Malati

Mathematics learning and teaching initiative

Geometry

Module 1

Interaction in Physical Space

Grades 4 to 7

Teacher document

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Overview of Spatial Module 1: Interaction in Physical Space

This module provide learners with opportunities to explore 3-dimensional objects in space. It is important that learners be given lots of experiences working with objects in space as this kind of activity forms a basis for the study of space in more abstract form. These are not the only activities related to the physical exploration of space – in each of the Malati modules the learners are provided with such opportunities.

The materials provide learners with a range of experiences appropriate for learners on the van Hiele visual level, for example, manipulating and sorting concrete objects, drawing and constructing figures and objects, and describing figures in their own words. Activities of this nature, together with guidance from the teacher, for example, by providing terminology where necessary, facilitate movement to the next Van Hiele level (analysis).

The aims of these activities are:

- 1. To examine the properties of different 3-dimensional objects. Learners are required to sort a variety of real life objects, for example, boxes and tins into different groups. They are likely to begin classifying according the overall shape and size, but the teacher can encourage learners to focus on the properties of these objects, for example, the number of faces, edges, corners, and curved surfaces and the shape of the different faces (rectangles, squares, circles, triangles etc). (See Sorting 1.)
- 2. To examine the properties of 2-dimensional figures, for example, by sorting given figures into different groups according to the number of sides, vertices etc. (See *Flat Figures.*)
- 3. **To distinguish between 2-dimensional figures and 3-dimensional objects**, for example, distinguishing between a rectangle and a rectangular prism
- 4. To learn, with the help of the teacher, the appropriate terminology for 3dimensional and 2-dimensional objects.
- 5. To be able to identify the 3-dimensional objects when represented in 2dimensional pictures. Learners are required to match the everyday objects to drawings of everyday and mathematical objects. Only the recognition of the appropriate figure is necessary at this stage, and the complexities of the representations are not explored (the actual nature of the representations is addressed in more detail in Spatial Module 2). (See *More Objects* and *More Pictures.*)
- 6. To explore the relative position of real objects and the perception of these objects from different places. Learners have to describe the position of a collection of objects on a table, for example, to the left / right, in front / at the back, and the relative size of the objects (objects far away look smaller than those close to you). (See *Where are the Objects 1?*)
- 7. To draw concrete objects from different perspectives. At his stage drawings are informal. Drawing conventions are introduced in the second module.

The teacher can provide rich experiences for the learners by varying the nature of the concrete objects used in the activities and encouraging learners to classify figures in different ways. More experiences with working with these objects is provided in

Spatial Modules 2 and 3 where learners explore representations of 3-dimensional objects in 2-dimensions, for example in drawings, nets and cross sections.

The representation of 2-dimensional objects in two-dimensions

When we represent 3-dimensional objects in drawings we use certain conventions. Learners who have no knowledge of these conventions will not be able to make the links between the real object and the picture. For example, a learner might know that a cube has six faces and twelve edges of equal length, but this does not look like this when we draw a cube in perspective!

In this picture one can only see three faces and the edges do not look equal.



It is important, therefore, that we assist learners in making the connections between real objects and the pictures that represent them and that we make the drawing conventions explicit.

There are a number of ways in which a 3-dimensional object can be represented in two dimensions. Consider for example the different representations of a cube:

1. A Plan / Orthogonal Projection: The plane cross section is drawn. This is what one would see when one looks at a cube "straight on".



objects are preserved, the opposite sides of each square in the cube remain the same length and parallel.

2. Non-Perspective Drawing (parallel projection): Distances between parts of the

3. *Perspective Drawing:* The sizes of the parts of the object are adjusted according to their distances from the viewer. The front of the cube is closer to the viewer so it is drawn bigger than the back of the cube. So the length of the lines and the parallelism is not preserved (although the distance and the parallelism in the plane parallel to the viewer is preserved). This is how objects appear in photographs.



4. Nets: The object appears to have been "unfolded":



In the first module we do not make the different representations explicit. Rather we require only that learners match real life objects with their representations. This gives us an idea of whether learners can recognise the representations. It is only in the second and third spatial modules that we explore the conventions in more detail.

In Spatial Module 1 all drawings of 3-dimensional objects are done in **perspective** as this is consistent with most representations learners encounter, for example, in photographs. In Spatial Modules 2 and 3 the differences between the different representations are made explicit.

It should be remembered that 2-dimensional drawings of objects are **scaled-down** representations of three dimensional objects. For example, this drawing of a tennis ball is not life-size:



As a result some learners might have problems recognising the correspondence between the representation and the real life object it represents.

We are not separating firstly, interaction with 3- and 2-dimensional figures and secondly, interaction with actual objects and their representations. It is felt rather, that there should be a dynamic relationship between each aspect of these forms of interaction. For example, a learner might find that they can recognise rectangles or squares in a real cereal box. Thus the study of 3-dimensional objects can enrich the study of 2-dimensional figures.

LIST OF ACTIVITIES

Various additional possibilities for the use of these core activities are provided in the teacher notes for each activity. The use of these possibilities is important in ensuring that learners are given rich experiences working with the two- and 3-dimensional objects.

As indicated below, some of the activities do not have learner handouts.

Sorting 1 (Only notes for teachers) Pictures More Pictures Fill in the table More Objects A Short Project

Flat Figures and Objects That Take Up Space Flat Figures Where are the Objects 1? (Only notes for teachers) What Can I See? (Only notes for teachers) Where are the Objects 2? Photos

The activity *Sorting 2* in which learners sort 2-dimensionla figures and drawings of 3dimensional objects has been included in Spatial Module 2. It can thus be done after the different forms of representation and the conventions associated with these have been made explicit.

Curriculum 2005

Specific Outcome 7 for Curriculum 2005 states that learners must "Describe and represent experiences with shape, space, time and motion, using all available senses". Specific Outcome 8 emphasises the study of 2- and 3-dimensional objects in natural and cultural contexts. The activities in the Malati Spatial Modules 1 to 3 provide learners in the intermediate phase with a range of appropriate experiences which can be done over an extended time period and will contribute to the achievement of the required outcomes by the end of the phase. Those areas for which these activities are shown:

Specific Outcome	Assessment Criteria
SO7 Describe and represent experiences with shape, space , time and motion using all available senses	 Assessment of the position of an object in space Descriptions of changes in shape of an object Descriptions of the orientation of an object Demonstrate an understanding of the interconnectedness between shape, space and time
SO8 Analyse natural forms, cultural products and processes as representations of shape, space and time	 Recognition of natural forms, cultural products and processes and their value Representation of natural forms, cultural products and processes in a mathematical form Generation of ideas through natural forms, cultural products and processes
SO2 Manipulate number patterns in different ways*	 Evidence that number patterns and geometric patterns are recognised and identified using a variety of media Completion and generation of patterns Exploration of patterns in abstract and natural contexts using mathematical processes
SO4 Critically analyse how mathematical relationships are used in social, political and economic relations	 Demonstrate knowledge of the use of mathematics in determining location

*In the activities in this module learners are required to compare 3-dimensional figures and to compare 2-dimensional objects. This process of classification involves looking for patterns. For example if a learner is comparing these 3-dimensional objects s/he could note that both the cube and the rectangular prism have six flat

faces, whereas the cylinder has a curved surface. In essence the learner is looking for patterns.







Sorting 1:

Teacher Notes

Have a collection of **different shaped** objects in the classroom: toothpaste boxes, cereal boxes, cheese boxes, "toblerone" boxes, tins, tennis balls etc. Variety in shape and size is important.

Ask the learners to sort the objects in different ways and to explain in each case why they have been sorted in this way. Stress that the designs on the objects are not important, but rather their shape.

It is important to note that learners on different van Hiele levels will respond differently. Responses should not be regarded as right / wrong, but only as an indication of the kind of experiences and instruction required by each learner. Some possible responses are:

- Grouping figures objects together according to their size, or those objects that "can be used for breakfast".
- Grouping the cylinders and spheres together because "they have curves" or "they are round"
- Distinguishing between a cylinder and a sphere, saying that a sphere is "round all over"
- A learner might group the rectangular prisms (toothpaste and cereal boxes etc) because they "are made up of rectangles". Those prisms with square faces might be distinguished from these.

When considering learners' groupings it is important to remember that, when classifying objects in mathematics we emphasise certain properties and ignore others. For example, consider the following objects:





In mathematics we would group the toilet roll and the tin of beans together, so we are in fact ignoring the fact that the ends of the toilet roll are empty. But we would not group the cup with these cylinders. The cup has an open and a closed end, but the size of the circular ends differs, so we place it in a different category to the cylinders. Learners might group the cup with the toilet roll as the ends are open – this is a perfectly acceptable grouping, but the teacher should make explicit what characteristics are taken into account in **mathematics**.

Learners are likely to use informal language – calling a cube a "box" or a "block". The discussion amongst learners in small groups and in a whole class is a useful opportunity for the teacher to gauge the use of language by learners. The teacher can introduce and reinforce the use of terminology where appropriate, for example, the names of the different objects (cube, rectangular prism, pyramid, sphere etc) and the terminology for the different parts of the objects (face, edge, vertex). Formal language should, however, only be introduced after learners have described the objects in their own words.

Some learners are likely to call a toothpaste box a rectangle. This is often a case of a lack of vocabulary as a learner might only know the word "rectangle". The teacher should focus the learner on the differences between a rectangular prism and a rectangle, that is, 3- dimensional and 2-dimensional figures.

Theoretically 1-, 2- and 3-dimensional objects can be distinguished as follows:

- A 1-dimensional object has length only, for example, a line
- A 2-dimensional object has area (length and breadth), for example, a geometrical figure such as a square
- A 3-dimensional object has volume (length, breadth and height), for example, a cube or cylinder.

One could assist young learners in distinguishing between two- and three-dimensions by considering whether the object can be filled with something like water, sand or sweets and considering whether the object can be 'covered' with a piece of paper.

The teacher can then alert the learners to the fact that the rectangular prism is made up of rectangles (and sometimes squares). This can lead to rich exploration of different geometric figures. "How many rectangles are there in the rectangular prism?". "How many rectangles / triangles are there in a triangular prism?"

The teacher has an important role to play in encouraging learners to focus on the properties of the objects (and not just on the overall shape). S/he should encourage learners to try different groupings. Some questions that can be asked are:

- "Which objects will roll on a flat surface?"
- How many vertices / edges / faces does the object have?
- What is the shape of the faces? Which faces are the same?

Further Activities:

• Learners could be required to identify other examples of the different objects in their everyday life. For example, the teacher can require that learner identify all the rectangular prisms in the classroom.

Pictures:

Teacher Notes:

The activity sheet "Pictures" on the next page provides 2-dimensional representations of 3-dimensional objects. The pictures have been selected to challenge common limiting conceptions, for example, learners may recognise that the drum in figure A represents a cylinder, but not the coin in figure G. Also that figure F (the brick) represents a rectangular prism, but not figure L (the present). The sandwich in figure E is a triangular prism, but the piece of cake in figure C has a curved surface. Learners should also be able to note the differences between figure E and figure C, figure N and figure B, and figure E and figure H.

The representations C, N and R have been included to encourage learners to compare the properties of the different representations. The objects that these represent do not have standard names, but learners can be encouraged to think of their own names, for example, a "wedge" for C.

The sheet can serve a number of purposes:

- Matching 3-dimensional objects with 2-dimensional objects: Have learners match the 3-dimensional objects from the previous activity, "Sorting 1" with these pictures of objects and give reasons for the groupings. So a tin of beans in real life could be matched with figures A and G, and a toothpaste box could be matched with figures F, K, and L (and possibly P).
- 2. Sorting 2-dimensional **representations** of 3-dimensional objects: Have learners sort these **pictures** of objects into groups and explain the groupings Encourage learners to give the objects in the pictures the correct names, for example, "Identify all the cylinders".
- 3. Matching 2-dimensional objects: Have learners match the pictures in this activity with those in the activity "More Pictures".

Further Activities:

Encourage learners to collect copies of photographs from newspapers and magazines and to try to identify the objects represented (see "Short Project").

Pictures



More Pictures



Pyramid

Teacher Notes: More Pictures:

This activity is designed to familiarise learners with the way 3-dimensional objects are represented for mathematical purposes. It also provides a useful opportunity for using and reinforcing the appropriate terminology.



- 1. Have learners match, giving reasons, the 3-dimensional objects used in the activity "Sorting 1" with these pictures.
- 2. Have learners match the pictures of the objects in the activity "Pictures" with those in this activity. Reasons for the groupings should be given.

Fill in the Table.....

Look carefully at the objects you have.

Fill in the information about each object on this table:

	Cube	Rectangular Prism or Cuboid	Triangular Prism	Square Pyramid	Sphere	Cylinder
How many faces?						
What shape/s are the faces?						
How many vertices (corners)?						
How many edges?						
How many curved surfaces?						

Teacher Notes: Fill in the Table...

During the sorting activity, "Sorting 1", learners should be encouraged to explore the properties of the different objects. For example, the teacher can ask:

- How many squares make up a cube? (the word "faces" of the cube can be introduced) How many edges? How many vertices (corners)?
- How many circles can you see on a cylinder?
- Does a sphere have any corners?

This activity provides a table in which this information can be written down. In some cases learners will not be able to fill in certain information about an object, for example, a cube does not have any curved surfaces and a cylinder does not have any vertices. This should be left open for learners to indicate where an object does not have any of the required properties.

Young learners might not be familiar with the mathematical terminology such as faces, vertices etc. The teacher should assist here. This is also a useful opportunity to introduce and / or reinforce terminology. Drawings of the different objects have also been provided to assist learners.

A column has been left open for the teacher to add other objects which might have been studied in class, for example, a triangular prism.

Further Activities:

- Learners are given the opportunity to explore the properties of a particular group of 3-dimensional objects (the platonic solids) in the activity "Platonic Solids" in Spatial Module 3.
- Group learners in pairs. One learner should describe a particular object to the other learner – this learner must then guess what is being described. For example, a learner with a toothpaste box could say the object has six faces, four of which are rectangles and two of which are squares. The language used will, of course, depend on the level of the learner.
- Have learners close their eyes and describe an object that is placed in their hands. This will require that learners explore the properties of the objects and use the appropriate terminology.

Short Project

You will need old magazines and newspapers for this activity.



Find pictures of 3-dimensional objects in the magazines and newspapers.

- Cut these out and paste them onto paper
- Give each object a name
- Explain how you know what the object is.

Try to find pictures of as many different objects as you can.

Here is an example:



You will be assessed on:

- Can you identify the objects in the pictures?
- Do you find pictures that have a variety of different objects?
- Can you name the objects?
- Can you write down your reasons?
- Can you explain your reasons to the teacher?
- Is your work neatly presented?

Teacher Notes: A Short Project

This is an idea for a short project. The teacher can use it to assess different aspects of a learner's performance – the teacher can follow these guidelines, or select certain aspects to assess.

By requiring learners to write down their answers as well as explain them to the teacher, the teacher can assess a learner's written and oral communication skills. The discussion can also help the teacher to clarify certain written responses where necessary.

		Response	Comments
Choice of picture	Selects pictures of a 3- dimensional objects Use correct mathematical terminology to name the object		
Written explanation	Is the responsible understandable to others?		
Oral communication	Is the responsible understandable to others?		
Presentation			

This is an idea for an assessment grid:

The teacher can use the categories attained / partially attained / not yet attained to classify the responses. Comments regarding instruction that a learner might need should also be recorded.

Flat Figures and Objects That Take Up Space

Look at the box on your desk and the picture your teacher has drawn on the blackboard.

The box **takes up space**. It has **thickness** and we can look at it from the front, the side and from above. We say it is a **3-dimensional object**.

The drawing of the box on the blackboard is **flat** and has **no thickness**. We call it a **2- dimensional object**.



Which of these objects are 3-dimensional? Which are 2-dimensional?

your body

a spoon

a tin of beans

a toothpaste box

this square:

a school shoe

this picture of a cylinder:

Can you think of some more 3-dimensional objects?

Teacher Notes: Flat Figures and Objects That Take up Space

The teacher must have an actual box, for example, a cereal box, in the classroom and must draw a simple picture of this box on the blackboard.

This activity is designed to proved learners with the social knowledge required to distinguish between 2- and 3-dimensional objects.

It is common for learners to call a 3-dimensional object cube a "square". This is often a case of learners lacking the appropriate vocabulary. Furthermore, learners are only exposed to 2-dimensional figures at school. The teacher needs to focus the learner on the differences between the two types. For it is precisely because of the differences between 2- and 3-dimensional objects that we cannot call a cube a "square"!.

It is important that learners distinguish between an actual box (3-dimensional), and a **picture** of that box which is 2-dimensional.

Flat Figures

Group the figures that you think belong together. In each case *give a reason* for the way you grouped them:



Teacher Notes: Flat Figures

This activity enables the learner to focus on the **2-dimensional figures**. Some learners might need to cut out the figures before grouping them.

The figures have purposefully been drawn in different orientations. It is hoped that, by presenting learners with figures in different sizes and orientations, it might prevent them forming limiting conceptions. For example, many learners will always recognise a square drawn like this:

But they will not recognise this as a square:



Such limiting conceptions can arise when we only present learners with figures in particular orientations.

Figures S, X and Y have been included to highlight the difference between open and closed figures and those that have straight sides and those with curves. In mathematics the closed figures with straight edges are called **polygons**. The closed 3-dimensional object formed with polygons, for example, cubes, prisms and pyramids are called **polyhedra**. The teacher can "drip" this terminology where appropriate in Modules 1,2 and 3.

The teacher should encourage learners to try different groupings and should use the discussion to reinforce the appropriate terminology.

Responses are likely to vary according to the van Hiele level of the learners.

For example, a learner on the visual level will group the figures according to their appearance as a whole, for example, they might select rectangles because they are shaped like "doors" or are "long and thin". These learners are likely to group figures together according to the number of sides, for example, placing all the triangles together, or distinguishing between the hexagon (figure H) and the pentagon (figure U).

Learners on the analysis level will be able to focus more on the properties of the different figures, for example, the rectangles will be grouped together because they have two long and two short sides.

It is important that the teacher focus learners' attention on the properties of the figures and encourage them to compare the figures. Learners should be encouraged to zoom in on particular groups of figures, for example, the triangles or the quadrilaterals or those figures with curves.

• For learners who place the rectangles together, the teacher should ask about figure M which also has two long and two short sides. Learners will then have to look at other properties such as the angles to note differences

- For learners who identify squares as those figures with all four sides equal, the teacher can get them to compare figure E which is a rhombus and also has all four sides equal.
- Figures F and J can be grouped together because they have curves, but how are they different?
- Learners should be encourage to compare figure B (an irregular quadrilateral) and figures T and V (trapeziums).
- Learners should note that all the sides of the hexagon (figure H) are equal so it is a regular polygon. Which other figures are regular polygons?

Where Are The Objects 1?

Teacher Notes:

The aim of the activity is for learners to explore the relative position and size of actual objects in space and to learn the required vocabulary. This is in preparation for later activities in which learners have to **visualise** the scene in a **2-dimensional representation** and do not have access to the actual objects.

In this activity learners will need to walk around objects placed on a table. It is suggested, therefore, that the teacher work on this activity with a group of learners at a time. The other learners could be engaged in enrichment in another mathematical topic.

Have learners place three objects on a table like this:



Ask the following questions:

- Have the learners look at one of the objects on the table, for example, the sandwich. Describe the position of the bottle and the box in relation to the sandwich on the table. Responses could be the bottle is in front and to the right of the sandwich, the box is behind the sandwich.
- Then ask the learners to change position and to describe the changes, for example, if a learner moves to the other side of the table as illustrated above, the sandwich will be on the left of the box. What would happen if you looked at the table from the other side? What would happen if you looked at the objects from the side (e.g. point A or point B)?
- Is there a point from which you could look at the table and not see the sandwich / bottle / cereal box? (learners will have to take the relative size, shape and position of the objects into account in this case).
- What happens when you move closer to the table / further away from the table? (Learners should note that the objects do not actually get bigger or smaller, but that they only look bigger or smaller because the relative position of the viewer has changed.)

The teacher can also require that the learners **predict** what they will see first and **then** actually move position to check the prediction. A task such as this is important in encouraging the learner to form a mental picture of the object/s and to adapt this picture.

The actual objects, position of the objects and number of the objects on the table can be changed to extend the activity.

What Can I See?

Learner Activity: This activity can be done using small objects in the classroom or exploring objects in large-scale space, for example the school buildings.

<u>Activity 1:</u> Select two suitable buildings in the school grounds. Let learners explore the conditions under which they can view the buildings. For example: Where must you stand to see the one building behind the other?

They should note, for example, that as one moves from point B closer to the small building one can see less and less of the large building (although the large box does appear bigger as one moves towards it!).

Activity 2: Place two boxes on the table, as for the activity "Where are the Objects 1?":



As for Activity 1 learners should be asked to consider where they must stand to see one box behind the other. Where must one stand so that the cereal box cannot be seen at all? And where must one stand so that the juice carton cannot be seen at all?

Teacher Notes: What can I see?

Α

The aim of this activity is for learners to explore the relationships between themselves and objects in space and between these objects themselves. Consider for example the following two boxes:



В

If one views the boxes from point A (the side view) one will not be able to see the smaller box because it will be hidden by the large box. If one looks at the boxes from point B (the side view), one will see the large box behind the small box. But if one moves from B closer to the boxes then at some stage the side view will only show the small box.

Learners should be provided with a number of experiences to explore the relative position of real objects. This is in preparation for activities exploring the relative position of objects as given in **visual representations** of the objects.

Where are the Objects 2?

Look at this picture of objects on a table:



Describe the position of the cereal box and the soccer ball in relation to the cooldrink can when you are looking at the objects from

- 1. point A (looking 'straight on')
- 2. point C (looking from behind)
- 3. point B on the right of the table
- 4. point D on the left of the table.

Teacher Notes: Where are the Objects 2?

This activity develops on the practical work done in the activity "Where are the Objects 1?"

In this case learners have to visualise and then describe the relative position of the objects using the 2-dimensional representation of the objects. It is important that learners have rich experiences with the concrete objects before attempting this activity. In some cases the use of the activity can be delayed until learners' visual skills have developed sufficiently – in the meantime the teacher can provide learners with more activities like "Where are the Objects 1?", but just varying the nature and number of objects on the table.

Photos



Choose an object such as cereal box or cooldrink can for this activity. Place the object on your desk.

You are going to pretend that you are a photographer. Your teacher will help you to make a camera.

- 1. Pretend that you are taking a photo from the front. Draw what you see when you take the photo.
- 2. Now move to behind the object. Draw what you see when you take the photo.
- 3. Now move to the side of the object. Draw what you see when you take the photo.
- 4. What would your photo look like if you were taking photograph from the top?
- 5. What happens to your picture when you move closer to the object?
- 6. What happens when you move away from the object?
- 7. Now take another photo of the object. Make sure that the photo is **different** to any of your other photos.

Teacher Notes: Photos

Learners will require objects such as cereal boxes, cooldrink bottles etc for this activity.

Learners can use matchbox lids or cardboard toilet rolls as cameras. They can also make cardboard 'frames' like this:



The teacher might like to demonstrate how learners can take "photos" of the object.

The activity is designed to give pupils an opportunity to explore the space in the classroom and the relationships between themselves and everyday objects in this space. Pupils should note that the size of objects in the picture changes when one moves closer or further away.

Learners drawings are likely to vary for example, when drawing a cereal box form the front, some learners will draw the box directly form the front:



Others will draw the box from an angle:



It is important to note that none of these responses are wrong – what is important is that learners can explain why the box has been drawn in different ways. It is only in Spatial Module 2 that we introduce conventions such as the "front view" and equip learners with drawing skills. The teacher should use this activity to gauge learners' drawing skills – it is important that the teacher talk to learners about their responses.

Some learners might concentrate on the detail in the drawings, for example, want to draw the label on a box: The teacher should, however, stress that this is not as important as the shape of the object.

Source of Ideas:

Gravemeijer, K. Realistic Geometry Instruction. In **Geometry Inservice Curriculum** *Materials*, AMESA Western Cape Region, 1995.