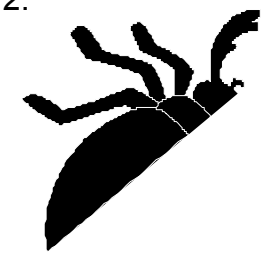
Completing Pictures

Only half of each of these pictures has been drawn. Draw the rest of each picture:

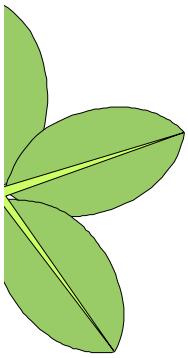
1.

2.



3.

4.



Teacher Notes: Completing Pictures

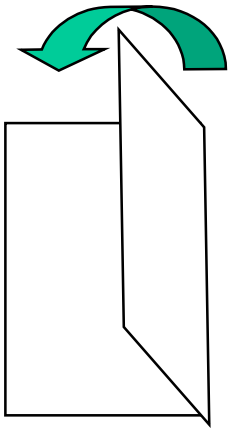
This activity uses learners' intuitive understanding of figures with line symmetry. The Pictures have been oriented in such a way that the lines of symmetry are vertical, horizontal or oblique (at an angle). It is hoped that by including this variety at an early stage, learners might not develop the limiting conception that lines of symmetry are either horizontal or vertical.

The teacher can ask learners how they knew what to draw – in this way they have to use their own words to describe the line symmetry. The word line symmetry can be introduced and then reinforced by the teacher – at this stage the description can be informal, for example, a figure has line symmetry if it can be folded on a line so that the one side lies exactly on the other. The teacher can also let learners use mirrors to look at the mirror images.

<u>Further Activities:</u> The teacher can require that learners identify (and possibly draw) patterns that have line symmetry. There is lots of scope for this, for example, leaves, flowers, logos, cross sections of fruit. Learners can also be challenged to consider whether there is more than one line of symmetry in any of the patterns.

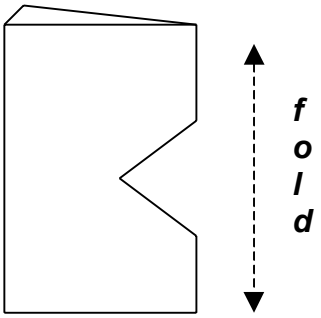
Creating Patterns 1

Fold a piece of paper like this:

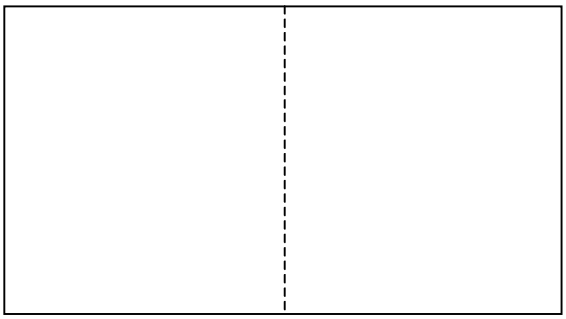


Now make a cut in the paper you have folded. Do not make it too complicated when you start out!

You could cut it like this:



Before you unfold the paper, draw what you think the paper will look like when you unfold it:



Now try again, but make more than one cut in your paper.

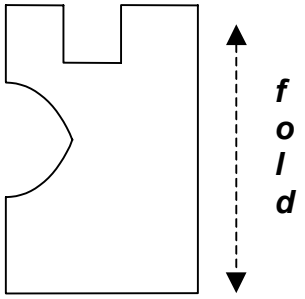
Teacher Notes: Creating Patterns 1

This activity requires that learners visualise what the three-dimensional folded paper will look like in two-dimensions when unfolded.

Understanding of these written instructions requires that learners interpret the visual information: The teacher might need to demonstrate the procedure to learners. Learners should be encouraged to begin by making ***simple*** *cuts.*

It is not necessary for all learners to cut. Rather, one folded paper can be used per group and then each member of the group can make a prediction. Learners can also be encouraged to compare their drawings before unfolding the paper. ***It is important that learners make a prediction before unfolding.***

Once learners have tried two or three patterns the teacher can extend the activity by drawing different possibilities on the blackboard, for example, the teacher could draw this:



Learners can then draw the unfolded paper. Opportunity should be provided for learners to compare drawings in small groups or in a whole class discussion: This can raise questions such as

- *Are the drawings the same or different? Why? For example, one might just be a reflection or rotation of another.*
- *How do we know this is the correct drawing?*

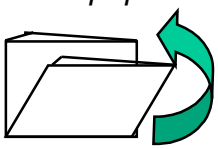
The teacher can also give a completed and unfolded piece of paper and require that learners identify how the paper was folded and what cuts were made to create this figure.

In order to answer such question learners will have to use visual skills, drawing skills, verbal skills and logical skills (in explaining why a figure should look as it does).

This is a useful context for introducing the notion of "line symmetry": What does the fold line do to the design that is created? Learners can be encouraged to draw in the fold line (a dotted line) to highlight the line of symmetry.

For those learners who can visualise patterns with one fold with ease, this activity can be extended: What will happen if you fold the paper differently?

- *What will happen of you fold the paper twice so that it looks like this?*



Fold

- *More questions can be asked about symmetry: How many lines of symmetry does your design have? Is there rotational symmetry? Is it possible to get rotational symmetry by folding paper in this way?*

Lines of Symmetry

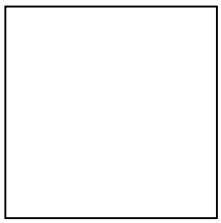
Which of these figures have lines of symmetry?

How many lines of symmetry does the figure have? In each case draw in the line/s of symmetry.

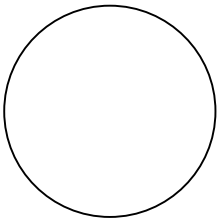
A.

B.

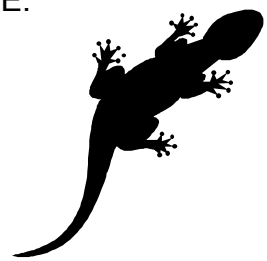
C.



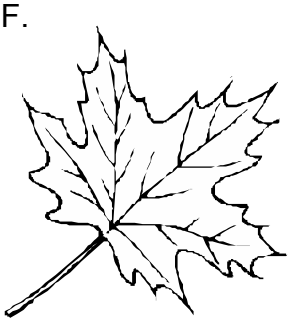
D.



E.



F.

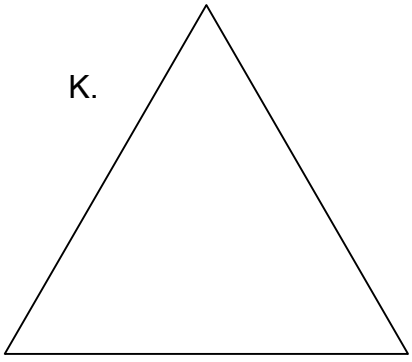


G.

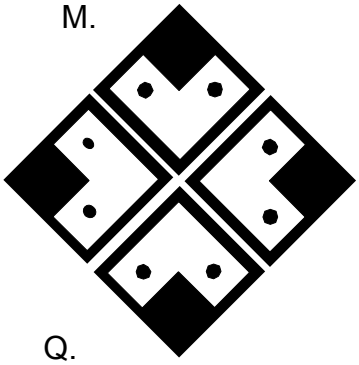
H.

J.

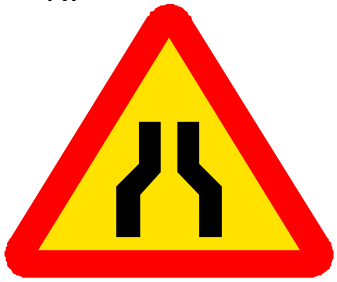
K.



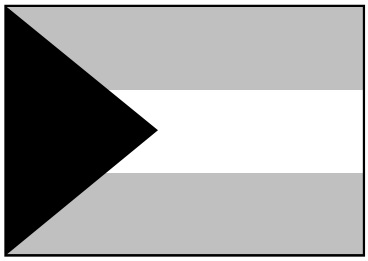
M.



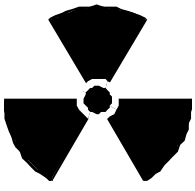
N.



P.



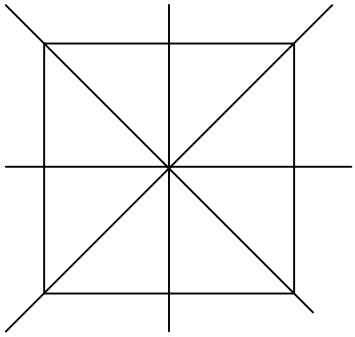
Q.



R.

Teacher Notes: Lines of Symmetry

This activity requires that learners identify lines of symmetry in different figures. Figures with different numbers of lines of symmetry have been included, as well as figures with oblique lines of symmetry. Learners might identify horizontal and vertical lines of symmetry in the square, but not the oblique ones:



Both mathematical figures as well as figures with patterns have been included. Learners might note that of one takes the pattern or the colours into account, the number of lines of symmetry might vary. For example, the rectangle in J has two lines of symmetry, but the flag in G has only one if one considers the pattern. If one considers the colours, the flag in P has no lines of symmetry! The teacher can indicate that in mathematics we do not take the shading or the pattern into account, but only consider the shape of the figure.

Further activity:

Encourage learners to find pictures of symmetrical figures and to identify the lines of symmetry.

Creating Patterns 2

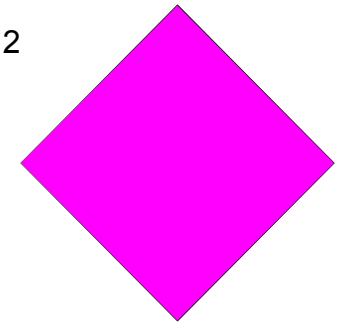
Anna says it is possible to fold paper and then cut it to make these designs.

Do you agree with her?

- If you do agree with Anna, show how you would fold and cut the paper to make the designs. Is there more than one way?
- If you do not agree, explain why it is not possible.

1

2



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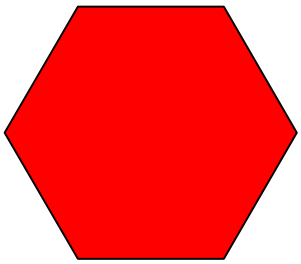

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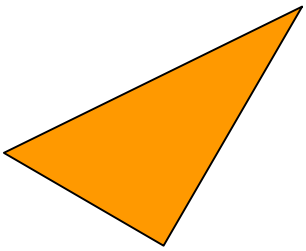

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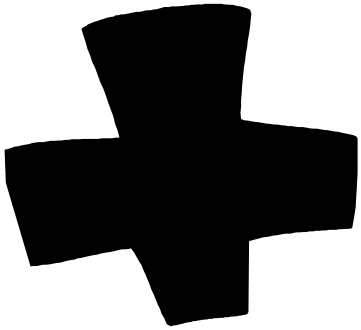

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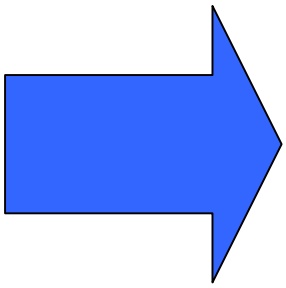

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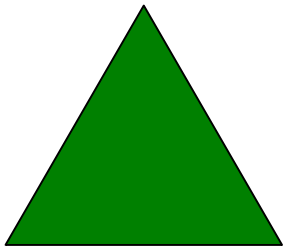

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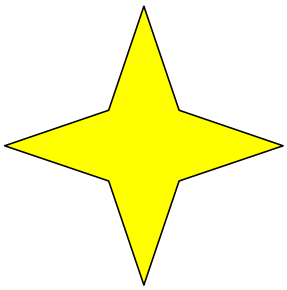
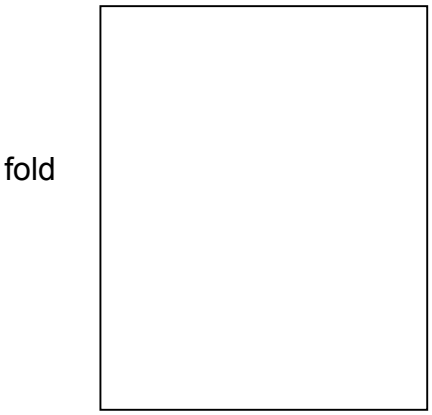


Teacher Notes: Creating Patterns 2

This activity requires that learners do the opposite to what was required in Creating Patterns 1: In this case a drawing of the final product is provided, and learners must indicate what folds and cuts were made originally. This will require that they identify the lines of symmetry.

Pupils do need some guidance on how to present the answer. They could draw something like this, and show where the fold is:

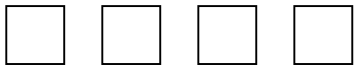


*It is also important that learners **talk** about the folds and the cuts and, in the case where the figures cannot be created, explain their reasons. Again, they should be encouraged to compare their solutions.*

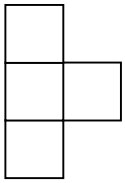
The following questions can be asked:

- *Which triangles can one make in this way?*
- *What geometric figures can be created by folding the paper once? And twice?*

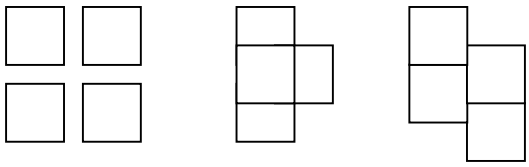
Patterns of Squares

Make patterns using four squares like this 

Here is an example:



You may not overlap the squares or the sides of the squares. Do not leave spaces in between the squares.



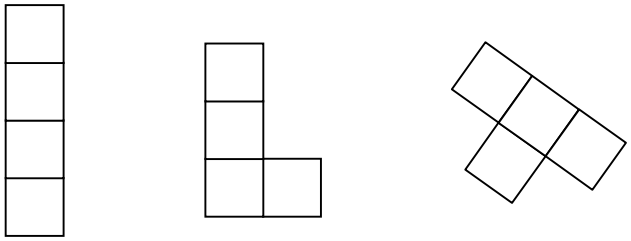
1. How many different patterns can you make using four squares?

2. Now create different patterns using **five** squares.

Teacher Notes: Patterns of Squares

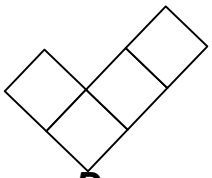
Some learners will need to physically manipulate and then trace actual squares, while others will be able to draw different figures. Square paper could also be used. In deciding what is a permissible figure, learners will need to analyse the figures given as non-examples.

Some possibilities with four squares are:



A ***B*** ***C***

It is important that learners compare their answers to confirm whether they have drawn all the possibilities. Some learners might, for example, include figure D below as an additional figure, while others will recognise that this figure is simply a rotation of figure B. Learners will use informal language, for example, "figure B has been turned".



D

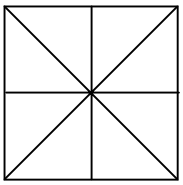
Some learners might need to cut copies of the completed figures and physically do the manipulations in order to recognise that one figure is merely a transformation of another.

*Patterns created with **five** squares are called pentominoes. There are twelve different pentominoes. Teachers should observe whether learners use systematic methods to construct the different figures.*

*<u>Further Activities:</u> Give learners a certain number of **cubes**, say five. How many different block buildings can one make with five cubes? Then increase the number of cubes.*

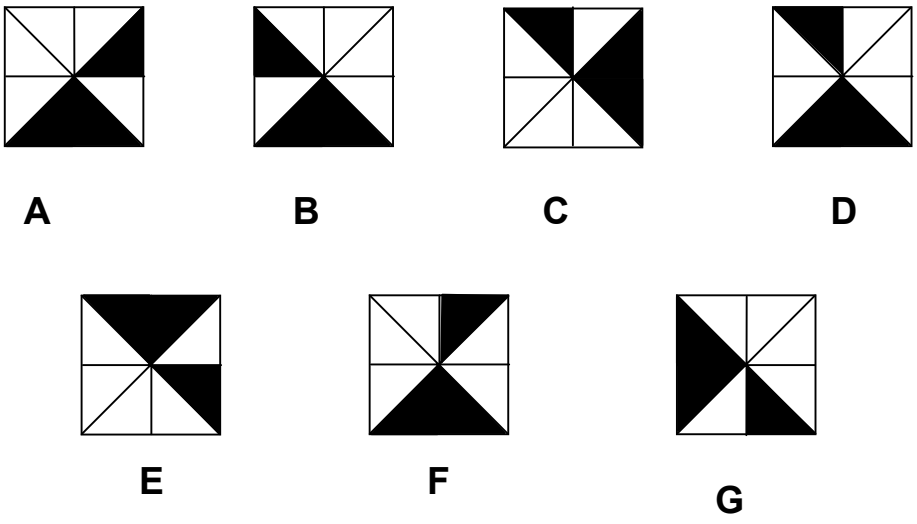
Same or Different?

Mrs Joseph has given the Grade 5 learners squares like this:

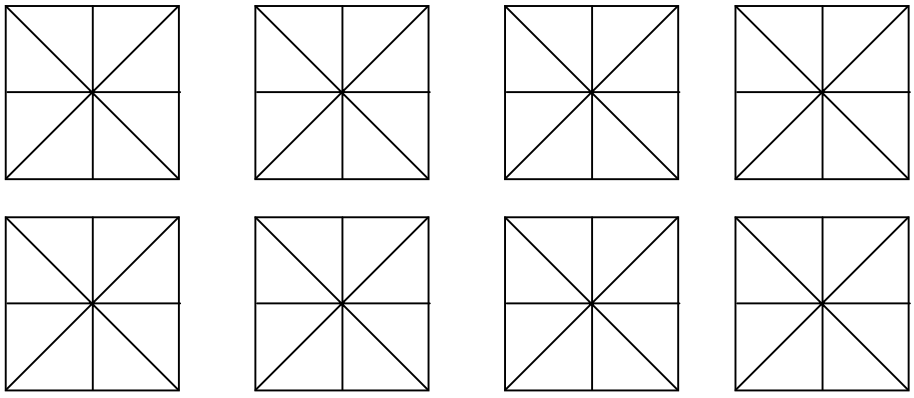


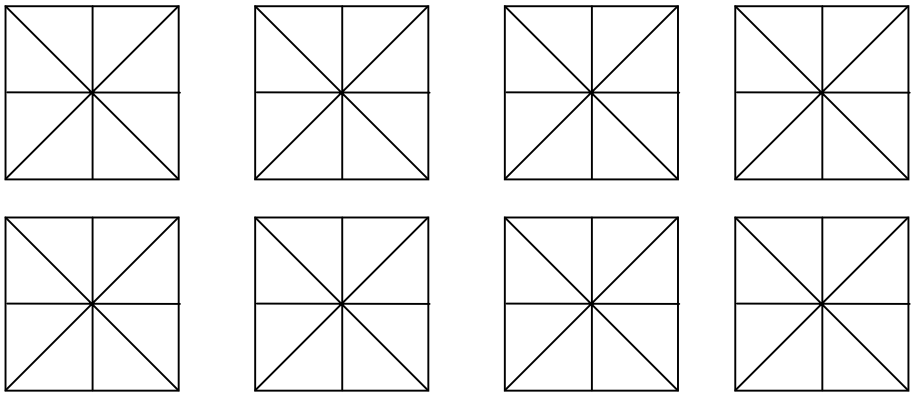
They must colour the square in as many **different** ways as possible.

The learners in Mpande's group have made these designs. Mpande says some of these figures are the same. Can you help the group to decide which of the squares are the same? Give reasons for your answers.



Now colour these squares to make designs that are different to Mpande's square. Make as many different designs as you can. Draw extra squares if necessary.





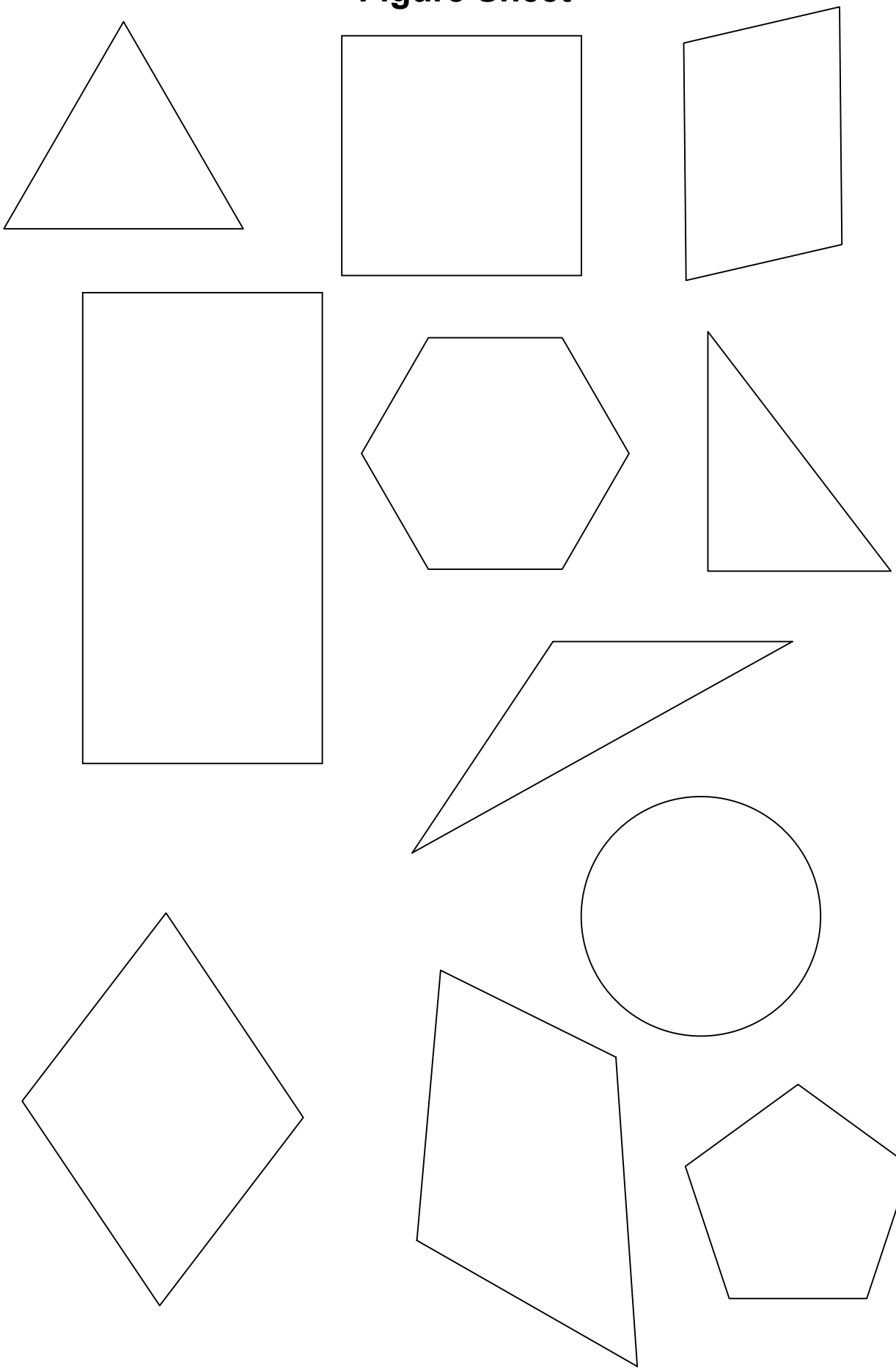
Teacher Notes: Same or Different?
As in the activity "Patterns with Squares, learners are required to reflect on what we mean by "different" in geometry. Only D and F are different to the others as they have been coloured differently. All the others are reflections or rotations of one another. Again, learners can use informal language such as flip and "turn" to describe these movements.

Some pupils might need to cut the figures and physically do the transformations, but pupils should be encouraged to perform these movements mentally.

In Question 2, sixteen squares have been provided for colouring. There are in fact thirteen different designs. This activity can be extended by using different divisions in a square.

Source of Ideas:
Bruni, J.V. and Silverman, H.J. (1977). ***Making Patterns in a Square*** *In:* ***The Arithmetic Teacher.***

Figure Sheet

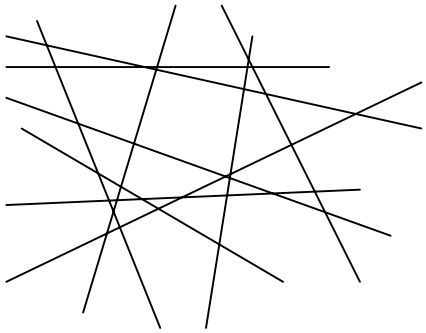


A Whole Lot of Figures!

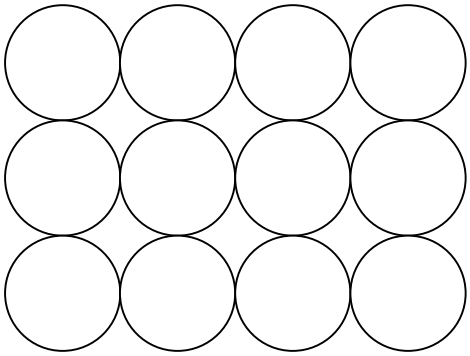
Look at these groups of figures.

1. What figures do you see in each group?
2. Try to describe each group of figures in words.
3. Compare the groups – what is the same or different about them?

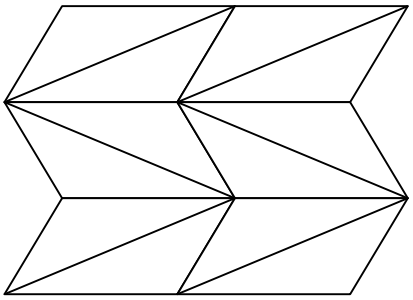
A.



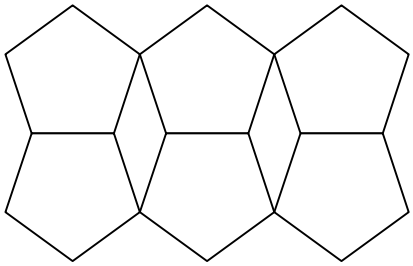
B.



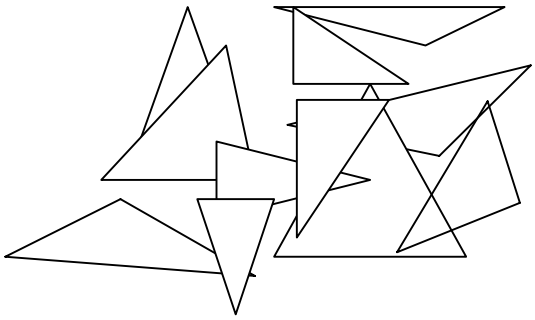
C.



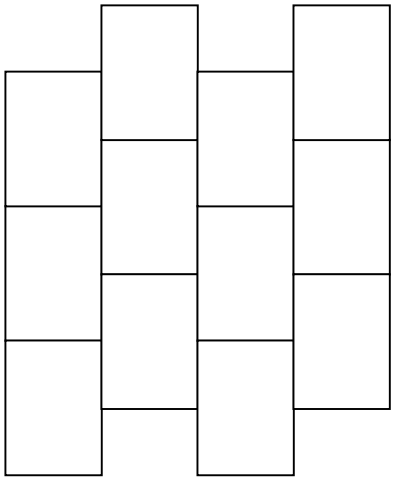
D.



E.



F.



4. Use the figures on your figure sheet to make groups of figures like B, C, D, E and F.
5. Now make different groups of figures. Describe each group of figures and draw what you have made in your book.

Teacher Notes: A Whole Lot of Figures!

This activity requires that learners focus on the different ways figures can be arranged, for example, overlapping, with a regular pattern, with gaps / no gaps. These are possible responses:

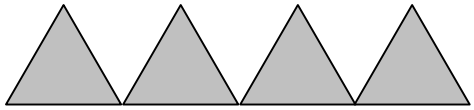
- *A and E have no regular pattern,*
- *figures in E are different sizes and shapes, figures in B, C, D and F are the same size and shape*
- *B and D have gaps in between (some learners will argue that this is not the case),C and F have no gaps*
- *There are no "overlaps" in B, C, D and F, but the figures in A and E "overlap" or "cross"*
- *In C and D the corners match, but this is not the case in E.*

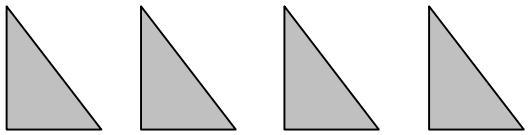
In order to create the given groups of figures in question 4, learners will have to select the correct figures from the figure sheet and then transform these into the correct orientation.
In question 5 they must make their own groups of figures – these can be random or have a pattern. It is important that learners draw and describe what they have made in words. Learners will be informally exploring which figures can tessellate.

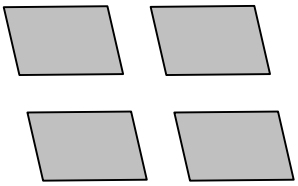
In this package we will be focusing on those arrangements that do have a regular pattern.

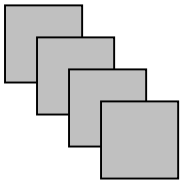
A Whole Lot of Patterns

Use the figures on your figure sheet to make patterns like these. In each case **describe how you moved the figure to make the pattern**.

1. 

2. 

3. 

4. 

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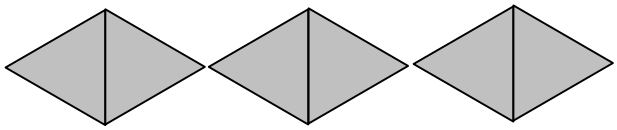
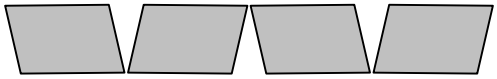
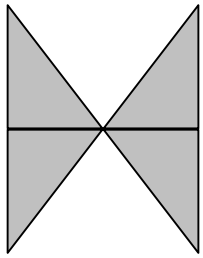
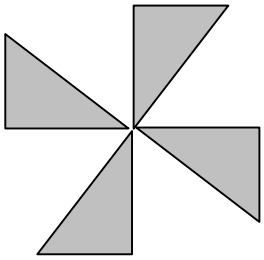
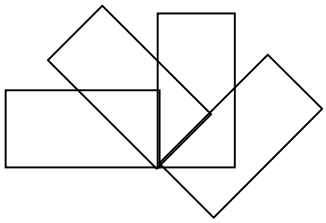
6. 

7. 

8. 

9. 

10. 

Now use another figure from your figure sheet to make some more patterns. In each case describe how you moved the figure to make the pattern.

Teacher Notes: A Whole Lot of Patterns

*In this activity learners informally explore **isometric transformations**, that is, transformations in which the shape and size of the figure is preserved ,but the position changes. It should be noted that in a transformation the figure actually moves – the patterns in this activity have been made by tracing around each figure before it is moved to the next position. The pattern thus consists of a collection of congruent figures.*

Learners will first have to select the appropriate figure from their collection of figures. The size of the figures in the patterns has deliberately been made different to that of the original figures in the figure sheet – it is an important skills for learners to be able to recognise the figures in different sizes.

Learners will use informal language to describe the figures, for example, "slide", "turn" or "flip". The teacher can "drip" the mathematical terminology in discussion.

*The first six patterns have been made using **translations** (slides). In a translation the figure is moved without changing its angle. The distance between each point and its image under the translation remains constant. Learners should note that the figures in a pattern can overlap, touch or have gaps in between them.*

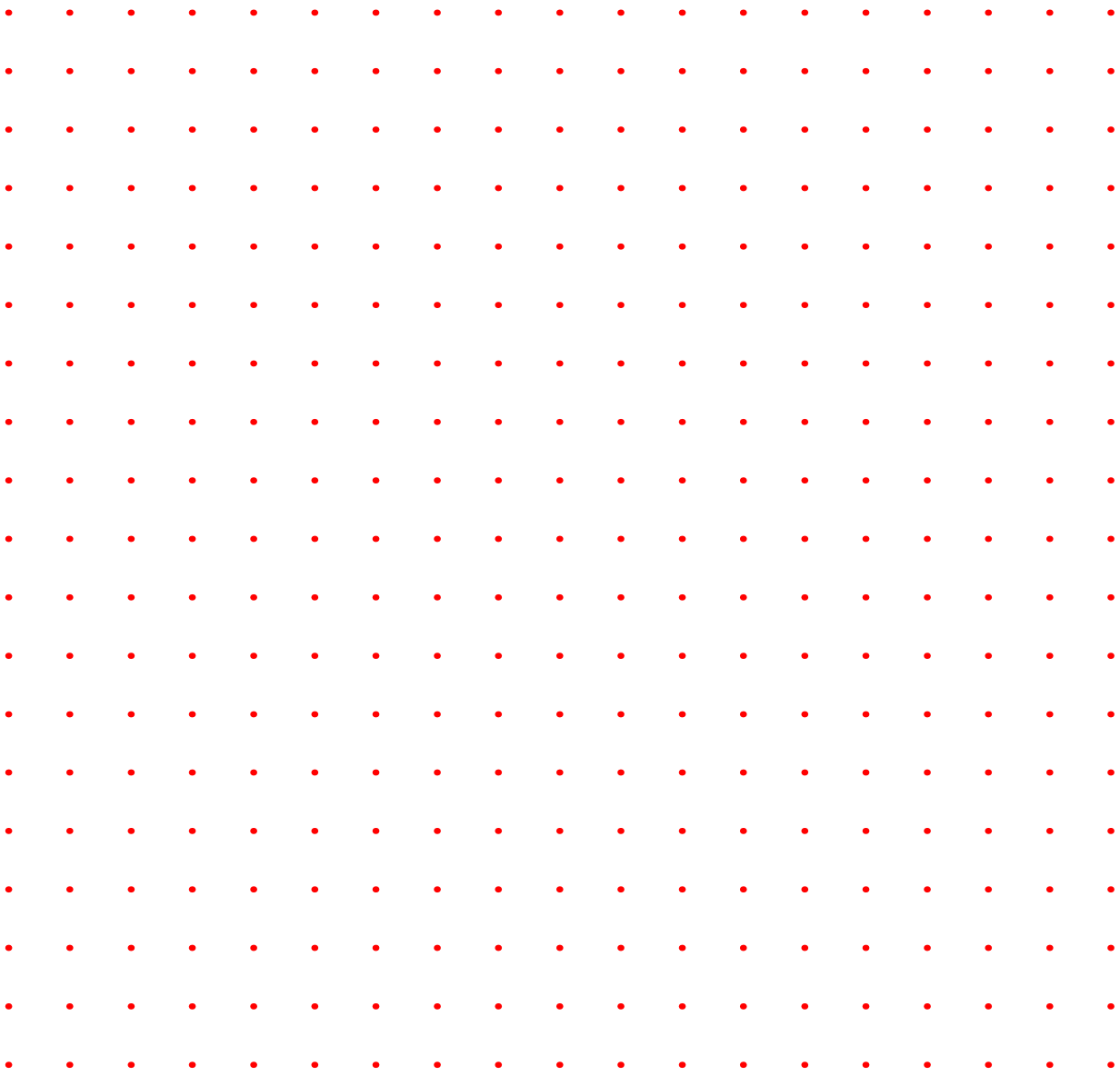
*Patterns 7 and 8 are **reflections** (flips) – each figure is reflected over a line (this can be vertical (as in pattern 7), horizontal (as in pattern 8) or oblique. Learners in the higher grades can be encouraged to identify the line over which a figure is reflected. This can be linked to symmetry, as the line over which a figure is translated is a line of symmetry in the pattern.*

*Patterns 9 and 10 are **rotations** (turns). A figure is rotated a certain number of degrees around a point. Learners in the higher grades can be challenged to identify the point around which the figure is rotated and the angle of rotation.*

It is important that learners describe the movements used to make their own patterns – this provides opportunities to reinforce the use of terminology.

Turning, Flipping and Sliding

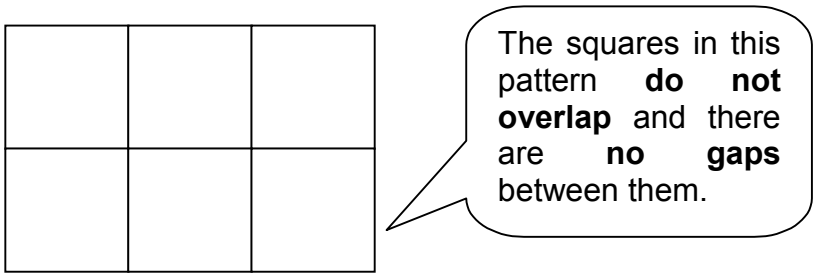
1. Draw a square on the dotty paper. Use **translation** (sliding) to create a pattern with your square.
2. Draw a triangle on the dotty paper. Use **reflection** (flipping) to create a pattern with your triangle.
3. Draw a parallelogram. Use **rotation** (turning around the same point) to create a pattern with your parallelogram.



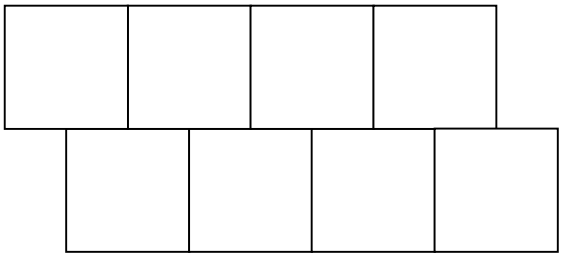
Tiling the Kitchen

The Zungu family is having tiles put on the floor of the kitchen in their new house. The tiles must cover the floor and there must be no gaps in-between the tiles.

1. Here is an example of how square tiles can be used:



This is another example.



(a) Compare these two patterns. What is the same and what is different?

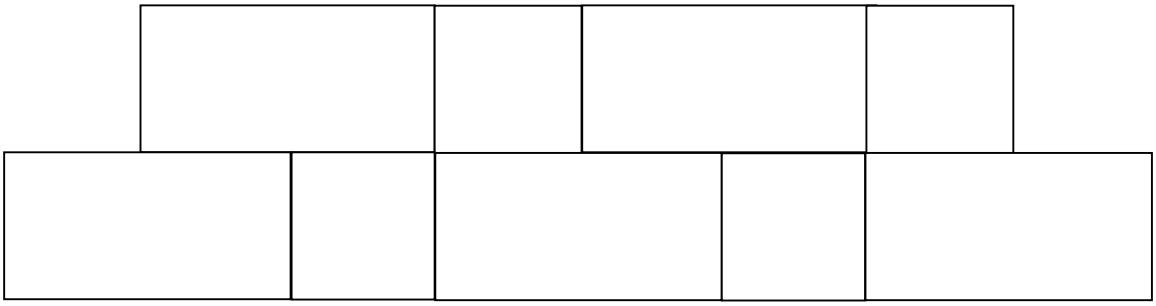
(b) Can you explain why this tiling works for a square?

2. Use the figures on your figure sheet to decide what shape the tiles in the Zungu's kitchen could be.

3. Can you explain why some of the figures can be used for tiling, but others cannot?

In the patterns we have been making the figures do not overlap and there are no gaps between the figures. We call a pattern like this a **tessellation**.

4. Lindiwe Zungu says she would like to have **two** different shaped tiles on the kitchen floor. She gives this example:



(a) What figures has Lindiwe use to make this pattern?

(b) What other patterns can Lindiwe make with two figures? Draw the patterns in your book.

Teacher Notes: Tiling the Kitchen

Each learner in a group should have a set of figures from the figure sheet. These can be shared within the group.

In this activity learners explore tiling (tessellations). In order to determine which figures tessellate, learners will have to reflect on the properties of the figures. Responses can be on different levels, for example, in explaining why squares tessellate, some learners will claim that this is because the figures fit together, while others will be able to use the size of the angles to explain. The teacher can reinforce terminology where required.

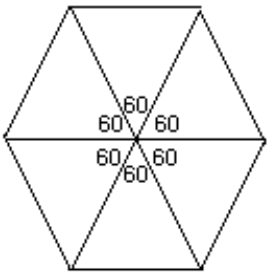
Tessellations also involve transformations – learners are moving the figures when making the patterns. Learners can be asked to describe how they moved the figures to make the patterns.

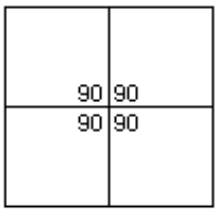
Learners should be challenged to consider whether all triangles will tessellate. Will all four-sided figures (quadrilaterals) tessellate? Again, these questions can be answered on different levels.

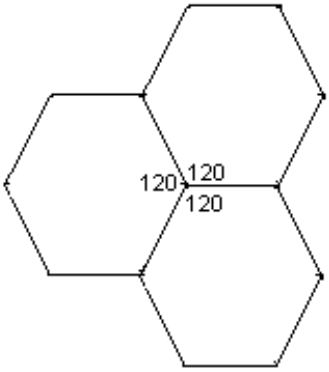
Question 4 requires that learners consider a di-hedral tessellation (a tessellation with one figure is said to be mono-hedral or regular, a tessellation with three figures is called a tri-hedral tessellation). As extension, learners can be challenged to make tri-hedral tessellations.

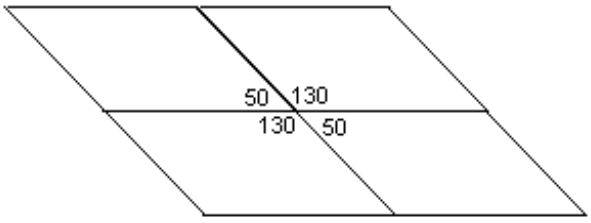
<u>Extension:</u>

Learners in Grade 7 should be challenged to use angles to explain the tessellations. Figures will tessellate if the interior angles at a point add up to 360º. Some examples are given below:





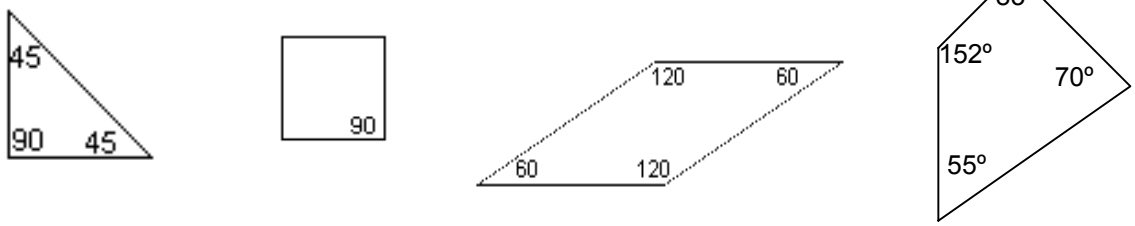




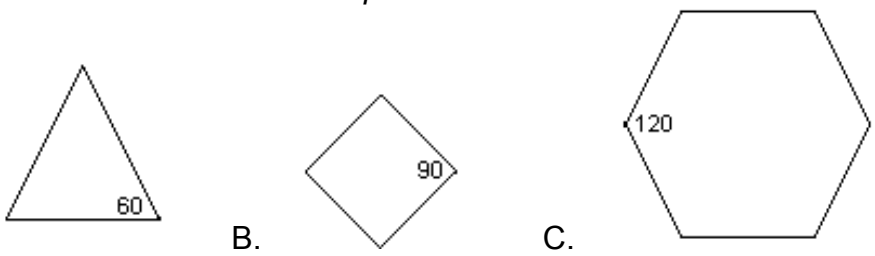
Learners can then be challenged to solve tessellation problems on a more abstract level, that is by considering the sizes of the angles of the figures rather than by physically manipulating the figures.
Consider these examples:

Can you tessellate each of these figures? Use drawings to help you.

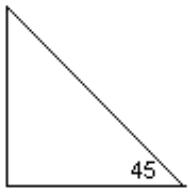
A. B. C. D.



Look carefully at each of these figures. How many of each figure will be needed to make a tessellation around a point?



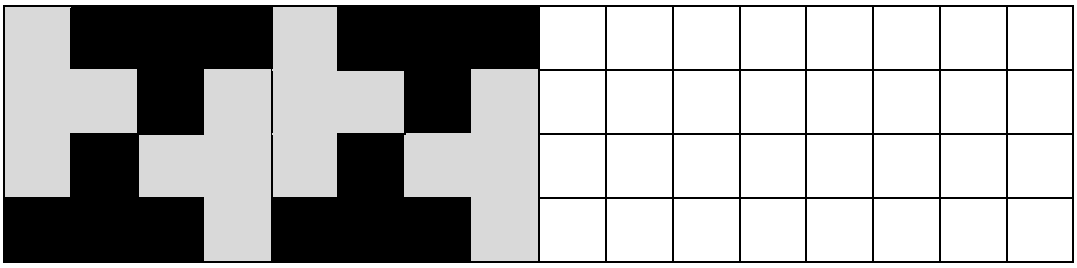
A. B. C.

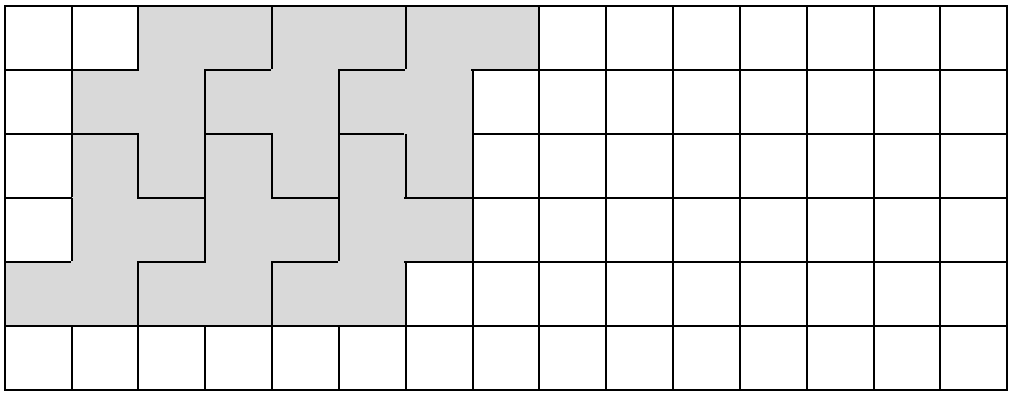


D.

Continue the Patterns

Delshe created these patterns. Can you continue her design?





Now make your own patterns on the square paper:

CONTINUE TO MODULE 5C