# HOW GRADE 3 & 4 LEARNERS DEAL WITH
# FRACTION PROBLEMS IN CONTEXT

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## ABSTRACT:

*Malati fraction materials make an appeal to primary school learners' intuitive knowledge of fractions and their ability to make sense of fraction problems when confronted with situations that are meaningful to them. This approach to fraction problems however asks for the realisation by teachers that learning is a social activity in which learners are engaged in discussion, invention, explanation, negotiation, justification, sharing, reflecting and evaluating. It is therefore necessary that teachers using this approach view learners as independent thinkers who are able to construct their own knowledge in a social atmosphere of reflection and sense making of tasks. Learners should be encouraged to use their own intuitive knowledge to solve these problems and to develop a sound concept of fractions over a period of time during which each learner's method is being valued and respected. Malati project schools implemented such an approach with co-ordinators monitoring the process through classroom visits and support to teachers. This paper looks at the outcomes of the approach in grades 3 and 4 in one Malati project school.*

## INTRODUCTION:

During my engagement as Malati project worker and co-ordinator of one of the primary project schools I had the opportunity to gather an enormous amount of evidence that illustrates that learners indeed use their intuitive abilities to solve fraction problems by using various strategies when allowed to work independently from the teacher. It was observed that they are able to work in this way effectively as long as they can relate to the specific problem situations.

Researchers are of the opinion that learners possess informal fraction knowledge just as much as they have intuitive notions of whole numbers (Murray, Olivier, & Human; 1996) which teachers should take advantage of. They involved learners in equal-sharing problems with remainders to be shared too, without the use of manipulatives. Their study resulted in the conclusion that young learners should be encouraged "to construct their own idea of fractions through their own actions" followed by discussion of different strategies and solutions and gradual introduction of the necessary "terminology and notation".

According to Empson (1995) teachers should spend enough time in developing concepts of equivalent fractions by using equal-sharing problems which may result in learners solving operations with fractions using their own methods "based on concepts that make sense to them". A problem-solving approach was implemented in her research where learners were never shown how to solve problems but were encouraged "to create their own representations" without the use of fraction manipulatives. The development of an understanding of equivalence should however be tackled over a long period and not only in isolated cases.

Hiebert & Hiebert (1983) state that older learners are capable of applying "routine skills or algorithms" but they do not have an understanding of the basic concepts of fractions which leads to low performance in basic fraction tasks (Carpenter, 1976).

This study intends to determine the outcomes of engaging learners in problems where they have to construct their own meaning of fraction concepts involving equal sharing over a

period of time using problems within their field of experience. Learners were encouraged to use their intuitive notions of fractions, mathematical tools and their own representations to deal with problems effectively instead of applying rules, algorithms and taking shortcuts.

## BACKGROUND:

Malati's aims are in concurrence with the present universal trends of change of reform of innovation in mathematics education at school level. The main focus is on "contextualised learning through personal and interactive reflection, an emphasis on problem solving as goal as well as a vehicle for learning, and a commitment to enable learners to make authentic personal sense of mathematics". (Malati, 1996).

Before trialling of materials in classrooms teachers were involved in several Malati inservice training sessions and workshops which offered further learning opportunities. Co-ordinators were afterwards readily available to give classroom support in developing an effective classroom culture needed for the efficient implementation of materials which is to "be measured in terms of positive transformations in teachers' conceptual understanding, their classroom practices and students' learning." (Malati, 1996).

The rationale for the fractions introductory materials design has its origin in the fact that grade 1 learners are able to make sense of fraction problems with frequent exposure to such problems. Operations involving fractions will only make sense to learners if they have a perceptible conception of what a fraction is. This is possible if such a concept is regularly revisited and developed through problem posing – not demonstrations, rules, recipes or definitions. Formal methods should be deferred for as long as possible. Learners should make sense of the purpose of equivalent fractions and problems, which were previously regarded as difficult, should not be delayed but introduced as the need arises (for example division with fractions). Learners should be exposed to collections of objects as the whole. (Lukhele, Murray, & Olivier, 1999; Malati, 1997; Van Niekerk, Newstead, Murray, & Olivier, 1999). Hiebert & Hiebert (1983) claim that there is a possibility that learners' difficulties with fractions in later school years could be related to an "incomplete" interpretation of basic "part/whole relationships."

Malati project workers assisted teachers in developing a classroom culture based on a problem-centred approach of learning through social interaction amongst learners in their endeavours to make sense of their own and others' constructions. (Murray, et.al., 1993). Learners are confronted with problems from their own field of experience – not to be solved by applying rules given to them by the teacher. They are given opportunities to be involved in discussions, arguments, justifications and feedback with the teacher as facilitator of the learning process.

## METHODOLOGY:

PARTICIPANTS:

All the teachers and pupils from grades 3 to 7 of the MALATI project school I co-ordinated were engaged in the MALATI fraction materials since the beginning of the second term during 1998. They started with the introduction package with great enthusiasm after the teachers were workshopped and supplied with intensive teacher notes for each activity in the package. This study concentrates on the results of a diagnostic test set up by the MALATI co-ordinator in conjunction with the teachers. The main purpose with the test was to use the results to enable teachers to differentiate amongst pupils during the following year. Another

reason was that teachers would be able to determine which fraction concepts still need attention in 1999.The results of 81 grade 3 pupils (51 Afrikaans-speaking and 30 English-speaking pupils, involving 2 teachers) and 117 grade 4 pupils (57 Afrikaans-speaking and 60 English-speaking pupils, involving 3 teachers) were analysed. The grade 4 teachers completed the whole package consisting of 33 activities while the grade 3 teachers worked through the first ten activities of the package.

During the classroom observations it was noted that, although the grade 3 teachers applied groupwork and allowed pupils to discuss the problems with each other, to share their ideas and to give feedback to the entire class, they still tended to demonstrate the teachers' methods to pupils, especially at the start of lessons. Only one grade 4 teacher had problems converting to an enquiry-based, co-operative learning approach while the other two grade 4 teachers made considerable progress in facilitating their lessons. Their learners were not spoon-fed; they were confident and liberal in their discussions with each other, their arguments, sharing of ideas and in giving feedback. They were allowed to use their own valued strategies without interference from the teachers.

All the teachers in these two grades were however, more than eager to adapt to a problem-centred approach after demonstration lessons by the co-ordinator (requested by the teachers), and ongoing classroom support and co-teaching.
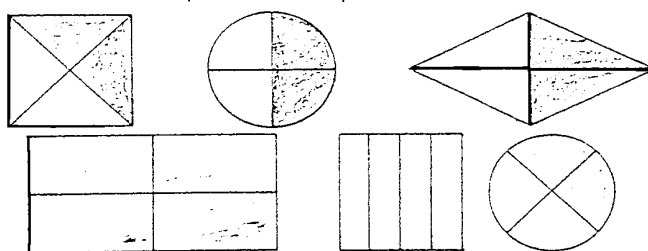
THE PROBLEMS POSED:

Before the MALATI intervention these teachers engaged their learners in fraction activities out of context as in the examples shown below.
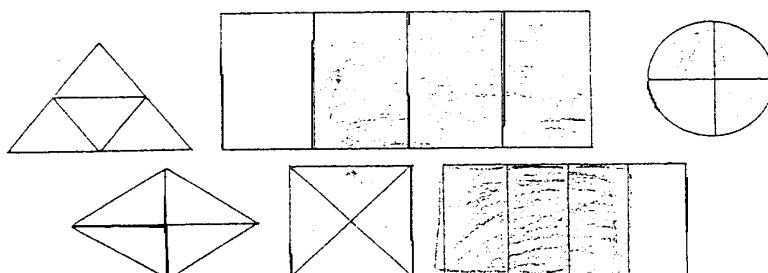


1. Colour in one quarter ($\frac{1}{4}$) of each shape.
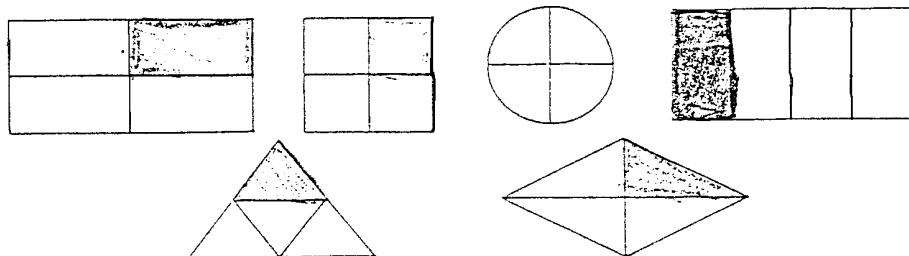
2. Colour in two quarters ($\frac{2}{4}$) of each shape.

3. Colour in three quarters ($\frac{3}{4}$) of each shape.

Although no evidence can be supplied of pupils' previous understanding of fractions before the intervention, it was obvious that these pupils were not previously exposed to this problem-centred approach, as also confirmed by the teachers. During informal discussions with the teachers it became clear that they experienced problems with the teaching of fraction concepts and that they found that learners' understanding of fraction concepts was poor. During the implementation of the MALATI fraction activities the teachers raised their appreciation for the positive and successful way their pupils dealt with equal-sharing problems. Some of the teachers even extended some activities for further enrichment of their pupils.

Five test items were adapted from the package focussing on equal sharing and comparison of the sizes of fractions. At this stage operations with fractions were not yet made explicit, but were handled informally. The pupils worked individually and they were encouraged to use their own methods in solving the problems.

OBJECTIVES FOR THE TEST ITEMS:

1. ***Zerick, Thurlow and Winslow have 19 Fizzers that they want to share equally so that nothing is left. Help them to do it.***

   - Equal sharing with a remainder that needs to be shared too.
   - To enable learners to share between two as well as amongst more than 2 people in order to overcome the barrier of just working with halves and quarters.
   - To draw problem situations to show their understanding of fractions.

2. ***Would you rather have $\frac{1}{2}$ of R1,00 or $\frac{3}{4}$ of R1,00? Explain why.***

   - To make a comparison between the sizes of fractions.
   - To be solved by direct physical comparison of fractions.
   - To handle and inspect fractions and think about the equivalence of fractions.
   - To deal with fractions as part of a collection of objects and not only as part of a single object (to use fractions as part of a numerical quantity).

3. ***A pizza is cut up into 8 slices and must be shared equally amongst 3 boys and 3 girls. How much would each one get?***

   - To divide a remainder of two into equal parts.
   - To use fraction parts other than just halves and quarters.
   - To determine whether $1\frac{1}{3}$ is equal to $1\frac{2}{6}$ or not.
   - To use the fraction wall as a learning aid without memorising equivalent fractions.

4. ***Which fraction is bigger: $\frac{5}{10}$ or $\frac{2}{5}$? Why?***
   - To make a comparison between the sizes of fractions.
   - To be solved by direct physical comparison of fractions.
   - To handle and inspect fractions and think about the equivalence of fractions.

5. ***The boys in the Handwork class make a small wire car from $\frac{1}{3}$ of a metre of wire. How many cars can they make from 5 metres of wire?***

   - A grouping division problem where the size of the group is a fraction.
   - To determine the number of lengths of $\frac{1}{3}$ m each to be cut from 5m.
   - To put fractional parts to form a whole together.

# RESULTS

The results of each of the five test items are presented in tables according to a coding scheme designed by the co-ordinator to provide information of how these grade 3 and 4 pupils deal with fraction problems in context after learners have repeatedly been exposed to such problems over a period of time. The problems are variations of those activities in the introduction package concentrating only on the first ten activities seeing that the grade 3 learners only dealt with those activities. The same problems were given to both grades. The results of each class are given separately in the table because there are considerable differences in the results of the different classes and in the different grades.

## CODES USED FOR MARKING:

**a**   Correct
**b**   wrong
**c**   uses thirds incorrectly / omit the fraction part
**d**   uses halves and quarters
**e**   $1\frac{1}{6}$ as an answer
**f**   no explanation given
**g**   no answer
**h**   use numbers to solve problems
**i**   use drawings as a solution strategy


ITEM 1:

| Grade | Number in class | a | b | c | d | e | f | g | h | i |
|-------|-----------------|------|-------|-------|-------|---|---|------|-------|-------|
| 3a | 51 | 72,5% | 0 | 21,6% | 3,9% | 0 | 0 | 2,0% | 3,9% | 78,4% |
| 3c | 30 | 76,7% | 3,3% | 6,7% | 6,7% | 0 | 0 | 6,7% | 13,7% | 70,0% |
| 4a | 57 | 49,1% | 24,6% | 8,8% | 17,5% | 0 | 0 | 0 | 14,0% | 82,5% |
| 4b | 28 | 42,9% | 21,4% | 10,7% | 21,4% | 0 | 0 | 3,6% | 3,6% | 60,7% |
| 4c | 32 | 65,6% | 12,5% | 12,5% | 9,4% | 0 | 0 | 0 | 12,5% | 71,9% |

74% of all the grade 3 learners handled this problem successfully. Only 1 learner got it completely wrong while 16% used thirds correctly with an incorrect whole number or omitted the fraction part of the answer. 5% of the learners used halves or quarters instead of thirds. 11% of these learners used numbers only in solving the problems while 75% used drawings to illustrate their processes and understanding.

In grade four 52% of all the learners were successful while 10% of them used thirds with incorrect answers or omitted the fraction part and 16% used halves and quarters instead of thirds. 74% of these learners used drawings to illustrate their processes and understanding.

EXAMPLES OF LEARNERS' STRATEGIES/SOLUTIONS:

1. **Zerick, Thurlow and Winslow have 19 Fizzers that they want to share equally so that nothing is left. Help them to do it.**

$6 \div 3 = 2$
$6 \div 3 = 2$
$6 \div 3 = 2$
$1 \div 3 = \frac{1}{3}$

$2 + 2 + 2 \frac{1}{2} = 2\frac{1}{3}$

$2\frac{1}{2} 2\frac{1}{2} 2\frac{1}{2} = 22\frac{1}{2}$

Each one gets $6\frac{1}{3}$ Fizz-pops

$18 \div 3 = 6 + 6 + 6 = 18$
$\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$
rem $1 \div 3 = \frac{1}{3}$

$19 \div 3 = 6\frac{1}{3}$
$3 + 3 + 3 + 3 + 3 + 3 = 18$
$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

Each one gets $\frac{1}{4}$

$6 \div 3 = 2$
$6 \div 3 = 2$
$6 \div 3 = 2$
$1 \div 3 = \frac{1}{3}$

$2 + 2 + 2 + \frac{1}{3} = 6\frac{1}{3}$ each

$6\frac{1}{3}$
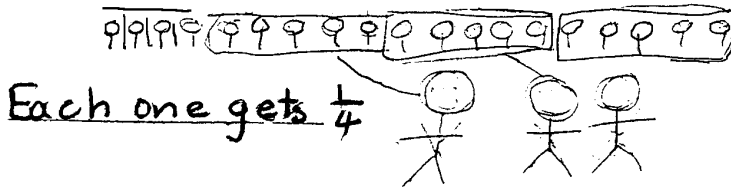
$6 \quad 6 \quad 6$ rem 1

each of them will get ½½c rem 1

ITEM 2:

| Grade | Number in class | a | b | c | d | e | f | g | h | i |
|-------|------|------|------|---|---|---|------|------|------|------|
| 3a | 51 | 0 | 92,2% | 0 | 0 | 0 | 7,8% | 0 | 0 | 0 |
| 3c | 30 | 6,7% | 43,3% | 0 | 0 | 0 | 43,3% | 3,3% | 3,3% | 13,3% |
| 4a | 57 | 50,9% | 26,3% | 0 | 0 | 0 | 14,0% | 8,8% | 33,3% | 14,0% |
| 4b | 28 | 14,3% | 32,1% | 0 | 0 | 0 | 50,0% | 3,6% | 7,1% | 7,1% |
| 4c | 32 | 25,0% | 25,0% | 0 | 0 | 0 | 46,9% | 3,1% | 15,6% | 0 |

Only 3% of the grade 3 and 35% of grade 4 learners handled this problem successfully. 21% of the grade 3 and 32% of the grade 4 learners gave no reasons for their answers.

EXAMPLES OF LEARNERS' STRATEGIES/SOLUTIONS:

2  **Would you rather have $\frac{1}{2}$ of R1,00 or $\frac{3}{4}$ of R1,00? Explain why.**

I will tack the $\frac{3}{4}$ because it is bigger

1/2 is bigger.
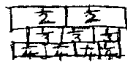
a half

I would take the $\frac{3}{4}$
It is more.
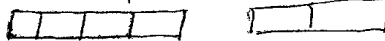
=R1,00    =R1,00
Ek wil 'n 75c hê

A $\frac{1}{2}$ of R1,00 becaus a $\frac{1}{2}$ of R1,00 is 50c
and a $\frac{1}{3}$ of R1,00 is only 33$\frac{1}{3}$c

R1,00÷$\frac{1}{2}$=50,    R1,00÷$\frac{3}{4}$=75,
Ek wat die 3/4 van dit is meer as die 1/2

1/2  Because 3/4 of a rand is 75c
$\frac{1}{2}$ of a rand is 50c

the half R1 of a the R1,00 is 50c    I would take the $\frac{3}{4}$ of R1,00
the a $\frac{3}{4}$ of a R1,00 =75c

ITEM 3:

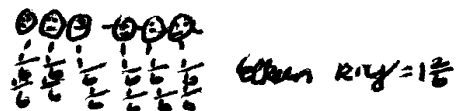| Grade | Number in class | a | b | c | d | e | f | g | h | i |
|---|---|---|---|---|---|---|---|---|---|---|
| 3a | 51 | 48,2% | 9,8% | 9,8% | 0 | 0 | 0 | 3,9% | 0 | 53,1% |
| 3c | 30 | 53,3% | 23,3% | 13,3% | 0 | 6,7% | 0 | 3,3% | 23,3% | 46,7% |
| 4a | 57 | 38,6% | 19,3% | 3,5% | 12,3% | 8,8% | 0 | 17,5% | 12,3% | 57,9% |
| 4b | 28 | 25,0% | 25,0% | 3,6% | 32,1% | 3,6% | 0 | 10,7% | 0 | 50,0% |
| 4c | 32 | 46,9% | 12,5% | 9,4% | 21,9% | 0 | 0 | 9,4% | 3,1% | 40,6% |

68% of the grade 3 learners were successful while 14% used thirds but the answers were incorrect. None of the grade 3 learners used sixths in their answers. 38% of grade 4 learners answered this question successfully. 5% used thirds with an incorrect answer and 20% used halves or other fractions instead of thirds. 6% gave $1\frac{1}{6}$ instead of $1\frac{2}{6}$ as an answer.

EXAMPLES OF LEARNERS' STRATEGIES/SOLUTIONS:

**3. A pizza is cut up into 8 slices and must be shared equally amongst 3 boys and 3 girls. How much would each one get?**
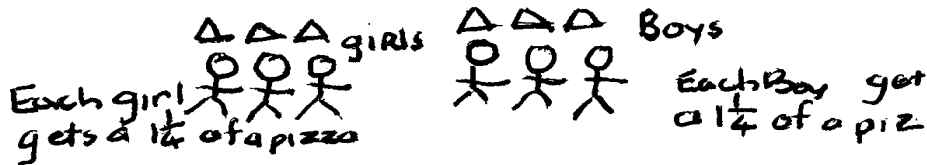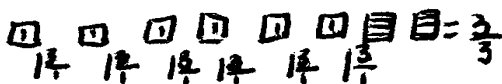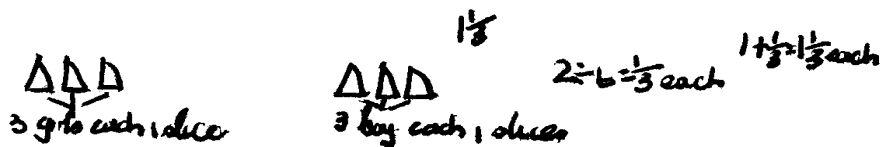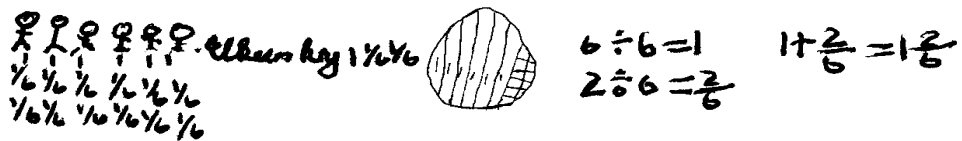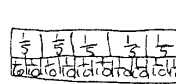
ITEM 4:

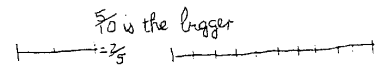| Grade | Number in class | a | b | c | d | e | f | g | h | i |
|-------|-----------------|------|-------|------|------|------|-------|-------|------|-------|
| 3a | 51 | 0 | 88,2% | 0 | 0 | 0 | 9,8% | 2,0% | 0 | 2,0% |
| 3c | 30 | 0 | 6,7% | 0 | 3,3% | 0 | 76,7% | 13,3% | 0 | 0 |
| 4a | 57 | 15,8% | 42,1% | 0 | 0 | 0 | 22,8% | 19,2% | 0 | 33,3% |
| 4b | 28 | 0 | 46,4% | 0 | 0 | 0 | 50,0% | 3,6% | 0 | 3,6% |
| 4c | 32 | 6,3% | 12,5% | 0 | 0 | 0 | 75,0% | 6,3% | 3,1% | 3,1% |

No one of the grade 3 learners was successful and only 9% of the grade 4 learners had correct solutions with reasons. 51% of the grade 3 learners and 44% of the grade 4 learners gave the correct answer but no reasons were given.
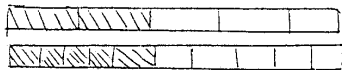
EXAMPLES OF LEARNERS STRATEGIES/SOLUTIONS:

**4. Which fraction is bigger: $\frac{5}{10}$ or $\frac{2}{5}$ ? Why?**

$\frac{5}{10}$ is bigger
The half of $\frac{10}{10}$ is $\frac{5}{10}$.

5/10 because $\frac{5}{10} = \frac{7}{14}$ and $\frac{2}{5} = \frac{4}{10}$
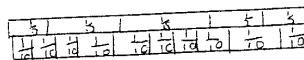
$\frac{5}{10}$ is the bigger

$\frac{2}{5}$ is die grootte

They are both equall

I will tack the $\frac{5}{10}$ because it is bigger

$\frac{5}{10}$ because $\frac{2}{5} = \frac{4}{10}$   $\frac{5}{10} = \frac{3}{6}$

$\frac{5}{10}$ is groter want dit is groter as $\frac{2}{5}$

$\frac{5}{10}$ or $\frac{2}{5}$ Because $\frac{4}{10}$ = to $\frac{2}{5}$

$\frac{5}{10}$ because it is a half

ITEM 5:

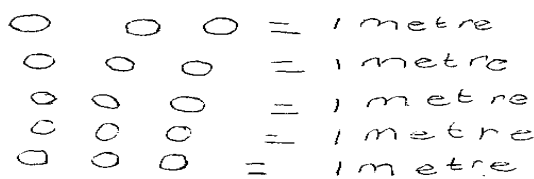| Grade | Grade totals | a | b | c | d | e | f | g | h | i |
|---|---|---|---|---|---|---|---|---|---|---|
| 3a | 51 | 90,2% | 9,8% | 0 | 0 | 0 | 0 | 0 | 54,9% | 39,2% |
| 3c | 30 | 93,3% | 6,7% | 0 | 0 | 0 | 0 | 0 | 93,3% | 3,3% |
| 4a | 57 | 28,1% | 56,1% | 0 | 0 | 0 | 0 | 15,8% | 7,0% | 12,2% |
| 4b | 28 | 35,7% | 60,7% | 0 | 0 | 0 | 0 | 3,6% | 0 | 3,6% |
| 4c | 32 | 53,1% | 40,6% | 0 | 0 | 0 | 0 | 6,3% | 3,1% | 6,3% |

91% of the grade 3 learners were successful but only 26% of them used illustrations. Most of these learners, almost all the learners in the English-medium class used an algorithm:
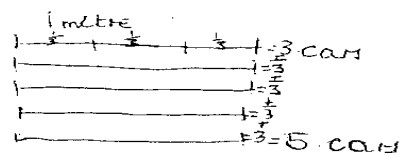$$3 \times 5 = 15.$$
37% of the grade 4 learners attempted this problem successfully while only 9% used drawings to illustrate their understanding.

EXAMPLES OF LEARNERS' STRATEGIES/SOLUTIONS:

5. **The boys in the Handwork class make a small wire car from $\frac{1}{3}$ of a metre of wire. How many cars can they make from 5 metres of wire?**

**DISCUSSION:**

During the classroom visits in these grades it was indeed a revelation for both the teachers and co-ordinator to observe and experience the learners' remarkable progress in the understanding of equal-sharing fraction concepts. The teachers followed the sequence of activities which aims to develop fraction concepts from equal sharing amongst two people. Teachers were given assistance in allowing learners to deal with the problems in an enquiry-based classroom atmosphere which most of them found to be very effective for pupils' learning. The teachers of the grade 3a and 4a classes experienced problems with the large numbers of pupils in their classes, but nevertheless tried their utmost to implement the approach. Different strategies were used in the endeavour to change their classroom practice and beliefs. This was done through demonstration lessons by the co-ordinator on their request, reading each other's lesson reports, watching each other's videotaped lessons and reflecting on it and participating in discussions during the subject meetings. The mathematics department was able to use one period every second week for discussions on materials implementation, classroom culture, etc.)

Some teachers started to realise the importance of social interaction and helping learners to develop into independent thinkers who are able to construct their own knowledge and use their own strategies to solve problems. It is evident from the variety of strategies and illustrations ( e.g. 75% of all the pupils in Item 1) used in pupils' individual attempts in the test that most of them have a basic understanding of equal sharing amongst two and more people. The majority of pupils overcame the barrier of only working with halves and quarters, especially the grade 3 pupils. 12% of all the pupils used halves, quarters or eighths. These learners might have become familiarised with meaningful concepts dealt with over a period of time.

Most of these learners, however, need much more exposure to the comparison between the sizes of fractions and equivalent fractions, which cannot be dealt with in one or two periods. (Empson, 1995). In Item 2 only 22% of all the learners were successful; 29% answered correctly without explanations. In Item four 6% of all the learners (only grade 4's) were successful while 40% of the total number of learners gave no explanation.

However, one should note that the increase in "the amount of time spent on teaching fractions may not result in better student achievement." (Carpenter, Coburn, Reys & Wilson, 1976.) The "when, how and what" of the fraction sequence need to be considered. Certain concepts may be introduced earlier or later and a different approach may be used. Ideas should be integrated so that it has "quantitative meaning to the student". The focus should not be on rules, but on the "total sequence of ideas" with the emphasis on the initial conceptual work with fractions. It is therefore disturbing to note that 91% of the grade 3 pupils dealt with Item 5 by only using an algorithm: $3 \times 5 = 15$ without illustrating their processes and understanding. Some grade 4 pupils attempted $\frac{1}{3} \times 5$ without success. The former rule might be an indication that it was supplied to them by the teacher without them making any sense of it.

A rewarding factor is the application of the fraction wall as a mathematical tool, e.g. in Item 3. This is an indication of some learners' level of understanding of the equivalence concept and their ability to apply previous knowledge. This is could be an indication of their outlook on fractions and the expansion of an awareness of the various aspects of fractions.

The fact that only 22% of the total number of learners (Item 2) and 6% of all learners (Item 4) could compare the sizes of fractions with success by solving it by direct comparison of fractions could be the result of traditional teaching of the rule that "the smaller the

denominator, the bigger the fraction (compare Lukhele, Murray & Olivier, 1999). The numerator was not taken into consideration at all. This limiting construction could be addressed by engaging pupils in a meaningful sequence of equivalent fraction activities.

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