THE "SLIPPERY" CONCEPT OF PROBABILITY: REFLECTIONS ON POSSIBLE TEACHING APPROACHES[1]

Kate Bennie

MALATI

There is a growing trend in South Africa and elsewhere to include the study of probability and data handling in the school curriculum. In response to a perceived need for curriculum development in this area, the Malati Statistics Group developed a package of probability materials which were trialled in grade 8 and 9 classes in early 1998. In this paper one of the developers describes the Malati experiences and reflects on the appropriateness of the theoretical framework chosen as the basis for the materials. This is used to suggest directions for future curriculum development in this area.

Background:

The move to formally include data handling and probability in the school curriculum in South Africa is a relatively new one. These topics are included in the Western Cape Interim Syllabus for grades 6 and 9, but it appears that they have been regarded as enrichment by most teachers and only studied if and when other topics have been completed. This work has tended to target more "able" pupils who often encounter related questions in mathematics competitions. In Curriculum 2005, however, one of the ten Specific Outcomes for Mathematical Literacy, Mathematics and Mathematical Sciences is *"Use data from various contexts to make informed judgements"*. This outcome contains two Assessment Criteria which refer specifically to probability, these being,

8. Evidence of knowledge of ways of counting

9. Understanding of concepts of probability.

Current mathematics education literature indicates a trend internationally to include probability and data handling in school curricula. Much of this literature has focused on the need for the inclusion of the topics, and not much is known about how pupils actually learn the topics or about the most effective pedagogical approaches.

The research that has been done on probability can be divided into two areas:

---

[1] The term "slippery" is used by Konold (1991) in describing the difficulties associated with the learning of probability.

Paper presented at the 4th Annual Congress of the Association for Mathematics Education of South Africa (AMESA), Pietersburg, July 1998.

- Research that attempts to describe how pupils think about probability. Much of this research points to the strong intuitions and the inadequate pre-requisite mathematical skills that pupils bring to the study of the topic at school.
- Research that attempts to determine the influence of instruction on these intuitions.

Hawkins and Kapadia (1984) refer to these two approaches as the psychological and pedagogical approaches respectively, and note that there has been little synthesis between the two approaches. Furthermore, much of this research seems to be based on pupil observation rather than on actual empirical research. Questions have also been raised about the nature of the research that has been done.

The trend to emphasise the study of probability in the school curriculum has been recognised by Malati, as well as the value of studying probability both in terms of its usefulness in everyday life and its intrinsic mathematical value, and the need for curriculum development and support in this area. In 1997 the Malati Statistics Group designed packages of probability materials for grades 5 to 9. This paper reports on the trialling process, relating the Malati experiences to the literature. The observations of fieldworkers, as well as comments by teachers in the Project, are used to reflect on individual aspects of the Malati approach, as well as on the wider theoretical framework in which this approach is set. It is hoped that these comments will contribute to the debate on materials development and teacher support in the field of probability as well as on curriculum development in general.

The Malati Project:

The Malati brief is to trial materials in Project schools and to provide support for the teachers, while at the same time reflecting on the process of change and constructing a model for curriculum development. The process of trialling a package begins with workshopping of teachers when they are provided with full packages of the materials containing rationale documents, student materials and teacher notes. Ongoing teacher support is provided in two ways: project workers attend lessons and organise regular discussion sessions with all participating teachers. Extensive field notes are written during this process. In the case of probability, the Malati materials were trialled at the beginning of 1998 and were used by six teachers in each of two high schools and two teachers in a third. The teachers were encouraged to use the materials simultaneously in grades 8 and 9 as it was felt that this might lighten the load of the teachers and allow for comparison and experimentation.

Malati Probability:

Before discussing observation made during trialling, it is necessary to give a brief description of the general Malati approach to the study of probability and the choices made during the development process.

<u>The Relationship Between Probability and Data Handling:</u>

The Malati approach to the study of probability and data handling was based on the developers' conceptual understanding of the two topics. Probability was presented before data handling as it was felt that an understanding of the concepts of chance and probability are necessary for the understanding of random and representative samples and the significance of statistical tests in data handling. It is in the testing of hypotheses and the determining of confidence intervals that probability and data handling come together and it was argued that pupils who have a sound understanding probability would be better equipped to make judgements and decisions regarding statistical data. The question that needs to be asked is whether this approach is, in fact, the best pedagogical approach. Garfield and Ahlgren (1988), for example, question whether this order is appropriate and warn of the danger of studying probability too early: they claim that the "intrusion of technical probability issues that are not likely to be understood will stall the learning process – and leave a distaste that could compromise subsequent instruction as well".

The design of the Malati probability materials was influenced by the Dutch approach to probability in that systematic counting was chosen as a "way in" to probability. Thus pupils are required to use tree diagrams, tables and graphs to represent possible outcomes and for calculating probabilities. This emphasis on systematic representation in problem solving is thought to be valuable not only in probability, but in other areas of mathematics, too.

The goal of the Malati Statistics Curriculum is to have pupils begin the study of probability in grade 5. Ultimately, therefore, pupils in grades 8 and 9 who have been through this curriculum should be familiar with systematic counting and the basic notion of chance. Pupils who were involved in the 1998 trials, however, had not been exposed to these ideas at school. The developers were concerned that these pupils 'catch up' on this work and thus the systematic counting component and the basic ideas of chance were incorporated into specially adapted activities for grades 8 and 9.

Contexts:

In selecting contexts with which to work, the developers encountered a conflict between selecting a context that would be familiar to pupils and creating an impression that probability is only related to the playing of games. It was decided to include games using dice and coins, but also other games with which it was thought pupils would be familiar, for example, those played in game shows such as 'Zama Zama'. The use of probability in other contexts, however, was also included, for example, activities required that pupils made choices between items of clothing, ice-cream flavours and political parties. It is interesting to note in this context that those teachers who did extend the Malati activities tended to use the context of marbles or coloured balls in a bag.

Design Principles:

An issue of great importance in the Malati philosophy is that of ensuring that the mathematics in the activities is made explicit. The developers found that there were many such opportunities in the grade 8 and particularly the grade 9 probability materials. As with all Malati activities, these probability materials were designed within the constructivist framework.

Classroom Culture:

A crucial aspect of the Malati philosophy is the development of a 'classroom culture' conducive to independent learning, that is, by encouraging among other things discussion, respect for one another's opinions, independent learning, a willingness to struggle and intrinsic motivation on the part of pupils. The trialling of the probability materials at the beginning of Malati involvement in schools was particularly interesting as teachers and pupils were coming to terms with new material as well as grappling with issues of class culture, many of which were new to many classes.

The Malati Interpretation of Probability:

The Malati approach to the study of theoretical and experimental probability was influenced by the suggestions made by Shaughnessy (1981) and de Jager (1992), that is, theoretical reasoning is used to predict outcomes and then an experiment is used as a form of checking. It was decided that the study of probabilities which can only be calculated experimentally would be addressed towards the end of the package.

Hawkins and Kapadia (1984) have identified four different definitions of probability and the Malati approach would be classified as “formal probability” in this framework, that is, probability is calculated using the mathematical laws of probability and reflects some of the assumptions of both ”a priori” and “frequentist probability”. The latter two kinds are defined as follows:

- “A priori/ theoretical probability”: The probability is obtained by making an assumption of equal likelihood in the same space.
- “Frequentist/ empirical/ experimental probability”: Probability is defined in terms of the limiting relative frequency of the occurrence of an event in an infinite, or near infinite number of trials (Konold, 1991).

These three forms are contrasted with the “subjectivist interpretation” in which probability is defined as the measure of belief in the truth of a proposition. In this approach people might assign different values to the probability of an event, but these initial probabilities are revised on the basis of new information, and the probabilities of different people will converge on the frequentist’s limit (Konold, *ibid.*)[2].

In the discussion that follows it will be argued that the responses elicited and the problems experienced when trialling the Malati materials could have been the result of the choice of the “formal interpretation” as the overall framework.

“Playing Games”

Malati staff were excited about the rather ‘different’ activities and the positive reaction received from colleagues during the development process. It was generally felt that these activities would appeal to pupils. It seems, however, that this “fun” aspect had an unexpected reaction from pupils: the questions “When are we going to do real maths?”, and “Are we still playing games?” were frequently asked. Many pupils did not take the work seriously and were not able to reflect on the thinking processes being used. The teachers, who were initially very excited about using the probability materials, seemed to be disappointed by the reaction of their pupils and were not sure how to respond. This issue of pupil reflection and motivation is an important one: Garfield and Ahlgren (1988) warn that we are not only dealing with cognitive problems in the study of

[2] Konold (1991:142) groups “a priori” and “formal probability” into one interpretation which he names the “classicial interpretation”. He notes that the use of this interpretation not only limits the study of probability to objects such as coins and dice, but that it is actually logically flawed in that the definition of probability is circular: probability is defined in terms of equally-likely outcomes, yet “equally-likely” outcomes actually means “equally probable”.

probability, but also what they term “affective obstacles”, that is, “faintness of motivation for learning what is believed to be a useless, forbidding, and even deceptive topic”.

Having noticed this response from pupils, the developers encouraged teachers to make explicit mention of the mathematics and thinking processes involved. This advice proved problematic to teachers, a number of whom confessed at the end of the package that they themselves were not aware of what to stress at the time! It also appears that much of what is mathematically important in the study of probability at the lower levels is not easily recognisable as it involves specific **thinking processes**, rather than mathematical formulae or calculations. Much of what can be recognised as formal mathematics was only included in the later activities for grades 8 and 9 which, due to time constraints, were not trialled.

Some pupils seemed to enjoy the challenge of convincing classmates that there were three possible outcomes for the tossing of a coin, these being, heads, tails or the side of the coin. Teachers soon began to grapple with how to focus pupils on what was important mathematically. Teacher A noted in discussions that he had difficulty knowing when to curb discussion – being unfamiliar with the content himself, he indicated that he was not sure where these discussions would lead and whether what came up could be used in the study of probability. Of course this is an important area of reflection for teachers in all areas of mathematics, but as will be suggested later in the discussion, is of particular relevance in the teaching of probability. For it appears that some of the intuitions about which pupils tend to argue so vehemently, might actually be of use in developing a sound understanding of probability at school.

Classroom Culture:

This issue of pupils reflecting on their thought process, however, is not restricted to the study of probability and should be regarded as a general class culture issue. Many problems related to class culture were identified in the classes: pupils were hesitant to share ideas and to listen to one another; they had difficulty verbalising their solutions and were reluctant to write these down; they took little care when reading instructions and questions; they rushed through problems; the completion of homework was a common problem in all three schools; and pupils tended to give up when they had comprehension or mathematical difficulties, or encountered large numbers in calculations.

Referring specifically to the study of probability, Shaughnessy (1981) notes that problems experienced can be the result of two things, namely, problems which have an underlying psychological origin, and those which occur when pupils simply have not thought about the problem. It could be argued that the latter problems are related to the culture that has been created in a classroom. Furthermore, the following observation suggests that problems with classroom culture could, in fact, aggravate certain problems peculiar to the study of probability.

In the "Zama Zama" activity designed to assess pupil understanding of the basic notion of chance, pupils were shown a diagram of a box containing three different coloured balls of equal size, and were asked which ball a contestant in the Zama Zama game was most likely to choose when selecting one ball with his/her eyes closed. A number of pupils chose one of the three balls initially, but following class discussion in which it was made clear that each outcome was equally likely, some responded that they knew that all along but felt that they had to choose one of the three options! Such responses could be related to the fact that pupils are accustomed to having to produce one answer for each problem. Fischbein (as quoted by Shaughnessy, 1981) suggests that such thinking is deeply rooted: he notes that deterministic thinking in science and science education in Western cultures emphasises the "necessary", while neglecting "uncertainty" and the "possible".

Terminology and Verbal Skills:

Teacher A commented that he had been surprised by the difficulty his pupils had had distinguishing between the use of the terminology in the everyday and mathematical contexts. For example, in the "Likelihood Scale" activity, pupils were required to place the following event on a likelihood scale: *"15% of Astros are blue. You choose a blue Astro from a full pack with your eyes closed"*. Rather than responding with the term "very unlikely" as expected, a number of pupils indicated that this event was "likely" as it could occur, that is, the next Astro would be blue or would not be blue. Pupils also seemed to equate the term "impossible" with "a very small chance".

Such responses are well–documented in the literature. Hawkins and Kapadia (1984) suggest that "the mismatch between linguistic and technical interpretations arises in other areas of mathematics, but is particularly acute in the area on probability". Green (as quoted in Shaughnessy, 1993) indicated that problems were frequently caused by pupils' inadequate verbal abilities. In his studies the word "certainty" was equated with "highly probable", and a "50%

chance” was interpreted as meaning that something might or might not happen. Shaughnessy (1992) reports similar findings.

How could this problem be addressed? Teacher A indicated that when teaching probability for the second time he would ensure that pupils were given an opportunity to reflect on the different usage of the terminology. Green (Shaughnessy, 1993) also proposes this as a possible solution. The classroom culture could also assist here: by providing an environment in which pupils discuss their work and have the confidence to question one another, opportunities will be provided in which different interpretations of questions could be discussed and the meanings clarified.

Discussing the level of cognitive development required for the successful study probability, Garfield and Ahlgren (1988) suggest that pupils might not have the general mental maturity and verbal ability to describe probabilistic situations. They suggest that it might be necessary to concentrate on concrete methods such as plotting and simple counting methods before pupils are able to deal with the abstractions necessary for probabilistic reasoning.

The “Outcome Approach”:

The problem encountered with the Astros question, as mentioned above, could also be attributed to what Konold (1991) terms the “outcome approach” in which pupils think they are being asked whether an event will occur, rather than quantifying how likely the event is. Pupils using this approach therefore do not see the result of a single trial as one of many such trials in an experiment, but regard the result in isolation. This could also be regarded as a language problem, for as Konold points out, this problem is related to the interpretation of the word “probable” as “likely to occur”. What is of particular interest to us in the design of our curriculum is the research by Konold, Pollatsek, Well, Lohmeier and Lipson (as quoted in Konold, 1996) on the effect of instruction on this approach: they concluded that the percentage number of pupils holding the “outcome-oriented view” was unrelated to earlier instruction in probability. This indicates that, although pupils might be able to calculate probabilities correctly, this does not mean that they have a sound understanding of the notion of probability.

Assessment:

Konold *et al*’s results have important implications for the nature of assessment in the study of probability. Konold (1991) warns that pupils incorrect responses might not be the result of poor understanding, but rather a result of problems with the interpretation of questions and Garfield and Ahlgren (1988) note a number of studies which suggest that an important factor in misrepresentation is the misperception of the question being asked. During trialling of the Malati materials, pupils were given assessments in the form of class tests designed by the teachers. As an important aspect of the Malati philosophy is the use of diagnostic assessment in planning instruction, teachers were given guidance on the detailed analysis of pupils written responses and were encouraged to combine this with their knowledge of individual pupils to plan remediation and extension where necessary. Perceived time constraints, and, it seems, fear, often meant that teachers did not undertake the suggested analysis and consequently no conclusions could be drawn about the cause those problems identified in the written tests.

It seems that the nature of the contexts and the "formal" framework in which the topic was framed, could have affected pupil responses. Hawkins and Kapadia (1984), for example, suggest that the "ball in the bag" questions used in the influential work of Green and Piaget, might not actually test what they are said to test. They note the range of concepts that these tests could be assessing, namely, relative frequencies, fractions, numbers, volumes, colour awareness, or personal preferences. They also note that pupils might respond to these according to what they have been told, rather than what actually happens.

Personal Preferences:

In the activity "Zama Zama" mentioned earlier, responses included "blue because the contestant is a boy", and "pink because it is my favourite colour". Pupils were thus basing their decisions on their personal preferences. This is confirmed in the research with younger children undertaken by Jones (as quoted in Shaughnessy, 1992): he indicated that colours on a spinner influenced the decisions of pupils in grades 1 to 3. He uses this to illustrate the fact that manipulatives can interfere with children's probabilistic thinking.

Distinguishing Outcomes:

One Malati activity required that pupils write down all the possible results (outcomes) of tossing two coins. Many pupils responded with HH, TT, and HT, and were not able to distinguish HT and TH as being different. Hawkins and Kapadia (1984), in fact, classify this as a "famous example" and note that mathematicians have assigned a probability of one-third to the possibility of getting one of the possible outcomes. During trialling it was found that systematic listing and classroom discussion during which pupils managed to convince one another, were useful in trying to counter this problem. The literature, however, provides a reminder that although these pupils might now respond with the correct mathematical answer, that may not, in fact, be convinced of its correctness!

The Rational Number Concept:

Both Green (in Shaughnessy, 1992) and Garfield and Ahlgren (1988) identify poor pupil understanding of ratio as one of the main underlying causes of poor pupil performance on school probability. Two related experiences stand out in the Malati experience. Firstly, pupils had great difficulty locating the probability of dice landing with a six on top on a probability scale owing to difficulty converting the $\frac{1}{6}$ to a percentage. This problem proved a major obstacle to the successful study of probability in both grades 8 and 9 and was the subject of debate amongst teachers and Malati project workers on the most appropriate form of support in this area. Secondly, during detailed analysis of pupil performance on the class tests, a link was noted between poor overall performance and poor performance on test items requiring use of fractions other than very basic fractions such as $\frac{1}{2}$, $\frac{1}{4}$ etc.

On the surface, it might appear that , by providing support on the ratio concept, performance on probability might be improved. This is a daunting task in itself, but Fischbein and Gazit (in Garfield & Ahlgren,1988) suggest that the solution is even less accessible: they claim that probability thinking and proportional thinking are based on two distinct mental schema and that progress as a result of instruction in one aspect, might not imply progress in the other. They do not, however, provide a solution to this dilemma.

Bramald (1994) suggests one approach which, while not addressing the poor understanding of the ratio concept, could assist in the study of probability. He notes that the choice of the “formal approach” to probability means that pupil are led too quickly to the manipulation of ratios, without providing an opportunity for reflection on reality. He provides two possible solutions: the first relates to the overall framework, that is, that pupils be given opportunities to explore probabilities based on statistical evidence rather than on assumptions of symmetry, and the second relates to classroom culture in that he suggests pupils work co-operatively in discussion.

“Judgemental Heuristics”:

Kahneman and Tversky (as quoted in Shaughnessy, 1993) expressed the opinion that people make estimates of the likelihood of events by using certain judgemental heuristics, some of which could be identified in pupils’ use of the Malati materials.

When carrying out experiments using the three balls in the “Zama Zama” activity, some pupils based their predictions on what had happened in the previous trials. So if a pupil had already withdrawn a certain number of balls of different colours, s/he might suggest that the next ball will be blue because s/he is ‘”due for a Blue”. This reasoning is what Kahneman and Tversky term “representativeness”, that is, decisions are made according to how well an outcome represents some part of the parent population. This term also covers what is commonly referred to as the “gambler’s fallacy”[3]

Another form of judgement is termed “availability”, that is, decisions are based on the ease with which a person can call to mind particular instances of an event. The example of a person’s experiences with car accidents is often used in the literature, but the Malati trial provided additional examples. In the “Zama Zama” activity, a pupil said the pink ball was most likely to be drawn because “they always draw pink in this game on TV”. In a test pupils were asked to classify the likelihood of a slice of bread spread on one side with butter and jam landing jam-side up when dropped on the floor, a number of pupils indicated that this was “very unlikely” because “it always lands jam-side down!”. Pupils also said that the likelihood of a light bulb which is expected to last for 300 hours blowing after 2 hours was “very likely” because “light bulbs never last for long”.

Kahneman and Tversky also mention the “conjunction fallacy” in which pupils regard the probability of two distinct events occurring simultaneously as greater than the probability of the individual events occurring. Unfortunately time restraints prevented us exploring this area in detail.

In determining how to deal with these approaches to decision-making, it is important to consider the claim by Kahneman and Tversky that these forms are, in fact, features of intuitive reasoning.

---

[3] Subjects using this reasoning will argue that, after a run of “heads” in tossing a coin, a “tails” is more likely to come up. Cohen (as quoted by Shaughnessy, 1992) notes that adult subjects tended to predict the outcome that was occurring less often (“negative recency strategy" or “balancing off"), but after a small number of trials they would switch to predicting the outcome that occurred more often (positive recency strategy).

How does this relate, and how should it relate, to what Garfield and Ahlgren (1988) call "correct statistical reasoning" which is studied at school? The following discussion could shed light on this debate.

Intuition:

Some researchers have criticised the distinctions suggested by Kahneman and Tversky and choose to classify pupils' reasoning processes differently. What is of interest in this context, however, is that pupils do seem to use the approaches mentioned in the literature and, as Well et al (as quoted in Hawkins & Kapadia, 1984) have indicated, different pupils will use different strategies. The Malati experiences certainly confirm Shaughnessy's claim that pupils do not approach the topic as blank slates, but have "firmly established beliefs about chance long before we teach any probability or statistics" (1993). Shaughnessy (1992) indicates, however, that these beliefs cannot be "checked in at the classroom door", that is, they conflict with 'school probability'.

The question is how we should deal with these intuitions/ beliefs in the classroom situation. Reflection on the Malati trialling process and further review of the literature suggests that the approach adopted might not have been the most appropriate: the developers were aware of the intuitions that pupils might bring to the study of the topic, but seemed very concerned about **replacing** these with "correct intuitions" in line with the "formal" approach to probability, rather than using the existing beliefs.

What is particularly challenging on reflection is that the approach used at Malati does not appear to be constructivist in nature as intended! It does not appear that what the pupils brought to the classroom was actually being used. In giving suggestions for an approach to probability from a radical constructivist position, Konold (1991) stresses that the conflicts between the classroom and the outside world should actually be welcomed by the constructivist teacher. It appears that the developers of the Malati curriculum were trying to avoid this conflict.

What alternatives exist for dealing with pupils' intuitions? In a review of the literature on the effect of instruction on pupils' intuitions, Shaughnessy (1992) notes some success in certain studies, but still attests to the difficulty in changing beliefs and conceptions. Carpenter, Corbitt, Kepner, Lindquist, and Reys (1981) note that some intuitions actually get stronger with age! Commenting on the findings of the Assessment of Performance Unit (APU) in England, Hawkins and Kapadia (1984) note that some pupils could give the correct theoretical predictions, but tended to revert to their original "hunches" when the results of experiments did not confirm the prediction. This was observed in the trialling of the coin and "Zama Zama" activities in the Malati package and points to the problems experienced when using the "frequentist interpretation" as a checking mechanism for "formal probability".

It is important to remember that these intuitions which some educators try to "retrain" can, in fact, be very useful in themselves. Shaughnessy (1992) notes examples of the use of "representativeness" and "availability" which prove useful in everyday life, but notes that problems occur when these are taken too far. It appears that these intuitions can also be useful in the learning of school probability: When introducing the "Zama Zama" activity, Teacher A used an approach which suggested that pupils were using their intuitions in the decision-making. He offered his pupils cokes if their made the correct prediction about which ball was most likely to be drawn. As soon as this real-life incentive was provided, pupils seemed more wary of jumping to conclusions and adapted their responses to the correct ones!

Fischbein's classification of intuition could be constructive in selecting a way forward: he refers to "primary intuitions" which are those pupils have before instruction, and "secondary intuitions" which are "restructured cognitive beliefs which are accepted and used as a result of experience or instruction. He stresses the belief that intuitions are adaptable. This approach suggests that intuitions should not simply be ignored, but should rather be used and adapted through appropriate instruction (as quoted in Shaughnessy, 1992).

Konold (1991) describes a classroom approach which could be useful in assisting pupils to adapt these intuitions: he suggests that pupils be encouraged to evaluate their intuitions according to the following three criteria: Firstly, do my beliefs agree or fit with the beliefs of others? Secondly, are my beliefs internally consistent? And lastly, do my beliefs fit with empirical observations? Of course, such reflection requires the appropriate classroom culture, but which once developed, could be valuable in all aspects mathematics learning.

The Way Forward:

It is clear from this discussion that the literature on the teaching and learning of probability provides the Malati Statistics Group with a number of possible strategies to deal with the problems noted. Some of these relate to general classroom culture, while others refer to the study of probability itself. The latter type of solutions can be framed within the "formal interpretation": Shaughnessy (1993), for example, proposes the following:

- that pupils be made aware of how beliefs and conceptions can affect decisions
- that simulation techniques be used so that pupils can confirm predictions
- that examples of misuse and abuse of statistics be discussed.

As mentioned, however, it appears that a number of the problems experienced during trialling were related to the use of “formal interpretation” of probability. What might be necessary is an adaptation of this framework. Hawkins and Kapadia (1984) propose an approach for the initial stages of the study of probability which merits further exploration. They suggest that a greater emphasis should be placed on “subjective probability” as this, they claim, is closer to pupil intuitions than “formal probability”. Furthermore, as it relies on comparisons of perceived likelihoods rather than on acquaintance with fractions, this interpretation should be more accessible to “less-mathematically sophisticated children” and at an earlier stage of education.

In this framework probability is assigned on the proviso that the individual assigning the probability is prepared to accept a bet on the basis of this decision. This approach is seen as valuable as it provides the opportunity for changes and for the individual to learn from experience. While proposing this approach Hawkins and Kapadia do stress the following:

- The importance of “coherence” in the assigning of probability, that is, that the choice is not going to result in certain loss for the decision-maker
- The need to study how “subjective probability” can be used for the study of “formal probability”. They note that the former will not spontaneously develop into the latter.
- The importance of developing an approach in which the “a priori”, “frequentist”, and “subjective” approaches play a role in providing pupils with an appropriate framework for the understanding of formal probability.

Conclusion:

A number of alternative approaches to the Malati interpretation have been suggested, but it is clear that more empirical research focusing on different methods of instruction is required The Malati Statistics Group hopes that the experiences and the discussion of related literature in this paper will contribute to the debate on nature of an appropriate probability curriculum for South African schools. The Group has a sense that the formulation of such a curriculum will be a challenging and extended process, but which, owing to the “slippery” nature of the topic of probability itself, will provide mathematics educators with exciting avenues for exploration and research.

References:

Bramald, R. (1994). Teaching probability. **Teaching Statistics, 16,** 85-89.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S. Jnr, Lindquist, M. M., & Reys, R.E. (1981) What are the chances of your students knowing probability? **Mathematics Teacher**, **74**, 342-344.

De Jager T. (1992). Probability a priori. **Pythagoras, 30,** 19-23.

Garfield, J., & Ahlgren, A (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. **Journal for Research in Mathematics Education, 19,** 44-63.

Hawkins, A.S., & Kapadia, R. (1984). Children's conceptions of probability – A psychological and pedagogical review. **Educational Studies in Mathematics, 15,** 349-377.

Konold, C. (1991). Understanding students' beliefs about probability. In Von Glasersfeld, E. (Ed.), **Radical Constructivism in Mathematics Education** (pp. 139-156). Dordrecht, The Netherlands: Kluwer Academic Press.

Konold, C. (1996). Representing probabilities with pipe diagrams. **Mathematics Teacher, 5,** 378-382.

Shaughnessy, J. M. (1981). Misconceptions of probability: From systematic errors to systematic experiments and decisions. In Schulte, A. P. (Ed.), **Teaching Statistics and Probability** (pp. 90-99). Reston, VA: NCTM.

Shaughnessy, J. M. (1992). Research on probability and statistics: Reflections and directions. In Grouws, D. (Ed.), **Handbook of Research on the Teaching and Learning of Mathematics** (pp. 465-494). New York, US: Macmillan.

Shaughnessy, J. M. (1993). Probability and statistics. **Mathematics Teacher, 86,** 244-248.