Malati

Mathematics learning and teaching initiative

Geometry

Module 3

Representations (nets, models and cross sections)

Grades 4 to 7

Teacher Document

Malati staff involved in developing these materials:

Kate Bennie
Zonia Jooste
Dumisani Mdlalose
Rolene Liebenberg
Piet Human
Sarie Smit

We acknowledge the assistance of Zain Davis, Shaheeda Jaffer, Mthunzi Nxawe and Raymond Smith in shaping our vision.

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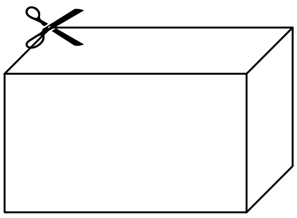
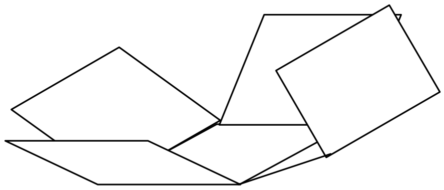
Unfolding Boxes 1

Take a cardboard box like this:



Cut the edges of the box so that you can open it up and lie it flat: and :

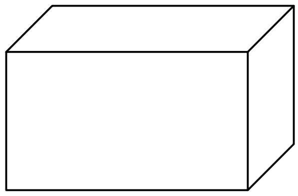


Draw the flat box in this space:

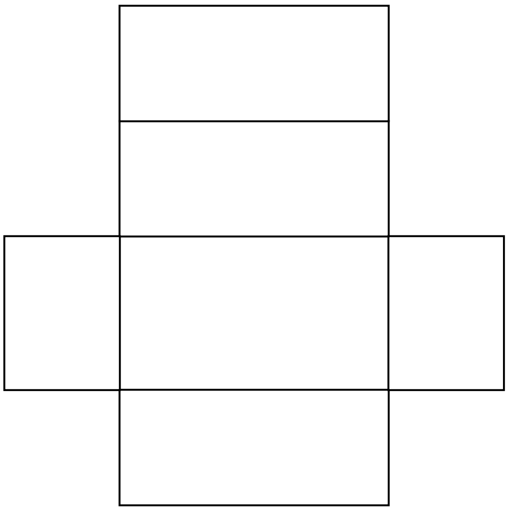
The flattened figure is called the **net** of the box.

Compare the net you have drawn to that of your classmates.

In the previous activities we learnt that we can use a drawing like this to represent a box on a flat surface:



A net is another way to represent a box on a flat surface:



Compare the number of faces that you can see on each representation.

Can you use the net of the box to count the number of edges and vertices (corners) of the box?

Teacher Notes: Unfolding Boxes 1

This activity introduces a new way of representing a 3-dimensional object in 2 dimensions, that is, in the form of a net. This notion of a net is a social convention and needs to be explained to learners.

When dealing with nets initially, learners should be given opportunities to cut up and unfold boxes and to cut and fold nets to form boxes. The experience gained doing the physical manipulation will assist learners to ***visualise*** *the folding and unfolding in later activities.*

The teacher might have to demonstrate the cutting process to the learners to get them started.

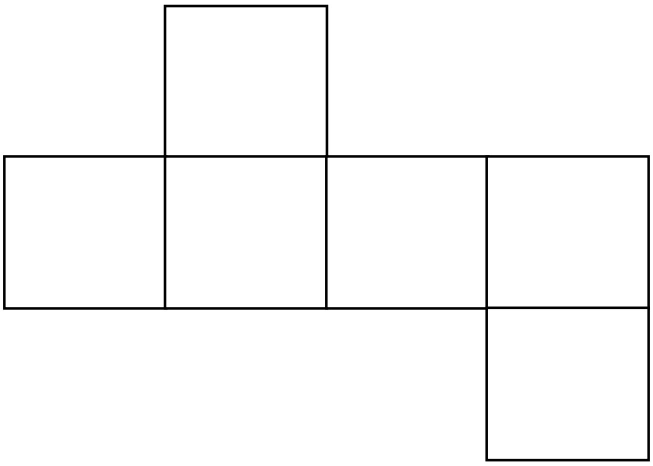
The teacher should ensure that learners use a variety of boxes to cut up – in this way nets consisting of different shaped and size polygons will be created for discussion. Different cuts will also result in different nets for the same boxes. Comparison and discussion is important here as it exposes learners to different nets of different objects.

Learners should note that the net is one way of representing a 3-dimensional object on a flat surface (in 2 dimensions). As in the case of the perspective drawing, not all the information about a particular 3-dimensional object can be deduced directly from the net – all the faces can be seen, but one has to refold the net to be able to count the number of edges and vertices.

Which Boxes?

These are nets for certain boxes. What kind of box can you make with each net?

1.



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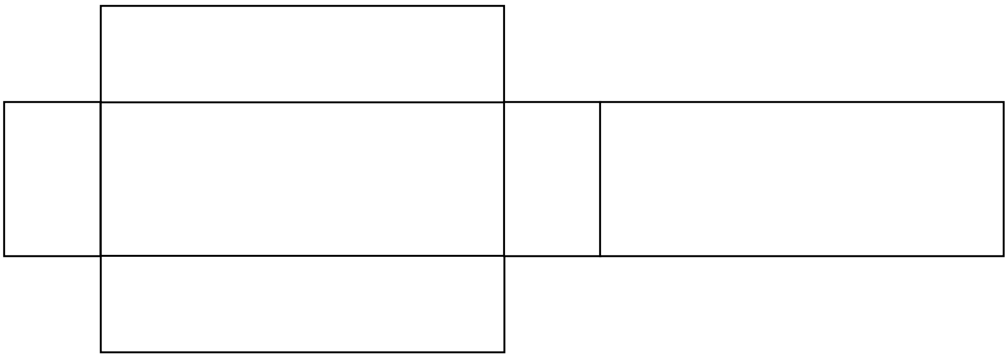

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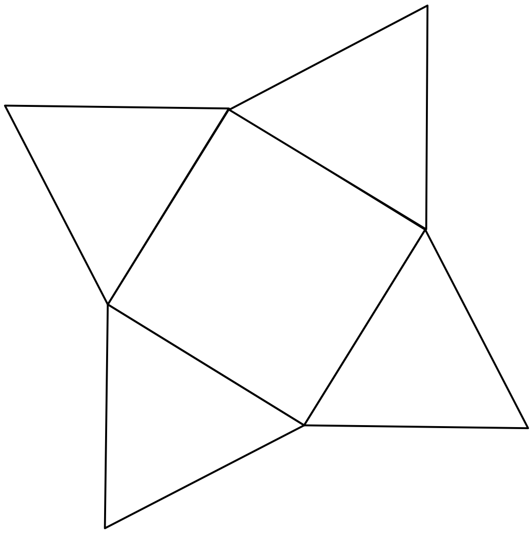

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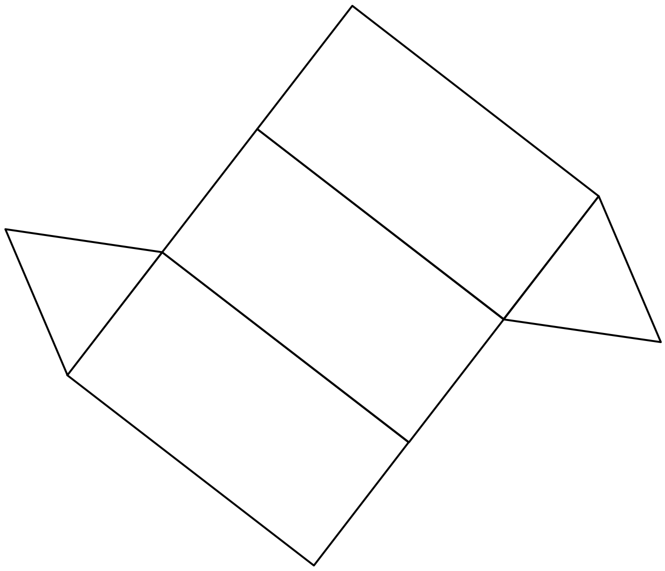

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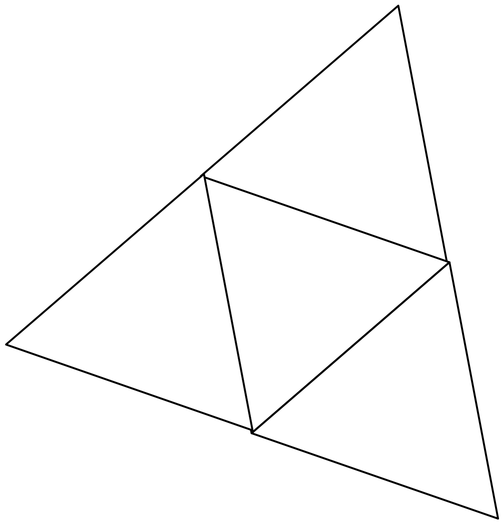

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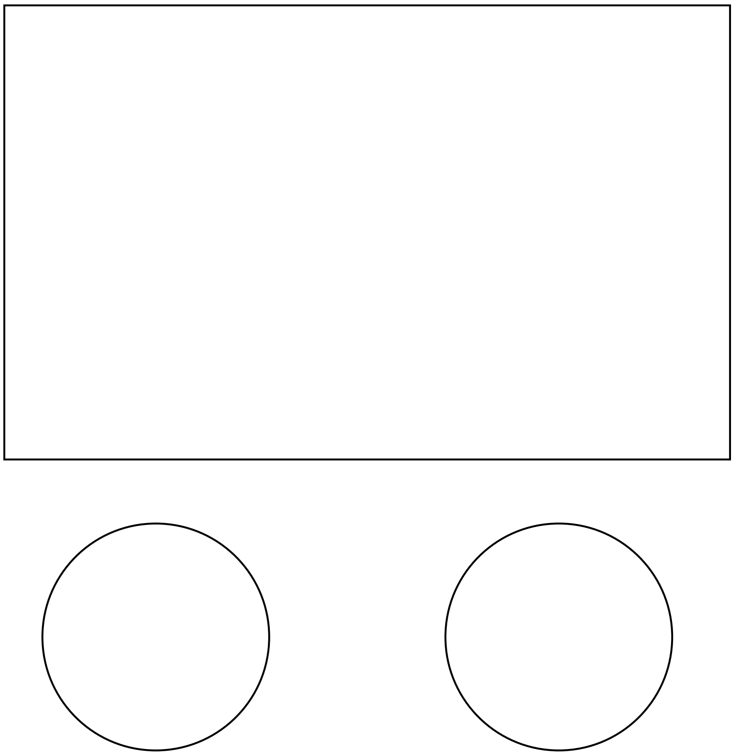


Teacher Notes: Which Boxes?

This activity provides nets of the 3 dimensional objects learners have been working with in this package. Learners are required to identify the 3 dimensional object that will be formed by folding each net.

Responses are likely to vary:

- *Some learners will be able to name the figure using mathematical terminology, for example, cube, rectangular prism, square pyramid etc. Others might use the properties – (1) has six faces; (4) has two faces that are triangles and three faces that are rectangles. This is a good opportunity for the teacher to reinforce mathematical vocabulary.*
- *Some learners will recognise the objects just by looking at the properties of the figures in the net, some will be able to visualise the folding, and others will need to cut and fold the nets.*

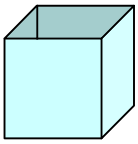
The teacher should be flexible about the cutting and folding of the nets. Some learners will need to perform this physical manipulation, whereas others will be able to visualise and will only need to cut and fold in order to check the answers. The objects can be held together using masking tape.

<u>Additional Activity:</u>

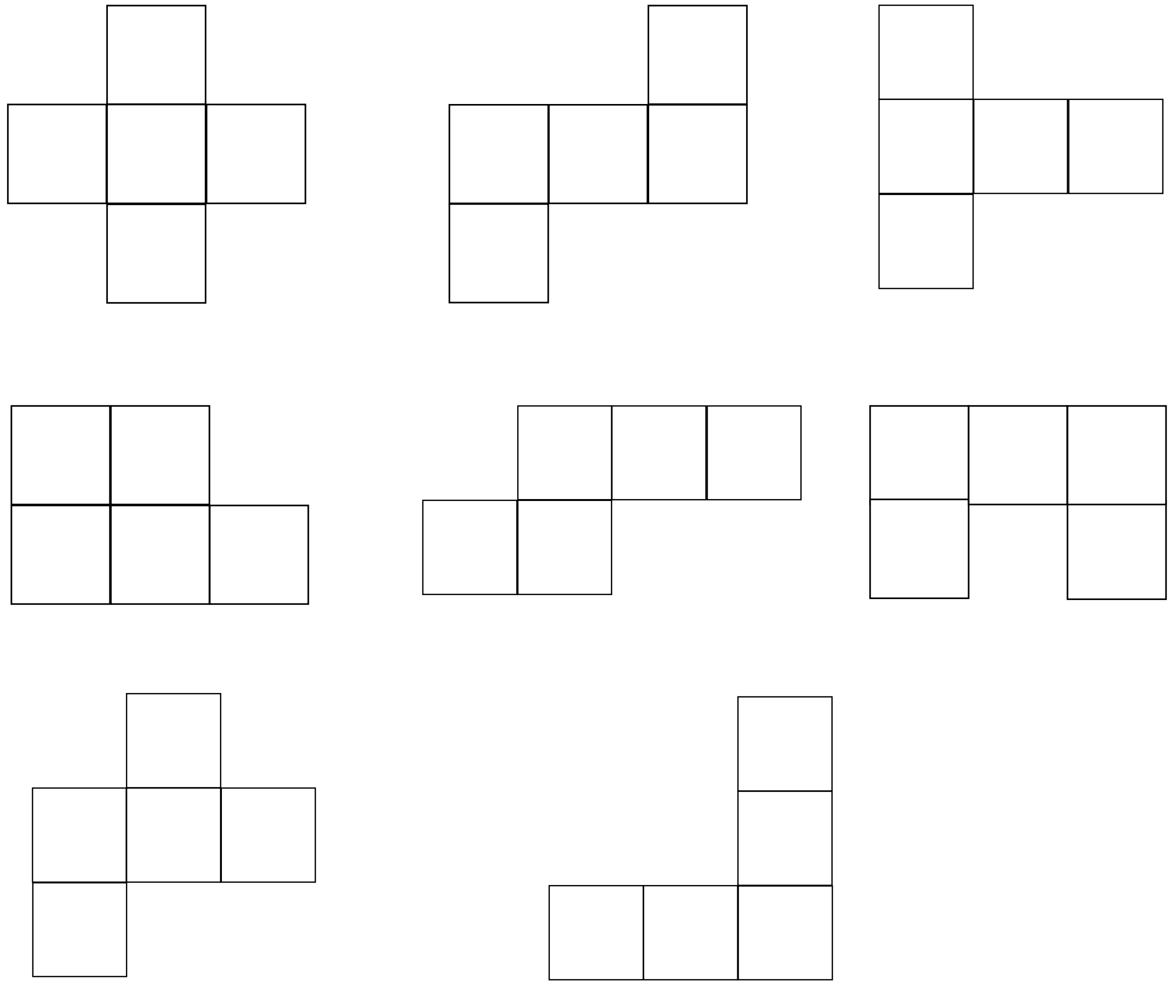
Learners could be required to draw different nets for each of the objects identified in numbers 1 to 7.

An Open Box

This is a drawing of a cube **without a top**.

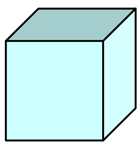


1. Which of the nets below can be folded to make this box?
   If the net **cannot** be used to make this box, explain why not.
   If the net **can** be folded to make the box, colour the square that will form the bottom of the box.



2. Try to draw a **different** net for this box and colour the square that will form the bottom of the box.

3. Now draw the net of a cubic box that **does have lid**.
   How many **different** nets can you draw for the box?

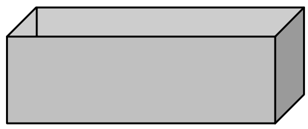


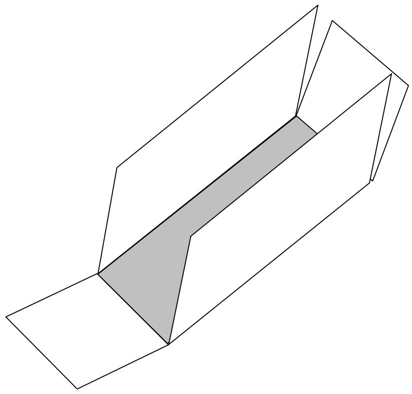
Teacher Notes: An Open Box
Different learners will require different assistance in this activity – some should be permitted to cut and fold the nets while others will be able to use visualisation only (in which case the cutting and folding can be used as a checking mechanism).

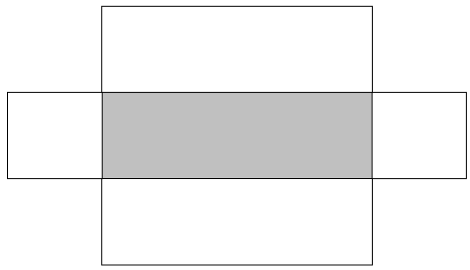
Pupils should be encouraged to explain why some of the suggested solutions are not nets for the given box. This will require the use of vocabulary such as next to, opposite etc.

More Nets

This box **without a lid** can be unfolded to form a net as shown:







1. Draw two **different** nets for the same box.

2. Draw a net which could be used to make a box **with a lid.**

3. How would you change the net to make a **cubic** box?

Teacher Notes: More Nets

In this activity learners are required to work with a rectangular prism (without a "lid"). In this case only one net is provided and learners must draw additional nets for the same box. Learners should be permitted to cut and fold to test their nets.

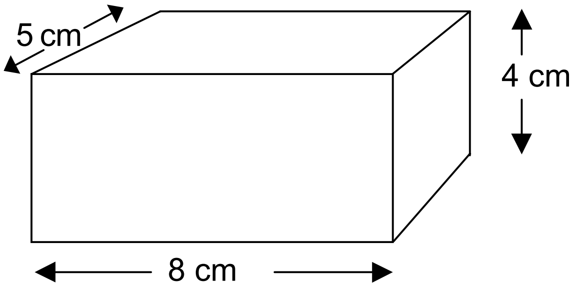
Learners should be encouraged to determine how many different nets there are for a box of this type.

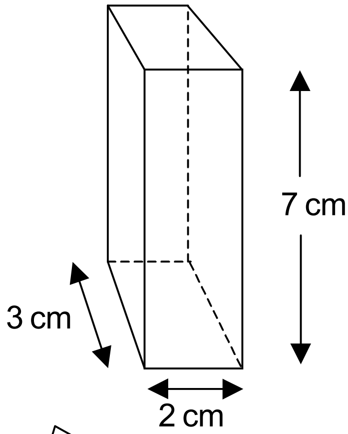
<u>Additional Activities (using measurement):</u>

- *You have been given a piece of A4 cardboard to make a box of this shape. What dimensions would you choose to make the box?*
- *Draw a net which could be used to make a box* ***with a lid.*** *What dimensions would you use to make this box out of A4 paper?*

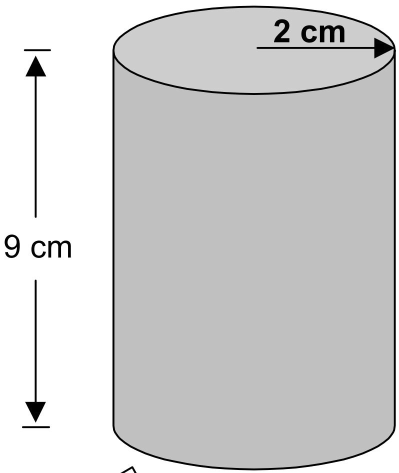
Unfolding Boxes 2

Draw nets for each of these boxes.
Fill in all the measurements on your diagram where it is given.

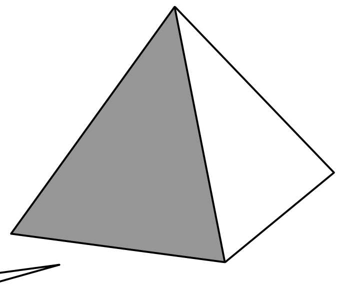




These two objects are called **cuboids (a rectangular block)**.



This object is called a **cylinder**.



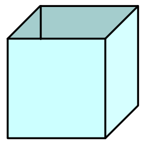
This object is called a **pyramid**.

Teacher Notes: Unfolding Boxes 2

Pupils' drawings need not be drawn to scale, but the lengths of the sides of the faces should be in proportion. The teacher can add additional activities using different objects.

Painted Boxes

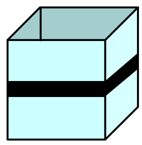
The Grade 7 Art class have made cubic boxes like this for storing their paints and brushes. The boxes **do not have lids**.



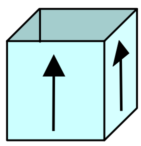
What net do you think they used?

Each pupil has painted the sides of his/her box differently.

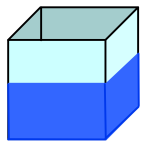
Tezi painted a stripe all around the sides:



Sizwe painted arrows on the sides of his box. All the arrows point upwards:



John painted the bottom of each side blue:



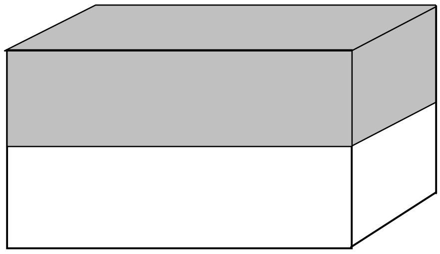
Show Tezi, Sizwe and John's markings on the net of their boxes.

Teacher Notes: Painted Boxes

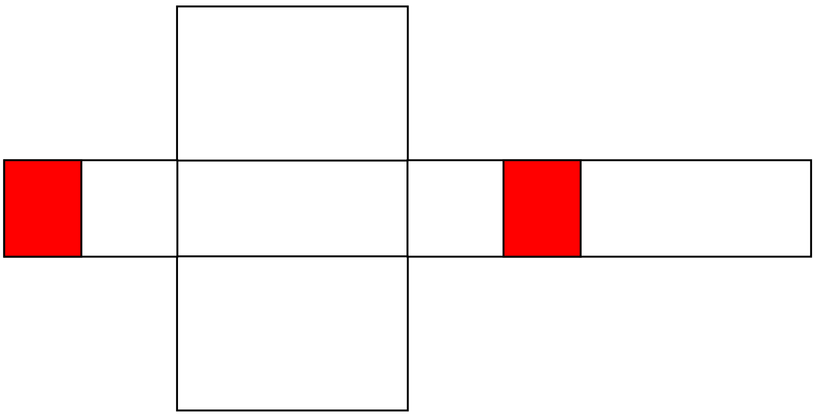
Different learners are likely to produce different nets for the box. Each learner should be encouraged to draw more than one net. Again, some learners might need cut and fold actual nets, whereas others will be able to visualise the nets.

The Unfinished Paintbox

This box as been partly painted as shown:



Jozua has drawn what he thinks is the net of the box, but he is having trouble showing which parts are painted.



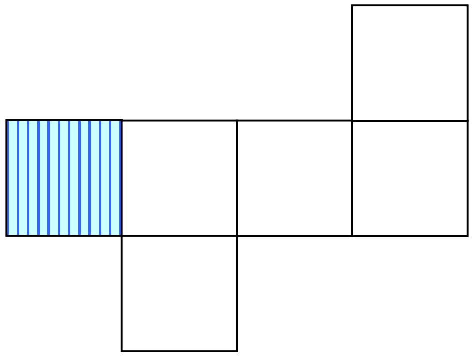
Will Jozua's net make this box?

If you think it is correct, show all the painted parts on the net.

If you think he is not correct, redraw the net correctly and show which parts are painted.

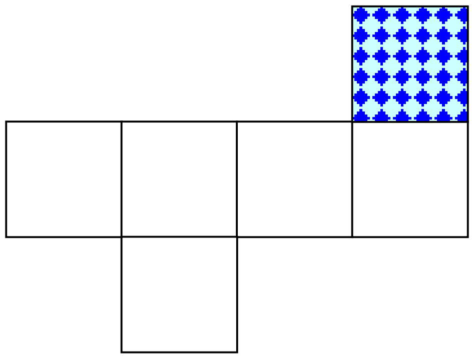
Folding a Net

Look carefully at this net of a cube:



1. Which face of the net will be opposite the striped side when the net is folded? Colour in face you have chosen.

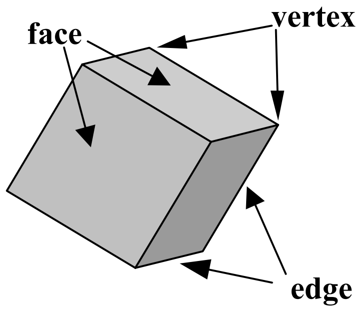
2. Which face will be opposite this dotted face when the net is folded? Colour in the face you have chosen.



3. Now draw a **different** net for a cube. Shade different faces and set questions similar to 1 and 2 for a classmate to answer.

Teacher Notes: Folding a Net

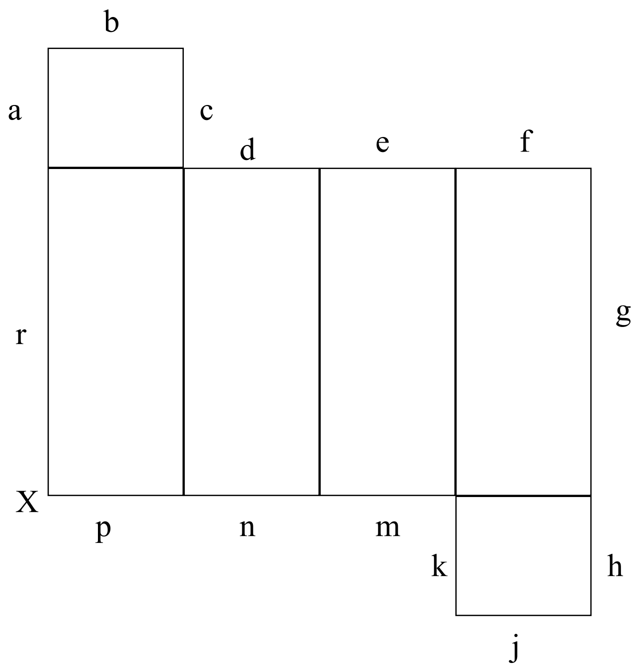
This activity uses the term "face" – the teacher might need to remind learners about the use of this term in mathematics:



Question 3 could be used as an assessment activity.

Matching Edges, Faces and Vertices

The diagram below shows the net of a rectangular prism. The edges are labelled with small letters.



1. Which edge of the net will fold onto edge g?

2. Which edge of the net will fold onto edge c?

3. Which edge of the net will fold onto edge n?

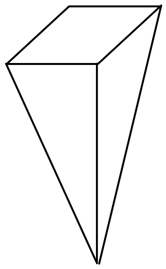
4. Which edges will meet at the vertex labelled X?

Teacher Notes: Matching Faces, Edges and Vertices

Learners can work together in pairs or groups to set one another additional questions using this net.

Which Net?

Match the objects with the correct nets:

1


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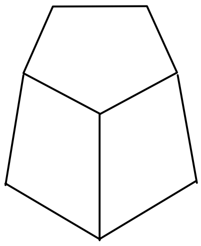

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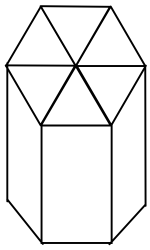

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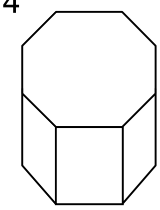

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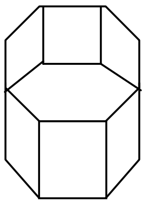

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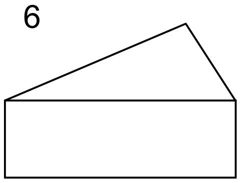

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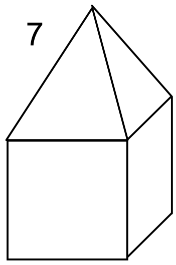

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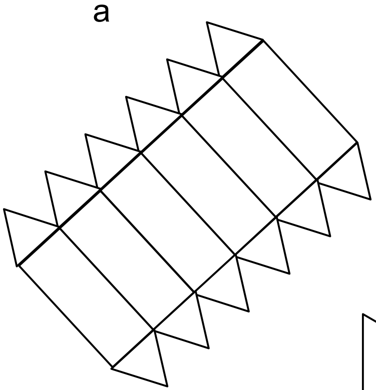

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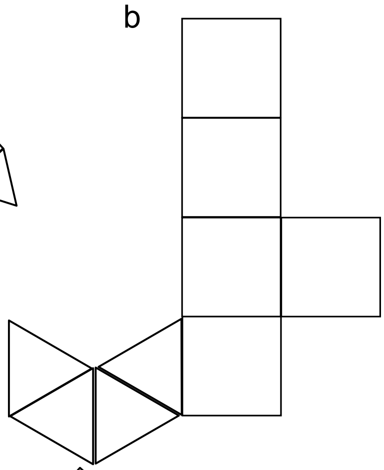

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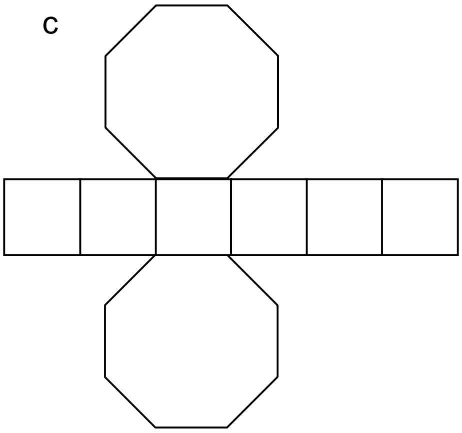

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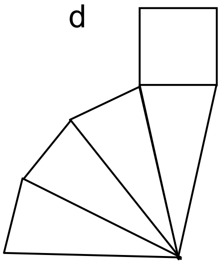

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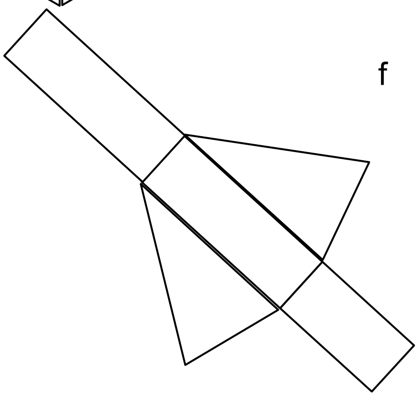

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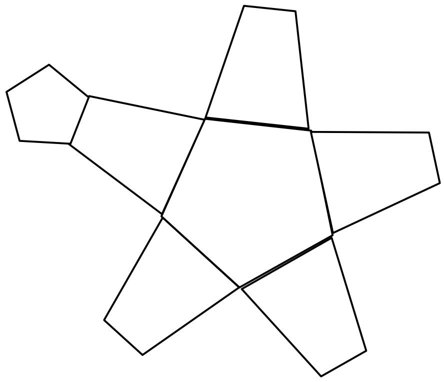

Drawing Nets

Draw the net of each of these objects. Name the object if you can.

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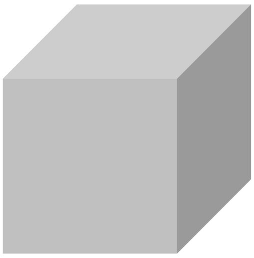

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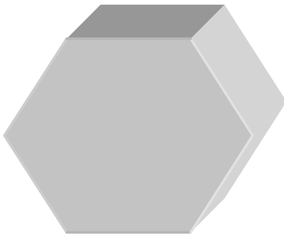

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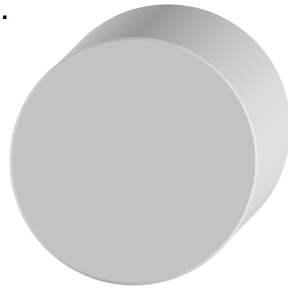

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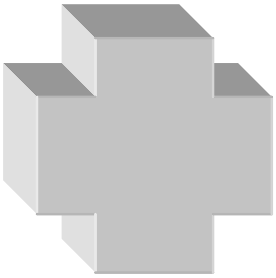

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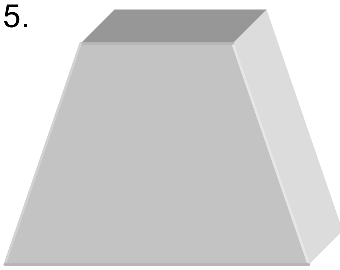

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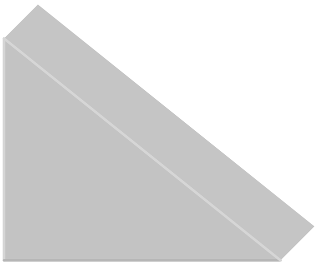

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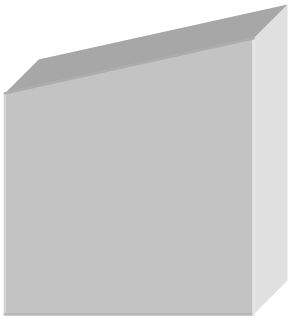
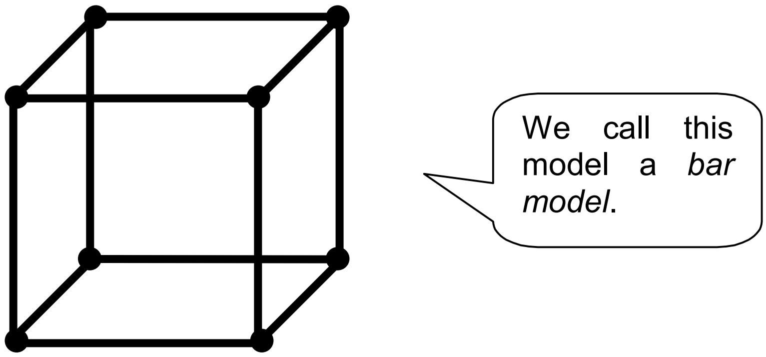


Making Models

For this activity you will need “bars” (use toothpicks, drinking straws, pipe-cleaners or matches) and “blobs” (use Jelly Tots, Prestik or plasticine).

1. Use your “bars” and “blobs” to build this cube:



   How many “bars” did you use? How many “blobs” did you use?

2. You must now build bar models of the following objects:

   (a) a triangular prism

   (b) a pyramid.

   Before you make each model, decide how many “bars” and “blobs” will be needed for the model.

3. How many “bars” are needed to make a pyramid?

Teacher Notes: Building Models

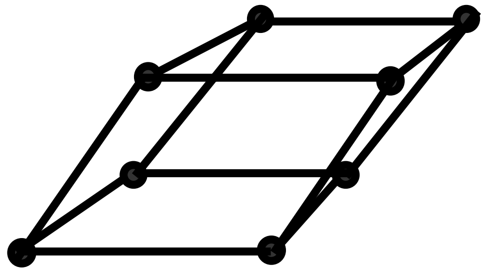
This activity enables the learners to revisit the properties of 3-dimensional objects. When deciding on the number of "bars" and "blobs" required, learners will need to consider the number of edges and vertices respectively. The teacher should ensure that the link between the "bars" and edges and the "blobs" and vertices is made explicit.

The teacher should try to vary the materials used for the "bars" – if only matchsticks are used for example, the learners will only be able to work with regular polyhedra. The use of different length pipe-cleaners could widen the scope of the activity – this will be required in the activity "A Strange Cube!". The teacher can also require the learners to construct additional models, for example, of the platonic solids (octahedron, icosahedron, dodecahedron).

In questions 2 and 3 the type of pyramid has not been specified – the shape of the base can vary, for example, a triangle, square or pentagon. In question 3 learners should note that for every side of the base, one needs one "bar" to form the pyramid. So if a pyramid has a base with n sides, then 2n "bars" are required to build the model.

A Strange Cube!

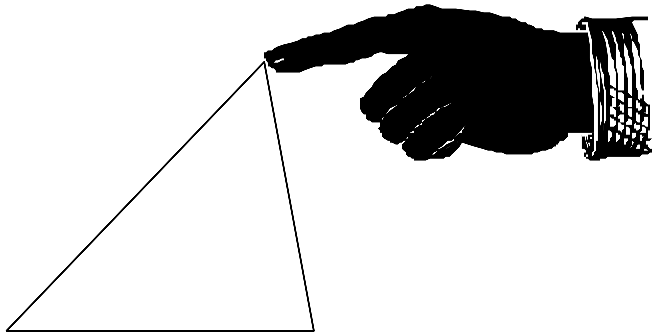
Bongani's group made this bar model of a cube, but someone put a book on top of it and look what happened!



Why did this happen? What could you do to the model to prevent this happening again?

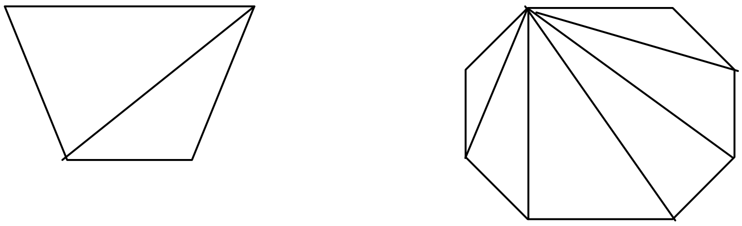
Could this happen the other bar models you have made? Explain!

The triangle is the only figure that does not change shape when pressure is applied to it:



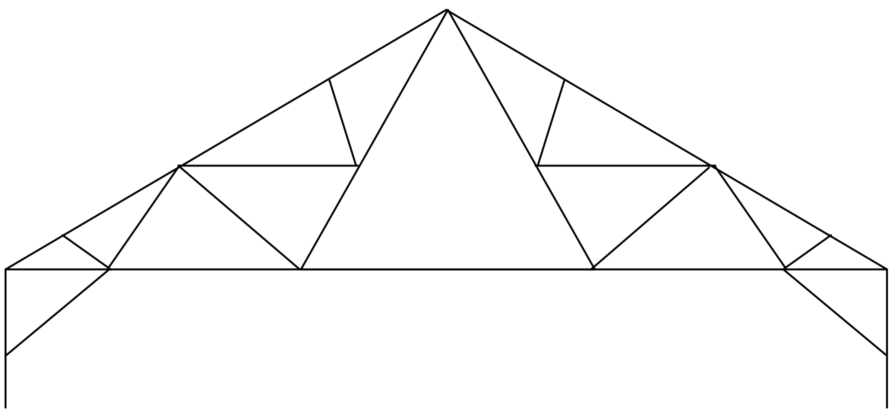
We say that a triangle is a *rigid* figure.

We can make other figures rigid by adding triangles to them:



The rigidity of triangles is used in construction – triangles are used in constructions to make sure that the objects are strong and can withstand pressure.

Look at this example of a roof truss. The triangles help to make the roof strong so that it can withstand the pressure of wind, snow and rain.

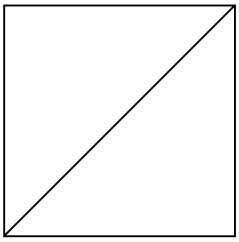


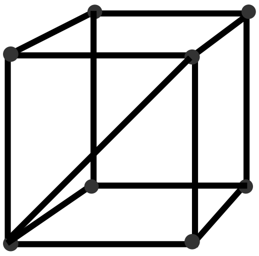
Give some other examples of constructions that use the rigidity of triangles.

Teacher Notes: A Strange Cube!

This activity requires the use of "rods" of differing lengths – "rods" of which learners can adjust the length according to their own requirements are ideal, for example, pipe-cleaners

*This activity explores the notion of rigidity of polygons. Learners might need to explore the rigidity of the individual polygons that make up the polyhedra – they can construct triangles and squares out of the "rods" and 'blobs". A triangle is rigid because pressure can be applied at any point and it will retain its shape. But the square will change shape when pressure is applied. The cube has altered shape because the square is not a **rigid** polygon. The cube can be made rigid by adding a diagonal to each square as shown:*

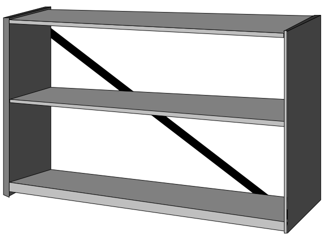


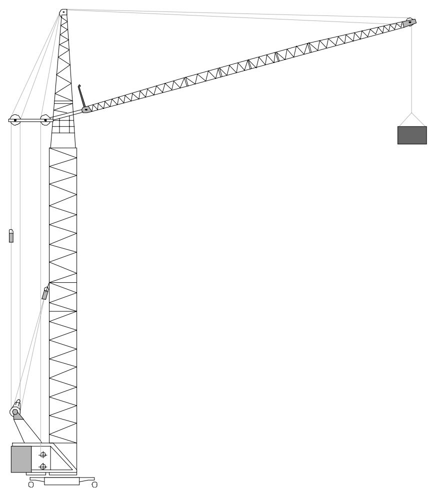


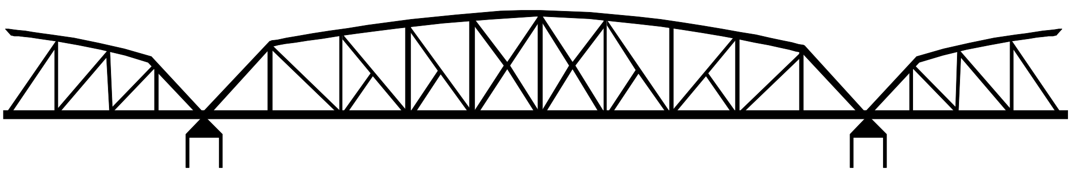
Learners should note that the triangular pyramid is rigid as it is made up of triangles. One "bar" will have to be added to the square pyramid to make it rigid.

Learners can be encouraged to explore the rigidity of other polygons, for example, rectangles, parallelograms, irregular quadrilaterals, pentagons, hexagons etc.

The concept of rigidity is an important part of construction – the use of triangles in constructions serves to strengthen structures. Examples can be seen in bridges, pylons and in furniture:



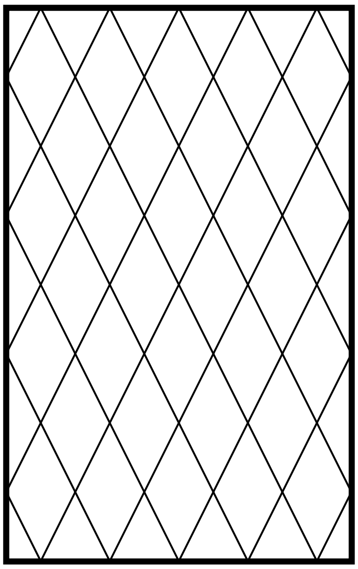




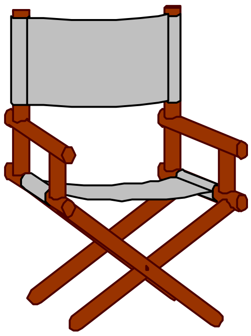
Folding Gates and Chairs

Look at the following objects:

An expanding gate:



A fold-up chair:



Explain why each of these objects are designed in this way.

Teacher Notes: Folding Gates and Chairs

This activity explores the notion of rigidity of polygons and the properties of quadrilaterals. It can be used as an assessment activity.

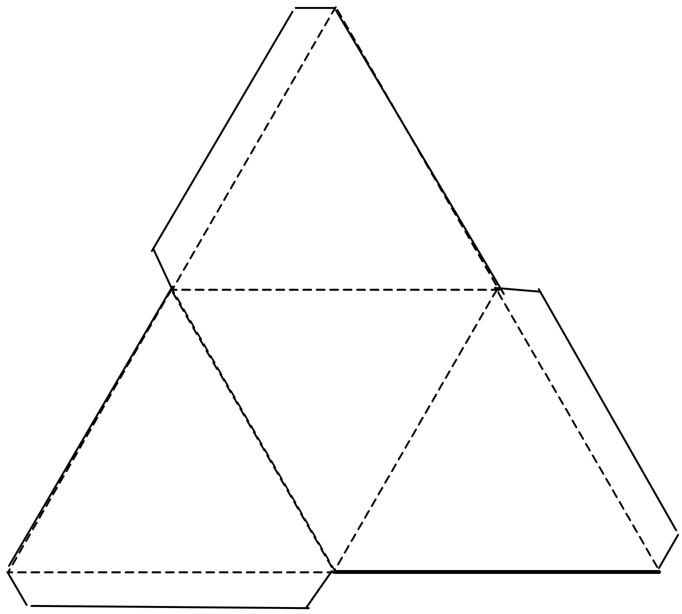
Learners should note that quadrilaterals have been used as these are not rigid and will “fold up” when pressure is applied to a vertex. The properties of the quadrilateral also play a role – in the above cases the adjacent sides of the quadrilaterals are equal so that the figure will fold “flat”. Learners should be encouraged to consider what would happen if a rectangle or a square is chosen.

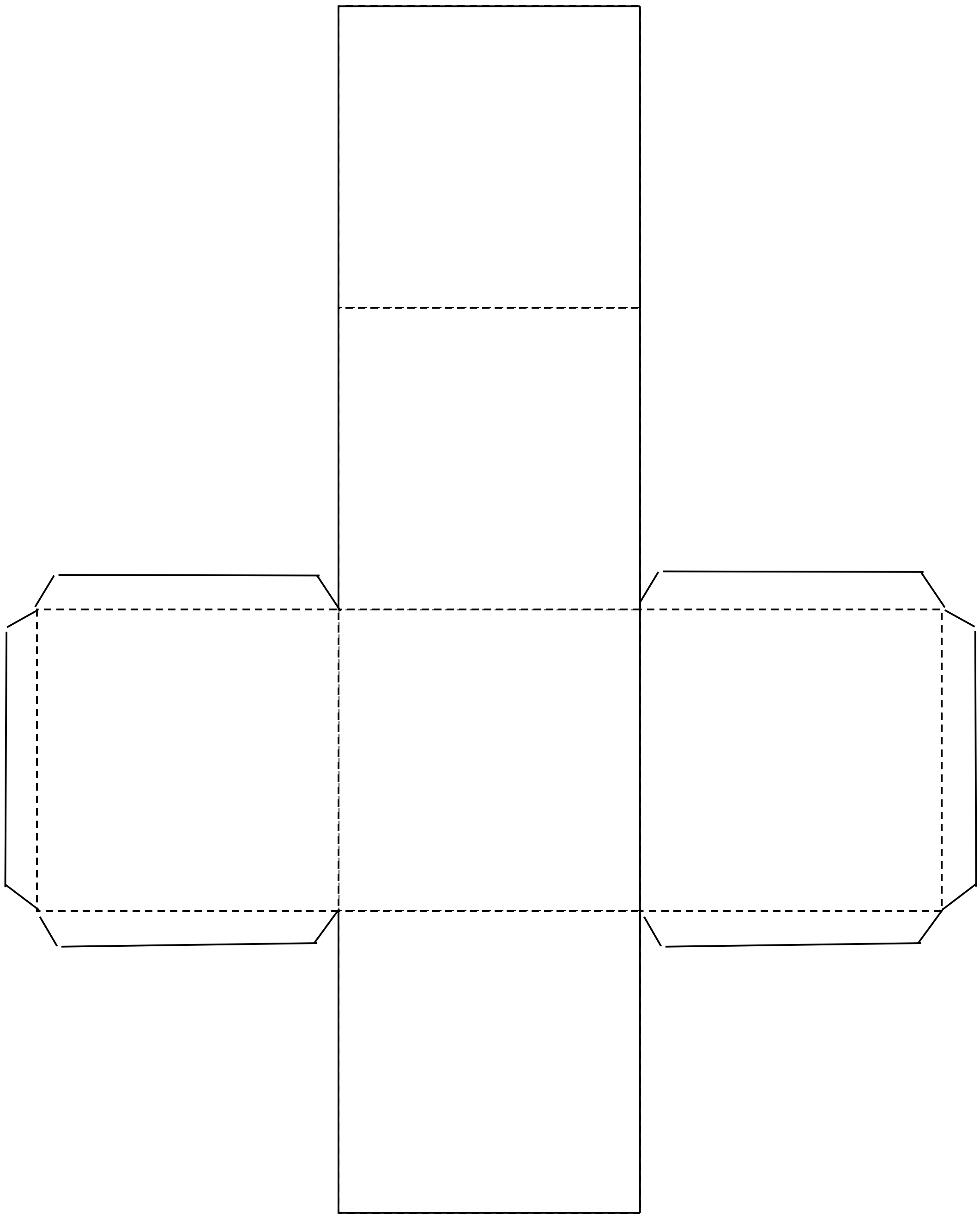
The Platonic Solids

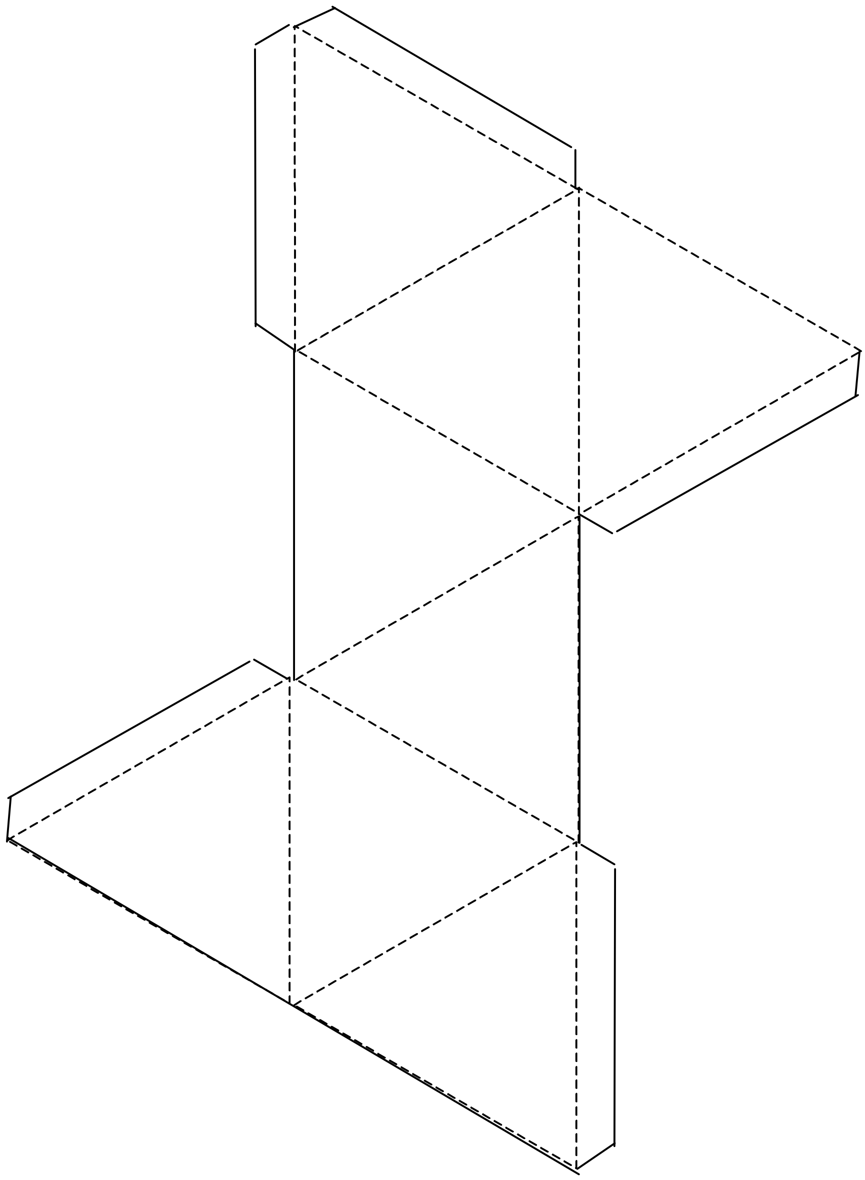
The nets for some three dimensional objects are given below (the “flaps” have been included to help you make the objects).

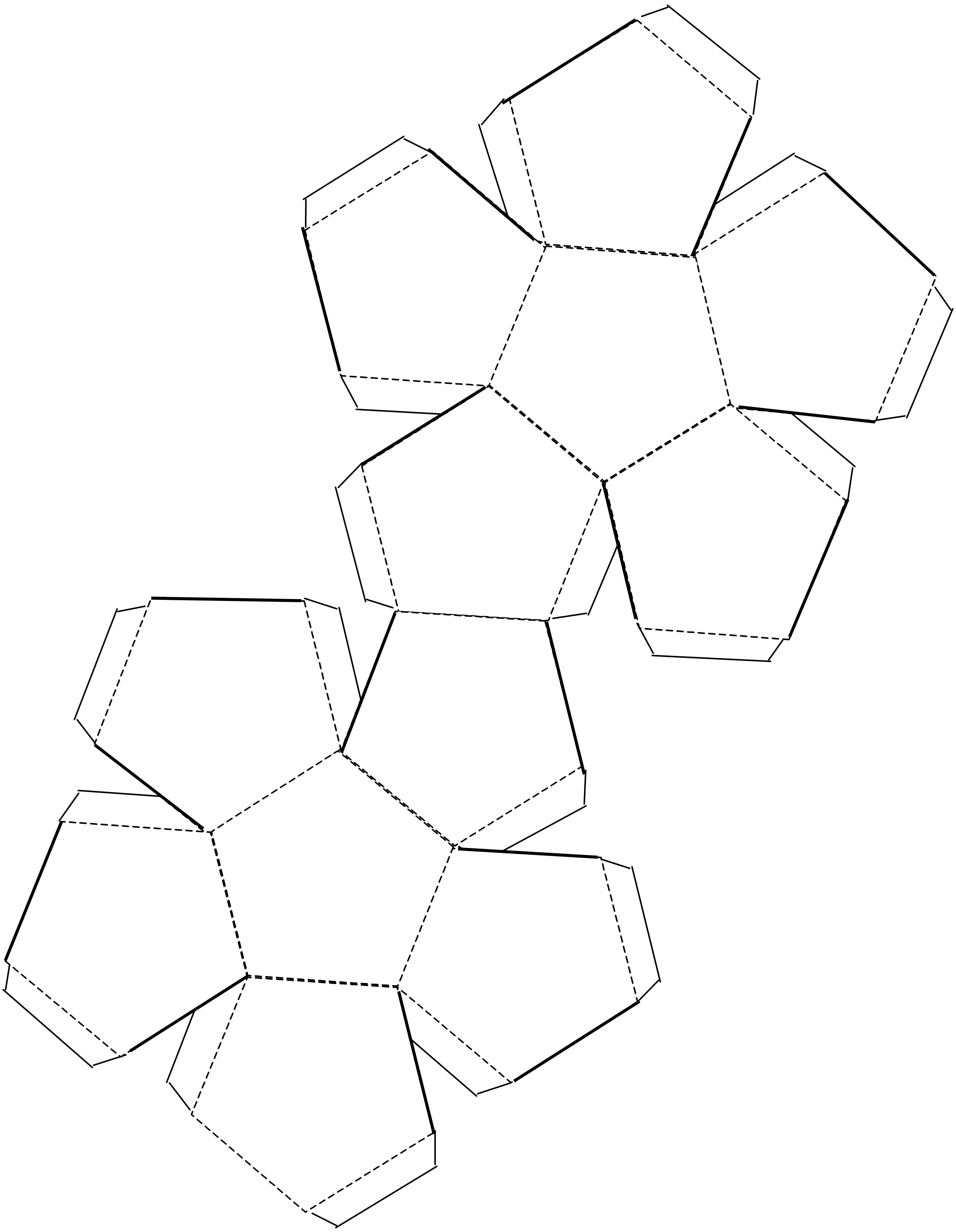
What do you notice about the figures that make up each of these nets?

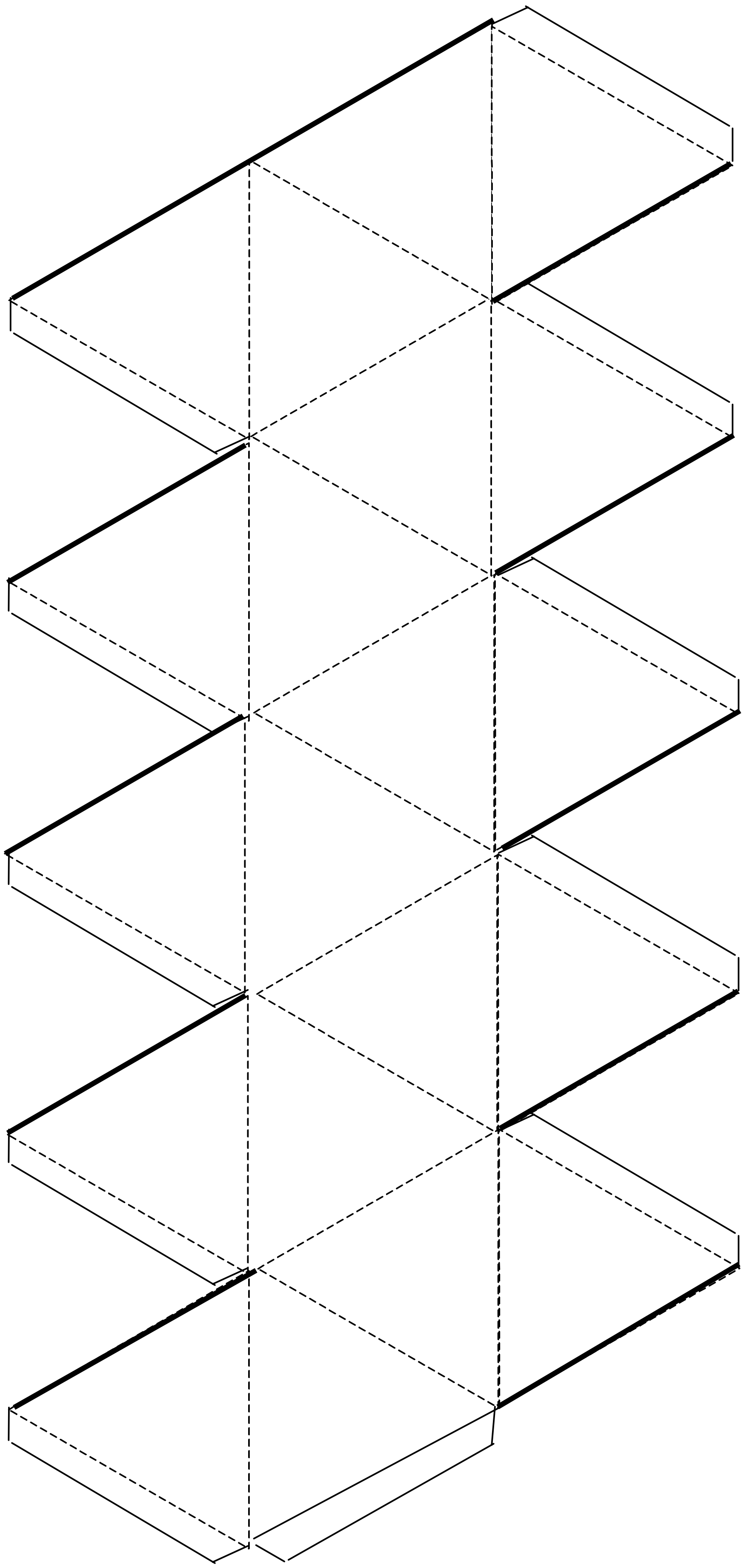
Now cut out the nets and fold them to make the objects.







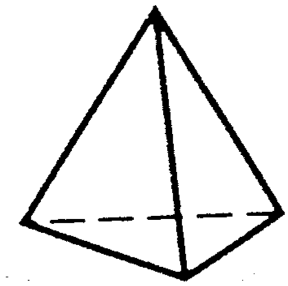




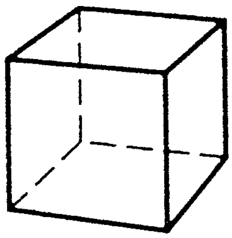
The objects you have made all have faces that are the same shape and size (congruent figures) and are regular polygons. There are the same number of faces at each vertex

There are five such polyhedra and we call them the **platonic solids**.

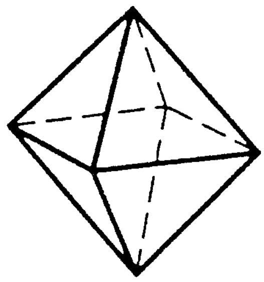
Match the objects you have made to these pictures of the platonic solids:



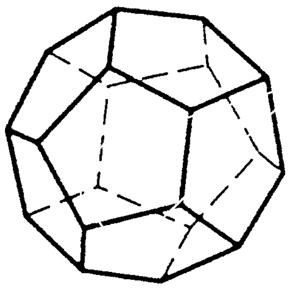
Tetrahedron



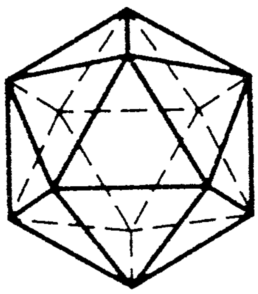
Cube



Octahedron

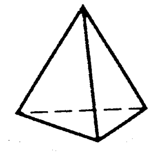
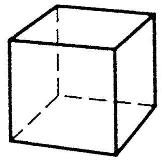
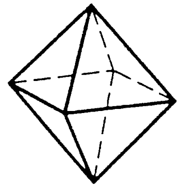
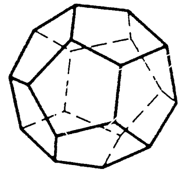
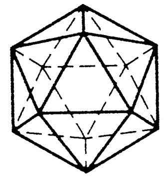


Dodecahedron



Icosahedron

Now use the objects you have made to fill in this table:

Polyhedron	tetrahedron	cube	octahedron	dodecahedron	icosahedron
					
Shape of face					
Number of faces					
Number of vertices					
Number of edges					

Look carefully at the numbers in your table. Can you find a relationship between the number of faces, vertices and edges in a platonic solid?

Does your formula work for the other three-dimensional objects you have studied?

Teacher Notes: The Platonic Solids

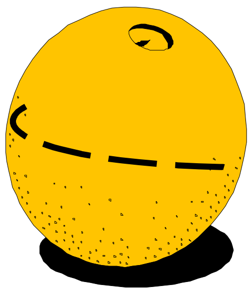
This activity provides learners with further practice in exploring the properties of polyhedra. The algebraic relationship between the faces, vertices and edges is also explored. Learners should also consider whether the formula works for other polyhedra, for example, a triangular prism or a square pyramid.

The term "platonic solids" is used to refer to the five regular polyhedra. A regular polyhedron has regular, congruent figures as faces and these faces meet in the same way at each vertex. The relationship between the faces (f), vertices (v) and edges (e) is given by $f + v - e = 2$. This is known as "Euler's Formula" as it was discovered by the Swiss mathematician Leonhard Euler in the 18^{th} century.

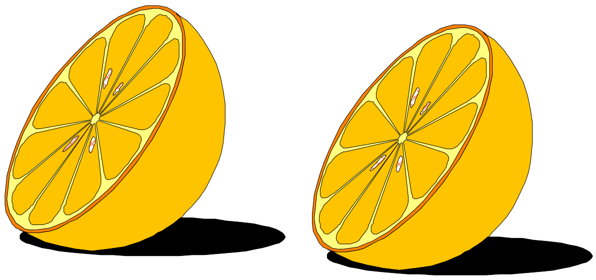
<u>Further Activity:</u> Learners can also be required to make the platonic solids using "bars" and "blobs". Before building each solid they should consider how many "bars" and "blobs" are required for each one.

Cutting Objects

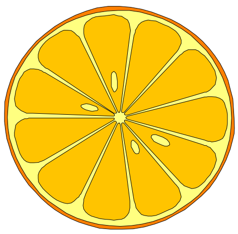
When we slice an orange in this way:

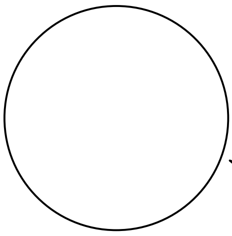


We get two pieces that look like this:



The end of the piece where the cut was made is a circle like this:

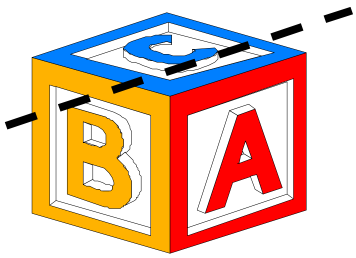




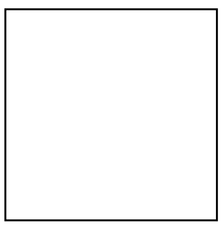
In mathematics we call this flat (two-dimensional) surface at the end the **cross section**.

Here is another example:

When we cut a wooden block like this we get two parts:



The cross section is a square like this:



Collect solid objects such as fruit or chocolate bars.
Imagine that you have cut these objects.

In each case explain what cut you made and draw the **shape** of the flat section at the end (the cross section). The detail is not important.

Teacher Notes: Cutting Objects
This activity is intended as a spatial exercise as well as an introduction to cross sections. The teacher should collect solid objects which can be cut. Fruit and vegetables such as bananas, apples, tomatoes, corn on the cob, peppers, kiwi fruit and onions can be used. Chocolate bars, sweets, loaves of bread, bread rolls, hard-boiled eggs, polystyrene shapes and sponges are interesting sources. Play dough is also a useful medium.

*It should be noted that the notion of a cross section is a social convention and refers to the **two-dimensional figure** formed when a cut is made in the solid. So the cross section of the orange is a circle. Learners are likely to want to take the three-dimensional object into account, but the convention should be emphasised. They should be reminded that the focus is on the **shape**, and not the detail of the cross section.*

***It is important that the pupils visualise the cross section before cutting** (the cutting should only be used for pupils who have tried and who cannot visualise the shape or for checking purposes).*

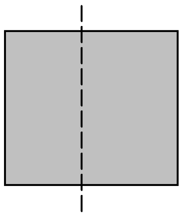
The teacher should start with vertical and horizontal cuts and make these terms explicit where appropriate. Pupils should also be encouraged to consider what happens if the horizontal cuts are made in different places – how will the cross section differ (similar/congruent)? The activity can also be extended by using cuts made at an angle.
Pupils should note that the shape of the cross section depends on the shape of the object and the way it is cut.

Source of Ideas:
*Sanok, G. (1997). **Mathematics in Nature: Patterns, Structure, and Functions** Workshop presented at AMESA Congress '97, Durban, July 1997.*

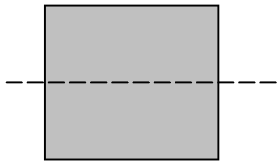
What Shape?

In these activities we will be using on only two types of cuts – horizontal and vertical cuts.

A vertical cut goes this way:



A horizontal cut goes this way:

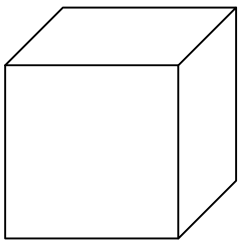


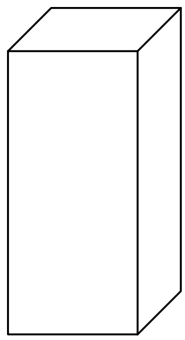
Choose an object in the classroom and show a friend how you would cut this object horizontally and then vertically.

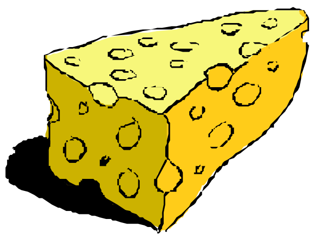
Imagine that you are cutting these objects (vertically and then horizontally).

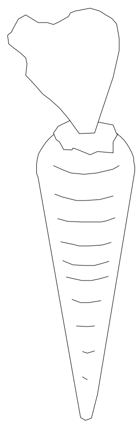
Draw the **cross section** for each cut. What shape is each cross section?

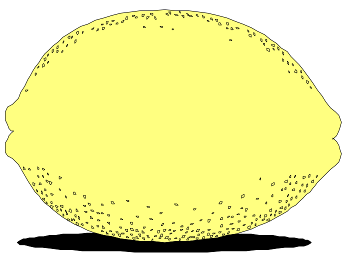
Remember that a cross section is a *two-dimensional* figure.

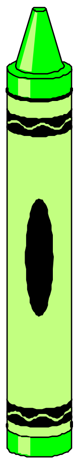
1. 

2. 

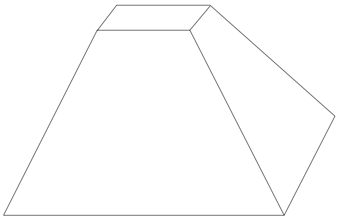
3. 

4. 

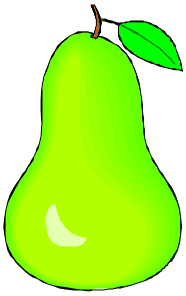
5. 

6 

7.



8.

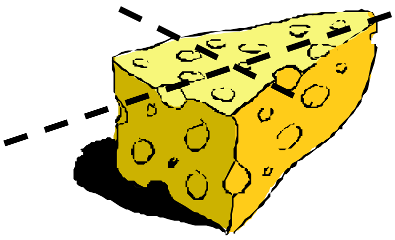


Teacher Notes: What Shape?

This activity requires that learners work with pictures of three-dimensional objects, rather than actual objects as in the activity "Cutting Objects". It is important that learners try to visualise the shape of the cross sections – the teacher can have the actual objects in the classroom, but these should only be used for checking purposes.

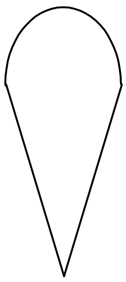
The drawings must be of two-dimensional figures. The following shapes should be drawn:

1. *Vertical and horizontal: both squares, figures are congruent irrespective of where the cut is made.*
2. *Vertical: rectangle; horizontal: square, figures are congruent irrespective of where the cut is made.*
3. *Vertical: both rectangles – depends on the cut as shown. Sizes of rectangles will vary according to where the cut is made.*



Horizontal: triangle, all congruent triangles

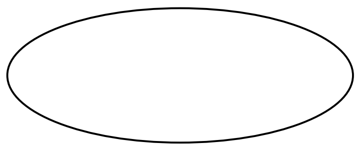
4. *Vertical:*



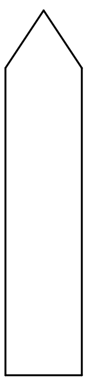
Horizontal: Different size circles

5. *Vertical: Different size circles*

Horizontal:

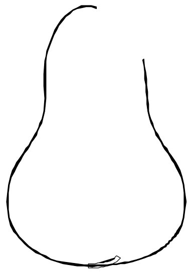


6. *Vertical:*



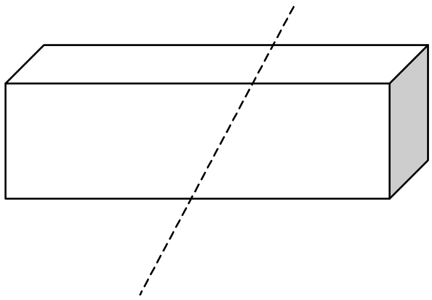
Horizontal: rectangle

7. Vertical and horizontal: different size rectangles

8. Vertical:  *Horizontal: different size circles*

Further Activities:

- *Do any of the objects have the same vertical and horizontal cuts? For example, the horizontal and vertical cross sections are both circles.*
- *Do any of the objects have horizontal cross sections that are the same shape, but different sizes? And for vertical cross sections? For example, the horizontal cuts in the carrot result in different size circles.*
- *Do any of the objects have vertical cross sections that are different shapes? And for horizontal cross sections? Can learners think of other three dimensional objects for which this might be the case?*
- *Learners can be required to draw cross sections for oblique cuts. For example, what is the cross section when we cut a rectangular prism like this?*



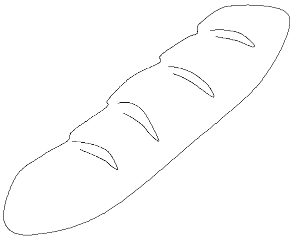
Match the Objects and Cross Sections

Some objects and drawings of **vertical** and **horizontal** cross sections are shown below (the drawings are **not** drawn to scale).

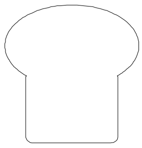
Match the cross section with the correct object.

If the cross section of the object is not drawn, draw the cross section.

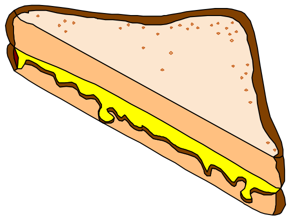
a



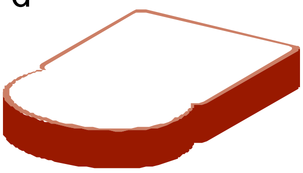
b



c



d



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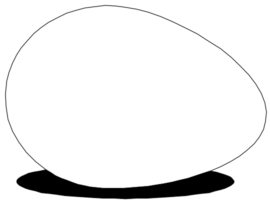
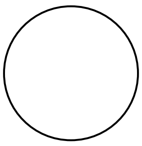
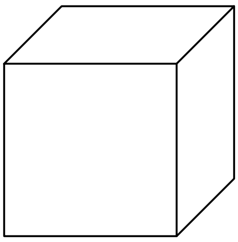


g



h



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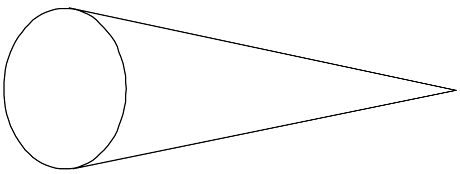

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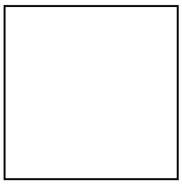

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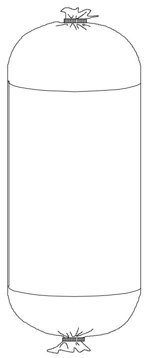

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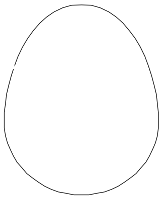
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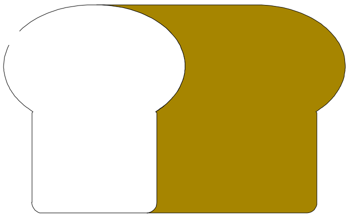
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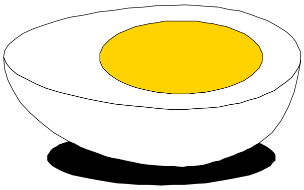
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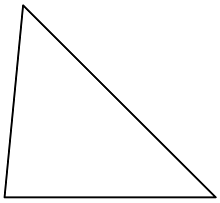
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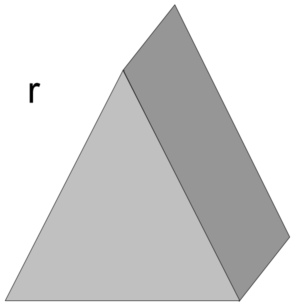

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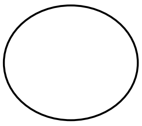
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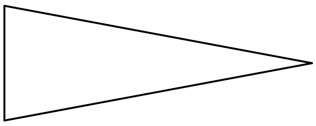
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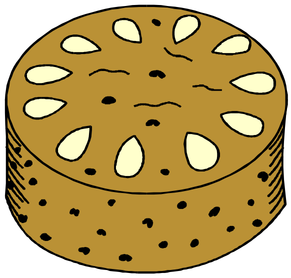


2. Group all the objects from the above list which have cross-sections with the following shapes:
   a) a circle
   b) a rectangle

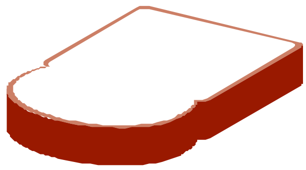
3. Which of the objects have the same shaped vertical cross sections. And the same shaped horizontal cross sections?

4. Can an object have the same vertical and horizontal cross section? Explain.

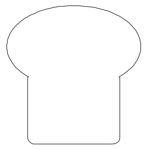
5. Can an object have horizontal cross sections which are different shapes? And for vertical cross sections? Explain.

Teacher Notes: Match the Objects with Cross Sections

This activity requires that learners match pictures of objects with the correct cross sections. Some objects have been included as distracters to address the misconception that a cross section can three-dimensional. For example, (d) is a slice of the loaf of bread, but it is not a cross section.



The vertical cross section is given in (b):



In fact (b) is a horizontal cross section of (d). Learners could also find the vertical cross section of (d). Cuts made in one pace will be a rectangle, but those made near the crust of the bread will be shaped as in (l):

Questions 2 to 5 are designed to encourage learners to reflect on the fact that the cross section depends on the shape of the object, the kind of cut (horizontal or vertical in this case) and where the cut is made.

Further Activities:

Pupils can be required to find cross sections resulting from oblique cuts of these objects.