Indispensable Mathematical Knowledge – IMK and Differential Mathematical Knowledge – DMK: Two Sides of the Equity Coin

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There are many pedagogical approaches that genuinely strive to foster equity in mathematics education. In this paper we first analyze why, although these approaches do acknowledge diversity, equity is not really accomplished. We then describe our research-based TAP (Together-And-Apart) approach that has been implemented in two projects in very different contexts, ISTAP in Israel and MALATI in South Africa. We describe how TAP achieves equity by both acknowledging diversity and ignoring diversity thus disarming school-mathematics of its traditional role as the gatekeeper of students' future. Finally we focus on a research site and one teacher's struggles and achievements in his attempt to accomplish TAP's goals.

Most mathematics educators accept, and even justify, using mathematics as a gatekeeper of further education. Nevertheless, they are aware that the current practice of mathematics education "contributes to the regeneration of an inequitable society through undemocratic and exclusive pedagogical practices..." (MEAS1 Proceedings, page 3). They suggest, however, that this inequity in mathematics education be dealt with via the employment of miscellaneous methods of instruction and class organization.

The most widespread approach for dealing with inequity and students' diversity is ability-grouping, either by setting up 'same-ability' groups within the same class or by placing students with different abilities in separate classes. Research indicates that teachers view ability-grouping as the best way of improving the scholastic achievements of all students and as the only 'fair' way for dealing with students of different ability-levels (e.g. Oakes, 1985). *"The question of how early some form of instructional grouping of students should occur...My response would be...as soon as the teaching and learning of mathematics occurs..."* (National Council of Teachers of Mathematics, 1998, p.8). Recent research, however, has clearly shown that the tracking systems contribute to the regeneration of an inequitable society. Studies of this sort have concluded that the placement of students in ability groups, in and of itself, increases the gap between students beyond what would be expected on the basis of the initial differences between them (e.g. Linchevski & Kutscher, 1998; Slavin, 1990).

Other attempts have tried to support equity by designing learning environments that permit and encourage different levels of mathematical knowledge and sophistication within the same community of learners. They suggest that the way to cope with within-class inequity is by developing learning environments that are sufficiently flexible to allow all students to show what they know and can do (MSEB, 1993). In our view, however, the latter attempts and its practices deal with only one aspect of the equity principle. What they actually do is legitimize different levels of mathematics without taking into account the gatekeeper-effect of mathematics.

What actually happens in most of the above-described systems is that diverse levels of mathematics are legitimized in the early stages of students' mathematics education. However, at a certain point in time (which may be different in different systems) certain specific mathematical knowledge is required in order for the student to be accepted into

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a prestigious learning trajectory, for instance allowing the student to study in a mathematics class leading to an 'accepted' high-school diploma. This filtering process occurs more than once during the students' learning career. The students who have learned in the lower tracks in the tracking system or in alternative 'tracks' in the heterogeneous system find themselves unprepared for these critical moments.

In this paper we describe our research-based TAP (Together-And-Apart) approach that was developed in Israel and has been implemented in two projects in very different contexts, ISTAP in Israel and MALATI in South Africa. We first describe the major assumptions and guidelines of TAP. We describe how this approach genuinely supports equity not only through appropriate learning environments, but also by providing learning interventions that prepare students for their mathematical crossroads. We then focus on one teacher's struggles and achievements in his attempt to accomplish TAP's goals.

TAP's major assumption

Tracking systems violate equity. We believe that equity in school mathematics can be achieved only when all learners are members of a fruitful, diverse mathematical community. We realize, however, that a rich learning environment in and of itself cannot guarantee each member genuine school-mathematics. We also know that such a community can be mathematically productive and endure to the satisfaction of all its members only if on the one hand its members have sufficient shared mathematical knowledge to make meaningful interaction possible, and on the other hand there is enough space for all members to express their mathematical diversity and to experience success. For this to be fulfilled the learning environment should guarantee each student's IMK and DMK:

- Certain essential mathematical knowledge (henceforth called 'Indispensable Mathematical Knowledge' or IMK) should be owned by all students notwithstanding the acceptance of diversity in other parts of their mathematical knowledge. Indispensable Mathematical Knowledge is that part of genuine school-mathematics that enables the heterogeneous mathematical community fruitful interaction to the satisfaction of all its members, culminating in open doors to higher education. It also enables the students to cope with future activities in mathematics and with society requirements. Equipping each student with IMK supports equity by enabling all students to be full partners in the heterogeneous mathematical community. In our view, it is the teachers' duty to identify IMK as well as to identify students whose IMK is insufficient, and to take responsibility for providing these students with repeated opportunities for acquiring it.

It is clear that the mathematics curriculum and thus IMK might vary among different educational systems. It is also clear that the choice of curriculum is one of the mechanisms certain systems use as a filtering device. However, for the purpose of this paper we assume that the curriculum-derived IMK is given and justifiable.

- Students are entitled and should be encouraged to fulfill their differential mathematical needs, abilities and preferences (henceforth called 'Differential Mathematical Knowledge' or DMK). This should lead to the construction of a learning environment that accommodates differences in the ways learners think about, construct and display mathematical knowledge and understanding. It should lead to the design of a teaching model that responds to students' diversity.

In our view the above-introduced requirements can be realized only if the learning environment is designed to concurrently recognize diversity in students' 'entry' points but also, at certain carefully defined points in the learning process, sometimes to 'ignore' diversity: In these cases TAP "does not accept" diversity in students' exit points. Ignoring diversity means that IMK should be owned by all students. Thus, ignoring diversity should lead to the design of a learning environment that guarantees students' acquisition of IMK (For more details see Linchevski & Kutscher, 1996 &1998.).

The research site: Stonehill High

As previously mentioned, TAP has been implemented in two different countries, ISTAP in Israel and MALATI in South Africa. A report and description of TAP's success in accomplishing equity in ISTAP as measured by students' mathematical achievements may be found in Linchevski & Kutscher (1998). In the current paper we report on the implementation of TAP by MALATI in Stonehill High School, South Africa.[1]

Stonehill High is one of seven schools participating in the MALATI Project in South Africa. This school is situated in a traditional black township and is, in many ways, typical of schools in disadvantaged areas in South Africa. The class-size at Stonehill High ranges from 40 to 50 students per class where students frequently have to share desks and seats. A considerable portion of teaching time at Stonehill is lost due to administrative reasons. For example, students' registration and time-tabling is finalized only at the beginning of the school year. At the beginning of the 1998 school year ten school days were used for the latter purposes. Teaching-time at Stonehill is disrupted on a regular basis mainly due to administrative issues and school events. More learning time is lost during the weeks that are devoted solely to examinations. All time lost is not compensated for.

Classroom practice at Stonehill prior to working with MALATI was typical of that in South Africa and elsewhere: Lessons were teacher-centred with whole class teaching the norm and dominated by low level questions and the mastery of procedural skills (Taylor and Vinjevold, 1999). Teaching was authoritarian with very little room for analysis or critique. Students at Stonehill were not accustomed to working in groups – they did not listen to one another and struggled to communicate orally or in writing. There was also little culture of doing homework at the school.

Assessment at this school was typical of the wider practice in South Africa. It was exam-driven, with "control tests" occurring at the end of a section or school term, and was used for reporting purposes (Niewoudt, 1998; Taylor and Vinjevold, 1999). The curriculum is divided into sections so that at the end of each quarter examinations were administered for each grade separately culminating in final, end-of-year examinations that assessed all the material learned throughout the school year. Promotion of students from grade to grade is not automatic; a considerable number of students in each grade at Stonehill are failed each year. These students, nicknamed "repeats", are required to repeat the entire year of schooling. The pass rate for mathematics in the final year (grade 12) at this school is very low.

Prior to the MALATI intervention, no IMK identification or consolidation[2] took place. No

[1] The name of the school has been changed.

[2] Process of guaranteeing IMK to students in need, after assessment indicated that their IMK was not yet acquired.

DMK diversification[3] between students was carried out. There was no attempt to follow up students' difficulties nor to take responsibility for bridging essential gaps. No analysis about what was and was not crucial for understanding subsequent topics was done. The teachers seldom used other forms of assessment such as projects or oral assessment.

Compounding these difficulties is the political background from which the Stonehill teachers stem. Complicated problems that are the product of the recent emergence of equality due to political changes in South Africa must all effect the teachers' grapple with MALATI's concept of education and, especially, equity. As members of a society previously discriminated in South Africa, these teachers had been part of the struggle for equality and democracy. Despite this, in their role of mathematics teachers they unwittingly continued to practice in their schools all the elements of undemocratic pedagogical practices, analyzed and discussed in this paper. And if in the past few years there has been more awareness (fostered through constant exposure to the state's new philosophy of a learner-centered, outcomes-based curriculum) that the school's pedagogical practice regenerates inequity within the school (and thus, eventually, jeopardizes its students' future) through its undemocratic methods, the teachers usually blamed outside forces for these problems, and expected external interventions to assist them in solving their problems.

The state's attempt to redress past inequalities in the distribution of human and physical resources has resulted in uncertainty that has led to an exodus of teachers from the profession and low-morale amongst those that remain. Unlike many South African schools, Stonehill has a mathematics department that has changed little during recent staffing changes. The six mathematics teachers at Stonehill have taught at the school for at least 7 years. Prior to MALATI, mathematics departmental meetings mainly dealt with administrative issues.

The decision to form a partnership with MALATI was taken by the whole school mathematics staff. This decision was facilitated by the fact that TAP is in line with the state's new philosophy of teaching and learning. The mathematics teachers seemed to be open to innovation and change, and participated enthusiastically in the MALATI project, forming a cohesive unit from its introduction.

MALATI supported the Stonehill teachers by providing learning materials, by its counselors' frequent visits to their mathematics classes and by weekly workshops where the teachers discussed appropriate strategies for cooperative learning, assessment and class organization in their heterogeneous mathematics classes according to TAP's principles.

In the context of the above factors and difficulties we now present a case study of Mr L and his attempts to implement TAP over a two-and-a half- year period. We will report in more detail on his first two years until his major break-through.

Mr L:

Mr L was a competent teacher with full command of his class. He felt most comfortable in his role as "center-star" in his teacher-centered classroom. But, as we will soon see, this

[3] In this article diversifying means organizing the class in homogeneous groups in order to cater for the differential needs of the students

quality interfered with his success in adopting and implementing more learner-centered environments. At this stage of MALATI, Mr L's perception of teaching mathematics meant demonstrating the solution process of an exercise and thereafter practicing it for a predetermined period of time. At the time he believed that: *"The answer is more important than the process"; "when a pupil can use a mathematical procedure correctly he understands it"; "if students methods are inefficient they* (these methods) *should not be encouraged"; "mathematics tasks can be solved only in one way"*. His main source for exercises was the textbook, usually inspired by the type of exercises these students would solve in their matriculation examinations - given they would reach this stage. This practice was dominant regardless of the students' grades, knowledge or success. The fact that many students failed, and that most students did not reach 'matric', did not trigger any process of reflection in Mr L other than devoting more preparation periods before end-of-term and end-of-year exams. Mr L did not believe in group-work. His guiding philosophy was: *"pupils cannot solve mathematical problems effectively unless they have been shown how to do them"*. He believed that assessments should take place at prearranged times that were decided on at the beginning of the year; it was not necessary to consider *"when pupils or teachers feel that the pupil is prepared".*

1997: First half year, July[4] - November: In the first half year, interaction between MALATI and Stonehill was limited to discussions in workshops.

1998: 10th January, Second year: Mr L's attempts at TAP started by organizing the class for group work. The children were grouped randomly because *"I feel I don't know them well enough yet."* Despite the groups, the learning was whole-class and teacher-centered.

Six weeks later: Mr L was trying to initiate improvements in the school. He had submitted a number of proposals for ways of stabilizing the timetable but to no avail. In the TAP spirit, after the first evaluation he reorganized his class into heterogeneous groups based on the test results. Although he went through the motions of fine-tuning the group composition as if readying pupils for 'real' group-work, he was unable to relinquish his role as 'center-star': The IMK consolidation was done in a whole-class setting.

9th March: Mr L reported that when marking the exam papers he noticed names of pupils he didn't recognize from class (even though they obviously attended his classes). And two of the latter students had outperformed his others pupils! He strode into this class, sought out these 'unknown' students, and then proceeded to lecture his class on their lack of motivation and hard work. The pupils had been introduced to the topic of "Probabilities with Dice" and since the MALATI tasks were inquiry-oriented and not procedure-and-drill, the students were not taking them seriously. Mr L commented to the pupils that some of them were complaining that the (mathematics) work they were doing was just a game. He assured them that this was still mathematics, with a new approach. Mr L's views of mathematics-learning were apparently starting to change.

Three months into the school year: Important TAP changes could be observed in Mr L's class. He was reviewing their control test. After giving them a pep talk on the importance of mathematics for future employment, he divided them into two groups[5] according to the information derived from this test's profile. Those students who needed IMK consolidation were divided into homogeneous pairs; the rest formed a small group at the

[4]The school year starts in January.

[5] This means that there are two different, concurrent learning-plans so that every small group is involved only in on of the learning-plans.

back of the class. The students moved willingly and quickly. He found teaching in this learning environment *"a scary process"*. While he was doing IMK consolidation, the other group continued with other activities and *"continued and worked very quickly, and then I was split again and I still have difficulty handling the different levels that the learners are at. I find that some people can finish their activity quite quickly, and then they have a negative input, or they become playful..."* His practice had undergone a major change: the tasks were not procedure-and-drill. However:*" They* (the children) *don't like it because it's 'easy' and 'different'. The kids are so used to struggling with maths that they don't know how to handle it"*. As was apparent on the 9th of March, we observe here too that Mr L had to contend not only with his own difficulties in the process of his changing views of mathematics learning, but also his students' difficulties, all having come from a completely authoritarian culture of learning in general, and steeped in a mathematics culture of procedure-and-drill in particular.

Two weeks later: The children were still sitting in their homogeneous pairs designed previously for IMK consolidation, although they had started a new topic – geometry. He handed out an activity and gave them five minutes to tackle it. Most of the children struggled (not having stronger students to confer with in their pairs) with the activity. Just when they got going, they were told time was up and Mr L initiated a whole-class discussion.

30th August: Slowly he became aware of his shortcomings and tried to be more conscious of the time devoted to group-work. He explicitly encouraged the students to work more independently in their groups. He was beginning to trust the students' abilities of learning: "*Many times I will leave them, but I will leave them with that doubt that I am not happy, so they will see where the problem is, if any, but I don't like guiding them in that direction."*

10th October: Mr L appeared quite comfortable with group-work. After assessment he again diversified the class for IMK consolidation. He prepared extra activities for those who performed well while he himself interacted with the others. Pupils got down to work quickly and continued so most of the lesson. When asked how he felt, he said: *"I think it helped the 'front'* (consolidation) *pupils. But I think I will have to assess them again to be more sure."* He was becoming more convinced of the benefits of group-work and of diversifying: *"In the smaller groups I find that I can be more attentive to them whereas the others who I feel don't need that much attention can go ahead."* He was gaining confidence in the ability of children to work on their own.

3rd November: Mr L was very frustrated by his lack of success in having the students do their homework and he constantly expressed his disappointment. He maintained group-work and some diversification but started to express dissatisfaction: *"I might be neglecting the stronger pupils and I need to work on this"*. His beliefs and attitudes seemed to have undergone major changes. He was aware that he was battling on three fronts: 1) the children's views of what mathematics is; 2) school and department regulations; 3) his old practices and beliefs.

1999: First week, January, third year: The school was still not organized for scheduled learning due to administrative reasons. Mr L arrived to attend his second lesson but an unplanned administrative session that morning took more than an hour so all classes were shortened. He said that he had proposed to the principal that periods should be no less than 50 minutes but the principal had some objections. Mr L felt he could handle the class in the TAP spirit and would need assistance only after the first assessment.

11th February: Once again learning time was wasted on administrative purposes thus the lessons were very short. The students were sitting in rows while Mr L was conducting a traditional teacher-centered class. As the lesson progressed he gradually encouraged the "quickers" to pair off or form groups which they did quite readily. He was concerned that *"the learners who work very quickly will get frustrated when working with slower learners"*. The counselor realized that Mr L was implementing mainly homogeneous groups. He suggested that the learners work in heterogeneous groups, at least for core activities, so that all could benefit from the interaction. Mr L seemed determined to work on it (and on himself).
9th March: Following the end-of-term exams Mr L maintained homogeneous groups even after IMK consolidation for a topic was completed. He indicated that the 'repeats' had benefited from IMK consolidation. The counselor urged him to implement heterogeneous groups especially for the core material.
One month later: Mr L indicated that the 'repeats' had given up on mathematics. So he decided to try to integrate some of them with the rest of the class[6].
22nd April: It seems that Mr L had made himself the commitment of adopting heterogeneous groups as his dominant class-practice. The class was organized in mixed-ability groups and Mr L moved from group to group struggling to get the learners to compare their answers. During class discussion the different groups had to report back. He had adopted the practice of sitting with a group for an extended period. This personal interaction with the students may explain why this year he knew not only the strategies used by the different groups but also by the different students, whereas at approximately the same time the previous year he barely knew his students' names.

From this point on Mr L's practice was focussed on fostering cooperative group-work in mixed-ability groups within a mathematical culture of inquiry and discourse. It was clear that Mr L was able to and indeed did successfully implement most of TAP's principles. He no longer saw himself as 'center-star': *"They can learn from one another, that is – is what I have learnt…they can also learn from me."* There is no better way to describe the change than using Mr L's own words: L: *"Perhaps I am scared because it worked…";* (Mr L and the counselor both laugh) C: *"Why are you scared?"* L: *"Because they don't need me"*. However, it was also evident that he was still oscillating between his old beliefs and his new experiences: He was not yet a full partner of TAP. He consistently needed proofs that TAP strategies were truly beneficial. At times he provided different groups with different activities but once the students were involved in these activities he began to have second thoughts, focussing on the negative aspects on what each group missed by not doing the others' activities instead of realizing what they had gained having had their specific needs addressed. At other times he would see a positive aspect and then seem to draw back as if to reinforce his original beliefs. He would declare that weak learners could benefit from working in mixed-ability groups: *"Uhm, what I'm finding is that in many groups certain people adopt those people that are not performing well…"* and in the same breath he could say that he was not sure that the weak students benefited from learning in mixed-ability classes. It was clear to him that mixed-ability group-work benefited the "strong" learners: *"I find that when they* (the "strong") *communicate in the* (mixed-ability)

[6] The traditional practice at Stonehill was to sit the repeats at the back of the class. They were usually physically much bigger than the 'regular' students in their class.

group they also learn some other skill – of speaking mathematics, which is of great help for them" - echoing Vygotsky (1986). And again a need to retract: *"I need proof that the strong learners would benefit from working in mixed-ability groups"*. If previously Mr L was concerned that he *"might be neglecting the strong pupils"* when they learned independently in the homogeneous groups, he now believed that *"within the group there is over enough intelligence to actually run through the activities."* But he still had a problem of *"a difficulty of the letting of one group go ahead."* This last difficulty was not only one of class-management and logistics that he was still experiencing. These expressions of contradictory beliefs were characteristic and representative of the way he expressed and exposed his inner conflicts with TAP's principles and practice.

When summing up, we can see that even under the objective difficulties – school culture, facilities, students' learning culture and the like – Mr L's practice underwent a remarkable change in terms of TAP. But from the many discussions and interviews, it was apparent that his beliefs did not undergo the same change. Why would a teacher with so much evidence, even hard data (*"looking at the results of last year versus the results that they obtained thus far… out of a class of 48 only five people have not improved on their mark of last year"*) and with a successful record of implementation, still cling to his old beliefs? Along with all the commonly recognized factors that affect beliefs, such as the change-agent's role, beliefs lagging behind practice, personality etc, one cannot ignore the social-context factor in which Mr L's change occurred. Most of the aspects in Mr L's old practice, such as teacher-centered lessons, end-of-term 'control' tests, 'failing' students becoming 'repeats' etc, had been shared also by previously privileged S.A. – and it seemed to work for them! Thus from this standpoint it might be reasonable to believe that with inequalities redressed and improved resources, most of the problems that the school experienced would disappear. Taking this perspective, it might be very difficult to be convinced that it was the old practice that posed the problem. Only honest reflection on the old practice will allow change where beliefs and practice go hand in hand.

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