SUCCESSES AND OBSTACLES IN THE DEVELOPMENT OF GRADE 6 LEARNERS' CONCEPTIONS OF FRACTIONS

Therine van Niekerk, Karen Newstead, Hanlie Murray and Alwyn Olivier

Mathematics Learning and Teaching Initiative (MALATI)

This paper reports on the development of some Grade 6 learners' conceptions of fractions during a yearlong intervention programme consisting of specially designed materials and a particular approach to learning and teaching mathematics. We highlight some gains in understanding, but also difficulties encountered in implementing the programme and suggest possible reasons for these difficulties.

INTRODUCTION

The content area of fractions has proved itself to be very complex and troublesome for children to master. Research shows many possible factors contributing to this poor understanding of fractions. According to Streefland (1991) there are two main sources of such problems:

- An underestimation of the complexity of fractions as a conceptual field, and
- The mechanistic way in which fractions are taught (based on rigid application of rules and complete detachment from reality).

Newstead and Murray (1998) summed up the factors that contribute to the poor understanding of fractions as follows:

- The initial presentation of fractions to children - both the way and the sequence in which the content is presented to them. For example, the use of pre-partitioned manipulatives and a restriction to halves and quarters only.
- A lack of opportunity in the class to resolve and monitor misconceptions (sometimes based on incorrect intuitions) that children might have.
- The tendency of children to apply their whole-number conceptual framework to fractions, interpreting a fraction as two whole numbers.

This paper reports on the development of some Grade 6 learners' conceptions of fractions in the course of 1998 during which specially designed materials and a specific approach were implemented. The theoretical framework underlying the materials and the approach is briefly described.

Paper presented at the 5th Annual Congress of the Association for Mathematics Education of South Africa (AMESA), Port Elizabeth, 5-9 July 1999.

THEORETICAL FRAMEWORK

Classroom culture is probably one of the most important factors influencing the development of any mathematical understanding. In our view, the classroom mathematical culture includes the nature of the tasks given to the children, the role of the teacher and the social culture of the classroom. In our materials design and in our work with teachers in workshops and in the classroom, we are guided by the following principles:

a) *Nature of the tasks*

Learners are presented with meaningful problems based on their everyday life. The children should experience these problems as *problematic*, not in the sense of difficult or frustrating, but in the sense of interesting and challenging. They should not be able to use drilled rules and formulae to solve these problems, but should be able to solve the problems using their intuitive knowledge and common sense.

b) *The role of the teacher*

The role of the teacher is primarily to facilitate the children's solution of problems and discussion of strategies. The teacher should not be the main source of mathematical information, but this does not exclude the teacher from sometimes giving necessary information (for example, mathematical conventions) and participating in the classroom discussion.

c) *The social culture*

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier and Human (1997) identified four features of the social culture in a problem-centred classroom. We consider these aspects to be essential for learners' mathematical development:

- All ideas are potentially important and should therefore be respected.
- Autonomy of methods should be encouraged. The need for every child to understand the method he or she is using must be respected. The children should also realise that a variety of methods can lead to the correct answer; therefore they should have the freedom to explore and share these methods with their peers.
- Mistakes must be seen as learning opportunities and should not be suppressed. They can lead to reasoning and discussion that might deepen learners' understanding of the problem.
- The authority for the correctness of the problem solution lies in the structure of the problem and not in the teacher or the other children. A method is not necessarily correct just because a popular child presented it.

CONTEXT OF THE STUDY

1. The school and the class

The Grade 6 class in this study was a class in a MALATI project school. They were provided with MALATI materials which had been designed for some important content areas (including fractions), and the teacher received regular workshops and classroom support. The school is situated in a large black township and many of the children travel long distances to school. This results in children arriving late at school or not coming to school at all. Teaching was mostly 'traditional' before the MALATI intervention - children worked individually and discussion of mathematical ideas was not encouraged.

There were 42 children in the class. Most of the children had been exposed to fraction instruction before, but our base-line research showed that serious limiting constructions existed (Newstead and Murray, 1998), so fractions were introduced from the beginning again.

2. The material

The fractions materials used in this Grade 6 class during 1998 were designed by the MALATI fractions working group based on the following principles for developing understanding of fractions:

a) When introducing fractions, the fraction symbol should be delayed until the fraction concept is stable. The fraction symbol can prove to be a confusing notation to children, so initially children should rather be encouraged to write the fraction names in words e.g. 3 quarters or three-quarters instead of $\frac{3}{4}$ (Mack, 1995; Newstead and Olivier, 1999).

b) Fractions should be introduced in the context of sharing situations with remainders which can be further shared. These sharing situations elicit the informal knowledge that the children bring with them to the learning situation and can be used successfully for introducing fractions (Mack, 1990; Murray, Olivier and Human, 1996; De Beer and Newstead, 1998).

c) Children should be introduced to fractions other than just halves and quarters very early in their work with fractions. If this is delayed, children can come to think that any 'piece' smaller than a half is a quarter (Murray, Olivier and Human, 1996; De Beer and Newstead, 1998).

d) No pre-partitioned manipulatives are used (Kamii and Clark, 1995) because such manipulatives can lead to misconceptions. For example, each of the 'pieces' can again be seen as a whole when regarded on its own.

The materials were in the form of worksheets that were handed out to the children from day to day. All the children completed the same worksheet during the same lesson. The teacher also received accompanying teacher notes, which provided suggestions on what the children might do and what they might learn as well as other critical aspects that the teacher might need to be aware of or need to address.

3. The teacher

This was the first time that the teacher had taught Grade 6 mathematics. At the beginning of the year, she used a very 'traditional' teaching approach, drilling methods and having the children repeat important words or conceptions orally. By the end of the year, some of the children were still not always willing to share their work because they were shy or because mistakes were not yet treated as potential learning moments (to be explained so that learners can attempt to understand the thinking behind the method). However, as the year progressed the teacher developed considerably; by the end of the year, she had created a classroom culture in which most of the children felt free to raise their ideas and discuss issues on which they disagreed as well as agreed. The teacher facilitated discussion without trying to lead the learners to the correct answer. At the beginning of each worksheet she helped the children to understand the problem. As the worksheets were not in the children's mother tongue, she clarified words and ideas for the children by asking them to explain to each other. The children were encouraged to work in groups, but were also allowed to work individually if they wished to. After the children had solved a problem, whole-class feedback sessions were held in which different methods and answers were discussed and clarified.

The classroom culture established by the teacher in this class was therefore in line with the culture which we consider necessary for mathematics learning to take place. This was not an easy task, given that the culture up to this point had discouraged children from talking to each other and sharing ideas, and that children were used to hiding their work and putting up their hands when finished so that the teacher could check their work.

4. Classroom support

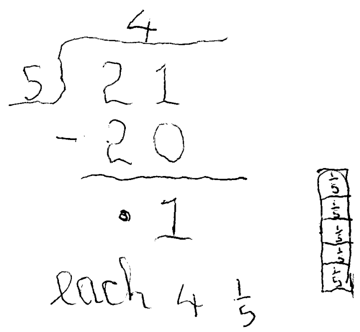
The researcher attended most of the fraction lessons in this class during the course of the year, not to teach but to help the teacher by making suggestions during the lesson and by discussing issues that arose during the lesson after the lesson. This included anything from discussing classroom culture issues to pointing out interesting responses from children. Workshops were held in which the material and underlying philosophy were discussed and in which teachers had the opportunity to talk to one another about the material and their experiences with the material in the classroom.

CHILDREN'S WORK

Most of the children appeared to have developed a fairly stable conception of fractions during the year. While some children started off with very traditional algorithms and methods, they gradually began to use methods that they understood and that helped them to solve the problems in practical ways, for example drawing. (Drawing was also encouraged if the children were 'stuck' on a problem). This development can be seen very clearly in the following extracts from one of the children's work.

In this sharing situation that was posed to Thobela, he tried to solve the problem with a traditional algorithm. When his algorithm could not help him beyond a certain point, he used a drawing. (26 of the 42 children could solve fairly difficult sharing problems by the end of the year).

Five friends want to share 21 chocolate bars equally. How must they do it?



He developed the concept of a fraction of a whole, where the whole is a collection of objects (20 of the 42 children could solve such problems successfully by the end of the year):

John's book has 88 pages. He says: 'I have read more than half of the book. I am on page 41.' Is it true? Explain.

False. Because the half of 88 is 44 and 41 is less than 44.

He also developed the ability to compare the size of fractions (27 of the 42 children could compare fractions successfully at the end of the year):

Which piece of a chocolate bar would you rather have? Why?

$\frac{1}{3}$ or $\frac{2}{6}$? I want them both. Because they are equal

$\frac{1}{2}$ or $\frac{3}{5}$? I want $\frac{3}{5}$. Because $\frac{3}{5}$ is bigger than a $\frac{1}{2}$.

He also understood the concept of equivalence (17 children appeared to have a good concept of equivalence at the end of the year):

The children were asked to decide 'Which of these pieces of chocolate do you think are the same size?' while examining a group of fraction symbols.

$$\frac{6}{18} = \frac{1}{3} = \frac{\cancel{5}4}{12} = \frac{5}{15}$$

$$\frac{2}{4}, \frac{3}{6}, \frac{1}{2}, \frac{4}{8}, \frac{6}{12}, \frac{9}{18}$$

$$\frac{5}{6}, \frac{15}{18}, \frac{10}{12}, \frac{2}{3}$$

By the end of the year, Thobela and many of the other children in the class had therefore developed a healthy basic concept of fractions as part of a whole (where the whole is one object or a collection of objects), and the ability to compare the size of fractions and find equivalent fractions.

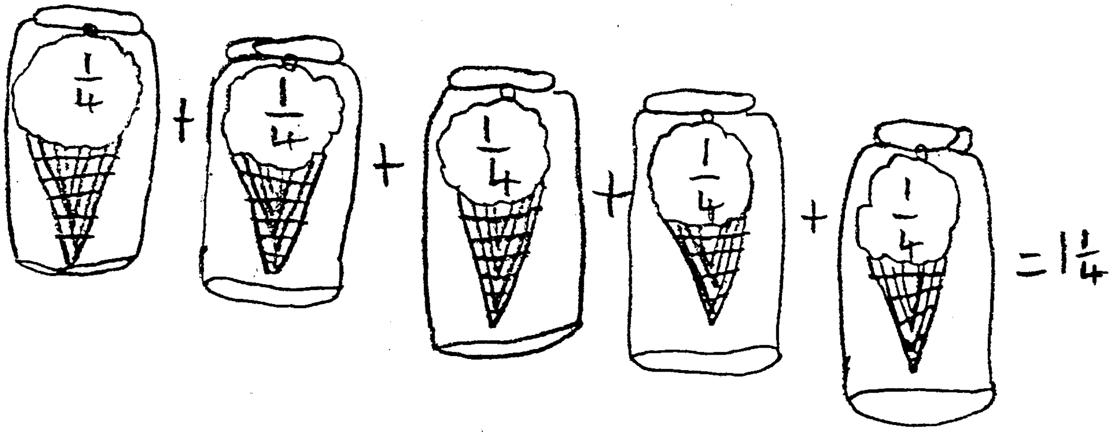
DIFFICULTIES

In spite of the research-based materials and intensive teacher support, several difficulties were observed to exist in the children's conceptions of fractions. Many of these could be seen to be persisting from previous experiences and instruction or from the teacher's difficulties with adapting to the new teaching approach. We will discuss some of these difficulties and try to give possible explanations for them.

1. **Difficulties that could easily be overcome with intervention from the teacher**

   (a) Children who were still drawing, even though they could solve the problem without drawing:

   Lisa wants to know what she has left over after a party. How much ice-cream does she have if she has 5 containers of ice-cream, each $\frac{1}{4}$ full.

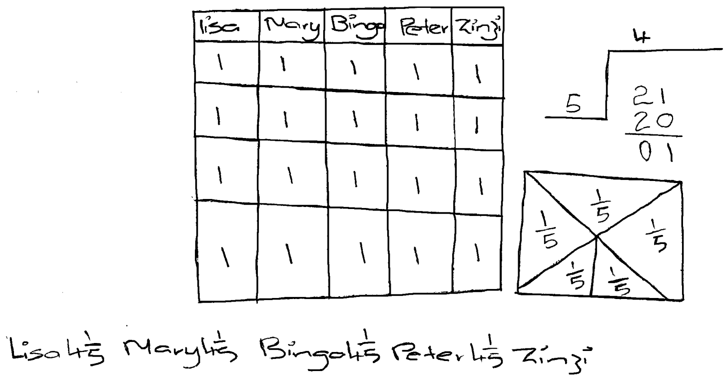


   (b) Difficulties with the notation of fractions. Some children could say the correct name of a fraction and give the correct explanation of their methods, but would write the symbol incorrectly. For example, they would invert the fraction, writing $\frac{3}{1}$ instead of $\frac{1}{3}$.

2. **The difficulty of equal parts**

For example, some children gave the correct numerical answer, but made incorrect drawings.

Five friends want to share 21 chocolate bars equally. How must they do it?



3. Mechanical errors

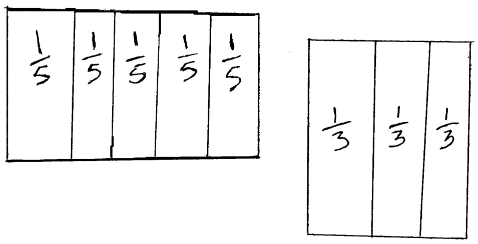
When children have been exposed to the same content for a while, they seem to over-generalize their self-constructed rules to other similar situations where it is not appropriate. For example, when some children were completing a ’fraction wall’ (where drawings of the same whole are given, partitioned into different sized pieces), they started with halves, thirds and just continued with quarters and fifths without counting the number of pieces.

4. Conceptual errors

(a) Some children made errors when comparing fractions, not always understanding that only fractions from the same ’whole’ can be compared.

What would you rather have, a third of a chocolate bar or a fifth of a chocolate bar? Why?

We want a fifth because its bigger than Third



(b) Some children made errors when carrying out the addition of fractions. In some cases they simply added the numerators and the denominators, treating the fraction as two different whole numbers (see also Lukhele, Murray and Olivier, 1999).

The children were given a recipe for one cake and were asked to work out how much of each ingredient would be needed for 5 cakes.

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{20}$

1egg + 1egg + 1egg + 1egg + 1egg = $\frac{1}{5}$

$1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = \frac{5}{10}$

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{20}$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{10}$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{10}$

$2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} = \frac{5}{20}$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{10}$

5. Classroom culture problems

The following obstacles to learning were observed in the classroom:

(a) *The perception that mathematics is not something that should make sense*. When given a problem, some children only used the numbers given to try to carry out some kind of (arbitrary) operation with them, without making sense of the problem.

(b) *Lack of problem solving skills*. Some of the children seemed afraid to begin, because they were afraid of making mistakes, because they were unfamiliar with problem solving or because they had difficulties reading or understanding the language. They were not always willing to try to make a plan to solve the problem. Some children were also not willing to share their work with their peers.

(c) *Teacher dependence*. Some children automatically put up their hands when they had finished so that the teacher could come and check if they were correct.

(d) *Time on task*. Some of the children took a great deal of time to organise themselves (finding pencils, etc.) before they began with the mathematics. The children were also very aware of the premium placed on neat work, so they would spend much time erasing their first efforts and colouring in.

POSSIBLE REASONS FOR THESE DIFFICULTIES

A few reasons can be suggested as possible causes for these problems:

- *Previous experiences and expectations of previous teachers*. It is clear from the classroom culture problems discussed above, that this was the first time that the children were expected to solve problems and work things out for themselves and not wait for the teacher to tell them what to do. In their previous experience, they expected the teacher to tell them whether they were correct or not and this was clear from the fact that they were observed to find it very frustrating at times when their current teacher did not do this. Therefore they found it difficult to take responsibility for their learning and were afraid of being laughed at by their peers.

  From the children's mechanical errors, it was also clear that the children were not accustomed to making sense of the mathematics and did not see this sense-making as a necessary part of mathematics. Mathematics was simply perceived as difficult.

  The previous teacher also expected neat work from the children, which caused them to waste a lot of time erasing and rewriting work. This could also have been a work avoidance strategy on the part of children who were unaccustomed to problem solving and therefore did not want to have to start with the problem.

- *Previous fraction instruction*. Some of the conceptual errors observed could be the result of previously-taught fraction knowledge which had no meaning to the child. An example of this is the problem of correct naming but incorrect drawing of fractions such as $\frac{1}{5}$, resulting from previous limited exposure to only halves and quarters. Some of the children already knew the 'recipes' and 'rules' for working with fractions and many misconceptions were already in place. This proved to be a great obstacle, especially with the use of the fraction symbol. In some cases, children confused the 'rules' that they had learned and applied them incorrectly or in inappropriate places. For example, some children treated addition of fractions in the same way as multiplication of fractions, leading to incorrect answers (Lukhele *et al*., 1999).
- *Absenteeism*. The fact that children were often absent and therefore did not complete all the worksheets and participate in essential discussions and reflection sessions could also have contributed to the difficulties.
- *Administrative problems at their school*. Teachers were very often absent or not in their classrooms. These problems led to a reduced contact time between the teacher and the children.
- *Difficulties with the material*. There was not enough emphasis in the material on, for example, developing the concept of fractions of collections of objects and the concept of fractions of different 'wholes'. These problems are currently being addressed during the revision of materials.
- *The level of difficulty of the concept of fractions*. As previously noted, the concept of fractions is difficult and the associated notation causes problems if children do not make the appropriate link between the words and the symbols. This was evident when children could use the correct fraction names but not write the fraction correctly.
- *The role of the teacher*. In making the difficult transition from a 'traditional' teacher to a facilitator of mathematical learning, the teacher obviously took time and encountered difficulties. For example, she did not always know when to intervene to give the children the necessary social knowledge to help them with the fraction notation. She may also have placed too much emphasis on drawing and telling children to draw when they were stuck. This may have resulted in the children thinking they *had* to draw, even though they did not need to anymore.

CONCLUSION

The success that we observed in the development of children's understanding of the fraction concept in this classroom was in line with a general positive trend in our project primary schools during 1998 (Newstead & Olivier, 1999). However, other than MALATI's intervention and support, there were many factors contributing to the success that we observed in this particular classroom. The teacher was very open and willing to change, and although the classroom culture which she has established is not yet perfect, she has successfully incorporated a great deal of the philosophy underlying our approach and has encouraged the children to talk to each other and share ideas, which we regard as very important. In other classes, where the classroom culture has not yet changed to this extent, we observed less success, even with the same materials.

However, it is clear that within the space of one year, along with the success, many difficulties were encountered. For the teacher to change her role and become confident in her new role will take much more than one year. It is also extremely difficult and time-consuming for the children to 'unlearn' their previous, unhelpful fraction knowledge, to change their previous beliefs about what it means to do mathematics, and to learn to see themselves as not only required to but also *able* to make sense of mathematics.

REFERENCES

De Beer, T. & Newstead, K. (1998). Grade 1 students' strategies for solving sharing problems with remainders. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education,* **4**, 327. Stellenbosch, South Africa.

Empson, S. (1995). Using sharing situations to help children learn fractions. *Teaching Children Mathematics,* **2**, 110-114.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Olivier, A. & Human, P. (1997). *Making Sense: teaching and learning mathematics with understanding*. Portsmouth: Heineman.

Kamii, C. & Clark, F.B. (1995). Equivalent fractions: Their difficulty and educational implications. *Journal of Mathematical Behavior,* **14**, 365-378.

Lukhele, B., Murray, H. & Olivier, A. (1999). Learners' understanding of the addition of fractions. Paper accepted for the 5th Annual Conference of the Association for Mathematics Education of South Africa. Port Elizabeth.

Mack, N.K. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research in Mathematics Education,* **26**, 422-441.

Murray, H., Olivier, A. & Human, P. (1996). Young students' informal knowledge of fractions. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education,* **4**, 43-50. Valencia, Spain.

Newstead, K. & Murray, H. (1998). Young students' constructions of fractions. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education,* **3**, 295-303. Stellenbosch, South Africa.

Newstead, K. & Olivier, A. (1999). Addressing students' conceptions of common fractions. Paper accepted for 23rd Conference of the International Group for the Psychology of Mathematics Education. Haifa, Israel.

Streefland, L. (1991). *Fractions in Realistic Mathematics Education: A Paradigm of Developmental Research*. Dordrecht: Kluwer Academic Publishers.