}{2}$ *or 50% or 0,5. The terminology such as '****sample space****' (all the possible outcomes),* ***chance****,* ***equal chance*** *and the various representations of chance should be used casually by the teacher <u>after</u> the meanings/concepts have been constructed and negotiated by the pupils. The teacher can talk about '****predicting****' outcomes, and the fact that the experiment to follow will test these predictions.*

Now Let's Test Our Predictions!

In the activity "Playing a Game with Coins", we found that Player A and Player B have an equal chance of winning – thus each player's chance of winning is 50% or $\frac{1}{2}$.

We also say that the ***probability*** of winning is $\frac{1}{2}$.

1. Complete the following *prediction*:

   We predicted the winner would be ………………

2. Find a partner, and decide who is Player A and who is Player B. Toss a coin each. Repeat this until you have done it 10 times. Record your results in a logical way.

   (a) Who wins the game? Is this what you expected? Discuss!

   (b) After this game, what do player A's chance of winning look like? And Player B's chances of winning?

> The result of the game or experiment is sometimes expressed as a *success fraction*. For example, if Player A won 6 times in this game, his success fraction could be expressed as $\frac{6}{10}$, where 6 is the number of *successful outcomes* and 10 is the total number of *possible outcomes*.

   (c) If player A won 6 times out of 10 in your experiment, we say his *success fraction* would be $\frac{6}{10}$. Express both Player A's and Player B's chances of winning as success fractions.

3. Now toss the coins another 20 times.

   (a) Who wins the game now?

   (b) Express Player A and Player B's chances of winning after 30 throws as a success fraction.

4. Examine your answers to questions 1, 2 and 3:

   (a) Complete the following:

   Before we played I predicted Player B's chance of winning would be …
   After we tossed 10 times Player B's success fraction was …
   After we tossed 30 times Player B's success fraction was …

   (b) Discuss how your prediction (probability) and the results (success fractions) of your 10-toss and 30-toss games were the same or different. What do you notice?

   (c) Estimate the success fractions for Player A and Player B if you tossed the coins 100 times. Explain.

5. Explain in your own words what you think the words 'probability' and 'success fraction' mean. What is the relationship between them?

What we *think/predict* will happen, based on *theory*, is called ***probability***.
The result when we *play* the game or *do* the experiment, is called the ***success fraction***.
Experiments can often help us to **check** what we have predicted. When do you think experiments will be useful in checking a prediction?

Teacher Notes: Now Let's Test Our Predictions

The prediction in Question 1 is the prediction from the previous activity…the winner could be either Player A or Player B. The teacher might need to make this link explicit. When pupils play the game, they should be encouraged to record the results systematically e.g. using a table or tallies or a histogram, depending on their previous experience with such tools.

To save time, groups can be split into pairs and their results pooled. The teacher can also ask the pupils to complete the 30 throws for homework, using the left hand as Player A and the right hand as Player B, for example.

Once again, the terminology introduced in this activity is used casually - there is no emphasis on the correct use of terminology until Question 5. This is deliberate – it is more important for pupils to grasp the concepts than to associate the concept with the definitions. The terms are:
Estimated ***probability*** *- this is based on theoretical predictions.*
Success fraction *- this is based on the result of an experiment.*

Pupils should be encouraged to record their results as success fractions for each player. Each group will have different success fractions and it is unlikely that they will all be $\frac{1}{2}$ *after only 10 throws. After 30 throws they might be closer to* $\frac{1}{2}$*, but teachers might want to add the results from the various groups together to explore the effect of number of a trials on the success fractions - the pupils should be explicitly aware that the success fraction tends towards the predicted chance of success (probability) as the number of events increases.*

Probability is defined as:

$$\frac{\text{number of possible outcomes which satisfy a specific condition}}{\text{total number of possible outcomes}}$$

Pupils should grasp this concept, without memorising the definition. They should also be able to explain how this differs from the success fraction.

Teachers should also make it clear that it is not always possible to predict exactly.

Can You Help?

1. There are ten marbles in a box. 5 of the marbles are yellow and 5 are red. When doing the Malati activities Mark learnt that the probability of removing a red marble without looking is $\frac{5}{10}$ or $\frac{1}{2}$.

   Mark decided to test this at home by removing a marble from the box 10 times. He found that he removed a red marble 7 times, and a yellow marble 3 times.
   Mark is now very muddled. Can You Help?

   (a) Explain to Mark why his results do not agree with what he learnt at school.

   (b) What could you do to convince Mark that the probability of removing a red marble is $\frac{1}{2}$? Explain why you have chosen this method to convince him.

2. The Grade 7 class have been using coins and dice to learn about chance. They have learnt that the probability of throwing "heads" with a coin is $\frac{1}{2}$, that is, there is an equal chance of throwing "heads" and "tails".

   Thokozile, who is a soccer fan, is very muddled. He says that Bafana Bafana has won the toss in five out of the last six matches.

   (a) Can you help Thokozile?

   (b) Thokozile's teacher said he should do an experiment to find out what happens. He has tossed the coin and filled in the results in this table:

Total Number of Tosses	10	20	30	50	70	100
Number of "Heads"	2	6	13	[redacted]	[redacted]	[redacted]
Number of "Tails"	8	14	18	[redacted]	[redacted]	[redacted]

   Unfortunately Thokozile's younger sister has torn off the corner of the page and he can not remember the results of his experiment. Can you help him?

Teacher Notes: Can You Help?:

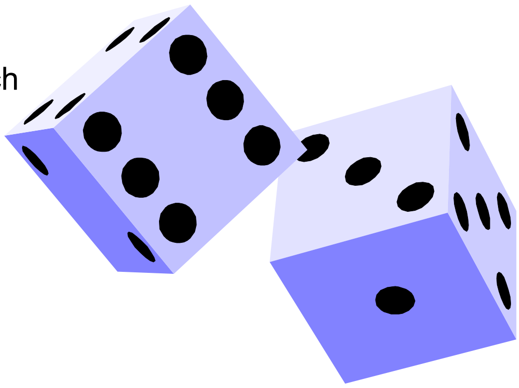
This activity is designed to reinforce the notions on theoretical and experimental probability raised in the activities "Playing a Game with Coins" and "Now Let's Test Our Predictions".

In question 1 pupils should note that the result of an experiment will not always agree with the theoretical probability. Mark should be advised to continue drawing marbles – as the number of balls drawn increases, so the success fraction will begin to approximate the probability.

Question 2 also raises the idea of using past history to give the probability. Pupils should note, however, that the tosses are independent of one another. In 2(b) pupils should choose numbers which indicate that the success fraction is tending towards the actual probability of $\frac{1}{2}$.

Now let's make some *predictions* about another game …

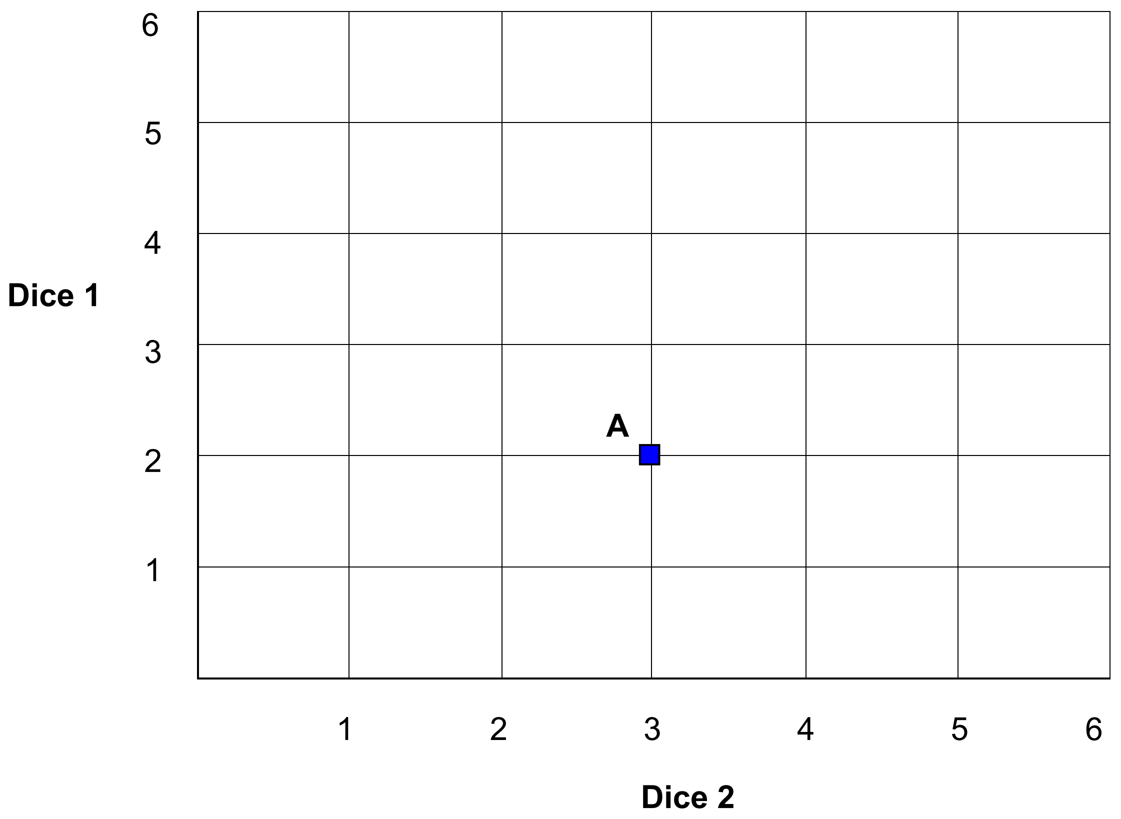
Playing a Game with Dice



This time you and your friend have two dice – each of you throws one of them, and then you add up the two scores.

Player A scores a point if the sum is **odd**.
Player B scores a point if the sum is **even**.

1. We can use a grid to represent all the possible results (outcomes) of throwing two dice. Use this grid to represent the results that would lead to Player A winning, and the results which would lead to Player B winning. One winning total for 'Player A' is already shown on the grid …



(a) Complete the grid.

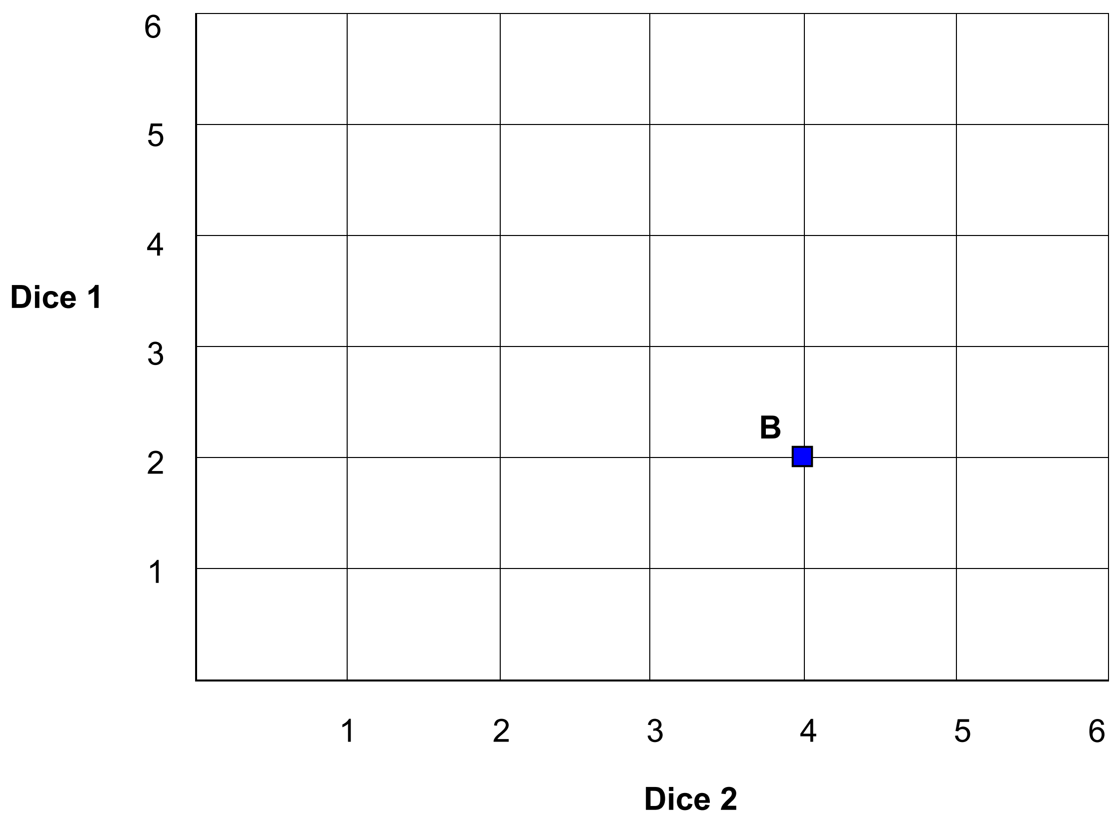
(b) Which player would you want to be if you wanted to win this game? Explain.

(c) What is your chance/probability of winning if you are Player A? And if you are Player B? Discuss.

2. Let's change the rules …

   Player A now gets a point if the *product* of the two scores (the two scores multiplied) is **odd**, and **Player B** gets a point if the product of the two scores is **even**.

   (a) Use this grid to represent the results that would lead to Player A winning, and the results which would lead to Player B winning. One winning total for Player B is already shown.



   (b) Is this a fair game now? Which player would you want to be to win this game? Explain.

   (c) What is your chance/probability of winning if you are Player A? And if you are Player B? Discuss.

3. Invent another game with dice which is *fair*, in other words in which their is *equal likelihood* of Player A and Player B winning? What would be the new rules of your game? How do you know that this is a fair game?

4. Now invent a game with dice in which Player B has no chance of winning …
   Player B’s winning must therefore be an *impossible outcome*. What would be the new rules of your game?

Teacher Notes: Playing a Game with Dice

This game is based on predictions only and dice should only be used if pupils are unfamiliar with the dice or if they do not understand that the chances of a dice landing on each side is equally likely. Pupils do NOT actually play these games.

It is essential that pupils read the rules of the game before commencing. One way of ensuring this is to ask pupils to explain these rules to each other.

In this activity it is necessary to record the outcomes of events systematically. Pupils are given the grid to do this, but they should be encouraged to also other systematic ways of representing the outcomes, if they feel more comfortable calculating probability in a particular way. It is, however, important that all pupils understand how the grid works. They should be encouraged to reflect on the grid in Question 1 and discuss why they think the "A" has been placed in that spot before completing it (it is also acceptable if they fill in the A or B in the blocks). The grid provides an important link with co-ordinate geometry. Please note that this grid is slightly different to the one used in the "Throwing a Dice' activity, in that each outcome is represented by a POINT. This is deliberate, and we would like teacher comments on whether pupils found this bridge to co-ordinate geometry too difficult or not.

Having completed the grid, the should count that there are an equal number of A's and B's. Both have and $\frac{18}{36}$ *or 50% chance of winning. Thus the players have an equal chance/likelihood of winning. In Question 2, the pupils will have to read the question carefully again. Having completed the grid, they will see that Player B has a greater chance of winning (*$\frac{27}{36}$ *as compared to Player A's* $\frac{9}{36}$*). It is thus not a fair game. The concepts possible/impossible events are introduced. Pupils should be encouraged to convince their friends that their games are suitable, by for example, using a grid to show possible outcomes. The teacher should encourage them to check whether their invented games are fair (Question 3) or give Player B 0% chance of winning (Question 4). Further activities for enrichment and consolidation (and/or homework) can be designed based on the pupils' own games, or on other games using, for example, the sum of the two numbers.*

Pupils should be encouraged to reflect on the use of the grid as a tool for working out probabilities.

<u>Source of ideas:</u>
Burns, M. (1992). ***About Teaching Mathematics: A K-8 Resource****. Math Solutions Publications.*

Another Game with Dice!

This time you have two dice, and you throw both of them. You really want to win this game. Which set of rules would you prefer?

1. You win if both the numbers thrown are even.
2. You win if at least one of the numbers thrown is even.
3. You win if only one of the numbers thrown is even.

What is the probability of each of these outcomes?

Ice-Cream Choices 2

Remember this ice-cream shop menu?

MENU

<u>Cones</u>

Plain Chocolate

<u>Flavours</u>

Vanilla Strawberry Fudge

<u>Toppings</u>

Nuts Jelly tots Banana

A customer can't make up his mind what ice-cream to have, so he closes his eyes and chooses a cone and a flavour by pointing blindly. The shop owner then presents the customer with an ice-cream with this cone and flavour.

You may remember that we represented all the possible choices using a *tree diagram*. Look carefully at the tree diagram you drew (or draw another one!), and answer the following questions (Hint: Count all the possible different ice-creams the man could order):

1. What is the chance (or *probability*) that he orders an ice-cream with a chocolate cone? Explain how you got your answer.

2. What is the probability that he orders a fudge-flavoured ice-cream? Explain how you got your answer.

3. What is the probability that he orders a plain cone with vanilla ice-cream? Explain how you got your answer.

Teacher Notes: Ice-cream Choices 2

The teacher should ensure that pupils have their tree diagrams in front of them to answer these questions, or the pupils should be asked to redraw the tree diagram. It is important that pupils use the tree diagram to answer the questions. The word 'probability' is used here, but pupils may need to discuss the meaning of this again. It refers to the number of outcomes/choices which satisfy a given condition over the total number of outcomes/choices.

This activity then helps pupils to use a tree diagram for finding probabilities, which most of them will do by simply counting branches of the tree diagram. It is a random choice because the customer is pointing blindly, so the probability in 1 is 1 out of 2 or 3 out of 6 or 9 out of 18 which all give $\frac{1}{2}$*. Pupils should share their various answers and should come to the conclusion that the answer is the same whether one counts ice-creams using the toppings and/or flavours or not. Similarly, the probability in 2 is 1 out of 3 or 6 out of 18 which both give* $\frac{1}{3}$*. If pupils count the ice-creams which satisfy the conditions in 3, their answer should be 1 out of 6 or 3 out of 18 which both give* $\frac{1}{6}$*.*

Assessment Activities

1. Nwabisa and Bonga are playing a game with two dice.
   They each throw a dice. Nwabisa gets a point if the number on her dice is bigger than the number on Bonga's dice. If the number on Nwabisa's dice is the same as or smaller than that on Bonga's dice, then Bonga gets the point.

   Is this a fair game? Explain.

2. Bongani has learnt at school that the probability of throwing heads when tossing a coin is 50%. He claims, however, that when he tossed a coin ten times he got three heads and seven tails.
   How would you these explain these unexpected results to Bongani?

3. Choose a game that you know that others play and discuss the role of chance in the game.
   You must explain how the game is played and why you chose the game for a discussion of chance.