# Reconceptualising the Teaching and Learning of Decimal Fractions

The teaching and learning of decimal fractions has been well researched, for example for example, Behr, Lesch, Post and Silver (1983), Brekke (1996), Davis (1991), Markovitz and Sowder (1991), Mitchell (1991), Sowder (1997) and Swan (undated).

The following issues were highlighted:

- Common misconceptions that learners harbour about decimal fractions, their underlying reasons and how they can be prevented and corrected.
- Which concepts and skills learners should already have developed to enable them to make sense of decimal fractions, and how lack of these give rise to the misconceptions mentioned above.
- What needs to be understood about decimal fractions and how learning programmes which limit learners' experiences not only prevent them from constructing a full understanding of decimal fractions, but also lead to misconceptions.
- The need for diagnostic teaching, therefore assessment instruments that enables the teacher to identify problem areas and most important, the reasons behind the problems.

The MALATI approach for the development of the concept of decimal fractions is based to a large extent on Swan's calculator-based approach.

## Prerequisite knowledge

In order to grasp the concept of decimal fractions, we believe learners need to have developed:

- a sufficient whole-number concept
- a stable concept of a common fraction
- a good understanding of equivalent fractions

We would therefore suggest that the teacher delays the introduction of decimal fractions until learners have been exposed to Phase 1 and at least half of Phase 2 of the MALATI common fractions materials.

## Introduction to decimal fractions

In the MALATI materials, decimal fractions are introduced using sharing problems, and the calculator is used to introduce decimals as an alternative *notation* for known common fractions:

## SHARING LIQUORICE (Worksheet 1)

1. (a) 11 strips of liquorice, all of the same length, are shared equally among 10 children. How much liquorice does each child get? Draw it.

   (b) What would you punch on the calculator to do this problem?

   (c) Check your answer with your calculator.

**Aspects of understanding – what needs to be understood?**

The MALATI approach addresses the following outcomes:

1. *Building a number concept for decimal fractions*

   - Developing the numerosities of decimal fractions. Learners should develop a feel for the size of decimal fractions in order to ensure a meaningful concept of decimal fractions. This is achieved with number concept development activities in the same way as in our common fractions approach. On the next page is an example of such a number concept activity:

## SNAKES WITH DECIMALS (Worksheet 7)

1. Complete the following diagram:
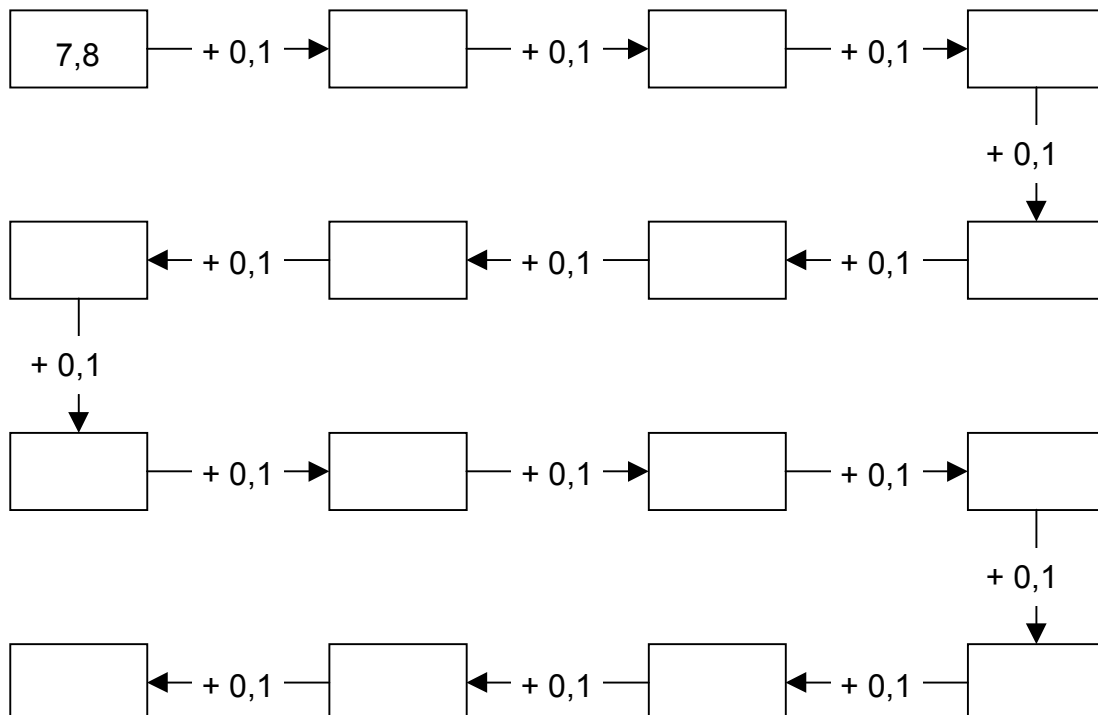
2. Calculate each of the following without using the multiplication button on your calculator.

(a) 3 x 0,1 =

(b) 6 x 0,1 =

(c) 10 x 0,1 =

(d)17 x 0,1 =

(e) 20 x 0,1 =

(f) 35 x 0,1 =

- Developing an understanding of the place value of digits in decimal fractions.
- Developing an understanding of why zero has to be used as a place holder. For example

## WHICH ONE IS THE BIGGEST? (Worksheet 12)

2. Sometimes we can take the zero away and it does not change the size of the number. At other times, we cannot take the zero away or the number will change.

In each case say whether or not we can take the zero away without changing the size of the number, and why?
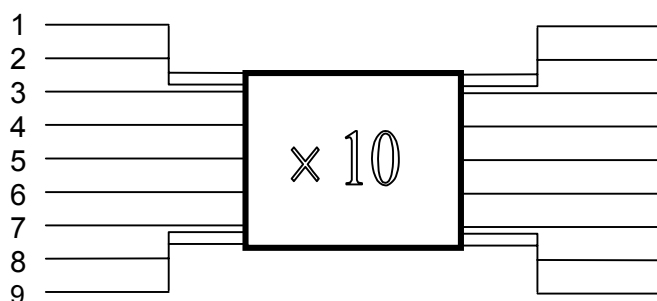
(a) 1,04

(d)   2,055

(b) 3,480

(e)   8,80

(c)0,42

- Beginning to explore the effect of multiplying and dividing numbers by powers of 10 and 100. This is also very important for number concept development. For example:

## THE WONDERFUL NUMBER 10 (Worksheet 5)

1. Complete the following:



*2. Linking common fractions and decimal fractions*

Learners should get the opportunity to experience decimal fractions as just another notation that can be used to express fractions. For example:

## NUMBER PATTERNS II (Worksheet 11)

1. In each case, start with the given number and do the operation in brackets at least 10 times.

   Then do the same pattern with corresponding common fractions. Check that your answers are the same.

   E.g.   $0,8 (+ 0,2) \rightarrow 1,0 + 0,2 \rightarrow 1,2 + 0,2 \rightarrow 1,4 + 0,2 \rightarrow$ ….

   $$\frac{8}{10} (+ \frac{2}{10}) \rightarrow 1 + \frac{2}{10} \rightarrow 1\frac{2}{10} + \frac{2}{10} \rightarrow 1\frac{4}{10} + \frac{2}{10} \rightarrow$$ ….

❶

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $6,4 (+ 0,3) \rightarrow 6,7 \rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | |
| $6\frac{4}{10}(+\frac{3}{10}) \rightarrow 6\frac{7}{10} \rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | |

*3. Estimating, measuring and scale reading*

As learners use the calculator, estimation becomes more and more important in order to be able to check their own calculations. Measuring and scale readings are very common everyday activities where children will come across decimal fractions. For example:

- Elementary use of the number line to three decimal places. See for example the activity on the next page on the use of the number line:

## DECIMAL FRACTIONS AND THE NUMBER LINE (Worksheet 16)

1. Counting in 0,2's. Complete the number line:
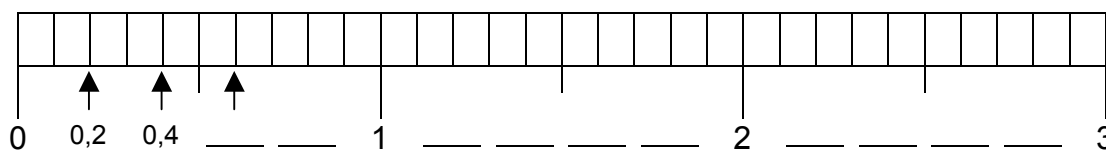


   (a) How many 0,2's in one whole?  _____

   (b) What common fraction is 0,2 therefore?  _____

- Developing a 'feel' for the size of units (cm, gram etc). For example:

## PAPER (Worksheet 13)

How thick do you think one sheet of paper is? Can you measure it with your ruler? Dumisani has a bright idea. He measures 100 sheets of paper. The stack is 14 mm thick.

1. Calculate how thick each sheet of paper is.

2. How thick will a document of 7 pages be?

3. If 245 copies of this document are printed and stacked on top of one another, how high will the stack be?

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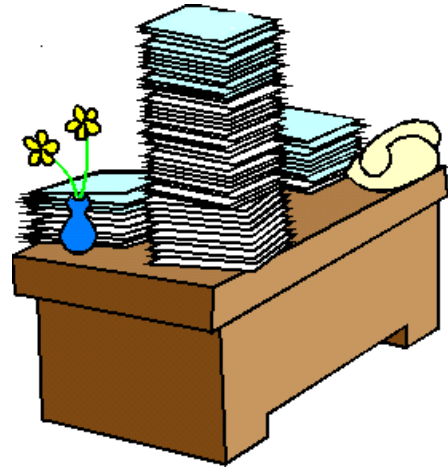
- Reading scales marked in tenths, fifths and twentieths. For example:

## SCALE READINGS (Worksheet 20)

1. Give the readings on the scales. Write your answers in the box next to the scale:

(a)

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0                    1                    2
|||||||||||||||||||||||
   ↑      ↑      ↑    ↑ ↑
   A      B      C    D E
```
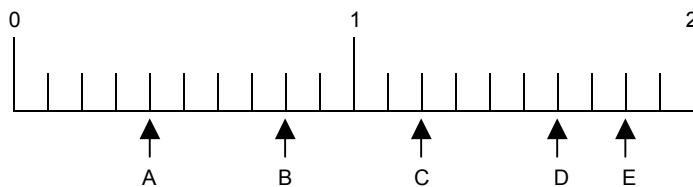
A = _____

B = _____

C = _____

D = _____

E = _____

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- Understanding the 'ten-ness' of decimals.
- Measuring activities in which learners experience the need for decimal fractions in order to be able to express the length of an object:

## MEASURING (Worksheet 18)

Here are two rulers with which we are going to measure in the following activity. Cut them out neatly and keep them safe:

**RULER 1**

| 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 | 1,7 | 1,8 | 1,9 | 2 |

**RULER 2**

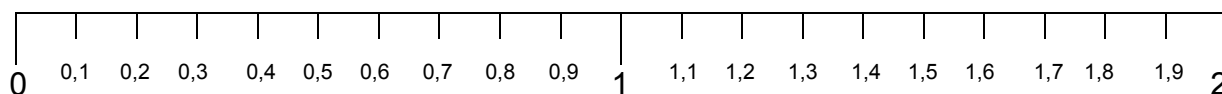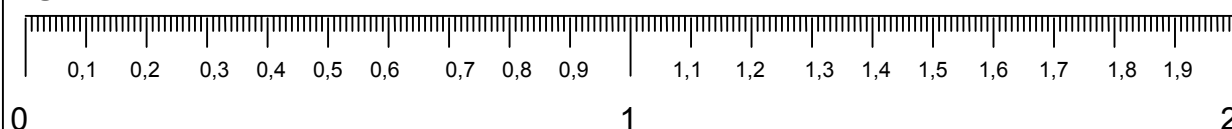| 0 | 0,1 | 0,2 | 0,3 | 0,4 | 0,5 | 0,6 | 0,7 | 0,8 | 0,9 | 1 | 1,1 | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 | 1,7 | 1,8 | 1,9 | 2 |

For each of the given lines:
Use ruler 1 to measure the line (estimate the length to at least two decimal places). Write your answer in column 2 of the table.
Now measure the line with ruler 2. Write your answer in the given column. (How close was your estimate?)

4. *Visualising a number line*

- Correct verbalisation of decimal fractions (for example, see Teacher Notes from Worksheet 10).

- Prevention/correction of the common misconception that a number like 3,78 is just two natural numbers separated by a comma. Unless this is carefully monitored in the classroom, learners very easily attach their whole-number schemes to decimal fractions.

- Developing a feel for the 'ten-ness' of decimals.

5. *Comparing decimals*

- Comparing decimals of different sizes.

- Correction of the misconceptions that 'longer'/'shorter' numbers are always bigger/smaller:

## WHICH ONE IS BIGGEST? (Worksheet 12)

1. In each case, say which decimal fraction you think is biggest and why?

(a) 0,03 or 0,3          (c)  3,5 or 3,412          (e)  0,76 or 0,760

(b) 5,31 or 5,13          (d)  4,09 or 4,1          (f)   0,89 or 0,089

6. *Addition and subtraction of decimal numbers*

- Performing simple calculations with decimal fractions. For example, repeated addition or subtraction as seen below:

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## NUMBER PATTERNS (Worksheet 23 & 24)

Use a pen to complete the following patterns. Add the first number repeatedly. Check your answers with a calculator after each sequence. If you find a mistake write down why you think if occurred.

❶   0,25 ; _____; _____; _____; _____; _____; _____; _____; _____; _____;

_____; _____; _____; _____; _____; _____; _____; _____; _____; _____; _____

❼ 2,67 (– 0,09) →

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- Linking addition and subtraction of decimal fractions to addition and subtraction of common fractions. See also for example Worksheet 11 (Number Patterns II) where the link between decimal and common fractions is made explicit.

7. *Understanding the meaning of multiplication and division*

- Learners should have a good number concept of decimal fractions after this package, which should enable them to make good estimations. Activities like the one below help to enhance these estimation abilities:

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## DECIMAL ESTIMATES (Worksheet 22)

Try to circle the correct answer **before** you use the calculator to check your estimate:

| Calculation | Estimate (Circle one) | | | Calculator answer |
|---|---|---|---|---|
| $5 \times 0,1$ | 5,0 | 0,5 | 0,6 | |
| $11 \times 3,3$ | 36,3 | 363 | 3,63 | |
| $0,1 \times 0,1$ | 0,1 | 1 | 0,01 | |
| $0,6 \times 2,1$ | 126 | 12,6 | 1,26 | |
| $0,8 \div 0,2$ | 0,04 | 0,4 | 4 | |
| $1,6 \div 0,2$ | 8 | 0,8 | 0,08 | |
| $4,4 \times 0,4$ | 17,6 | 176 | 1,76 | |
| $4,4 \div 0,4$ | 1,1 | 11 | 0,11 | |

Fill in >, < or = to make the following true: (without using a calculator)

$4,8 \times 0,3$ _____ $7,2 \div 0,9$

$4,8 \div 0,3$ _____ $7,2 \times 0,9$

$4,8 \div 0,3$ _____ $7,2 \div 0,9$

$4,8 \times 0,3$ _____ $7,2 \times 0,9$

- Understanding the grouping meaning of division by decimals, in other words for a divided by b "How many b's in a?". The learners need not even be aware that they are doing division and the teacher should not tell them. The following extract from Worksheet 21 illustrates division as grouping:

## DECIMAL FRACTIONS AND THE NUMBER LINE II

1. Counting in 0,05's. Complete the number line:



| 0 | 0,5 | 1 | 1,5 | 2 |
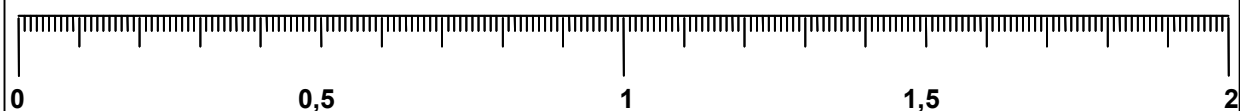
(a) How many 0,05's in one whole? _____

(b) What common fraction is 0,05 therefore? _____
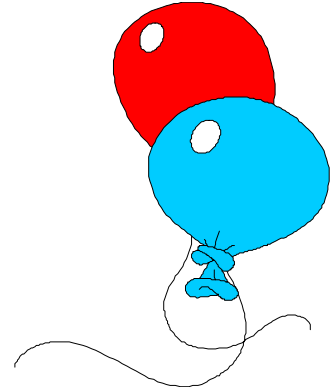
- Halving and doubling, as seen below in this extract:

## NUMBER PATTERNS IV (Worksheet 24)

❸ 112,64 (halve) → 56,32 →

- Correction of misconceptions that:
  - division makes smaller or multiplication makes bigger. See for example Worksheets 22 and 25 (Decimal Estimates).
  - a smaller number cannot be divided by a larger number.

8. *Rounding off*

## BALLOONS (Worksheet 17)

A party box of 100 balloons weighs 250g and costs R 6,79.

(a) What is the mass of one balloon?

(b) Complete the table:

| Number of balloons | 1 | 2 | 3 | 4 | 5 | 10 | 15 | 25 | 50 |
|---|---|---|---|---|---|---|---|---|---|
| Mass (g) | | | | | | | | | |

(c) What is the price of each balloon?

The calculator gives 6.79 ÷ 100 = **0.0679** which is R0,0679 or 6,79c.

Is this a *sensible* answer? How do you know?

Sometimes we have to *round* decimal fractions *off* in order to get sensible answers. For example, when we are working with money, it does not make sense to talk about R0,5214 so we would *round* this *off* (or *round* it *down*) to R0,52 or 52c. In the same way, it does not make sense to talk about R0,5281, so we would *round* this *off* (or *round* it *up*) to R0,53 or 53c.

How would you round off R0,5551?

**Mathematical and social knowledge**

The MALATI approach gives pupils opportunities to develop a 'feeling' for the value of the numbers after the comma. It is intended that this takes place while they make predictions, examine the calculator response, reflect and discuss with peers. Decimal fractions express all fractions as tenths, hundredths or thousandths and these chosen denominators have an effect on the numerators. The process of making sense of this special notation may take some time and discussion, and the teacher should have patience. However, the fact that $\frac{1}{10}$ = 0,1 is in fact social knowledge and if, after the first two or three worksheets, pupils have not grasped this from interacting with the

calculator and their peers, the teacher should intervene and share this convention with them (see for example the Teacher Notes for Worksheet 8).

## The role of the calculator

Calculators should always be available in the classroom while working with decimals. Calculators fulfil three important roles in our approach:

- As a checking device to give immediate feedback.

- To introduce the decimals as an alternative *notation* for common fractions:

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### SHARING LIQUORICE III (Worksheet 3)

1. 27 strips of liquorice of the same length are shared equally among children at a party. For each of these cases write your answer as a common fraction and then check your answer with the calculator. Write the common fraction and the calculator answer in the table below:

| Number of children | Amount of liquorice – written as a fraction | Amount of liquorice – calculator answer |
|:---:|:---:|:---:|
| 2 | | |
| 3 | | |
| 5 | | |
| 8 | | |
| 10 | | |
| 12 | | |
| 20 | | |

2. Are the answers in the two columns different? Why?

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- Decimal number concept development. Calculators can be used to help learners with counting in decimals. For this calculators can be programmed with certain operations (See Teacher Notes of Worksheet 7 on how to programme the calculator to perform different operations).

## The role of prediction

Predicting and estimating decimal answers (for which the calculator then gives feedback) is an important part of the MALATI approach once the concept has been introduced (see Teacher Notes for Worksheet 4). Calculations with decimals are considered of secondary importance, because in everyday life calculations are almost

always carried out on calculator. For this reason, pen-and-paper multiplication and division of decimals are not included in this package. However, as with the common fractions materials, realistic problem situations are presented which, together with a good decimal number concept, prepare the ground for understanding of the basic operations with decimals.

**The role of games**

Games provide a pleasant, relaxed and supportive environment for the development of decimal number concept and concepts related to operations with decimals. Some examples are provided below, but teachers are encouraged to try different variations.

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## DECIMAL INVADERS (Worksheet 15)

Procedures to play the game:

1. Two players need one calculator
2. Player 1 enters any decimal number e.g. 43,598. This number must be 'shot down' (replaced by 0 by subtracting).
3. Players take turns to 'shoot down' a digit. (One at a time.)
4. The player that ends with 0 wins.
5. If a player changes the number on the screen but does not shoot down a digit, the other player gets two turns.

**43,598**

Example:

| | Press | Number on screen |
|---|---|---|
| Player 1: | 43.598 | 43.598 |
| Player 2: | $-$ 0.5 $=$ | 43.098 - The '5' has been shot down. |
| Player 1: | $-$ 40 $=$ . | 3.098 - The '4' has been shot down. |

Repeat this with different numbers!

**43,⃝98**

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**The language of decimals**

To avoid confusion, teachers should be careful to pronounce the words 'tenTHS', 'hundredTHS' and 'thousandTHS' differently to the words 'tens', 'hundreds' and 'thousands'.

The correct name for a decimal like 3,75 is "three comma seven five" and not "three comma seventy-five". The latter encourages the misconception that 3,75 is **two** natural numbers separated by a comma, instead of **one** number (see Teacher Notes for Worksheet 10).

The conventions regarding decimal commas and points should be discussed with the learners. In South Africa, the convention is to use a comma but the calculator shows a decimal point (and some shops write prices without either, for example $3^{99}$). In some countries like the USA (but not in South Africa), the comma denotes thousands, and learners may have come across this. This is social knowledge which needs to be clarified so that learners can all share the same meaning of the decimal point/comma. It is important that this is done early to prevent possible misconceptions (also see Teacher Notes of Worksheets 1 and 2).

**Money as a context**

The use of money as a context for introducing decimal fractions is discouraged, as it also encourages the misconception that the decimal number is **two** sets of **whole** numbers (rands and cents) separated by a comma. Therefore we have not included any activities with money as a context.

**Diagnostic teaching**

In the content area of decimal fractions, misconceptions can easily arise and become deeply entrenched. For example, pupils may come to believe that a decimal fraction containing hundredths is always smaller than a decimal fraction containing tenths, while this is not true, for example in the case of 3,8 and 3,45. Teachers must be constantly aware of pupils' responses and identify explanations that are based on half-heard or half-remembered rules. Where possible, these should be gently challenged using counterexamples and/or discussion with peers.

Diagnostic assessment should be used to diagnose such misconceptions as early as possible. An activity like the one given below (and, for example, Worksheet 12) can be used as a diagnostic assessment to identify learners' misconceptions and to help to resolve them.

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**MARKING HOMEWORK (Worksheet 19)**

The following worksheet was given to Zanele for homework. Mark the work, correcting all the mistakes.

Decimals:                                                    Name: _Zanele_

1. Write 0,2 as a common fraction:    $\frac{1}{2}$

2. Write 3,5 as a common fraction:    $\frac{3}{5}$

3. 3,6 + 0,3 = 3,9

4. 4,8 + 4,3 = 8,11

5. 0,7 - 0,1 = 0,6

6. 0,27 - 0,1 = 0,26

Write down the next three numbers in each pattern:

7. 0,2 ; 0,4 ; 0,6 ; _0,8_ ; _0,10_ ; _0,12_          (Adding 0,2's)

8. 1,2 ; 0,9 ; _0,6_ ; _0,3_ ; _0_          (Subtracting 0,3's)

9. 0,34 ; 0,36 ; _0,38_ ; _0,40_ ; _0,42_          (Adding 0,02's)

10. 0,5 ; _0,10_ ; _0,15_ ; _0,20_          (Adding 0,05's)

11. 0,25 ; _0,50_ ; _0,100_ ; _0,200_          (Doubling)

12. 0,8 ; 0,4 ; 0,2 ; _0,1_ ; _0,½_ ; _0,¼_          (Halving)

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### Sources for tasks

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