4. TEACHERS' MATHEMATICAL EXPERIENCES AS LINKS TO CHILDREN'S NEEDS

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Circumstances have forced us to design a two-day workshop programme for teachers with very weak mathematical backgrounds and very rigid and instrumental perceptions of the nature of mathematics and mathematics teaching. The aim of such a workshop is both to change these perceptions in teachers and to equip teachers for radically different classroom practice in line with their new perceptions. The technique of posing problems that are challenging to the teachers themselves and then encouraging reflection on their experiences has proved to provide a driving force and a network of connections that enable us to cover a number of major issues in a limited time.

Introduction

There are different ways in which an INSET programme may attempt to address these two issues, depending on which perceptions and which skills are addressed. We believe that, for mathematics teachers from kindergarten to twelfth grade, the perceptions that radically influence their classroom practice concern

- the nature of mathematics
- the way mathematics is both learnt and applied in life
- children's mathematical thinking
- the aims of school mathematics
- how children best learn mathematics, given particular aims.

The necessary skills are clearly those that enable the teacher to create and sustain on a daily basis the learning environment which will support the type of learning in children the teacher has come to accept as desirable.

Our own perspectives of the above matters are based on a socio-constructive view of knowledge, and on our continuing research on young children's thinking and on environments which seem to support their thinking. We try to implement these ideas in the classroom through a problem-centered approach to mathematics learning and teaching, where students are presented with problems that are meaningful to them but which they cannot solve with ease using routinized procedures, and where students are expected to discuss, critique, explain and justify their interpretations and strategies (Murray, Olivier & Human, 1993).

INSET programmes with views similar to ours on mathematics education may use a particular technique as part of their programmes: Such programmes expose teachers to doing mathematics at their own level in the hope that this will encourage teachers to reflect on the nature of mathematics and mathematics learning (e.g., Davidson, Weisglass & Robertson, 1990; Simon & Schifter, 1991; Hadar & Hadass, 1990; Corwin, 1993). However, much like the Educational Leaders in Mathematics (ELM) Project (Simon & Schifter, 1991), we take it one step further: We actively use the mathematical experiences to which we expose teachers as the core around which we construct the rest of the programme. Because of practical considerations, the format for INSET programmes which is usually requested by teachers and school principals has been a two-day workshop, in some cases with the expectations from both teachers as well as supervisors (school principals included), that such a workshop should bring about not only a paradigm shift in the teachers' perceptions about learning and teaching, but also equip the teacher to establish and maintain on a daily basis a completely different classroom culture.

Although we therefore use techniques that are common to other INSET programmes, we labour under a severe time constraint, which makes essential a careful selection of the main issues to be covered and the most economical way in which they can be covered. For the purposes of this paper, we limit our discussion to two-day INSET workshops for lower elementary teachers with a very low perception of their own mathematical abilities, and indeed only limited skills and little explicit understanding of basic whole number arithmetic. Where knowledge has been retained in a formalised fashion by some teachers, it is mostly of a highly instrumental nature.

It could be anticipated that these teachers would find it even more problematic to broaden their perceptions about the nature of mathematics and how mathematics is learnt than better educated teachers, yet this is only partly the case, as we shall indicate later. It is, however, the case that the time constraint has a much more severe (and negative) impact on these teachers than on other teacher groups, mainly because more time has to be spent on basic teaching skills such as classroom organisation and making work cards.

The workshop format that we use at present has remained constant for the past six workshops of this type, and has provided us with valuable insights. We shall, however, have to start planning INSET programmes that are less dependent on a particular workshop presenter and that can easily and safely be expanded to reach larger numbers of teachers.

Organisational Information

Each workshop runs for two consecutive days from eight o'clock in the morning to about a quarter past three in the afternoon, with breaks for tea and lunch. The number of participants has varied from 34 to 47, and consisted mainly of K-3 teachers, with a sprinkling of upper elementary teachers who function as subject heads for mathematics. Most of the teachers had only had school mathematics up to 9th grade, and some had left school after tenth grade. They all had at least a three-year teachers' diploma. During their school and college years they had been exposed to quite rigidly traditional views of mathematics as a series of set formulae which had to be memorised and then applied to the appropriate word problems. Although it appeared during the workshops that especially the K-3 teachers possessed strong intuitive powers for solving problems they could not identify as school-type problems, these thinking skills had never been sanctioned. Initially most teachers were embarrassed to explain their reasoning processes, saying that their strategies were not mathematically "correct".

One advantage is that with this group of teachers we were only dealing with K-3 mathematics, and we have much anecdotal evidence from other K-3 teachers who found that their own number sense and general arithmetic skills developed simultaneously with those of their students when they started to follow a problem-centered approach to mathematics teaching and learning. For the K-3 workshop, therefore, we believe that it is sufficient to attempt only the following:

1. Addressing teachers' perceptions about the nature of mathematics and how mathematics is learnt and practiced (used).

2. Addressing teachers' perceptions about their own mathematical abilities and how they (can) do mathematics.

3. Describing and justifying a problem-centered approach to mathematics learning and teaching.

4. Sharing information on some basic guidelines for establishing a problem-centered learning environment in the classroom. This includes dealing with matters that are seldom perceived as issues in classrooms where transmission-type teaching is the norm, but which in our longitudinal research have shown to be crucial for problem-centered classrooms.

According to our basic technique of providing teachers with their own mathematical experiences, the activities that address teachers' personal views (points 1 and 2) also supply us with direct links to children's thinking and children's needs (which we present by means of many examples of children's work and videos of children solving computational problems), which lead directly to points 3 and 4. We now briefly elaborate on the main ideas to be covered under points 3 and 4.

A problem-centred approach

In brief, the approach implies that the teacher regularly (i.e. on most days) poses problems to her students that they do not experience as routine problems and that they cannot solve with ease. Students are expected to construct solution methods for the problems with the tools that they have available (theorems in action, number knowledge at different levels of development). Students are also expected to share ideas, to discuss, justify and explain among themselves. Although students may (and should) experience classroom events as informal and child-centered, the teacher has to plan the classroom activities and tasks in accordance with a simple but important set of guidelines that we have been able to formulate through monitoring the development of students' computational strategies in selected classrooms over the past nine years (e.g. Murray, Olivier & Human, 1994).

The guidelines consist of:

- Certain simple but powerful activities that help students to develop a flexible number knowledge, which directly influences the solution methods they construct

- A variety of word problems that suggest different computational methods – if some problem types are omitted, certain methods may not be constructed. Teachers therefore choose their word problems from a list of basic problem types so that the different meanings of the four basic operations and fractions are all covered

- Students mainly learn through voluntary interaction with each other, and not through listening to the teacher, but the teacher has to know that social-type information still has to be supplied to her students (e.g. recording skills, and knowledge involving the measurements). The ability to distinguish between the logic of a solution method and the way in which it is recorded is essential for a teacher.

The problems posed to teachers in the workshops

When we use the teachers' own mathematical experiences and their reflections on these experiences as a laboratory to provide clues to (or empathy with) children's needs, the basic assumption is that adults' and young children's responses to novel mathematical situations are sufficiently similar to use in such a way. Simon and Schifter state this categorically: "Teachers' learning can be viewed in much the same way as mathematics students' learning." (1991:312). Although we accept this as probably generally true, it is not only obvious but also clear from experience that teachers (and other adults) only respond in ways that can be used as departure points for children's thinking when the mathematical problems posed are of such a type that the adults involved cannot solve them automatically (or mechanically), but actually have to construct solution strategies for them.

The choice of problem for a particular audience is therefore crucial. It is important that the problem situation makes sense, even though it may be ambiguous (this is referred to later). We have never used a puzzle-type problem or investigations, since we do not know whether a situation that has no clear connection with any syllabus content will have the same powerful effect on the teachers. We know of INSET programmes where investigations have been used very effectively, but the informal lore among the teachers themselves has it that when teachers only experience enjoyment with problems that they do not relate to a syllabus, they view these experiences as add-ons, "something you do every Friday." We cannot afford this to happen, since there is no time to correct this impression.

Furthermore, it should be possible to solve at least the initially-posed problem(s) by direct modeling, i.e., by drawing or a sketch, because direct modeling enables a person to easily resolve an incorrect choice of a drilled method at a deep level. At this stage in the workshop, a logical refutation only serves to strengthen existing beliefs about the nature of mathematics! It should be kept in mind that the problems used create powerful situations only because they suit these particular audiences and other audiences will need other problems.

An example to show how problems are used

In the workshop, the problem is always presented to the group as a whole. Teachers are encouraged to consult with each other, to leave their seats and move around if needed. The presenter moves around, trying to maintain a very low profile, but identifying a variety of different conceptualizations of the problem. Different teachers are then requested to explain on the overhead projector how they had conceptualized and then solved the problem. This is followed up with a general discussion, eliciting from the teachers the links that need to be established to future topics, or illuminating and emphasising points that will be referred to again. It must be emphasised that these discussions are very thorough and that the teachers really share not only their mathematical thinking but also especially their feelings and fears; i.e., all the factors which could have inhibited or supported their thinking.

The apple tarts

Mrs. Daku bakes small apple tarts. For each apple tart she uses 3/4 of an apple. She has twenty apples. How many apple tarts can she bake?

This is posed as the very first problem, immediately after the opening and welcome on the first day. The most common solution methods generated by the teachers are:

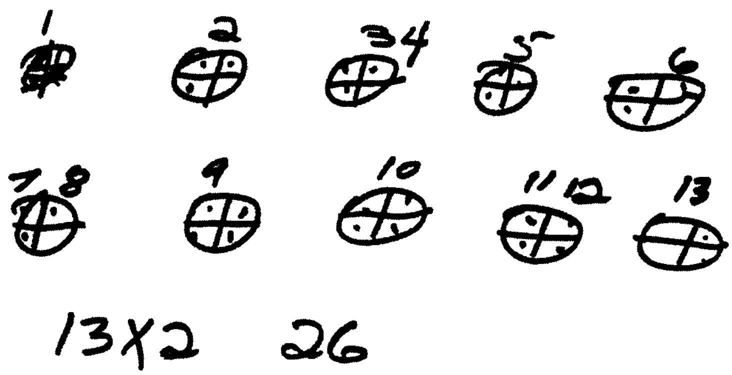
1 Incorrect choice of a drilled method.

$$20 \times \frac{3}{4} = 15$$

2 Direct modeling of the situation.

Twenty apples are drawn and each is divided into 3/4 and a 1/4; the 3/4 pieces are counted (giving twenty tarts), then the remaining quarters are grouped. Sometimes the remaining quarters are grouped into threes, giving another six tarts with two quarters left; sometimes the remaining quarters from each group of three apples are immediately dealt with.

Lillian solves the problem in this way, having first reduced the problem to ten apples and afterwards doubling the answer.



3. Numerical approaches which closely model the problem structure.

Cassius: "Twenty apples give me at least twenty tarts. With the twenty quarters I make five apples. Five apples give me at least five tarts. With the five quarters left I make one apple. There are two quarters left. I have twenty plus five plus one tarts, and half an apple left."

Sibongile: "Three apples give four tarts. How many groups of three in twenty? Six groups of three is eighteen. So eighteen apples is equal to 6 * 4 tarts. That is twenty-four. There are two apples left. That's another two tarts and half an apple left."

Beauty: "I thought about the kitchen. First I cut all the apples into quarters and then I find out how many groups of three I can make. So I do 20 * 4 = 80; 80 ÷ 3 = 26 remainder 2."

Beauty knew she could make twenty-six tarts, but she needed prolonged discussion and an inspection of one of the direct modeler's drawings to decide what the remainder of two signified.

These discussions generate a great deal of excitement among the teachers, especially when they are informed that from the formal point of view the problem involves division with a fraction, which is only specified in the local seventh grade school syllabus.

The following important perspectives arise naturally out of the whole episode:

1. Attempts to classify the problem type and choose an operation made the problem more difficult for some teachers, and an incorrect choice of operation prevented some teachers from solving the problem. Teachers who simply responded to the structure of the problem itself and who tried to make sense of the situation, using the tools they had available and felt confident with, were invariably successful.

   This very important perspective serves as a link to the next session during which examples of young children's responses to problems are studied, and comparisons made between the child's view and the adult's view of problems which seem quite routine to adults. Mistakes that children make when they feel forced to "choose an operation and apply a procedure" are discussed extensively.

2. The tools that were used to solve the problem are identified: a knowledge of fractions, of whole numbers and of some recording skill. This is elaborated on during a session where the development of young children's number concept is studied, and a practical demonstration, with some teachers acting as children, and videotaped classroom scenes give some ideas of suitable number concept development activities. The role of the teacher regarding the "social knowledge" component of mathematics, and the development of communication skills, both verbal and written, are also discussed.
3. The teachers are asked to reflect on how they went about it when they solved the problem: when did they talk to one another, about what did they talk, what was the main effect of these discussions on their thinking processes and why, etc. These issues touch on the classroom culture, the didactical contract between teacher and students, the nature of knowledge and how knowledge is constructed (individually as well as socially), some characteristics of a good learning environment for mathematics, research-based information on young students' own perceptions of what constitutes a good learning environment, etc. These topics take up most of the morning of the second day, and four or five video episodes of children interacting are shown and discussed.

The other problems posed to the teachers in the course of the workshop introduce other important issues. A proportional sharing problem involving three workmen generates discussion on the role that the context of the word problem plays, as regards the ambiguity of interpretation, its influence on the level of complexity of the problem, etc. Teachers are asked to reflect on how students may respond to some word problems, and are then asked to make up word problems for particular problem types, keeping in mind variables that affect young children's understanding of a particular word problem. Consensus is reached that different cultures and different backgrounds should not be ignored in the mathematics classroom, but should actually be subjected to discussion. Other problems lead to the idea that children themselves can generate mathematical ideas that are sufficiently rich to initiate and support discussions about new topics, given appropriate problems to think about.

Evaluation

Free-format evaluations invited from teachers at the end of every workshop were unanimously positive and enthusiastic. Most of the teachers mentioned that they now "knew where to begin" in their own classrooms, but that they desired a follow-up workshop in approximately six months' time. About 10% of the teachers involved suggested that the workshop be spread over three days, not to deal with more issues but to give them more opportunity for discussion and reflection. About half the teachers responded in person as well, stating that the workshop was the most meaningful and radical training experience that they had ever had. We mentioned in the beginning that workshops with any chance of lasting influence probably need to address the two main issues of beliefs and skills. Since the teachers' free comments mentioned both these issues extensively, there is at least the possibility that the workshops were to some extent successful. It is, unfortunately, the case that no workshop can really be evaluated until its effects on classroom practice can be observed. Changed classroom practice is yet again heavily dependent not only on the quality of the workshops given, but also on factors like peer, principal and supervisor attitudes and support.

Conclusion

It has been proved possible to identify some problems, which when posed to teachers during a workshop, will supply them with mathematical experiences that can serve as links to both the basic principles of a problem-centered approach, as well as to the practicalities of classroom organisation and the flow of classroom activities. The teacher's own experiences when solving problems can encourage reflection on what mathematics is, how mathematics-related learning takes place, and the factors that encourage or hinder such learning. These reflections can then help the teacher to understand her students' needs.

We would also like to draw attention to a point that we regard as potentially even more important than the above. It was stated earlier that these teachers perceived themselves as mathematically weak, and had been exposed to years of formalised mathematics instruction, and that it may be hypothesized that this background would make it more difficult for them to broaden their perspectives. It may be that these teachers actually respond very readily and at a very deep level to a workshop that is geared to providing them with personal experiences of their own thinking, sense-making abilities. We would therefore suggest that personal feelings of incompetence and anxiety which have been caused by formalist mathematics teaching may be turned to good account if handled correctly, and need not be a liability at all. We believe that these teachers can actually be brought to identify with and understand children's needs more deeply and truly during such a workshop than some of the more advantaged teachers.

Concluding on a personal note, we quote Marilyn, who at the beginning of the second day of a workshop, came up to the presenter and said with great excitement: "I thought and thought right through the evening and almost couldn't sleep last night. I have learnt the one thing I think is the most important thing I have learnt as a teacher; that is that if you show somebody how to do a problem, you stop him thinking. This is what happened to my mathematics."

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