"Draw What You Would See From Behind": An Analysis of Grade 5 and 6 Learners' Strategies for Solving Spatial Ability Activities[1]

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This paper reports on one aspect of a research project designed to obtain some clarity on the notion of "spatial ability". Questions which arose from an extensive review of literature in the field and attempts to build a model of "spatial ability" for use in mathematics education were investigated empirically. One theme explored was the strategies used by grade 5 and 6 learners to solve tasks which require that learners view representations from different perspectives. Five different strategies used on a variety of tasks were identified. The results are used to reflect on a preliminary model of "spatial ability" and to suggest areas for further research in this field. Certain issues relating to the methodology are also discussed.

Introduction:

Recent innovations in mathematics curricula have included calls for classroom activities that promote or require the use of "spatial skills". For example in Curriculum 2005 in South Africa, one of the ten Specific Outcomes for Mathematical Literacy, Mathematics and Mathematical Sciences focuses on the study of shape, space and time. Furthermore, research in the past few decades has produced varying results on the links between "spatial ability" and performance in geometry and in mathematics in general.

In attempting to develop a "vision" for the teaching and learning of geometry at school, staff at Malati have formulated the following objectives for the study of geometry from grades 1 to 12: to develop "spatial skills"; to learn to use a number of tools to solve problems; and to develop a sense of the structure of mathematics.

As one of the materials developers charged with the creation of tasks for the development of "spatial skills" in the intermediate phase, and having little experience working in this phase, I undertook an extensive review of the literature on "spatial ability". A number of dilemmas and questions arose out of this, namely,

- There is little consensus on the nature of "spatial ability" and on how to test / identify the phenomenon.
- Most of the work done in this area has been done by developmental psychologists and factor analysts. Recent reviews of the literature in these fields have attempted to synthesise the research for use in mathematics education. The work of van Hiele, Lesh, Krutetskii, Skemp, Bishop, Mitchelmore, Wattanhawa and Fennema has been identified as making a contribution in this regard (Bishop,1980; Clements 1983).
- Certain questions regarding specific aspects of the existing research need to be investigated and clarified, for example, what strategies do learners use when required to view "objects" from different perspectives, and do these strategies differ for "small-scale" and "large-scale" space?[2].

[1] Funding for the Master of Philosophy (Mathematics Education) Degree of which this study forms part was obtained from the Centre for Science Development (CSD).

[2] Views on the nature of "space" and "objects" within this space vary. I am adopting the view that "shape" and "space" result from interaction between the individual and "objects" in space. These "objects" obtain their properties as a result of their relative position to other "objects" and to the individual. In this sense, therefore, one

My review of the literature revealed that the task of designing "spatial tasks" as part of the Malati curriculum was going to be more complicated than expected. Bishop (1983) describes the task for mathematics educators: "It is clear (to a mathematics educator, at least) that there can never be a 'true' definition of spatial ability: We must see definitions and descriptions of abilities and processes that help us to solve our own particular problems". With this daunting task in mind, I resolved to use the existing literature as well as my own empirical research to build my own model of "spatial ability" which could be of use in the development of "spatial tasks" at Malati. The resulting research report is also to be submitted as a Masters dissertation. This paper focuses on one theme explored in this research project.

Preliminary Model of "Spatial Ability":

In order to place the theme under discussion in context, it is necessary to describe briefly the preliminary model of "spatial ability" developed following the literature review. This review revealed that, within the variety of fields interested in "spatial ability", there is some consensus on a general definition of the phenomenon, that is, that it involves the ability to:

(a) form and retain a visual image[3]

(b) mentally manipulate this image or oneself in relation to the image in problem solving.

I would suggest that a combination of these two abilities is necessary for the accurate perception of and interaction in the visual world. Piaget's use of the word "spatially mature" is useful in describing a learner with these skills (in Clements, 1983). For it seems that the process of attaining this level of interaction is a developmental one: A number of researchers suggest that learners move from being able to interact with concrete "objects" in the everyday world, to being able to form visual images of these "objects", to being able to use this visual imagery in problem solving (even when the "object" is no longer present).

There is not, however, consensus when it comes to identifying the specific strategies used in the problem solving in (b) above. Some of the dilemmas I faced when engaging with the literature in this regard were selected for further research. One of these themes is described below.

Viewing "Objects" from Different Perspectives:

Two aspects of "spatial ability" which recur in the literature are

(a) the ability to view objects from different perspectives

(b) the ability to mentally manipulate objects.

A number of researchers classify different "spatial tasks" according to the two strategies thought to be used in solving these tasks. For example, Tartre (1990) uses the work of McGee to classify two different kinds of tasks. She claims that each of the two categories are composed of several tasks and can be differentiated on the basis of what is to be moved, namely the object or the perspective of the viewer. Each is described as follows:

- Spatial Visualisation Tasks: These require that "the viewer mentally move the representation of the indicated object, either turning it, as in mental rotation, or rearranging

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does not distinguish between "objects" and their representations, as all "objects" can be regarded in some way as representations.

[3] There is a debate in the literature regarding the nature of "visual imagery", for example, are visual images "pictures in the mind" which are isomorphic to the referent, or are they propositional in nature? I am not in a position to comment on the actual nature of these images, but I do find Presmeg's definition useful: She defines a visual image as "a mental scheme depicting visual or spatial images" (1994).

it, as in mental transformation"[4]

- Spatial Orientation Tasks: In these tasks "nothing need move at all except the perspective of the person" doing the task. "Spatial orientation skill requires that the subject understand a visual representation or comprehend some change that has taken place in two representations".

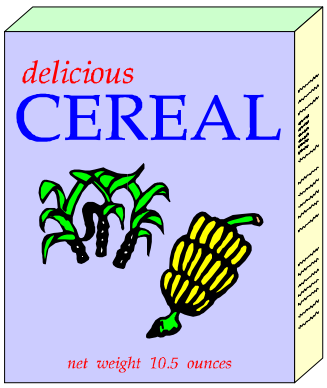
French (as quoted in Eliot and Hauptman, 1981) used much of the work of factor analysts to identify three spatial factors, two of which are

- The Orientation Factor: "requiring a person to remain unconfused by varying orientations in which a configuration may be presented"
- The Visualisation Factor: "ability to comprehend imaginary movements in 3-dimensional space or to manipulate objects in the imagination".

The distinction between tasks which require mental manipulation of objects and those involving the orientation / perspective of the viewer is also evident in the work of McGee (as quoted in Clements, 1983) and Thurstone (as quoted in McFarlane Smith, 1964).

But are these two skills distinct? Will learners use only the suggested skill on a particular task? Consider for example the following task in which learners are required to view the representation of a box from different perspectives[5]:

Look carefully at this box:



*(a) Draw what you would see if you were looking at the box from **behind**.*
*(b) Draw what you would see if you were looking at the box from **above**.*
(c) *Draw what you would see if you were looking at the box from **the side.***

It seems possible that this single task could be solved in two ways:

- By the viewer mentally rotating the box so that it faces a different way. ("Spatial Visualisation" in Tartre's scheme).

[4] Tartre claims to be using the classification of forms of manipulation suggested by Kersh and Cook: They divide spatial visualisation into two categories, namely, "mental rotation" (the whole object is moved) and "mental transformation" (this involves "different operations on separate parts of the mental image").

[5] It is important to note that this picture is a two-dimensional representation of a three-dimensional object so is not in fact "a box", but a picture of a box. Although such drawings serve as a mediation / representation for a spatial situation, we often tend to forget that the "reading-off" of these drawings is not self-evident. Does it not require some sort of "spatial skill" to interpret such a representation? Doudy (1998), for example, suggests that the ability to read two-dimensional representations of three-dimensional objects requires both an ability to build mental images as well as some knowledge of two-dimensional geometry at a certain level.

- By the learner re-orientating his / her position in relation to the box, and visualising a movement in his / her own position in order to view the "box" from a different perspective. This would be classified as a "Spatial Orientation" task in Tartre's scheme.

Other questions which arise when considering problems of this type are:
- Do learners employ any other strategies on tasks of this nature?
- Do strategies differ when learners have to visualise two or more "objects" from different perspectives? (Such a task would require that a learner attend to the relative position of the "objects" as well as the relative position of himself / herself in relation to these "objects".)
- Does the presentation of the task influence the selection of a strategy? For example, does a learner use different strategies when given possible answers (as in a multiple choice format) to those used when required to draw the solution as in the above example?

Methodology:
The literature indicates that factor analysts have tended to use time-limited written tests and that developmental psychologists have used clinical interviews for collecting data on "spatial ability". The question we need to ask is, "What are these tests or data collection methods actually measuring?" Questions have been raised regarding the appropriateness of these methods for research in the field of mathematics education. Furthermore, research into the relationship between drawing skills and "spatial ability" sounds warnings about the requirement that learners present their answers in the form of drawings: Mitchelmore (1980) suggests that "representational ability" can lag behind "spatial perceptual development" and reminds us that different cultures have different preferred drawing styles. Clements (1983) lists a number of researchers whose work supports the idiosyncratic nature of performance on spatial tests. These results have been used to argue that learners be assessed on a variety of tasks in different contexts.

The data collection methods adopted for this study were determined by this debate on the methodology in this field, the nature of the questions (arising from my literature review) which required that I explore the strategies used by learners, and the circumstances of my work at Malati.

<u>The Activities:</u>
These were selected from the first draft of one the Malati "Spatial Skills" Packages. The assumption is thus being made that these activities are "spatial" in nature. These activities require that learners visualise the view of "objects" from different perspectives. These tasks can be classified as follows:
- Visualising from different perspectives: one "object", for example, a cereal box; solution required in the form of a drawing (Activities 1 and 4): Each item uses only one object, for example, a cereal box, a tin of beans, an egg, a sandwich and a bottle. Learners are required to draw what they think the objects would look like from behind, above, and the side.
- Visualising from different perspectives: two or more "objects", for example, two boxes; solution required in the form of a drawing (Activity 3): This activity requires that learners draw what they would see if the looked at the two-dimensional representations of two boxes of different sizes from above, below, a point each on the left and the right of the "objects", and from behind.
- Visualising from different perspectives: two or more "objects", for example, two boxes; possible solutions in the form of drawings are provided (Activity 2): In this activity

learners are presented with a picture of a flower pot, a plate and a cup. A drawing of an eye is used to show views of these "objects" from different perspectives. Learners much match the view from these four different perspectives with four given drawings of the scene and explain their reasoning.

The Data Collection:

This was collected while learners were completing the activities during their ordinary geometry lessons. In each class I sat with a specific group of learners so that I could observe the process of completing the activities. I audiotaped the discussions between pupils and with myself, wrote detailed field notes recording these discussions and learner gestures, and collected the written responses.

Comparing clinical observations to classroom observations, Bishop (1980) questions whether the classroom context might alter behaviour. It is, of course, possible when using the data collection employed in this study, that what the teacher says or what other learners in the group might say could influence the performance of specific learners. For this reason such interactions were noted in the field notes.

The Sample:

The selection of the sample was determined by the choice of the data collection method discussed above and by the context in which I work at Malati. The school at which the research was conducted in one of two primary schools involved in the Malati Project in which instruction is conducted exclusively in English. The other such school experienced a number of staffing changes during the year which it was felt might affect the data collection process.

The choice of grades and particular classes in each of these two grades was influenced by the need for me to work closely in the classroom context with the teachers of these classes. Colleagues assisted me in negotiating with teachers who were already co-operating enthusiastically in the Malati project. One class in each of grade 5 and grade 6 was selected.

The nature of the data collection process as described above required that I select from each class a group that was functioning well and in which all pupils were participating in the discussion.[6] Initially the two teachers were asked to identify possible groups. I subsequently spent one lesson in the grade 5 class and two in the grade 6 class observing the interaction in the pinpointed groups and testing the use of the recording equipment with each group. On the basis of this a group was selected in each class:

Grade 5: Four boys – Ashton, James, Ryan and Stuart.

Grade 6: One boy – Trevor; Three girls – Cindy, Belinda and Gaylene.

The requirements of a Masters dissertation meant that the sample was restricted to eight learners.

[6] The following teaching approach is advocated by Malati where appropriate and was used by these two teachers for the research tasks: The learners sit in groups. When issued with a printed activity, each learner is required to read and then attempt the task as an individual (learners do discuss problems with comprehension if necessary). On completion of aspects of an activity learners in a group discuss and compare solutions. At a time deemed as appropriate the teacher conducts a whole class discussion so that different responses can be shared and discussed.

Analysis:
A thorough review of the transcripts, the field notes and the written responses of learners was used to identify the strategies used by learners on the different tasks. A "profile" on each learner's performance on the set of tasks as a whole was also compiled.

Owing to the nature of the data collection process, I was not able to question each learner on his/her response to every item. Thus it was not always possible to determine, for example, which strategy each of the eight pupils used when solving a particular task. I am thus not able to comment on the extent to which each strategy was used. Rather, I have used the presence of a particular strategy in the data as evidence to suggest that this is one way in which the task in question can be solved.

Results:
Preliminary Remarks:
When solving the tasks, a number of learners lifted the page on which the activity was printed or moved themselves in relation to the page. It is difficult to determine whether this was linked to a movement of the "object" or the individual. As a result, these responses were not taken into account in the analysis.

With reference to my preliminary model of "spatial ability" in which the ability to form and retain a visual image was identified as one aspect of spatial ability, it appears that one learner, Cindy, is not able to form such a visual image. For example, when required to draw a tin of beans from above, she responded by drawing a rectangle, and when required to draw an aerial view of the classroom, she was not able to draw the grouped desks in the correct positions, but drew these in straight lines along the walls of the room. She was not able to describe the mental processes used. Attempts by myself and Cindy's classmates to assist her by providing concrete objects as models for the tasks did not appear to help. Some of her responses were correct, but it appears that she might have been copying her classmate's drawings (she admits to this in one case), mimicking their verbal responses, or reacting to the type of questions I asked. As a result of my confusion in this regard, I did not consider her responses when identifying the strategies used on the tasks.

One "object"; solution required in the form of a drawing (Activities 1 and 4):
In this case there is evidence of four strategies, these being:

1. The learner visualises his/her own movement in relation to the "object". This was identified in the responses of Gaylene, Ashton, James, Ryan, and Stuart. For example, commenting on how he pictured what a toilet roll looks like from above, James said, "I imagined a real toilet roll and looked around it". Stuart claimed that he could picture what a suitcase looks like "from above": "If you go around the suitcase like this you get the same as you have here".
2. The learner visualises the movement of the "object" itself. This was identified in the responses of Trevor, Gaylene, Belinda and James. Trevor, who was able to describe his thinking with ease explained: "…Miss, I picture the box in my mind, miss. Then I just turn the box…put the box down, miss, and look at it from the top".
3. The learner does not necessarily visualise a movement in the specific context of this activity, but relies on visual memory from a similar situation. This was identified in the responses of Trevor, Ashton, and Belinda. Regarding the item with the tennis ball, Trevor said, "…once I looked up in a book that if you look at it (a sphere) from all directions, it

will still be the same". Ashton refered back to a previous activity: 'You know the other one we did with the vase? The vase and the…cup, miss. The handle changed side as the eye changes".

4. One learner developed a short argument in explaining the left / right discrimination when viewing an object from "behind". This argument is not necessarily reliant on visual images. Referring to the picture of the "factory", Trevor commented, "It was on the left side, Miss, but I swopped it around so it is going to be on the right side".

The use of the third and fourth strategies described above suggests that learners might change their strategies as they become more accustomed to doing activities of this nature. Bishop (1989) has, in fact, suggested that increased facility with "spatial" tasks might lead to learners moving away from visual methods. He stresses that the strategy used by a learner on a task depends on, among other things, memory of visualisations and the ability to recall or generate appropriate visualisations.

Interestingly, some of the learners adopted different strategies for the different items, that is, the different "objects". Gaylene, for example, used strategy 1 for the cereal box, but strategy 2 for the tin of beans and the sandwich. Trevor, James and Ashton also varied their strategies on different items. It would be interesting to consider whether something specific to the different "objects" determines whether strategy 1 or 2 is used. Furthermore, what determines whether learners use the more "abstract" strategies 3 and 4?[7]

<u>Two or more "objects", solution required in the form of a drawing (Activity 3):</u>
These items require that learners reflect on the relative position of two or more "objects" as well as the relative position of themselves to these "objects". It is possible that different strategies could be used to those used in the case above in which only one "object" is presented.

In the activity with the two boxes only the use of strategies 1 and 4 could be identified. The following explanations suggest that the learners were visualising a movement in their own position:

> Ashton: *So if you are looking from above, you should actually be pretending…a helicopter, whatever going, looking from above*
> James: *I imagine I am a radar dome…But it gives you a view…round things…it walks around*
> Ryan: *I think of a glass coffee table, and I am lying under the coffee table*

Trevor's explanation suggests the use of strategy 4: In explaining his drawing of the two boxes from the side he said, "Miss, I am only drawing one box here, Miss, because if you are looking from the left this will be behind it. So I just need one box".

<u>Two or more "objects"; possible solutions in the form of drawings are provided (Activity 2):</u>
This item was included to compare the strategies used by learners when presented with possible answers in the form of drawings with those used when they are required to draw the

[7] "Abstract" in this case is being used in the sense that the learner might not be relying on the visual image in strategies 3 and 4 to the same extent as is the case when using strategies 1 and 2.

solution. It was also felt that the use in the activity of drawings of "eyes" to show the different angles from which the objects were to be viewed might influence the strategy used.

Interestingly, the use of the strategies 1, 2, 3 and 4 mentioned above were identified in the responses.
Strategy 1: Stuart suggested that he was picturing the movement of his body: "I imagined…that I could see from that side and I could see only the handle of the cup"
Strategy 2: Ryan wrote and spoke about how he would move the picture: "I made like I turned it around…I turned the picture around"; "I pretended the picture was real. Then I turned it around".
Strategy 3: This was only identified in Ashton's response: "…when we did the cup in the other activity…so I used that…
Strategy 4: Both Trevor and James used short arguments in explaining their reasoning. For example, Trevor explains: "…the plate is on the left side when you are looking at it…when you look at it from here it is on the right. So I say it is "B", Miss". His written response is: "…because the cup you can see is hiding away behind this because it is bigger (pointing to the flower pot) and you can only see the ear". James describes how he uses the relative position of the "object": "I saw that this, the plate will be on the right because that is opposite from here…because that will be on the right ……and this behind, the jug will be behind the flowers".

It appears that Gaylene and Ryan used a combination of strategies 1 and 2. Furthermore, Ryan's response suggests that he was employing an additional method (strategy 5) for he appears to have used the given solutions (a) to (d) as a guide in determining his answer: "First I thought number one was (c) because I looked at it from this side and so I saw the handle and then I looked at (b) and so I saw this side".

Discussion:
Methodology:
The decision to collect the data for this research in the classroom context did have its problems in that

- In one instance a learner appears to have 'mimicked' responses and copied the written responses of her classmate
- In one instance a teacher used one of the research activities without my knowledge and an additional activity of this type had to be designed for use with this class
- At times different learners were absent from school and I had to work with them on an activity at a later date and not in the original group context
- I was not able to question each learner on his/her strategy on every item. This is not, however, a problem in the light of how the data was used to identify the use of different strategies
- Some learners volunteered more information to me and initiated conversations with other learners more readily and frequently than others. Thus more information was obtained on these learners.

It is felt, however, that this method was useful in gaining a range of valuable information in that

- The nature of the classroom culture in the selected classes meant that there was discussion amongst learners which provided useful information on learners' strategies without my

having to question them .[8] Certain learners engaged in constructive arguments with one another and after a short while openly shared their thinking with me

- I could assist in clarifying problems that arose due to the fact that the activities were in draft form
- I could clarify problems arising from the visual conventions and vocabulary used in the activities
- I could question learners about their strategies where necessary
- I could question learners about their drawings in an attempt to establish some shared meanings.

Spatial Ability:

Obviously a study involving a much larger sample would be required to confirm the use of and determine the extent of the use of the five strategies identified in this research. It is felt, however, that the actual identification of these strategies is useful for the following reasons:

- In terms of building a "model" of "spatial ability" these results point to the possible problems with a model that attempts to classify "spatial ability" tasks according to specific skills. Rather, it seems more appropriate to identify a range of different strategies which could be used on a variety of tasks which could be classified as "spatial tasks". For it appears that learners will use different strategies or a combination of strategies on different tasks. The five strategies identified here, along with others noted in the wider research project, can be used to compile a list of strategies that learners might use for performance on problem solving activities as mentioned in section (b) of the preliminary model of "spatial ability".
- An important aspect of the Malati model for teacher development is to encourage teachers to reflect on the strategies used by learners. The results of this research could be useful in assisting teachers in the identification of individual learners' strategies as part of assessment and subsequent planning of instruction. The results of this research have, for example, been used at Malati in the design of an "assessment schedule" for use by teachers in building an "overall picture" of each learner's spatial ability.
- In terms of materials development in mathematics education, an analysis of these results could assist in the design of a wide range of tasks in the hope that the variety might promote the development and use of the range of different skills which have been identified.
- The size of this sample meant that I was able to compile a detailed "profile" of each learner's overall approach and performance on all the tasks. These profiles are useful in determining certain trends in individual learners' strategies for solving "spatial" tasks. For example, one of these learners did not seem to be able to visualise at all; two learners could progress on the visualisation tasks once they had been given support with concrete "objects", while another only needed such "objects" as a checking mechanism; some struggled consistently with left / right discrimination; and some paid careful attention to shape and scale in drawing. "Profiles" of this nature could be useful in developing learning programmes for individual learners.

[8] One important aspect of the Malati philosophy of teaching and learning mathematics is the development of a classroom culture in which pupils take responsibility for their own learning. Features of such a culture are that learners work independently, work spontaneously and effectively in small groups, are motivated by finding a solution to a problem, are willing to persist and to struggle, can communicate verbally and in writing, feel safe to argue and to question, respect one another's opinions and listen to one another, and are not unduly disturbed by mistakes.

Furthermore, this aspect of the research project has pointed to a number of areas for research in the field of "spatial ability":

- Does increased facility with "spatial" tasks influence the strategy used?
- Does the nature of the "object" for example, a cereal box or a tin of beans, influence the strategy used?
- Trends that emerged in the 'profiles" of these eight learners suggest that there are possibly certain identifiable "stages" in a learner's progress to "spatial maturity". This could be explored in more detail.

The results of this research into the strategies used by learners when solving tasks which require that the learner view representations from different perspectives, as well as the results of the investigation into other aspects of "spatial ability", have confirmed my initial view that the field is far more complex than expected. But I feel that this small study has been valuable: Firstly, I have been able to use these results in the creation of materials for the development of "spatial skills" and in interaction with teachers and, secondly, it has alerted me to the richness of this field as a source for research in mathematics education.

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