Malati

Mathematics learning and teaching initiative

Fractions

Phase 1
Grades 4, 5, 6 and 7

Teacher Document

This package is based on materials by Hanlie Murray.

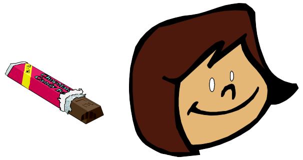
Malati staff involved in the adaptation of these materials and the development of additional materials:

Karen Newstead
Therine van Niekerk
Bingo Lukhele
Agatha Lebethe

We acknowledge the valuable comments of Amanda le Roux.

December 1999

1. Lisa Shares Chocolate



1. Lisa and Mary have 7 bars of chocolate that they want to share equally between the two of them so that nothing is left. Help them to do it.

2. Lisa, Mary and Bingo have 7 bars of chocolate that they want to share equally among the three of them so that nothing is left. Help them to do it.

3. Lisa, Mary, Bingo and Peter have 13 bars of chocolate that they want to share equally among the four of them so that nothing is left. Help them to do it.

Teacher Notes:

There are various ways of introducing fractions to learners. In this task an equal sharing situation with a remainder that can also be shared out is used. It is important that problems 1 and 2 are both attempted by all learners so that learners have to share among three friends as well as sharing between two friends. If the possibility of dividing an object in three equal parts is not met very early, learners are so caught up in halves and quarters that these function as barriers to the other fractions.

Depending on their previous experience, learners may respond to these problems in many different ways. Many learners will draw the problem situation and the answers. Most learners will share out as many whole bars as possible and then cut the remaining bar into two (or three or four) pieces.

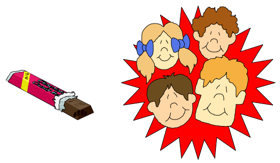
The following can also happen:

- Very weak learners or learners with no experience of fractions may cut all the bars into smaller pieces, or slice some bars into smaller pieces, and then share out the pieces. Sometimes the pieces may be of unequal size (e.g. whole bars, halves and thirds are treated as if they were the same) and the learner only focuses on the number of pieces each friend has to get.
- Some learners may through their drawings show that they have solved a problem correctly, but then misname the fraction part, e.g. draw a third, but call it a quarter.
- Some learners may solve a problem on a completely numerical level and obtain a correct or incorrect answer.

The teacher should ask the latter learners to draw their answers as well; even learners who may have written the correct answer may not know what it means.

Leave enough time for learners to show their ideas and discuss them. Especially learners who have sliced up more than just the remaining bar(s) should be able to inspect the other learners' ideas. There is ***no need*** to talk about the names of the fractions during this session.

2. Equal Sharing



1. Five friends want to share 11 chocolate bars equally. How must they do it?

2. Five friends want to share 21 chocolate bars equally. How must they do it?

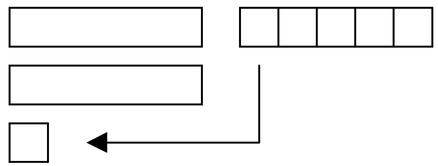
Teacher Notes:

This task is a continuation of the previous task. The teacher can expect to see similar responses from the learners as for the previous task. This task gives further opportunity for discussion and comparison of methods.

The only danger (but a very real danger) is that some learners will confidently state that the remainder should simply be written above the divisor for the correct fractional part, e.g. eleven shared among five is 2 rem 1, which is $2\frac{1}{5}$.

This is not acceptable - learners *must be able to draw their answers to prove that they understand it.*

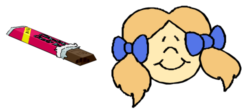
Therefore, even if a learner can give the answer to problem 1 immediately, she should still show classmates that the answer looks like this:



Of course this will no longer be necessary later on.

If there is time, the teacher can start discussing the names of the fractions during this session. It is dealt with explicitly in the next task.

3. Giving Names



When we divide something into 2 equal parts, we call these parts halves

When we divide something into 3 equal parts, we call these parts thirds

When we divide something into 4 equal parts, we call these parts fourths or quarters

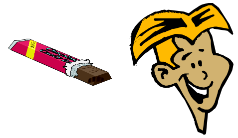
When we divide something into 5 equal parts, we call these fifths

1. What would you rather have, a third of a chocolate bar or a fifth of a chocolate bar? Why?

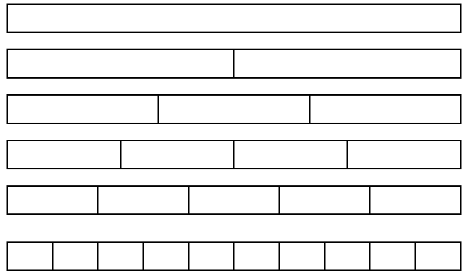
2. What would you rather have, a fourth of R1 or a third of R1? Why?

3. How many cents is a half of R1? A fourth of R1?
   What would you rather have, a half of R1 or two fourths of R1? Why?

4. Putting Pieces Together



Look at these big chocolate bars. They have all been cut into different equal pieces. Give the pieces names (halves or thirds or whatever you think). Now see if you can put together some smaller pieces to form a bigger piece with another name.



Write down what you have found.

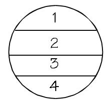
Teacher Notes (Worksheet 3 and 4):

These tasks are very important and the teacher may have to spend two days on it with some learners. However, learners need not find all the possibilities for Worksheet 4.

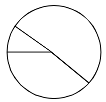
These two worksheets have three purposes:

Firstly, the teacher discusses the names of the fractions with the learners, as well as the important idea that fractions imply *equal parts*.

For example, this circle is divided into four parts, but the parts are not equal. However, parts 1 and 2 together form a half.

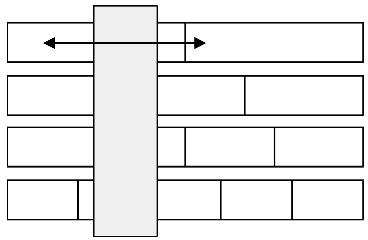


Also, these are not equal parts.



Because it is so difficult to divide round objects into equal parts (except into halves, quarters, etc.), "long" objects are mostly used in these worksheets.

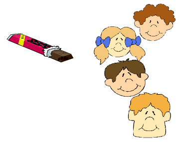
The second purpose of the worksheets is to start learners thinking about the equivalence and non-equivalence of fractions. Equivalent fractions is a crucial idea in operations involving fractions. Worksheet 4 especially initiates thinking in the direction of equivalent fractions.



The teacher should encourage learners to use any way they may think of to identify the equivalent fractions; a ruler slowly moved from left to right across the fraction wall is one good idea.

The third purpose of these worksheets is to introduce fractions as parts of collections of objects and not only as a part of a single object, e.g. a fourth of R1, which can only be represented as a fourth of 100c, thus 25c. This use of fractions (as part of a numerical quantity) occurs repeatedly in these worksheets.

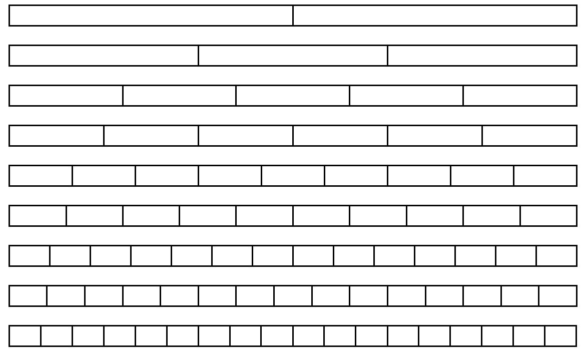
5. Fair Sharing



1. Six friends want to share 7 chocolate bars equally. How must they do it?
2. Six friends want to share 8 chocolate bars equally. How must they do it?
3. A chocolate bar is cut into two equal pieces. What do you call each piece?
4. A chocolate bar is cut into 7 equal pieces. What do you call each piece?

6. Making Bigger Pieces

Look at these chocolate bars which have been cut in different ways:



1. How many sixths of a chocolate bar must you put together to make a third of a chocolate bar?
   How may sixths to make two thirds?
   How many tenths to make a fifth?
   How many tenths to make a half?
   How many ninths to make two thirds?

2. Write down all the bigger pieces you can make from
   ninths fifteenths eighteenths

Teacher Notes (Worksheets 5 and 6):

These worksheets cover many of the same concepts as the previous worksheets, with the exception of problem 2 on Worksheet 5. It is absolutely essential to repeat the problems in this way because concepts take a very long time to develop and even longer to become stable. Also, learners very seldom meet fractions in ordinary life, except for the words half and quarter, words which many young learners misunderstand anyway. They often regard a half as ”any bigger piece", and a quarter as “any piece smaller than that".

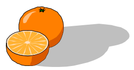
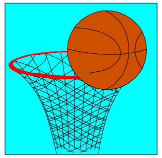
Problem 1 of Worksheet 5 is an equal sharing problem with a remainder of one, which has to be cut into six equal pieces and named correctly.

Problem 2 is a new situation. This time there is a remainder of two chocolate bars that still have to be shared. Learners who have heard too much about halves and too little about the other fractions tend to resort to halves and then get stuck. To prevent this from happening, the previous worksheets as well as problem 1 of Worksheet 5 involved the other fractions heavily. At this point learners would rather think of slicing each of the remaining chocolate bars into six pieces, giving each of the friends a piece from each chocolate bar. Each friend would then get one and two sixths of a chocolate bar. The teacher need not show learners how to write $\frac{2}{6}$ rather keep to “two sixths". The fraction notation is introduced in the next worksheet.

Other answers are also possible, e.g. if you cut each of the remaining chocolate bars into three equal pieces, each friend receives one chocolate bar and a third of a chocolate bar.

Learners are now confronted with the problem of deciding whether $1\frac{1}{3}$ and $1\frac{2}{6}$ are the same or not - the whole of problem 1 of worksheet 6 involves questions of this type. Learners can answer these questions by sliding their rulers over their fraction walls as shown for worksheet 4. These equivalent fractions *should **not*** be memorised to be recalled quickly: whenever learners need to check for equivalent fractions, they should try to resolve the problem physically, e.g. by consulting their own fraction wall. Take worksheet 6 very slowly and discuss the ideas thoroughly.

7. Food for the Netball Team

A short way to write a half is $\frac{1}{2}$

A short way to write a seventh is $\frac{1}{7}$

A short way to write a twentieth is $\frac{1}{20}$

1. Two netball teams play a game. There are 14 children all together. The sports teacher wants to give each child $\frac{1}{2}$ of an orange. How many oranges does she need?

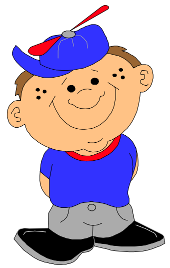
2. One of the parents brings a bag with 35 chocolate bars to share among the 14 players. How much chocolate bar does each player get?

Teacher Notes:

Problem 1 introduces a very important concept, that of *putting together* fractional parts to form wholes. It is possible that some learners may want to draw the halves and then circle or link pairs of halves to find the number of wholes; other learners may use more numerical methods.

Problem 2 seems more difficult than it is. It is an equal sharing problem with a remainder. If 35 chocolate bars are shared among 14 players, each player will get 2 chocolate bars and there is a remainder of 7. In this case the shortest solution would be to divide each of the remaining 7 chocolate bars in half to obtain 14 halves, one half for each player. Learners who, according to the way they solved the other problems, cut each remaining chocolate bar into 14 pieces and then give 7 pieces (one piece from each chocolate bar) to each player, will obtain an answer of $2\frac{7}{14}$. These two possible answers, $2\frac{1}{2}$ and $2\frac{7}{14}$, should then be reconciled. The fraction wall for worksheet 6 includes a representation of fourteenths that can be compared with halves.

8. Name and Share (Assessment)



1. A chocolate bar is cut into 9 equal pieces. What do you call each piece?

2. A chocolate bar is cut into 5 equal pieces. What do you call each piece?

3. Three friends want to share 7 bars of chocolate equally between them. How much chocolate does each friend get? Draw a picture to show how you got your answer.

4. Three friends want to share 8 bars of chocolate equally between them. How much chocolate does each friend get? Draw a picture to show how you got your answer.

Teacher Notes:

This activity is intended to diagnose problems that the children might have with the naming of fractions and with sharing problems.

Two kinds of sharing problems are addressed:

- Where one whole is left to be shared among the friends, and
- Where *more* than one whole is left to be shared among the friends.

It is important that the specific problems or misconceptions which children might have are addressed immediately after this activity and aret left to become deeply entrenched. If there is a need for consolidation MORE SHARING and SAUSAGES can be used.

9. Cake for the Netball Game

Mrs Jerimiah bakes five cakes for the party after the netball game.

For one cake she needs:

$\frac{1}{4}$ cup margarine	$\frac{1}{2}$ cup sugar
1 egg	$\frac{1}{2}$ cup milk
$1\frac{1}{2}$ cups flour	$2\frac{1}{2}$ teaspoons baking powder
$\frac{1}{4}$ teaspoon salt	$\frac{1}{2}$ teaspoon flavouring

(Kook en Geniet, 17th edition, 1964)

Work out how much she needs for five cakes.

Teacher Notes:

The first problem of the previous worksheet required putting together halves to form wholes. This worksheet presents a more difficult version of the same problem. Since eight different ingredients have to be dealt with, learners should be allowed sufficient time to complete this problem.

This problem touches on the addition of fractions, a topic which causes many problems and leads to many severe errors in higher school grades. One of the most common errors is the following: $\frac{3}{8}+\frac{3}{8} = \frac{6}{16}$. The errors indicate very strongly that learners need *extended* exposure to *practical* situations where fractional parts have to be added, as well as, of course, a very stable concept of fractions themselves (for example, that a third is a bigger part of a whole than a fifth).

This problem is conceptually so important that learners should be encouraged to represent the quantities involved as fully as possible - if they tell the teacher that they would rather do the problem physically with cups, etc., using water and sand to represent the ingredients, the teacher should try to arrange this.

10. Preparing Porridge

Peter and Anna prepare soft porridge for breakfast. For each bowl of porridge, they use $\frac{1}{3}$ of a litre of milk.

1. If they make 6 bowls of porridge, how many litres of milk do they need?

2. They have 5 litres of milk. How many bowls of porridge can they prepare?

3. They have $2\frac{1}{2}$ litres of milk. How many bowls of porridge can they prepare?

Teacher Notes:

Problem 1 reinforces the idea in the previous worksheet: that fractional parts can be put together to form wholes. This task may be a little easier than the previous task - this is also intentional.

What the problem really asks is how many wholes are needed for six thirds. It is therefore a repeated addition problem, i.e. one of the multiplication types. Learners will probably solve it by putting together groups of three thirds to form wholes.

Problem 2 is the opposite of problem 1. Learners must work out how many thirds are there in five wholes. This is a grouping problem, i.e. how many thirds (groups of size a third) can be taken out of five? Learners will probably think like this: one litre gives enough milk for three bowls of porridge, so five litres will give enough milk for 3 + 3 + 3 + 3 + 3 bowls of porridge.

Problem 3 is exactly the same, except that learners also have to decide how many thirds can be taken out of half a litre.

FORWARD TO FRAC1(2)