# THE USEFULNESS OF AN INTENSIVE DIAGNOSTIC TEST

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**Abstract**

*This paper describes the design of a diagnostic test that was used to investigate the errors that pupils make in simplifying very elementary algebraic expressions. The purpose for designing this test was to see whether it could reveal difficulties that might have been overlooked in tests of a more extensive nature. The process of test item selection is explained to illustrate the intensive nature of the test and to make the distinction between intensive and extensive testing. Through an analysis of error patterns and interview responses an evaluation is made of the effectiveness of such a test as a diagnostic tool.*

## Introduction

Pupils at all levels have a lot of difficulty in manipulating algebraic expressions. At a very elementary level, when pupils are asked to simplify an expression like $2x + 3x$, some of the most common mistakes observed are $6x^2$, 6x and $5x^2$. What makes it so difficult for pupils to simplify $2x$ + 3x to $5x$? Is this kind of difficulty related to the pupils' understanding of the concept of equivalence ? I have observed that while pupils may recognize that $2x + 3x$ is not equivalent to $6x^2$ when asked to compute the value of these expressions for a particular value of $x$ ,they still make the above mistakes when asked to simply the expressions. Is this difficulty perhaps related to a poor understanding of the distributive property, in numerical contexts, for example, 2x4 + 3x4 = 4x(2+3)? Is the difficulty that of being unable to see the structural match between 2x4 + 3x4 and $2x + 3x$? Would a better understanding of the concept of a variable have eliminated this kind of difficulty? These are some of the questions that come to mind in trying to probe the possible underlying sources of the difficulties that manifest in the common errors we observe.

It is not the purpose of this paper to provide answers to these questions but merely to point out how important this kind of reflection is in trying to understand the difficulty of the pupils. The results of this exploratory research, based on an analysis of the errors made, has revealed three categories of difficulties. An analysis of these difficulties is the focus of this paper.

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**Designing The Test**

I will define an extensive test, in the context of simplifying algebraic expressions, to be one that includes test items with all the operations and more than one variable.

In designing an intensive test I therefore considered the following factors :

1. the numbers of variables
2. the nature of the exponent of the variable
3. the number of terms in an expression
4. the number of operations in an expression

The test items were restricted to one variable $x$ with exponent one only, expressions with either two or three terms involving only addition and subtraction.

The test items constructed were all of the type

$ax \pm bx \pm cx$, where a $\in$ single-digit integers and b, c $\in$ single-digit counting numbers.

The items were categorized into four groups based on the nature of a, b and c as follows :

Group 1: $|a| = b$ ; $c = 0$

Group 2: $a \neq b$ ; $c = 0$

Group 3: $|a| = b = c$

Group 4: $a \neq b \neq c$

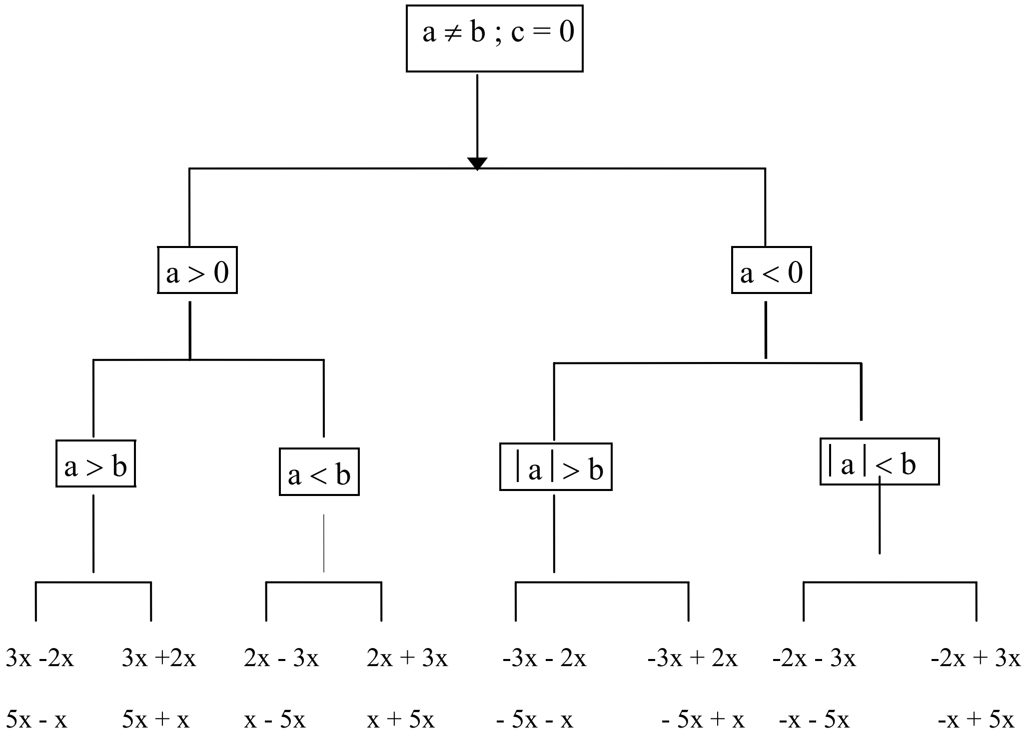
The criteria for categorizing the expressions into the four groups were to see whether item difficulty would be influenced by:

1. increasing the number of terms in the expression
2. the nature of the number coefficients

Each of the groups were divided into several subgroups to investigate whether the following criteria would influence item difficulty :

1. the order of the number coefficients
2. the nature of the signs in the expressions
3. the position of the signs in the expressions
4. 1 and -1 as coefficients ( in group 2 only )

The following diagram illustrates how the above criteria were used to select items for group 2:




In order to increase the reliability of the test, two items were constructed for each sub-group in the four groups. This also made it possible to distinguish between consistent errors ( i.e. incorrect responses to both items in the sub-group) and random errors ( i.e. an incorrect response to one or the items in the sub-group). This resulted in a large number of test items, a total of 80. The items were randomized and the test was divided into two parts , each having 40 items. The test was given in two parts because its intensive nature as well as the length of the test. These two factors could have affected the concentration span of the pupils , hence influencing the occurrence of careless errors.

A detailed tree-diagram of all the subgroups can be found in the appendix 1.

This test was administered to pupils in grade 9 and in grade 10 in a comprehensive school in England. The total sample size was 40, 24 pupils in grade 9 and 16 pupils in grade 10. I decided to use these two groups to see whether the errors made by the pupils in the two groups were similar and whether there was a reduction in error responses in those who were engaged in this kind of algebraic manipulation for a longer period. In order to obtain more information about the strategies that pupils used in simplifying the algebraic expression, a total of five pupils who produced consistent errors were interviewed. In the interviews the pupils were simply asked to explain how they arrived at their answers. The pupils were given no indication that their responses were incorrect.

**Analysis of the Error Patterns**

Three categories of difficulties could be identified on the basis of the errors that occurred.

1. **Misinterpretation of symbolic notation**

The absence of a visible numerical coefficient in $x$, led to its interpretation as "$0x$" instead of $1x$. This is evident in the following responses:

$$- 2x + 2x = x$$
$$x + 4x = 4x$$

2. **Difficulty with the subtraction concept**

Here many pupils had not understood the non-commutativity of subtraction:

$$2x - 3x = 1x$$

3. **Difficulty in operating with the integers**

In this category most of the difficulty seem to be due to some kind of interference between the "+" and "-" signs as operators for addition and subtraction or designators of positive and negative number.

Here five kinds of interference were observed:

**A. "- "sign ascribed a dual role**

This error can be explained in that the second "-" sign is given a dual-role :$-2x - (-2x) = 0$

$$-2x - 2x = 0$$

This kind of confusion is often evident in the language that pupils use to describe an expressions. These are some of the ways in which pupils described the expression $-2x$ $-2x$:

negative two $x$ negative two $x$

*negative two x subtract two x*

*negative two x minus two x*

*minus two x minus two x*

*negative two x subtract positive two x*

*minus two x subtract two x*

From these descriptions it appears as if the pupils regard the word "negative" and "minus" to have synonymous meaning which indicates that they do not adequately distinguish between the operation sign and the sign of a number. Teachers often do not make this distinction in their own teaching.

The verbal descriptions also suggest two ways of seeing the expression :

-2x -2x    or    -2x - 2x

A.    B

The relationship between the visual image and the mental interpretation of this image was not investigated. Is a visual image of type A, for example, more likely to produce an incorrect response? Is a visual image of type A, for example, more likely to produce a response of $4x^2$? This kind of error was not observed in this study but has been observed particularly where pupils are familiar with rules such as $x \times x = x^2$.

From the interviews it also appeared that none of the pupils converted subtraction to the addition of its additive inverse. The above strategy could have been used by pupils who produced correct responses but was not investigated here.

**B. <u>- sign loses its role as operator - (operation-elimination and operation interference)</u>**

Here it appeared as if addition was carried out and the multiplication rule for integers applied. The role of the - sign as an operation for subtraction is eliminated:

$$\boxed{\begin{array}{l} -\ 3x - 2x = 5x \\ -\ 6x - 5x = 11x \end{array}}$$

**C. <u>+ sign ascribed a dual role - ( operation -interference)</u>**

Here the operations addition and multiplication is carried out:

$$\boxed{\begin{array}{l} -3x + 2x = -\ 5x \\ -\ 6x + 5x = -\ 11x \end{array}}$$

It is interesting to note that where operation interference occurred , addition is always carried out , even in the absence of the + sign as in category B.

**D. <u>+ sign loses its role as operator ( operation -elimination)</u>**

Here the pupil associates the + sign with the $x$ and multiplies [ $5x\ (+x) = 5x^2$ ]:

$$\boxed{5x + x = 5x^2}$$

**E. <u>operation interference caused by the adjacent $x$'s</u>**

The incorrect responses in the different subgroups below were induced by the adjacent $x$'s.

| | | |
|---|---|---|
| $5x + x = 5x^2$<br>$4x + x = 4^2 x$ | $5x - x = 5$<br>$4x - x = 4$ | $-5x - x = -5$<br>$-4x - x = -4$ |

The $x$'s were grouped [ $(x + x)$ ; $(x - x)$] and the answer was then added to the coefficient of the first term. A pupil's response to $5x + x = 5x^2$ and $4x + x = 4^2x$ was:

**"***I added the x's and got two x, but I was not sure where to put the two***"**

**<u>Discussion</u>**

An analysis of the error patterns seem to suggest that most of the difficulties highlighted here stem from a poor understanding of integers. Pupils who lack confidence in operating with negative and positive integers often just ignore the negative sign, for example in $-2x + 3x - 4x = 1x$:

*"I add two and three , got five and then subtracted"*

In $-2x + 2x = -4x$ the observed dual-role and operation - interference effects appear to be due the multiplication rules, is evident from the following response:

*"I add them together and because of the minus and the plus it changed to a minus"*

From my experience I have observed that pupils who were able to add and subtract integers, often made this kind of error after multiplication of integers was taught. The responses of pupils in the interviews seem to support Ranney's hypothesis that the operational characters "provide the loci" from which the expression is manipulated. (Ranney,1987, p. 39).The interviews showed that pupils use several methods for simplifying algebraic expressions with three terms but there was no interference of the multiplication rules. Expected responses like $2x + 3x - 4x = 24x$ or $24x^3$ was not found. The rules for the multiplication of integers are often learnt and reinforced with two factors, for example a negative number multiplied by a negative number is equal to a positive number.

If the rules were reinforced in a different way, for example multiplying an odd number of negative numbers equals an odd number, would the interference still manifest only in expressions with two terms?

**Concluding Remarks**

Most of the errors observed here have been revealed in extensive tests. The intensive nature of this diagnostic test did however enable one to diagnose the particular difficulty of individual pupils which could have been overlooked if the test were of a more extensive nature. This is best illustrated in the results of a pupil who produced only one consistent error and three random errors. A closer examination of the items in the incorrect responses shows that they all involve the same sign structure, where two " - " signs appear in succession. [ $-4x - x$; $-x - 4x$; $3x - 3x - 3x$; $9x - 9x - 9x$; $8x - 6x - 7x$ ]. The initial response of this pupil in the interview to $3x - 3x - 3x = -9x$ [*"Well, the minuses, so I added them and put the minus*"], seems to suggest that the visual impact of the two successive -signs was a significant factor .The intensive nature of the test also highlighted the absence of the interference of the multiplication rules when simplifying expressions with three terms and also made enabled one to predict a possible cause for the interference.

Restricting the test item to a single variable could have enabled pupils who have mastered the operations with integers to produce correct responses without having a real understanding of the nature of the variable. If one analyses the verbal responses of the pupils it appears as if they operate on the integers and then simply " attach " the "*x*" to the answer. Here the letter is interpreted as a concrete object in its own right. Thus a pupil who is successful at simplifying algebraic expressions of this type could still be operating at the procedural level. For effective diagnosis to take place we need to be aware of these limitations.

The notion of an intensive diagnostic test is nevertheless a diagnostic tool that needs to be researched more extensively as it forces one to focus on a very specific area. Furthermore it is necessary for us to develop a culture of teacher researchers in this country and hence we need to engage in this kind of exploratory research to inform our teaching practice.

**Reference:**

Ranney, M. (1987). The role of structural context in perception: Syntax in the recognition of algebraic expressions. Memory and Cognition vol.15. no.1 pp. 39-40.