No More Little Boxes: The Need for Integrated Mathematics (for Teachers)

Karen Newstead, Reddam House Preparatory School, Cape Town[1]

This paper tells the story of a teacher who embraced the Critical Outcomes and encouraged learners to solve problems using their own strategies, and to share these strategies with their peers. In the process of solving a fractions problem, one boy came up with a strategy that was difficult to interpret within the context of the problem. Within the context of equivalence, there could be no doubt that the solution was correct. However, the response was unexpected in the context of fractions and caused the teacher to experience a critical moment in which she simply did not know. Her knowledge of fractions was more than adequate, as was her knowledge of algebraic equivalence. She could not, however, retrieve and integrate the necessary knowledge from its compartment at the moment when the incident arose in the classroom. In the discussion that followed, the teacher and project workers reflected on the implications of the Critical Outcomes for the teacher's role and for his/her own integration of mathematical knowledge.

Introduction

The critical outcomes of Curriculum 2005 have many implications for teachers. If learners are now to become independent and responsible decision-makers who are encouraged to make sense of mathematics by reasoning individually and as members of a group, the role of the teacher and the nature of his/her classroom management must be transformed. Teaching mathematics in this way requires teachers to reconceptualise their own fundamental notions of teaching, learning and mathematics (Goldsmith & Schifter, 1997). This paper illustrates the need for a different understanding of what mathematics is. Compartmentalised boxes of mathematical knowledge will not be useful to teachers if we are encouraging learners to make sense of the mathematics in their own way, because we can no longer anticipate the kind of mathematical knowledge that may need to be addressed in any particular lesson. Learners may surprise us with strategies that require access to unexpected compartments of knowledge. Our own mathematical knowledge needs to become more integrated so that, in information processing terminology, we can retrieve the necessary information as required. If "teachers' learning can be viewed in much the same way as mathematics students' learning" (Simon & Schifter, 1993), integration of knowledge within the Learning Area of Mathematics must be an objective for teachers as well as learners.

[1] Previously MALATI (Mathematics Learning and Teaching Initiative) project.

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Background

This paper tells the story of a 'watershed' moment for a teacher called Feroza[2]. Feroza was a teacher at one of the project schools of the Mathematics Learning and Teaching Initiative (MALATI) teacher and curriculum development project. This was a three-year project of the Open Society Foundation that provided teachers with research-based materials for the development of learners' mathematical content in some crucial areas such as fractions. Extensive workshopping and classroom support were also provided.

The MALATI approach required a classroom culture that originates from a theoretical orientation that students construct their own mathematical knowledge irrespective of how they are taught. Cobb, Yackel and Wood (1992) state: "… we contend that students must necessarily construct their mathematical ways of knowing in any instructional setting whatsoever, including that of direct instruction," and "The central issue is not whether students are constructing, but the *quality and nature* of these constructions" (p. 28, my italics). In order to ensure that the constructions are based on sense-making, the approach made use of carefully-selected problems, supported by a learning environment that encouraged reflection and social interaction. Teachers did not demonstrate solution methods for problems, but expected students to construct their own strategies, and depend on peer collaboration for error identification and the development of more powerful strategies.

Feroza is a young, capable, highly motivated Grade 6 teacher who enjoys mathematics and has always considered herself to be a competent mathematician and teacher. She is open to change and improvement, and participated enthusiastically in the MALATI project. At the time of the story described in this paper, Feroza had been part of the project for approximately 14 months.

The Classroom Challenge

A problem was given to Feroza's Grade 6 class:

Diane and James work in the garden. Diane is older and works faster than James, but they work the same number of hours. James gets paid $\frac{3}{4}$ of what Diane gets paid. If James gets R18, how much should Diane be paid?

Feroza told the learners to solve the problem on their own and then share what they had found in their groups. She selected certain strategies (including one that resulted in an incorrect answer) to be presented to the class in the whole-class discussion that followed.

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2 Teacher and learner names have been changed.

Learners presented the following strategies:

Maria's strategy:
□ ÷ 4 x 3 = R18
R18 ÷ 3 x 4 = R24

R24 ÷ 4 x 3 = R18.

Hendrik's strategy:
$\frac{1}{4}$ = R6,00
$\frac{2}{4}$ = R12,00
$\frac{3}{4}$ = R18,00
$\frac{4}{4}$ = R24,00

These two strategies evidently represented what Feroza had expected, and she praised the learners (particularly Maria) for their clear working. She pointed out that Maria had checked her response.

Kyle's group had reached the answer of R22,50. They said that they had divided R18 into 4 quarters, then added another quarter. The teacher praised them for logical reasoning, but made it clear that this was incorrect by saying "But where did you get the 18÷4?". This was not resolved, and Kyle was unsure where their mistake had occurred.

Sitting at the back of the classroom was a boy called Rashiq. The girl sitting next to him called attention to his work by saying "Miss, Rashiq's got another method". Rashiq was then asked to come and show his method on the board.

Rashiq's method:
R18 x 4 = R72
R72 ÷ 3 = R24

At this stage, it was clear that Feroza could not understand why this method resulted in the correct answer. Within the context of this problem, it did not make sense to her. Kyle tried to reason that "Rashiq's method first goes up and then comes down". Hendrik suggested that the class tried this method with different numbers to determine whether it would "always work". Feroza agreed that this would be a good strategy, and suggested that the class do this. However, most of the class had lost interest by now.

Our analysis
Fortunately for us, two project workers were present in the classroom to observe this interesting variety of strategies. We were collecting data for research purposes, and therefore could not intervene, but were able to interview the teacher after the lesson.

We ourselves were unsure of how Rashiq had reached his response within the context of this problem. Possibly, he had observed that his group had found the answer to be R24, and had then devised a clever (but random) way to reach the same answer. It is also possible that instead of beginning the solution of this problem by finding a quarter of the amount, Rashiq reasoned as follows[3]:

$\frac{3}{4}$ of the total amount is 18

If we take this amount and multiply it by 4 then we have $\frac{12}{4}$ which would be equal to 72.

72 is therefore three times the amount. The amount is therefore 72 divided by 3.

Rashiq was unable to explain his strategy to the class, or (at a later stage) to the project workers.

Feroza's dilemma

In an interview afterwards, Feroza admitted awareness of her uncertainty in response to this unexpected response:

> *F: …I think I was shocked there, because I felt that at that point I wasn't equipped to tell him that his method was correct or his method was incorrect. I didn't know myself. He got the correct answer, but he wasn't going according to what he was supposed to do as far as I…I didn't think that he could switch those numbers around and do it that way…*
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> *F: …What, is this acceptable?…I mean we know that all methods are acceptable, nowadays it's not just one method, it's how he got to his answer. You know I was worried, is that going to work for all cases? Was this a special case, that it worked for that?*
> *K: So you were actually unsure whether it was true for all cases?*
> *F: Yes. And that is where I came and I said right, OK, now try it with something else*
> *K: So you actually were trying to convince yourself as well…*
> *F: Myself as well, yes (laughs). As well as the rest of the class.*

For Feroza, this was a critical incident in her teaching:

> *F: You know at that point, I think that I, at that point, I felt as though I, it was the first time I was caught off guard. I was really caught off guard and I…and I felt that I needed to sit alone first and work it out on my*

[3] I am grateful to my colleague Rolene Liebenberg for her analysis of this strategy and for her general reflections.

own. You know, so, I mean he was talking about the number getting higher and the number getting lower and I was sitting there and I was thinking to myself you know, I actually need to sit with this on my own first and then, you know, feel more confident about the sum, or about the method, or about what he did. And that is the, that is the type of thing that you sit up with in the classes, I mean children challenge you nowadays. I mean that was a challenge.

We asked her whether she had since resolved the problem in her head:

F: Well, the examples that they gave worked. I was actually too scared to actually go and try it out at home again.

Feroza reflected further on her role as the "mathematical authority" (Goldsmith & Schifter, 1997) for determining right and wrong, and expressed her contradictory beliefs in this regard:

F: I...would love to encourage in kids, not to just sit there and accept everything. I mean the teacher doesn't always, there was a typical example that you don't know everything.

F: ...as you both could see, that lesson, that part of the lesson really took me by surprise. I think I, but also you know sometimes I think, I feel that, you know kids always expect you to know everything. In a, and ***I felt, in a way that I was letting them down somehow, that I didn't have the answers****. But then again on the other hand, I want them to know also that I don't know everything, you know?...I just feel I would have done so much more harm as if I had made as if I knew what was going on, and said but 'No, but that can never be done, you know, that is not on, this is the way you do it and…'. I rather admitted to the fact that I wasn't sure about what was happening.* ***I think they, sort of gain more respect for you in that way, if you sort of bring yourself down to their level as well****.*

Feroza's resolution

During the interview that followed, we were able to focus her attention on the mathematics involved:

R: Does it bother you that two computational procedures that are equivalent and I mean like why, it gave the same answer. Do you feel something, do you feel any, you know unsettled by it, now even?
F: I think I still feel a bit unsettled by it. It's, you know I…
R: You feel it's something that's not, that hasn't been quite resolved.
F: Yes.
R: Is that how you feel at the moment?
F: Yes. Definitely. I think it hasn't been…that is why I said, something like that, we're going to have to go back and do more examples like that.
K: So you're still not sure…

F: *I'm still not sure if it works.*
K: *…why, you're not sure if it will always work. (shows her 18 ÷ 3 x 4 and 18 x 4 ÷ 3 written underneath each other)*
F: *…that's 24, and that was, 18 times 4, I can't think of that answer now…*
K: *Ja, they both gave you an answer of 24.*
F: *24. Ja. (silence) It's the inverse operations, so it shouldn't really be a problem, I mean if you're thinking of the laws where that is concerned.*
R: *So now you're seeing it with different lenses.*
F: *(laughs) Yes.*

Although the teacher used the wrong terminology ('inverse operations'), it is clear that she finally resolved the issue for herself during the interview and was relieved. In the context of a fractions problem (of which the structure was $\frac{3}{4}$ of   = 18), she was simply not expecting to encounter the number laws.

Discussion

From a teacher development perspective, the interview described above highlighted some areas in which it was still necessary for us to challenge and (simultaneously) support Feroza. For example, we were concerned about the fact that she commented that Rashiq was not working according to "what he was supposed to do". The Critical Outcomes imply that teachers cannot arrive in the classroom with a preconceived notion of how learners will/should solve the given problem. On the other hand, they also cannot adopt the philosophy that "all methods are acceptable nowadays". Goldsmith and Schifter (1997) highlight the importance of striking a balance, and the importance of the teacher's own mathematical knowledge for this purpose:

> Teachers changing their practice must find a balance between valuing students' individual constructions of their mathematical understanding and guiding them toward the shared understandings, principles, and structures that make up the domain of mathematics. The goal of mathematics education is not for each student to develop his or her own idiosyncratic view of mathematics. It is not a matter of "anything goes"…Each student will necessarily develop a personal understanding of mathematics and an individual relationship to it, but these should fall within the bounds created by a shared understanding of the domain. *Teachers who have developed clear and reasoned aspects about the essential aspects of the discipline will be in a far better position to guide students productively through the mathematical terrain.*
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> p. 35, my italics

If teachers are to take advantage of "teachable moments" (Goldsmith & Schifter, 1997), they need to have a solid and integrated understanding of the underlying mathematics. They need to be equipped to respond to the strategies that learners may come up with, however unexpected. I recommend that any teacher development project should include the following:

- a component that addresses teachers' perceptions about the nature of mathematics and how mathematics is learned and practised (Murray, Olivier & Human, 1999);
- a component that improves teachers' understanding of the mathematical content and the links between previously compartmentalised areas of mathematics (MALATI, 2000); and
- adequate support for the teacher (Etchberger & Shaw, 1992; MALATI, 2000) as s/he attempts to integrate his/her mathematical knowledge while simultaneously implementing a different teaching approach.

Taking all this into account, there will still be incidents in which the teacher is challenged by an unexpected response and will need time to make sense of a learner's strategy. Hopefully the teacher will be excited to discover that s/he underestimated his/her students, and take the problem out of the classroom situation in order to reflect, persist, struggle, and collaborate with peers (other teachers). Some digging in various unexpected little boxes may be required, but is this not what we would expect from learners in this exact situation?

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