"SHAPE AND SPACE"
AN APPROACH TO THE STUDY OF GEOMETRY IN THE INTERMEDIATE PHASE

Kate Bennie

MALATI

The Malati Geometry Group has identified the importance of spatial skills in the study of geometry and has developed a Spatial Skills package for use in grades 4 to 7. During this workshop participants will be given an opportunity to engage in detail with selected activities in this package. Responses from the group, as well as examples of pupil material, will be used to explore the Malati approach to spatial skills and geometry, the assessment of these topics, and the alignment of the Malati approach with Curriculum 2005.

Background:

When organising a geometry workshop for teachers, a Malati project worker received the following response from a grade 7 teacher:

Teacher: *They [geometry workshops] are always the same component.*
Project Worker: *And what's that?*
Teacher: *Geometry… Always cutting and folding angles. Always.*

This comment sums up the attitude of many teachers to geometry, and sadly it seems, to some geometry workshops. It is clear from the little research that has been done on the teaching and learning of geometry in South African primary schools, and from general observations of the situation, that there is a great need for improvement. INSET workers have noted that some primary teachers even ignore the presence of geometry in the curriculum. As a result of these observations, geometry has been the focus of a number of recent INSET initiatives. The above quote suggests, however, that some teachers still have a negative attitude to the teaching of the topic.

There is also general concern about the study of geometry in the high school, where the study of the topic has traditionally focused on the preparation of pupils for, and the study of, formal Euclidean geometry. Many teachers can attest to the problems experienced by both teachers

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and pupils with the teaching and learning of this topic. Mathematics educators such as the van Hieles would argue that the problem lies in the approach taken at both primary and secondary school[1]. Furthermore, Bishop indicates that there is a lot more to geometry than the traditional formal approach:

> *Geometry is not the study of proofs! Geometry is the study of spatial relationships that can be found in the three-dimensional space we live in and on any two-dimensional surface in this three-dimensional space.*
>
> (Bishop, 1983)

Malati Geometry:

At Malati the importance of the study of geometry at all levels of school has been recognised, as well as the need for materials development and teacher support in order to improve on the current situation. The following objectives for the study of geometry at school have been identified:

- To develop spatial skills
- To learn to use a number of tools to solve problems
- To develop a sense of the structure of mathematics.

The first objective has been identified as the priority in the primary school. Extensive review of existing literature and discussion with educators in the Western Cape and elsewhere has been used in the development of a Spatial Skills package for use in grades 4 to 7. These activities are designed for pupils on the van Hiele Visual level and movement towards the Analysis level is encouraged.

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[1] According to the van Hiele theory, a pupil can move through five (later revised to three) hierarchical levels of geometric understanding. The first three levels are appropriate for the teaching and learning of geometry at the intermediate and senior phases:

- Recognition/ Visual Level: The pupil identifies, names and compares geometric figures on the basis of their appearance as a whole.
- Analysis Level: The pupil analyses figures in terms of their properties, establishes the properties of figures empirically, and uses the properties to solve problems.
- Informal Deduction/ Ordering Level: The pupil understands the relations within and between figures, gives informal deductive arguments, and formulates and uses definitions.

The van Hieles argue that a pupil cannot perform with understanding on one level if s/he has not passed through preceding levels.

Malati Spatial Skills:

There is clearly no consensus in the literature on the classification and naming of spatial skills: terms such as spatial sense, spatial perception, spatial insight, spatial visualisation and spatial orientation have been used to describe this area of thinking. The Malati Geometry Group has chosen to use the term "Spatial Skills" to refer to a range of related skills necessary for the interaction with shape and space. The materials require that pupils work with two-and three-dimensional objects as well as transformations between the two dimensions.

Consider, for example the following activity from the Malati package:

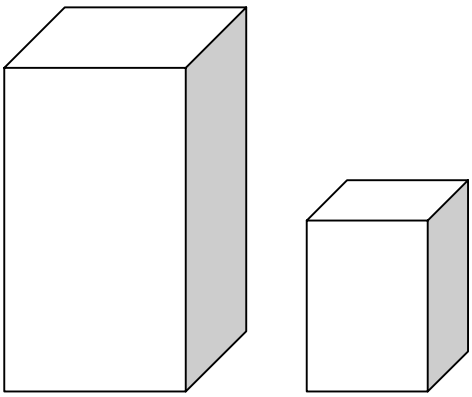
Draw what you would see if you were looking at these two boxes from

(a) above

(b) behind

(c) point A on the left of the boxes

(d) point B on the right of the boxes.



Some primary pupils will able to work with this two-dimensional representation of the three-dimensional boxes, but others will need to work with the actual boxes. It is, of course, expected that different pupils will use different strategies in completing this task, but it could require the use of the following skills:

- physical exploration of space and the relationship between the viewer and objects in this space
- interpretation of two-dimensional representations of three-dimensional objects

- exploration of relationships between objects in two-dimensional representations of three-dimensional objects
- visualisation of objects from different positions (small objects)
- mental transformation of three-dimensional objects: rotations and reflections
- left and right discrimination.
- drawing of three-dimensional objects in two-dimensions.

Other skills required in the Malati activities are:

- location of places from different orientations
- transformation three-dimensional objects into two-dimensions and vice-versa.
- mental transformation of two-dimensional objects: rotations and reflections
- visualisation of objects from different positions (larger scenes, for example, the whole classroom or school).

During the trialling of these materials, Malati staff are carrying out research into precisely what skills pupils use when completing the activities.

Spatial Skills and Curriculum 2005:

The Malati activities have been designed in alignment with Curriculum 2005 which emphasises the following spatial skills in various Specific Outcomes for the intermediate phase:

- facility with geometric patterns in two- and three-dimensions.
- exploration of tessellations and transformations
- determining of location , that is, drawing and reading of maps
- description and understanding of the representation, position, orientation, and changes in the shape of objects in space.

Rationale for Spatial Skills

What is the value of pupils completing a package on spatial skills? Smit (1998) stresses the importance of these skills:

> *Without spatial sense it would be difficult to exist in this world – we would not be able to communicate about position, relationships between objects, giving and receiving directions or imagine changes taking place regarding the changes in position and size of shapes.*

Pupils are surrounded by spatial settings and the ability to perceive spatial relationships is important for:

- everyday activities, for example, reading maps, playing sport.
- technical and scientific occupations
- The study of mathematics itself: Most teachers can attest to the problems pupils have when studying surface area and volume of three-dimensional objects in grades 8 and 9. Mathematics educators' interest in the relationship between spatial ability and the development of geometric concepts is a recent one, but there is a growing body of data that indicates that pupils can perform on geometric tasks when these are related to their spatial abilities. There is also increasing interest amongst mathematics educators in the relationship between the use of visual imagery and general mathematics achievement.

The study of spatial skills in the Malati curriculum is thus not just regarded as preparation for the study of more formal mathematics in the secondary school, but is also seen to have value in its own right in that it provides pupils with an opportunity to interact in the changing space in which they live.

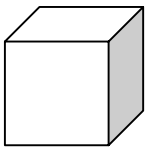
Where is the Mathematics?

Spatial activities can be used in many school subjects, for example, in geography, art and physical education. But why include a spatial skills package specifically in the **mathematics** curriculum? Bishop provides a motivation for this:

> *Geometry is the mathematics of space, and mathematicians approach space differently form artists, designers, geographers, or architects. They search for mathematical interpretations of space. Mathematics educators, therefore, are concerned with helping pupils gain knowledge and skills in the mathematical interpretations of space.*
>
> (Bishop, 1983)

But where is the mathematics that Bishop talks about? Van Niekerk points out that geometrical figures are, in fact, hidden in spatial objects, thus spatial skills activities provide excellent opportunities for dealing with two- and three-dimensional figures. For example this activity requires that pupils use visualisation skills, but also requires that pupils study the three-dimensional figure (a cube) as well as its two-dimensional net:

*This is a drawing of a cube **without a top**.*



1. *Which of the nets below can be folded to make this shape?*

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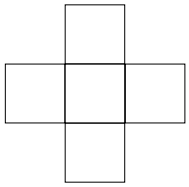

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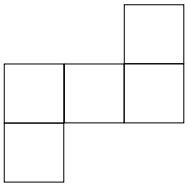

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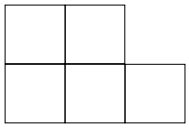

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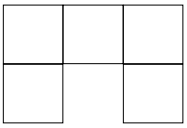

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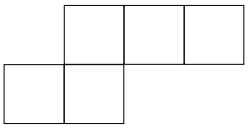

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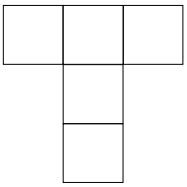

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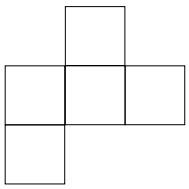

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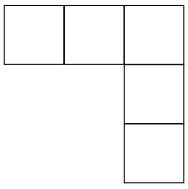

2. *Colour the square that will form the bottom of the box on each net you choose.*
3. *Try to draw a different net for this box and colour the square that will form the bottom of the box.*
4. *Now draw the net of a cubic box **which does have lid**.*

Workshop Outline:

Workshop participants will be given the opportunity to engage with selected activities from the Malati Spatial Skills package. Responses from the group, as well as actual examples of pupil work, will be used

- to highlight certain aspects of the Malati approach to mathematics teaching and learning in general
- to highlight the Malati approach to spatial skills, some of which has been described above
- to identify actual spatial skills required for the completion of the activities and to make a preliminary report on the findings of research into the skills used by pupils
- to discuss the use of Malati materials for the implementation of Curriculum 2005
- to discuss possible forms of assessment of pupil performance on activities of this nature.

References:

Bishop, A. J. (1983) Space and geometry. In Lesh, R., & Landau, M. (Eds.), **Acquisition of mathematical concepts and processes**. New York, US: Academic Press.

Department of Education (1997*)* **Intermediate phase policy document**. Pretoria, South Africa.

Fuys, D., Geddes, D., Lovett, C. J. & Tischler, R. (1988) The van Hiele model of thinking in geometry among adolescents [monograph number 3]. **Journal for Research in Mathematics Education.** Reston, VA: NCTM.

Smit, S. (1998) **An introduction to geometry through shape, vision and position**. Unpublished manuscript, University of Stellenbosch, Stellenbosch, South Africa.

Van Niekerk, R. (1995) From spatial orientation to spatial insight: A geometry curriculum for the primary school. **Pythagoras, 36,** 7-12.