THE INFLUENCE OF DIFFERENT REPRESENTATIONS ON CHILDREN'S GENERALISATION THINKING PROCESSES

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This is a second report on our ongoing research into students' thinking processes in generalisation situations. In this study we varied the representation of the activities we presented to children along several dimensions, namely in the type of function, the nature of the numbers, the format of the tables, and the structure of the pictures. Our results show that varying these dimensions has little effect on children's thinking – as before, few children tried to find a functional relationship between the variables, except in two simple cases, but persisted with using the recursive relationship between function values. While using recursion was successful in extending number patterns to nearby values, students find it tedious for finding larger function values. They then mostly attempted to adapt their recursion strategy in some way, but made many logical errors in the process. Our biggest concern is not so much the fact that students make many errors, but that they do not feel the need, or do not have the know-how, to verify their methods or answers against the given data.

INTRODUCTION

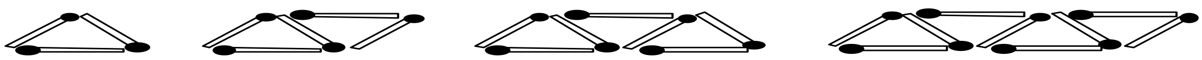
Number patterns, the relationship between variables and generalisation are considered important components of algebra curricula reform in many countries. It is evident that South Africa shares this view as can be seen in the curricular changes to the Junior Secondary syllabus (Syllabus of Western Cape Education Department, 1996). South Africa's new curriculum plans (Curriculum 2005) also emphasises the importance of generalisation as is evident from the following specific outcomes for Mathematical Literacy, Mathematics and Mathematical Sciences:

> "*Use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes.*
> *Use various logical processes to formulate, test and justify conjectures*".

There have also been suggestions to use generalised number patterns as an introduction to algebra. However, there is insufficient research that deals with the cognitive difficulties children encounter and the feasibility of such an approach. Much of the available research on children's thinking processes in generalisation reports on children's strategies in abstracting number patterns and formulating general relationships between the variables in the situation (e.g. Garcia-Cruz and Martinon, 1997; Orton and Orton, 1994; Taplin, 1995).

In a previous study (Linchevski, Olivier, Sasman & Liebenberg, 1998) we presented grade 7 students with problems like the following:

(C3): Matches are used to build pictures like this:



Picture 1 Picture 2 Picture 3 Picture 4

The table shows how many matches are used for the different pictures. Complete the table.

Picture number	1	2	3	4	5		20		100		*n*
Number of matches	3	5	7	9							

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We found that most students' generalisations and justification methods were invalid, because they are not aware of the role of the database in the process of generalisation and validation. We also found that children worked nearly exclusively in the number context and did not use the structure of the pictures at all. Also our interviewees did not view their answers as hypotheses that should be validated. For example, they did not, and seemed unable, to verify their justification against the given data pairs (1 ; 3), (2 ; 5), (3 ; 7), (4 ; 9).

Few children managed to construct a function rule to find function values. Rather, they focussed on recursion (e.g. $f(n + 1) = f(n) + 2$ in problem C_3 above), which led to many mistakes as they tried to find a manageable method to calculate larger function values. The most common, nearly universal mistake was to use the proportionality property that if $n_2 = k \times n_1$, then $f(n_2) = k \times f(n_1)$. For example, in problem C_3 above, from f(5)[1] = 11 they deduced that $f(20) = 4 \times 11 = 44$. Although this property applies only to functions of the type $f(n) = an$, children erroneously applied it to any function. It is possible that our choice of numbers might have triggered the proportional multiplication error, i.e. that our use of "seductive numbers" in a sequence like $n = 5$, 20 and 100 *stimulated* the error (we regarded these numbers as seductive from a multiplicative point of view).

Based on the above we viewed the following as questions for further research:

- whether the use of non-seductive numbers will *prevent* children from making the multiplication error, also when they encounter seductive numbers in other problems
- whether the visual impact of the table, as for example shown in problem C_3 above, also contributed to the persistence of the proportional multiplication error
- whether pictorial representations in which the function rule is "transparent" will encourage children to use the structure of the pictures to more easily find function rules.

In this paper we report on some first findings on these three questions.

RESEARCH SETTING

The activities

We designed a series of eight generalisation activities in which we varied the representation of the activities. Four activities were formulated in terms of numbers only (in the form of a table of values), and four were formulated in terms of pictures only (in the form of a drawing of the situation). Each pictorial representation had a corresponding numerical representation.

The numerical tables of values were presented in different formats: "continuous" (e.g. I_T below, in which input values for which the corresponding function values had to be calculated were included) and "non-continuous" (e.g. II_T, where the input values were not given, but were presented verbally by the interviewer). The tables were presented in both vertical and horizontal format.

The pictorial representations of the activities were chosen to be either "transparent", i.e. the function rule is embodied in the structure of the pictures (e.g. in I_P below), or "non-transparent", i.e. the function rule is not easily seen in the structure of the pictures (e.g. in III_P). As with tables, pictures were presented in both "continuous" and "non-continuous" format.

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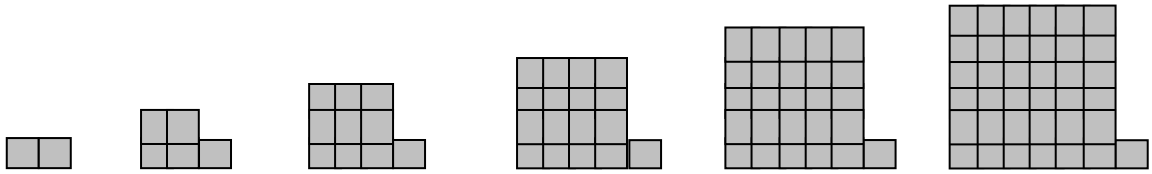
[1] Formal functional notation was not used in the actual problems or in communications with the students. It is merely used here for reporting on the students.

The questions in each activity were basically the same, namely given the values of f(1), f(2), f(3), f(4), f(5) and f(6), we asked students to first find f(7) and f(8), and then the function values of certain further input values and to explain and justify their answers and strategies. These input values were both “seductive” (e.g. 20, 60) and “non-seductive” (e.g. 19, 59). Two of the functions were linear functions of the form $f(n) = an + b$, and two functions were simple quadratic functions. We supply below a selection of the activities.

I_P [2 3]

Blocks are packed to form pictures that form a pattern as shown below:



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5 Picture 6

I_T

Tiles are used to build pictures to form a pattern. The table below shows the number of tiles in a particular picture.

Picture number	1	2	3	4	5	6	7	8	…	20	…	60	…	*n*
Number of tiles	2	5	10	17	26	37								

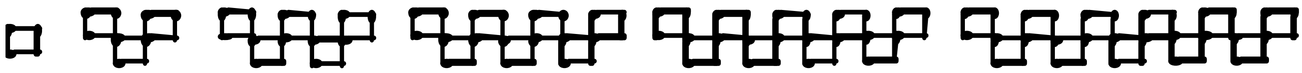
II_T

Matches are used to build shapes to form a pattern. The table shows the number of matches used to build a particular shape.

Shape number	Number of matches
1	4
2	12
3	20
4	28
5	36
6	44

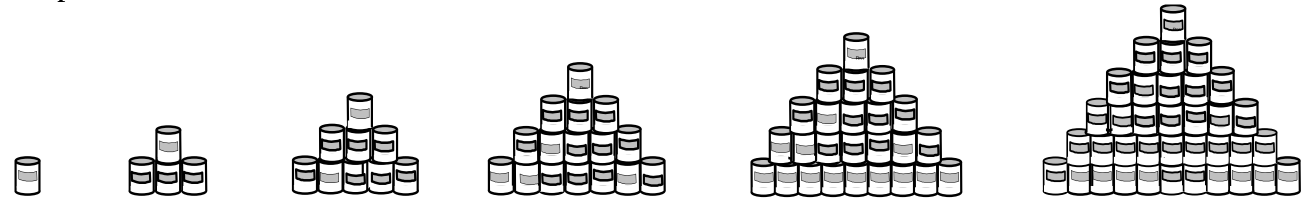
II_P

Matches are used to build shapes. A different number of matches is used to build each shape.



Shape 1 Shape 2 Shape 3 Shape 4 Shape 5 Shape 6

III_P



Pyramid 1 Pyramid 2 Pyramid 3 Pyramid 4 Pyramid 5 Pyramid 6

[2] The subscript P indicates that the problem was presented in a spatial context in the form of a pictorial representation of the situation and the subscript T indicates the problem was presented in a numerical context in the form of a table of values.
[3] All the drawings were presented to students in vertical format, but is here given horizontally due to space considerations.

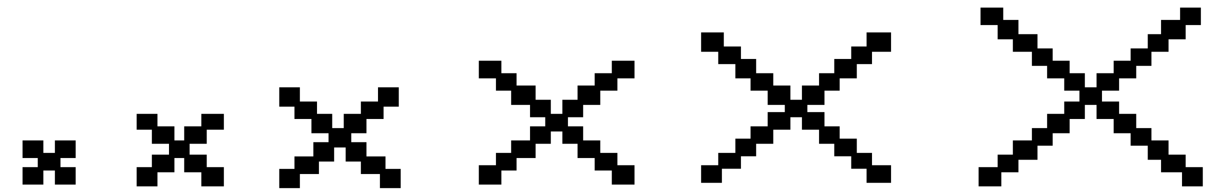
III_T

Peter uses blocks to build pictures that form a pattern. The table shows the number of blocks he needs to build a particular picture.

Picture number	Number of blocks
1	1
2	4
3	9
4	16
5	25
6	36

IV_P

Tiles are arranged to form pictures like this:



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5 Picture 6

Methodology

We interviewed ten grade 8 students at one of our project schools in a historically disadvantaged area of Cape Town before they had received any instruction on patterns, sequences or algebra. The students were selected by the teacher so that they were representative of the grade 8 class. Each student was interviewed three times in 45-minute sessions by either two or one of the researchers. The interviews of each student took place 5 to 7 days apart. In the first two interviews each student was presented with three activities and they were asked to explain or clarify their answers or strategies, but were not challenged in any way. We wanted to ascertain what they did spontaneously. The pictorial and numerical activities were presented to the children on different days. In the third interview they were given two further activities and then asked to reflect on some of their previous solutions and to justify their answers. Based on their responses the researcher asked questions to create cognitive conflict. All interviews were videotaped. In addition to the video protocols, written transcripts of the subjects' verbal responses as well as their paper-and-pencil activities will be used in the analysis. The analysis will be used to design a teaching intervention aimed at addressing the cognitive difficulties children have in the processes of generalisation.

RESULTS AND ANALYSIS

Most students had no difficulty finding f(7) and f(8) in any of the activities – they either found and used the function rule correctly, or used recursion correctly for these nearby values. However, in trying to find a manageable strategy for finding further-lying function values, children invented a variety of different strategies, both correct and incorrect. These strategies and their frequency are summarised in Table 1. We will refer back to the table in our analysis.

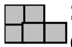
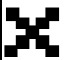
The nature of the function

Finding function rules

It is interesting to note from the data in Table 1 that more than half of the students found and used the function rules in activities I_P and II_T. These both represent simple quadratic functions. One could be tempted to conclude that students easily recognise such simple quadratic function rules.

Table 1

Number of students using each strategy per activity

Activity number and representation format	Recursion, counting-on	Proportional multiplication error	Decomposition of input value, e.g. f(n) = f(a) + f(b) + f(c) where $a + b + c = n$	Difference method f(n) = $n \times d$	Extended recursion: f(n) = ($n - k$)d + f(k) (d is common difference)	Function rule	Other
I_P  Transparent picture, continuous Seductive input values (20, 60, n) Quadratic function (f(n) = $n^2 + 1$)	2		1		2 wrong variations	5	
I_T Horizontal continuous table Seductive input values (20, 60, n) Quadratic function (f(n) = $n^2 + 1$)		4		1	1 wrong variation	3	1
II_P  Transparent picture, continuous Non-seductive input values (19, 59, n) Linear function (f(n) = 8n – 4)	1		1	3	1 2 wrong variations	2	
II_T Vertical non-continuous table Seductive input values (20, 60, n) Linear function (f(n) = 8n – 4)		4		3	1 2 wrong variations		
III_P  Non-transparent picture, non-continuous Non-seductive input values (23, 79, n) Quadratic function (f(n) = n^2)	3		1	1	2 wrong variations	2	1
III_T Vertical non-continuous table Seductive input values (29, 87, n) Quadratic function (f(n) = n^2)	1	1			1 wrong variation	6	1
IV_P  Transparent picture, non-continuous Seductive input values (20, 60, n) Linear function (f(n) = 4n + 1)	1	2		2	2 2 wrong variations	1	
IV_T Horizontal continuous table Non-seductive input values (23, 117, n) Linear function (f(n) = 4n + 1)	1		3	1	2 2 wrong variations	1	

One immediately, however, also notices the marked differences in students' responses for the same functions in the picture and the table contexts. I activity I_P the picture is transparent, but students find it much more difficult to recognise the same function rule from the equivalent table in activity I_T. In activity III on the other hand, children easily find the rule in the table, but not in the non-transparent picture.

It is clear that students found it much more difficult to formulate function rules for linear functions. From our interviews it seems that children try to construct simple multiplication (proportional) structures, but when it does not fit the database, they quickly give up and then invent all kinds of error-prone recursion strategies.

Recursion
When students focus on recursion patterns, however, they find the constant difference between consecutive terms in linear functions much easier to handle than the changing (increasing) difference in quadratic functions, leading to many errors. We describe these strategies and errors in the following sections.

Seductive vs. non-seductive numbers
The proportional multiplication error
In our earlier work with grade 7 students we found a persistence with the erroneous proportional multiplication error. In this study six of the ten students interviewed used it at least once in the series of activities. For example:

Interviewer:	How many tiles in Picture 20? (*in* IV_P)
Peter:	OK, I am using 5 (*meaning n = 5; f(5) = 21 in the picture*) to get to 20. So 21 times 4 is 84, because 5 times 4 is 20.
Interviewer*:*	How many tiles do you think we'll use for Picture 60?
Peter:	(*Pause ... looking at the numerical "table" he had prepared from the given database*) ... Picture 10 is 41 tiles, so 41 times 6 … the answer is 246.

This erroneous strategy was used only with what we call "seductive numbers". Vergnaud (1983) argues that this is an over-generalisation of the many direct proportional relationships that students are intuitively aware of from an early age. Fischbein et al (1985) posit that children generalise the way they were initially taught in school before they develop a critical attitude and that some mental behaviours tend to act beyond any formal control because these behaviours shape the ideas and the facts at hand in a meaningful way.

When students could easily find the function rule the nature of the input values was immaterial, i.e. they did not make the multiplication error, even for seductive numbers.

Extending recursion
A few students managed to adapt their focus on recursion to a manageable strategy for finding further-lying function values. This extended recursion method is symbolised by $f(n) = (n - k)d + f(k)$, where (d is the common difference between consecutive terms. Here is an example:

Interviewer:	OK, Shape 59? (*How many matches in Shape 59 in* II_P*?*)
Hamid:	So first I subtract 19 (*he had previously calculated f(19) = 148*) by 59 and then you get your answer of 40 and then I times it by 8 (*the common difference between terms*) and then I get my answer and then I add it by 148, that is Shape 19's answer.

Some children used this method also in the case of seductive numbers.

Some children also worked with this method, but they often seemed to lose track of all the details. This was mostly because they worked verbally, and did not write down information or their strategy. In this example Voda correctly calculates $(n - k) \times d$, but then does not add f(k):

Interviewer: Shape 60? (*how many matches in Shape 60 in* II_T)
Voda: (*works on calculator*) 320
Interviewer: Please explain to us
Voda: I subtracted 20 by 60 *(he means 60 – 20)* and then I times 40 by 8.

While the extended recursion method is correct for linear functions, many students also erroneously applied it to or adapted it for the quadratic functions. For example:

Interviewer: Ok, and then Picture 20? (*How many matches in Picture 20 in* I_T*?*)
Harold: I subtract 20 by 8 (*he had previously calculated f(8) = 65*) . . . I subtract 8 by 20, then I get 12 … with that 12 I times by 2 is equal to 24 … then I add 24 by 15, is equal to 39 then I add 39 to 65, is equal to 104.
Interviewer: Just explain the 15 please
Harold: That's the 15 I added by 50 (*f(7)*) to get 65 (*f(8)*).

Decomposition of input value

The introduction of "non-seductive numbers" gave rise to other inappropriate strategies when students could not find a multiplicative relationship between the non-seductive numbers. For example:

Interviewer: How many cans do we need to build Pyramid 23? *(activity* III_P*)*
Errol: [*Long pause … staring at the database. In the previous problem (activity* I_P*) he prepared a "table" of values using recursion up to f(20) and then used the proportional multiplication method to predict f(60). He presses on the calculator 64 + 64 + 49 + 30*]…ya .. it's ..207.
Interviewer: Can you explain how you got your answer please?
Errol: Ya, … 8 (*referring to Pyramid 8*) is 64 and 7(*referring to Pyramid 7*) is 49 … so I add 64 + 64 + 49 and then another 30.
Interviewer: Why did you add those numbers?
Errol: … Uhmm … because if I take … 8 + 8 + 7 = 23 … so I take the number by 8 (*referring to f(8)*), then I add it to itself and then I add the number by 7 to it.
Interviewer: Okay I understand, but where does the 30 come from?
Errol: I minus the 8 by 23 (*he means 23 minus 8*), so I get 15 … so I multiply by 2, then I get 30.
Interviewer: Can you explain to me why you did that?
Errol: … the difference in between is growing by 2 every time …

This method, generally symbolized by $f(n) = f(a) + f(b) + f(c)$ where $a + b + c = n$, was also used by other students. It seems that this strategy is born out of students' inability to find factors for numbers such as 19, 23, 59, 117. One student calculated f(60) = f(20) + f(20) +f(20). This method could be seen as a variation of the proportional multiplication error, rooted in the close relationship between repeated addition and multiplication

The difference method

The erroneous difference method, symbolised by $f(n) = n \times d$ was invoked with both "seductive" and "non-seductive" numbers.:

Interviewer: Ok, how many in Picture 23? (*How many tiles in Picture 23 in* IV_T*?*)
Linda: (*works on calculator)* . . . 92
Interviewer: Just explain please?
Linda: It will take too long to add 4 every time (*she previously found a constant difference of 4 between the terms of the sequence*). So I just said 23 times 4.

It could be argued that the inclusion of a direct proportional example where $f(n) = an$ would have presented children with the conflict situation which would then perhaps limit both the proportional multiplication and difference method errors. However, Orton and Orton (1994) included such an example, but these errors still persisted.

The visual impact of tables

From Table 1 it is clear that the visual presentation of the numbers in a table format for the function did not impact on the errors children made. The table in activity I_T was horizontal and "continuous" whereas the table in II_T was vertical and "non-continuous". Four children made the proportional multiplication error in both these examples. One student committed the difference method error in I_T whilst 3 students committed the error in II_T. The way we presented the questions as "continuous" or "non-continuous" in the picture activities also did not effect children's strategies. This can probably be explained by the fact that when the input numbers were not presented in writing, children made their own "continuous" "tables", so the visual distraction remained.

"Transparent" vs. "non-transparant" pictures

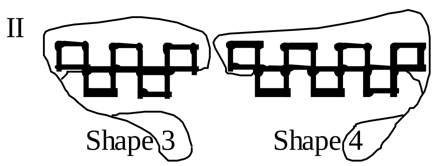
In I_P five students successfully recognised the function rule from the structure of the picture. Two other children, when counting the number of tiles in each picture aloud, used the structure of the picture : " … 4×4 + 1 = 17 , … 5×5 + 1 = 26 , … 6×6 + 1 = 37 …" but did not reflect on the structure they had verbalised and thus could not find the function rule.

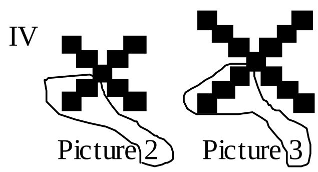
In II_P most students recognised that 2 squares (8 matches) were being added but then converted to numerical mode, constructing their own "table" of values, e.g. " … 1 = 4, 2 = 12, 3 = 20, etc.". Only two children described the function rule from the structure of the pictures, namely as (n + n – 1) × 4 and n × 4 + (n – 1) × 4 respectively.

No child could recognise the function rule of using the picture as the database in III_P. Two students found the function rule once they reverted to the number context.

Only one student used the structure of the picture in IV_P to identify the function rule.

It seems that these students do not have the necessary know-how of *how* to use the structure of a picture to find a functional relationship. If one wants to find a function rule in a table, one necessarily takes some specific value of the independent variable (input number) and tries to construct a relationship between this input-output pair. In the case of pictures, few children seem to intentionally take a specific input number and try to see this number in the structure of the picture, as illustrated in the following diagram:





Of course, it further requires a rich number sense, e.g. in II to see a further relationship in the numbers (2 is one less than 3, and 3 is one less than 4) before one can formulate the function rule $[n + (n - 1)] \times 4$. In IV one must see the multiplication or equal addition structure before one can formulate the rule $4 \times n + 1$. A weak number sense will therefore also contribute to students' difficulties in using the structure of pictures to see the general in the particular required to formulate function rules.

Most students could see and use the structure of the pictures in a *recursive* way, e.g. in II students used the structure that 2 squares (8 matches) are added each time, and in IV they used the structure that 4 tiles are added to each successive picture. However, this did not help them to find the function rule, and students mostly then constructed a table of these values and then used the numbers in the table inductively. Of course, one could use the extended recursion method to use this recursive structure to formulate the function rules as $4 + 8(n - 1)$ and $5 + 4(n - 1)$ respectively. It is interesting that all cases of the use of the extended recursion method were in the context of tables and none in the context of pictures.

In all activities where students identified a function rule, most of them described their rule in words rather than using symbols. A distinction can also be made between those who could verbalise the rule using general language (e.g. in I_P "..times the number by itself and add 1") and those who could only verbalise the rule in terms of a specific number (e.g. "…if I know the number, say if it is 100,.then I times 100 by itself and add 1).

Verification of strategies

Consider the following protocol:

Interviewer: Ok, Shape 19? (*How many matches in Shape 19 in II_P?*)

Peter: (*Peter successfully found f(7) and f(8) by counting the number of squares and then multiplying by 4 to get the number of matches. Now he starts making a systematic table of the number of squares in each Shape, using a recursive pattern:*

7	8	9	10	11	12	13	14	15	16
13	15	17	19	21	23	25	27	29	31

He then stops and goes back to looking at the pictures again.)

OK, I realised if I do this it is a bit of a hassle, so I looked at the pattern (*in his database*) and I figured the difference (*between f(n) and n*)

I took here (*pointing at f(5)*) . . . the difference between 9 (*f(5)*) and 5 (*n*) is 4 and by number 6 it is 5 . . . yes (*he checks again*) . . . 5. And by number 7 it is 6 and by number 8 it is 7. So I just tried it out. So I said to myself OK it is right and it will take too long to do it like this (*referring to his table of values*). So Shape 19 is 19 + 18, is 37, so 37 blocks times 4 gives you . . . 148 (*using the calculator*).

Interviewer: OK, and in Shape 59?

Peter: OK, its 59 + 58 is equal to 117, that is the number of blocks and then I take 117 (*enters on the calculator*) times 4 which gives the number 468, that is the number of matches in Shape 59.

Clearly, Peter has constructed an efficient rule, which we can symbolise as $[n + (n - 1)] \times 4$, based on a sound analysis of patterns in the given and extended database, and he verified that his pattern holds against the database several times. He was convinced and he could use the method with assurance.

However, this style of working stands in stark contrast to most students' approach to such generalisations. While the students who used a function rule necessarily deduced the rule from the database, the other strategies reported in this paper are mostly not based on the database – students did not find the methods in the database, nor did they check it against the database. This applies to correct as well as to incorrect strategies. Students seem not to realise the need to validate their generalisations, and seem not to have the know-how of how to validate a generalisation against the database.

DISCUSSION

As in our previous study, students worked nearly exclusively in the number context and not with the pictures, favoured recursion methods, had difficulty in finding function rules and made many errors, including the proportional multiplication error. There is, however, one marked difference, namely the variety of strategies used by the students in the present study in comparison to the previous study. Of course this could be attributable to the differences in the subjects, who are at a different grade level, and from a different socio-economic background. We would argue, however, that the difference is mainly attributable to the introduction of non-seductive numbers in our activities.

It is for this reason (the greater variety of strategies), that we plan to extensively use non-seductive numbers in our planned intervention. However, as is evident from the examples in this paper, the use of non-seductive numbers will probably not prevent the ubiquitous proportional multiplication error when students encounter seductive numbers, nor will it prevent the other erroneous strategies reported here. For that we believe we should address two more fundamental issues, viz.

1. The development of an awareness of the need to view any strategy as an hypothesis that should be validated against the database, and a focus on skills of how to do it.
   When one looks at the variety of strategies used by children, one can probably safely say that they have the ability and flexibility to find many patterns and relationships between numbers. The problem, however, is that most children are finding *random* relationships between the numbers without reference to the given database. We are struck not so much by the frequency and persistence of children's errors, but by their lack of an essential aspect of "mathematical culture", namely to view any strategy as an hypotheses that should be justified or verified against the given database. It seems that students lack simple *strategic knowledge*, e.g. to test an hypothesis against special cases.
2. A more explicit study of the properties of different function types, and a comparison of such properties to become aware which properties apply to which function types.

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