Malati

Mathematics learning and teaching initiative

Decimal Fractions

Grades 6 and 7

Teacher Document

Malati staff involved in developing these materials:

Therine van Niekerk
Amanda le Roux
Karen Newstead
Bingo Lukhele

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1. Sharing Liquorice

1. (a) 11 strips of liquorice, all of the same length, are shared equally among 10 children. How much liquorice does each child get? Draw it. *(1 $\frac{1}{10}$)*
   (b) What would you punch on the calculator to do this problem? *(11 ÷ 10)*
   (c) Check the answer with your calculator. *(1,1)*

2. (a) Without using a calculator, work out how much liquorice each child gets if 6 strips of liquorice, all of the same length, are shared equally among 5 children. Draw what each child gets. *(1 $\frac{1}{5}$)*
   (b) Check the answer with your calculator. *(1,2)*
   (c) What do you think the number on the calculator means?

2. Sharing Liquorice II

1. (a) 23 strips of liquorice of the same length are shared equally among 10 children. How much liquorice does each child get? *(2 $\frac{3}{10}$)*
   (b) Check the answer with your calculator. *(2,3)*

2. (a) Without a calculator, work out how much liquorice each child gets if 27 strips of liquorice of the same length are shared equally among 10 children. *(2 $\frac{7}{10}$)*
   (b) Check your answer with your calculator *(2,7)*

3. (a) Without a calculator, work out how much liquorice each child gets if 12 strips of liquorice of the same length are shared equally among 5 children. *(2 $\frac{2}{5}$)*
   (b) Check your answer with your calculator *(2,4)*

4. (a) How is $2\frac{3}{10}$ shown on the calculator? *(2,3)*
   (b) How is $2\frac{2}{5}$ shown on the calculator? *(2,4)*
   (c) Which is bigger, $2\frac{3}{10}$ or $2\frac{2}{5}$? *(2 $\frac{2}{5}$)*

Teacher Notes:

The purpose of both these tasks is to enable children to assign meaning to decimals as an alternative notation for fractions, a concept that should be stable by now. Don't, however, expect the children to understand decimal fractions after this activity. This activity makes them aware of the existence of something like decimal fractions and it is enough if it encourages them to think about it. Very few children will be able to make sense of this on the first day.

Remind the children that the decimal point on the calculator is written as a decimal comma in South Africa! This is social knowledge that the children learn from others by listening, watching, reading and asking questions etc. The pocket calculator is used in a lot of the decimal fraction activities. We need to remember that effective use of the calculator has become a necessary life skill in our society. Effective and sensible teaching of decimal fractions should therefore include activities to prepare learners for using calculators effectively and interpreting calculator displays sensibly.

If the children ask what the calculator answers mean, the teacher should say that that is the way the calculator expresses (writes) fractions. The children **should not** be told the 'logic' of the decimal notation (i.e. that the digit after the comma means tenths etc.) at this stage. This will come later.

It is also important that both question 1 and question 2 in Sharing Liquorice 1 are done on the same day.

Understanding of equivalent fractions is necessary for children to find, for example, the decimal equivalent of $2\frac{2}{5}$. If the children merely regard '0,4' as an alternative notation of $\frac{2}{5}$ it is in order. It is **not necessary** that they grasp the place value (that 0,4 actually means 4 tenths) at this stage. It is maybe too early for this. This concept will stabilize in due time.

When the children check the answers with their calculators, they need to know, for example, that 11 strips shared equally among 10 children means $11 \div 10$. Question1b in Sharing Liquorice 1 addresses that and should be clarified by the children through discussion.

What learners may do:

- The learners should solve the problem on paper
- They should also do the calculation on the calculator

What learners may learn:

- It is very important that the children *experience* the calculator answers as a different notation for their own common fraction answers.
- That decimal fractions are merely another notation for fractions - This prevents the misconception that decimal fractions are 'different numbers'.
- That sharing is one way of viewing division.
- The meaning of a fraction as an operator.

3. Sharing Liquorice III

1. 27 strips of liquorice of the same length are shared equally among children at a party. For each of these cases write your answer as a common fraction and then check your answer with the calculator. Write the common fraction and the calculator answer in the table below:

Number of children	Amount of liquorice – written as a fraction	Amount of liquorice – calculator answer
2	$13\frac{1}{2}$	13,5
3	9	9
5	$5\frac{2}{5}$	5,4
8	$3\frac{3}{8}$	3,375
10	$2\frac{7}{10}$	2,7
12	$2\frac{3}{12}$	2,25
20	$1\frac{7}{20}$	1,35

2. Are the answers in the two columns different? Why?

Teacher Notes:

This task is to provide further experience with decimals.

The children should complete the table row by row, so that they can get the chance to reflect on the different notations. If they complete the table column by column, the whole idea of creating conflict, by reflecting on the different notations, is lost.

As in the previous task the children will only be able to use the calculator if they interpret equal sharing as division. Therefore $\frac{1}{20}$ of 27 = 27 ÷ 20 = $\frac{27}{20}$ = $\frac{1}{20} \times 27$

Teachers should make sure that there is enough discussion to ensure that learners reflect on the various notations.

What learners may do:

- The learners should work out the common fraction.
- They should also do the calculation on the calculator.
- Work column by column. Don't allow this - see above.

What learners may learn:

- The idea that decimal fractions are merely a different notation for fractions is enforced.

4. Liquorice Predictions

1. (a) 47 strips of liquorice are shared among 10 children. How much liquorice will each child get? (Find your answer without using a calculator) *(4$\frac{7}{10}$)*
   (b) What do you think the calculator's answer for 47 ÷ 10 will be? Write down your prediction.
   (c) Do 47 ÷ 10 on your calculator and check your prediction. *(4,7)*
2. (a) 47 strips of liquorice are shared among 5 children. How much liquorice will each child get? (Find your answer without using a calculator) *(9$\frac{2}{5}$)*
   (b) What do you think the calculator's answer for 47 ÷ 5 will be? Write down your prediction.
   (c) Do 47 ÷ 5 on your calculator and check your prediction. *(9,4)*
3. (a) 43 strips of liquorice are shared among 10 children. How much liquorice will each child get? (Find your answer without using a calculator) *(4$\frac{3}{10}$)*
   (b) What do you think the calculator's answer for 43 ÷ 10 will be? Write down your prediction.
   (c) Do 43 ÷ 10 on your calculator and check your prediction. *(4,3)*
4. (a) 35 strips of liquorice are shared among 10 children. How much liquorice will each child get? (Find your answer without using a calculator) *(3$\frac{5}{10}$ / 3$\frac{1}{2}$)*
   (b) What do you think the calculator's answer for 35 ÷ 10 will be? Write down your prediction.
   (c) Do 35 ÷ 10 on your calculator and check your prediction *(3,5)*
5. (a) 17 strips of liquorice are shared among 4 children. How much liquorice will each child get? (Find your answer without using a calculator) *(4$\frac{1}{4}$)*
   (b) What do you think the calculator's answer for 17 ÷ 4 will be? Write down your prediction.
   (c) Do 17 ÷ 4 on your calculator and check your prediction *(4,25)*

Teacher Notes:

The purpose of the predictions of the calculator answers is to force the children to think about the decimal notation. It doesn't matter if the predictions are poor, nor does it matter if they simply guess. Discussing and sharing thoughts are very important here. A lot of reflection is necessary for reconciling the two notations.

If the teacher watches carefully, this is a good diagnostic activity.

If you think that it will help to structure the children's thoughts, they can use a table to keep track of their answers. For instance:

Problem	Common fraction	Prediction	Calculator answer
1. 47 ÷ 10			

If they use a table, they must complete it row by row. (Same reasons as in Sharing Liquorice III)

In this activity the calculator is used as a feedback device and a teaching aid to introduce new concepts.

What learners may do:

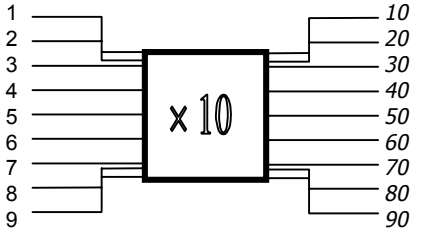
- They can predict, for example, that 2$\frac{1}{2}$ =2,2 or that 2$\frac{2}{5}$ =2,2 or 2,5. This activity is designed to identify those misconceptions before they become permanent.
- Think about decimals and the decimal notation.

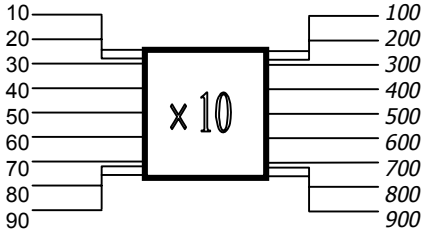
What learners may learn:

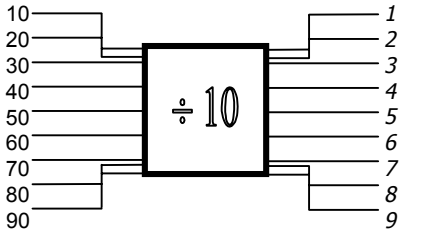
- That the remainder is not the same as the digit after the comma.
- That neither the denominator nor the numerator of the common fraction is the same as the digit after the comma.

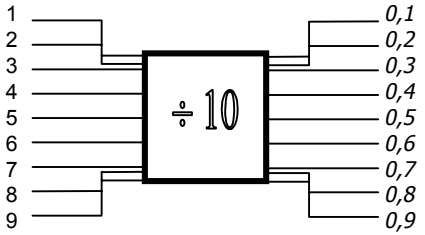
5. The Wonderful Number 10

1. Complete the following:









2. Complete the following tables:

Number	2	16	88	40	130	9	199	11	21
20 x Number	*40*	*320*	*1760*	*800*	*2600*	*180*	*3980*	*220*	*420*

Number	5	9	45	12	120	15	30	60	100
30 x Number	*150*	*270*	*1350*	*360*	*3600*	*450*	*900*	*1800*	*3000*

Number	6	13	36	9	110	11	5	17	29
700 x Number	*4200*	*9100*	*2520*	*6300*	*7700*	*7700*	*3500*	*11900*	*20300*

Number	4	10	25	38	19	17	47	320	100
4000 x Number	*16000*	*40000*	*100000*	*152000*	*76000*	*68000*	*188000*	*1280000*	*400000*

Number	221	10	100	44	56	180	29	88	712
100 x Number	*2210*	*1000*	*1000*	*4400*	*5600*	*1800*	*2900*	*8800*	*71200*

Teacher Notes:

What learners may do:

- If they understand the calculations they are doing, they may consciously multiply by the multiples of ten, for example 10 x 20 = 200 (and **not** just simply 1 x 2 = 2 followed by 'adding' of the zeros).
- They might however realize that they could get the answer by simply multiplying the numbers before the zeros and then adding the zeros. The teacher should **not** point this out to the learners. If they come up with it on their own, the teacher can ask them why they may do that.
- Learners are likely to have difficulty in formulating their thoughts for question 2. The teacher should not expect perfect and concise answers, but must ask for explanations. It is, however, important that the learners discuss this and reflect on what they were doing.

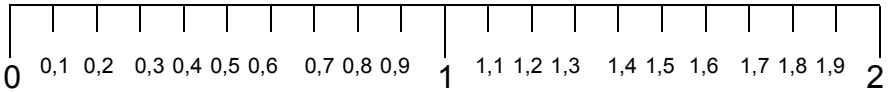
What learners may learn:

- The first and third questions are intended to develop comprehension and skills in multiplying with multiples of 10.
- Question two offers learners the opportunity to try to explain logically why it is easy to multiply by 10. We can also see this as a place value activity.

6. Estimates

Here are two rulers with which we are going to measure in the following activity. Cut them out neatly and keep them safe:

RULER 1



RULER 2

For each of the lines below:
Use ruler 1 to measure the line (estimate the length to at least one decimal place). Write your answer in column 2 of the table.
Now measure the line with ruler 2. Write your answer in column 3 of the table. (How close was your estimate?)

LINES:

A
B
C
D
E
F
G
H

Line	Reading on ruler 1	Reading on ruler 2
A		1
B		2
C		0,3
D		0,6
E		0,1
F		1,7
G		1,3
H		0,2

Which of the rulers would you prefer to use? Why?

Teacher Notes:

Again it is important **not** to let the children complete the table by completing column by column. This defeats the whole purpose of the activity. The use of ruler 2 is a way of checking the estimate of ruler 1, which should help children to improve their estimation skills.

Comparing the reading of ruler 2 to the reading of ruler 1, gives learners an opportunity to reflect on the ruler 1 reading. The learners should therefore complete the table row by row.

Teachers should show the children how to cut out the rulers, as it is very important that they must be able to use the rulers.

The teacher should also make sure that the children understand the terminology of one decimal place. This should not be introduced in terms of tenths, hundredths etc. The teacher can rather tell the children that this refers to one number after the comma.

Before they start measuring, it is important to tell the learners that ruler 2 has been constructed by subdividing each part of ruler 1 into 10 **equal** parts.

What learners may do:

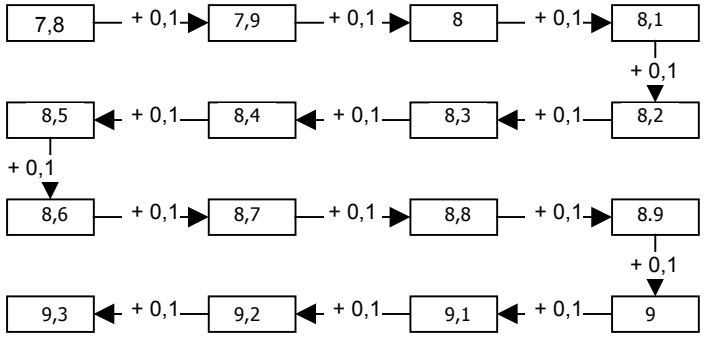
- Learners should measure each line with the two rulers in turn.

What learners may learn:

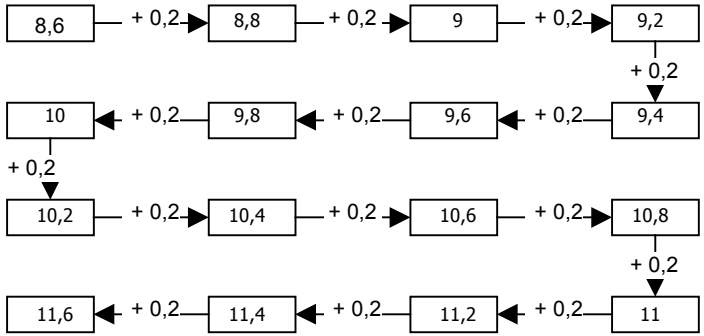
- To read scales which are calibrated in units and tenths.
- To estimate readings on a scale.

7. Snakes with Decimals

1. Complete the following diagram:



2. Calculate each of the following without using the multiplication button on your calculator:
   (a) 3 x 0,1 = 0,3 (b) 6 x 0,1 = 0,6 (c) 10 x 0,1 = 1
   (d) 17 x 0,1 = 1,7 (e) 20 x 0,1 = 2 (f) 35 x 0,1 = 3,5
3. Write 0,1 as a common fraction $\frac{1}{10}$
4. Complete the following diagram:



5. Calculate each of the following without using the division button on your calculator:
   (a) 1 ÷ 0,2 = 5 (b) 2 ÷ 0,2 = 10 (c) 2,6 ÷ 0,2 = 13
   (d) 5 ÷ 0,2 = 25 (e) 10 ÷ 0,2 = 50 (f) 54 ÷ 0,2 = 270
6. Write 0,2 as a common fraction $\frac{2}{10} = \frac{1}{5}$

Teacher Notes:

The learners can use calculators to help them calculate the answers in question 1 and question 4. It would be good if the teacher can help the learners to programme the calculators to count in 0,1's. Different calculators need to be programmed differently. Eg. 0,1 [+] [=] or 0,1 [+] [+] [=]. Let the learners play with this for a while, so that they can get used to how their calculator works, before they start with the activity.
This activity can be used as a diagnostic activity and to remedy misconceptions. If the teacher finds that some of the learners still do not understand, more 'snakes' can be given (see clean template).
If the learners can not complete the whole activity in one period, questions 4 - 6 can be done on the next day as a new activity.

What learners might do:

- There are two ways in which calculators can be used as an operator. The first method is *say first then press*. This is used for calculations that the learners can do already. In this method the calculator gives immediate feedback. The learner knows that his answer is right/wrong and can act accordingly. The other method is *press first, think later.* This method is for calculations that the children can't yet do well. The learners can then reflect on the answer that they get from the calculator. The intention is that they use the first method (*say first then press*) in question 1. If the learners are, however, not familiar enough with decimal fractions, the second method can be used to improve their knowledge of decimals.
- In question 2 the intention is not to formalize multiplication with decimals, but rather to let the learners interpret the multiplication of a decimal with a whole number as repeated addition. This may lead to learners counting 0,8 ; 0,9 ; 0,10 etc. This can be resolved by letting them add on the calculators and reflecting on the answers.
- If children understand the grouping meaning of division, question 5 might be interpreted as "how many 0,2's are there in 1?". They might answer this by counting in 0,2's and keeping track of the number of 0,2's using the calculator or the 'snake' they completed in question 4.

What learners may learn:

- Develop their number concept of decimal fractions.
- See multiplication as repeated addition.
- The relationship between common fractions and decimal fractions.
- See division as: 'how many do I need to make up …'

8. The Wonderful Number 10 (Cont.)

1. Complete the following table:

Number	800	50	6	77	64	3,6	4,8	1,23	$\frac{7}{10}$
10 x Number	8000	5000	60	770	640	36	48	12,3	7

2. (a) Why, do you think, is 10 x 345 = 3450
   (b) Why, do you think, is 100 x 345 = 34500
   (c) Why, do you think, is 10 x 3,4 = 34

3. Complete the following table:

Number	200	160	800	40	600	90	190	7	11,3
Number ÷ 10	20	160	8	4	600	9	190	0,7	1,13
Number ÷ 100	2	16	0,8	0,4	60	0,9	19	0,07	0,11

Teacher Notes:

This activity uses the same concepts as The Wonderful Number 10 (I).

What learners may do:

- If they understand the calculations they are doing, they may consciously multiply by the multiples of ten, for example 10 x 20 = 200 (and **not** just simply 1 x 2 = 2 followed by 'adding' of the zeros).
- They might however realize that they could get the answer by simply multiplying the numbers before the zeros and then adding the zeros. The teacher should **not** point this out to the learners. If they come up with it on their own, the teacher can ask them why they may do that.
- Learners may begin to formulate a rule about moving the commas in the case of multiplying (or dividing) a decimal by 10. Once again, should **not** point this out to the learners.
- However, if learners incorrectly formulate a rule that one simply 'removes the comma', they will reach the conclusion that $1{,}23 \times 10 = 123$. The teacher can challenge this by asking whether they think this is a reasonable answer.
- Learners may obtain incorrect answers like $\frac{70}{100}$ or $\frac{7}{100}$ for $\frac{7}{10} \times 10$. Once again, the teacher should ask them to reflect on what a sensible answer might be, taking into account the meaning of $\frac{7}{10}$.
- Learners are likely to have difficulty in formulating their thoughts for question 2. The teacher should not expect perfect and concise answers, but must ask for explanations. It is not an acceptable response to simply say that 'when you multiply by 10 you add a zero'. It is important that the learners also discuss and reflect on why this is the case. They should be encouraged not to use the word 'add' in this context, as it is incorrect.

What learners may learn:

- The first and third questions are intended to develop comprehension and skills in multiplying with multiples of 10.
- Question 2 offers learners the opportunity to try to explain logically why it is easy to multiply by 10. We can also see this as a place value activity.
- Hundredths are introduced informally for the first time. If it is clear that learners have no understanding of what 1,23 means, the teacher should tell them that the two places after the decimal comma signify how many hundredths there are.

9. More Calculators

1. Fill all your answers in on the table below:
   (a) 5 friends share 11 bars of chocolate equally. How much chocolate does each of the five friends get? What does the calculator say?
   (b) How much chocolate will two of the friends have together? What does the calculator say?
   (c) How much chocolate will three of the friends have together? What does the calculator say?
   (d) How much chocolate will four of the friends have together? What does the calculator say?
   (e) How much chocolate will they all have together? What does the calculator say?

	How much chocolate – as a fraction	How much chocolate – calculator answer
Each friend	$2\frac{1}{5}$	2,2
Two friends	$4\frac{2}{5}$	4,4
Three friends	$6\frac{3}{5}$	6,6
Four friends	$8\frac{4}{5}$	8,8
Five friends	11	11

2. Draw your own table (like the one above) to write the following answers as common fractions and calculator answers:
   (a) 10 friends share 21 chocolate bars equally among themselves. How much chocolate does each child get? ($2\frac{1}{10}$ / 2,1)
   (b) How much chocolate will two of the friends have all together? ($4\frac{2}{10}$ / 4,2)
   (c) How much chocolate will three of the friends have all together? ($6\frac{3}{10}$ / 6,3)
   (d) How much chocolate will four of the friends have all together? ($8\frac{4}{10}$ / 8,4)
   (e) How much chocolate will five of the friends have all together? ($10\frac{5}{10}$ / 10,5)
   (f) How much chocolate will six of the friends have all together? ($12\frac{6}{10}$ / 12,6)
   (g) How much chocolate will seven of the friends have all together? ($14\frac{7}{10}$ / 14,7)
   (h) How much chocolate will eight of the friends have all together? ($16\frac{8}{10}$ / 16,8)
   (i) How much chocolate will nine of the friends have all together? ($18\frac{9}{10}$ / 18,9)
   (j) How much chocolate will all ten of them have together?(21)

Teacher Notes:

Again the children are forced to think about decimal notation. They are also in a situation where they can compare decimal fractions and common fractions and should be encouraged to do so.

What learners may do:

- Add incorrectly by 'forgetting' about the whole numbers or by adding the fractions incorrectly e.g. $\frac{1}{5} + \frac{1}{5} = \frac{1}{10}$
- Finish the one column and then start on the other one – **Do not** allow them to do this. One of the purposes of this activity is that the learners should compare the decimal fraction to the common fraction. This is not achieved if they complete the columns separately.

What learners may learn:

- Adding fractions (common fractions and decimal fractions)
- Getting the feeling of the 'muchness' of decimal fractions

10. Number Patterns

Use a pen to complete the following patterns. Add the first number repeatedly. Check your answers with a calculator after each patterns. If you find a mistake, write down why you think it occurred.

❶ 0,2 ; 0,4 ; 0,6 ; 0,8 ; 1,0 ; 1,2 ; 1,4 ; 1,6 ; 1,8 ; 2,0 ; 2,2 ; 2,4 ; 2,6 ; 2,8 ; 3,0 ; 3,2 ; 3,4 ; 3,6 ; 3,8 ; 4,0 ; 4,2 ; 4,4 ; 4,6

❷ 0,3 ; 0,6 ; 0,9 ; 1,2 ; 1,5 ; 1,8 ; 2,1 ; 2,4 ; 2,7 ; 3,0 ; 3,3 ; 3,6 ; 3,9 ; 4,2 ; 4,5 ; 4,8 ; 5,1 ; 5,4 ; 5,7 ; 6,0 ; 6,3 ; 6,6 ; 6,9

❸ 0,4 ; 0,8 ; 1,2 ; 1,6 ; 2 ; 2,4 ; 2,8 ; 3,2 ; 3,6 ; 4 ; 4,4 ; 4,8 ; 5,2 ; 5,6 ; 6 ; 6,4 ; 6,8 ; 7,2 ; 7,6 ; 8 ; 8,4 ; 8,8 ; 9,2

❹ 0,5 ; 1 ; 1,5 ; 2 ; 2,5 ; 3 ; 3,5 ; 4 ; 4,5 ; 5 ; 5,5 ; 6 ; 6,5 ; 7 ; 7,5 ; 8 ; 8,5 ; 9 ; 9,5 ; 10 ; 10,5 ; 11

❺ 0,6 ; 1,2 ; 1,8 ; 2,4 ; 3 ; 3,6 ; 4,2 ; 4,8 ; 5,4 ; 6 ; 6,6 ; 7,2 ; 7,8 ; 8,4 ; 9 ; 9,6 ; 10,2 ; 10,8 ; 11, 4 ; 12 ; 12,6 ; 13,2

Teacher Notes:

This activity should first be done by hand and checked **afterwards,** with a calculator. Again the calculator can be programmed to do this. (see 'Snakes with Decimals')

What learners may do:

- By checking their answers with the calculator, they might discover their own misconceptions and try to resolve them. The teacher should discourage them from simply copying the calculator answers.
- They might want to do all the patterns before checking on the calculator. Don't allow this. Let them talk about the first pattern and the answers they found before they carry on. The calculator will help to give immediate and correct feedback.

What learners may learn:

- That if they write about their own mistakes, it can help them to resolve misconceptions very early. Common misconceptions include treating the decimal point as if it were merely a point between whole numbers: 2,8 + 0,2 = 2,10
- Correct verbalization of decimals. Something like nought point ten doesn't exist. It is the same as nought point one and therefore smaller than nought point eight.
- Number concept of decimal fractions.

TO NEXT