Malati

Mathematics learning and teaching initiative

Geometry

Module 7

Area

Grade 4 to 7

Teacher Document

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Rationale for the learning and teaching of Area

Research background on the understanding of the concept of area

The research on the conceptions that learners have of the concept of area show that many have no mental image of area at all, depend on memorized formulae and incorrectly use linear measures for area. It has also been noted that learners often generalise changes in linear dimensions to changes in area. For example, in responding to the effect of halving or doubling the sides of a square, most learners say that the area is also halved or doubled.

Tierney et al. (1990) has shown that primary teachers often use perimeter instead of area to compare the size of figures. They suggest that for these learners the perimeter is the only "**perceptual clue**":

> *We believe that many of them cannot focus on the **region whose area they have to compare or find**, and they have learned to associate perimeter with a formula for area. An often expressed view is that the area of the region is the "amount of stuff in that region." This idea leads naturally for many people to a conviction that if the length of a boundary of a region is kept fixed as the shape of the boundary is changed then the area stays constant because: "the amount of stuff hasn't changed".*
> Tierney et al., 1990, pp. 313.

This research has also shown that many learners are not able to relate the process of finding the area by counting units with the formula for finding the area. Tierney suggests the need to understand the mental processes involved in dealing with these different aspects of understanding the area concept. We believe that learners need to be provided with experiences that make these relations explicit.

We have taken these research findings as well as the Van Hiele Theory into consideration in developing the concept of area.

In the following section we describe the way in which the concept of area can be explored at the Van Hiele visual and analysis levels.

Visual Level

At the visual level the concept of area is developed as the *covering of space* through activities in which the areas of different figures are compared. We build on learners' intuitions about the notion of largeness as a means of distinguishing between the concepts of area and perimeter. The learners are then provided with experiences in which they have to identify the geometric figures in larger figures. Learners who are not able to mentally visualise and transform figures at this level must be given experiences in which they cut figures and rearrange them in order to compare the areas of different figures. The emphasis is not on the actual measurement of the dimensions of figures at this level. Learners are, however, given experiences in which they have to draw different figures with the same area on grids, and to compare the areas of different figures drawn on grids. These activities are designed to reinforce the notion of area as a covering of space.

Analysis level

At this level the activities focus on the area in relation to other properties of the figure. Empirical methods such as working on grids and the dissecting and transforming of figures are still used to establish important generalisations such as the area formulae. These generalisations are re-enforced in activities that focus on the calculation of the area of the figures. Learners at this level should be familiar with the properties of polygons such as the quadrilaterals and the triangles and need to use the properties to make the necessary dissections for transforming one figure into another. We recommend that these aspects be revisited or integrated.

Area and Curriculum 2005

Curriculum 2005 recognises the importance of measurement as a skill for communication. It is required that learners are familiar with the following appropriate skills of measurement:

- working with relevant units of measurement
- issues of accuracy

The following recommendations apply specifically for the study of area:

- construction of 2D and 3D figures with a given area
- calculate the area of regular and irregular figures using decomposition and estimation
- apply the formulae used in measurement of regular polygons and polyhedra
- discover patterns in measurement and generalise procedures to solve problems involving areas and surface area
- use visualisation and symmetry to solve problems involving classification and sketching

Recommended activities:

Primary grades: Visual level activities (Activities 1-12, included in Prim 07a)
Secondary grades: Analysis level activities (Activities 13-21, included in Prim 07b and c)
Of course, teachers should

- where appropriate, give primary learners some analysis level activities as extension
- where necessary, give secondary learners some visual level activities as preparation.

References and Sources

Tierney, C. (1990). Prospective primary teachers' conceptions of area. In G. Booker, P. Cobb & T. Mendicuti (Eds.), **Proceedings of the Fourteenth International Conference on the Psychology of Mathematics Education: Vol. 2.** (pp. 307-314). Mexico.

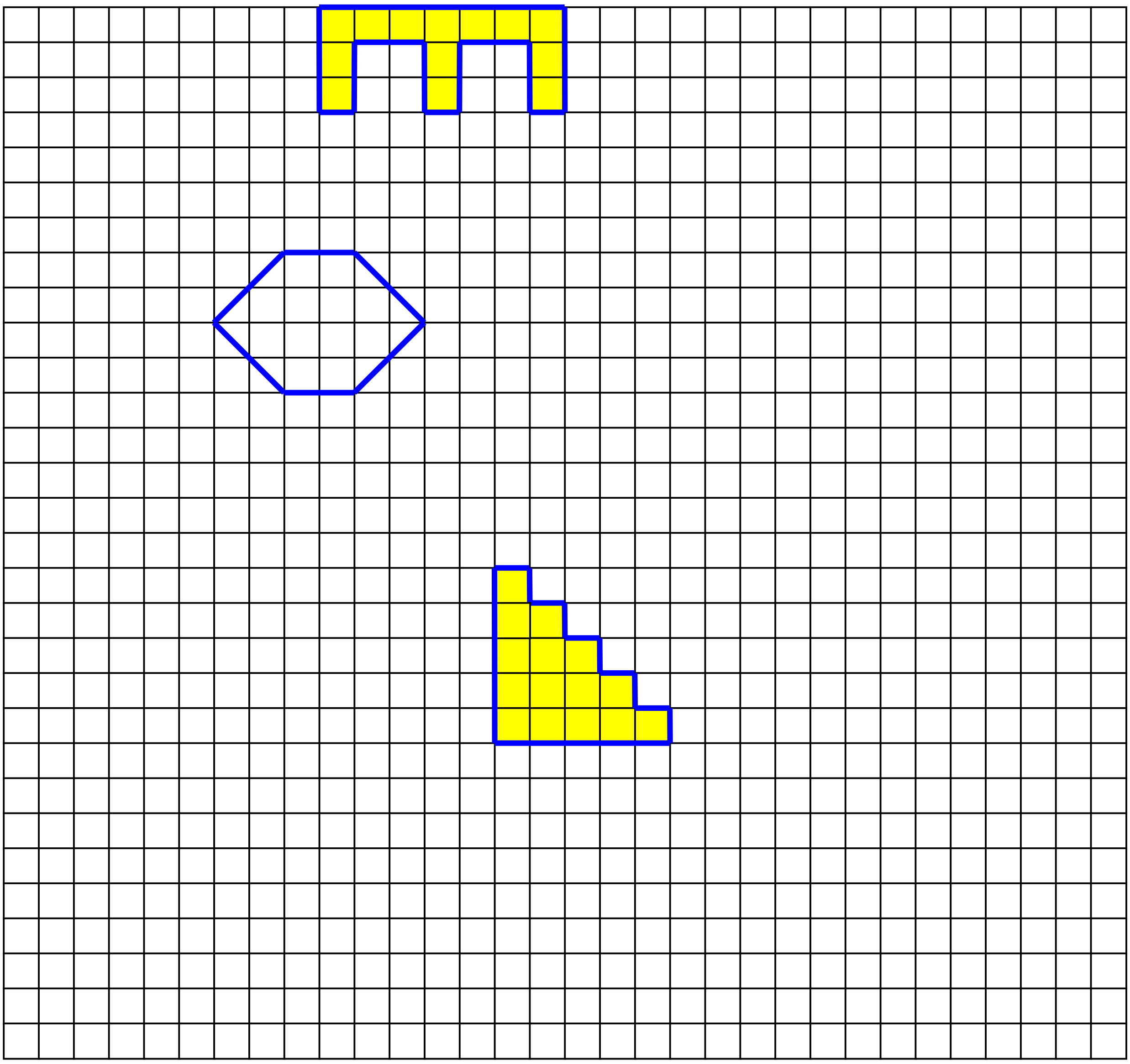
Connected Geometry (1997). **The Cutting Edge: Congruence, Area and Proof**. Education Development Centre.

Activity 1

SHAPES AND SIZES

Arrange the figures below in order of size, from the smallest to the biggest.

Explain *how* you decided to arrange the figures in order of size.



Teacher Notes: Activity 1

The aim of this activity is to establish the learners' intuitive understanding of the notion of largeness. Some learners may, for example, use the concept of perimeter to compare the figures while others may use the concept of area.

We enclose the additional activity "How big is the rondavel?" to address the issue of using the area or the perimeter to express the size of two-dimensional objects.

Source:
Human, Olivier and Associates (1999). ***Basic Numeracy Course. Class Exercises 3****. Parow-East: Ebony Books CC.*

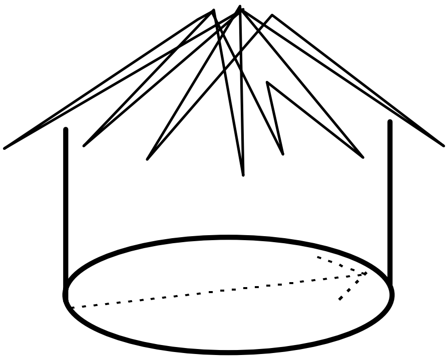
HOW BIG IS THE RONDAVEL?

Pule marries his childhood sweetheart on the farm and then they move to the city. He promises her that he will build her a house which *is not smaller than* her traditional rondavel at home.

How can Pule measure how big the rondavel is?

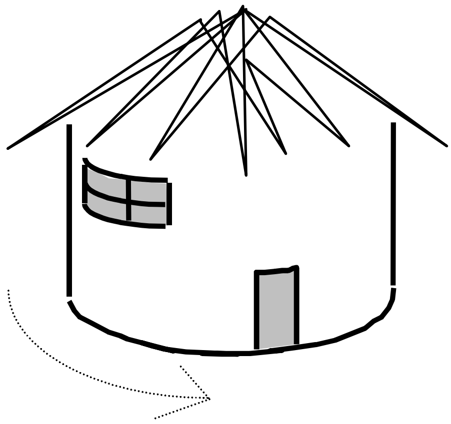
1. Pule can measure the size of the rondavel in two ways:

He can measure the distance from the wall across the floor to the other side:



or

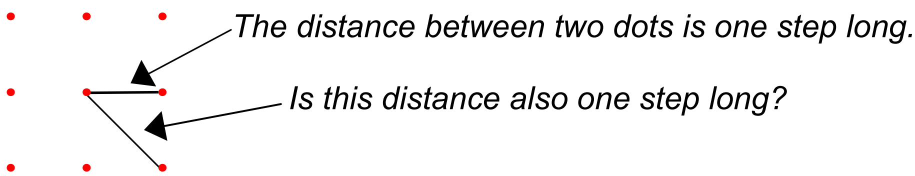
He can measure the distance *around* the rondavel:



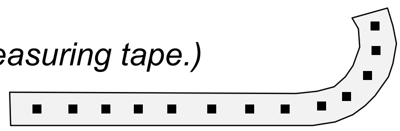
Which is the best way? Why?

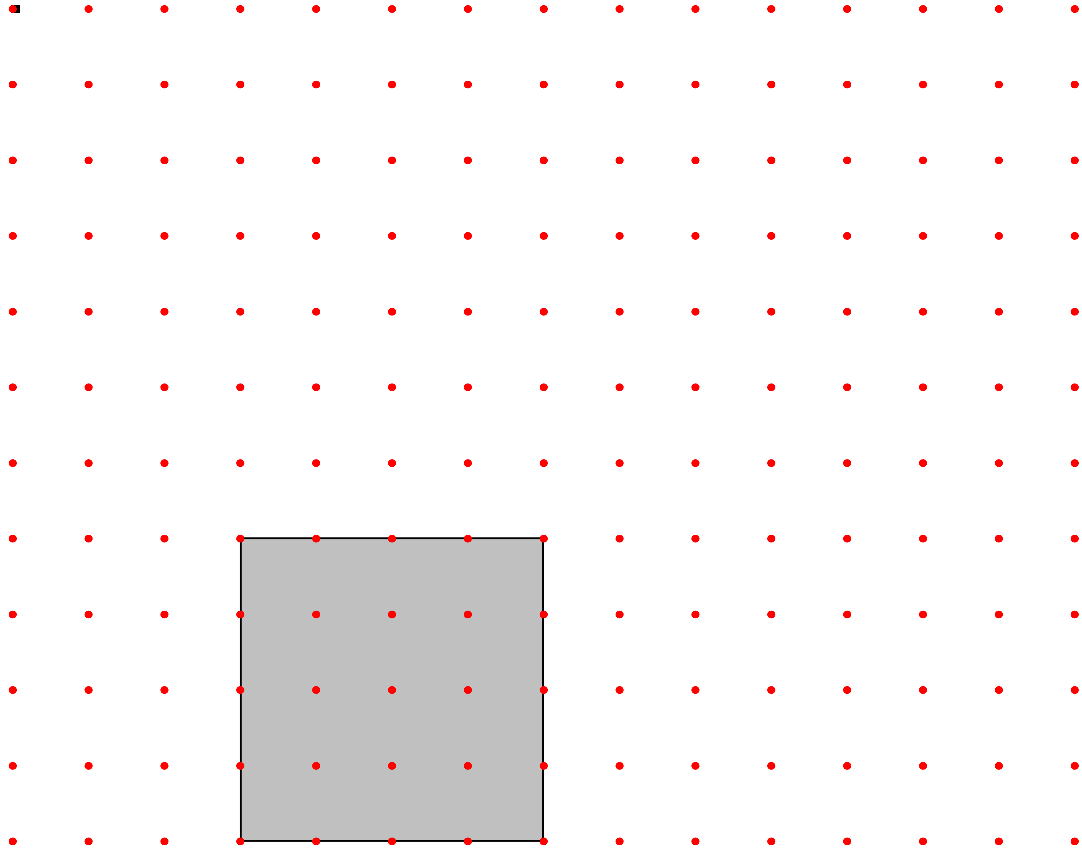
2. Pule walks through the rondavel and finds that it is 4 steps big.
Use the dotty paper on the next page to draw a floor plan for the rondavel, showing how big it is.

Note:



3. How many steps is it around the rondavel?
(Cut out one row of dots and use it as a measuring tape.)



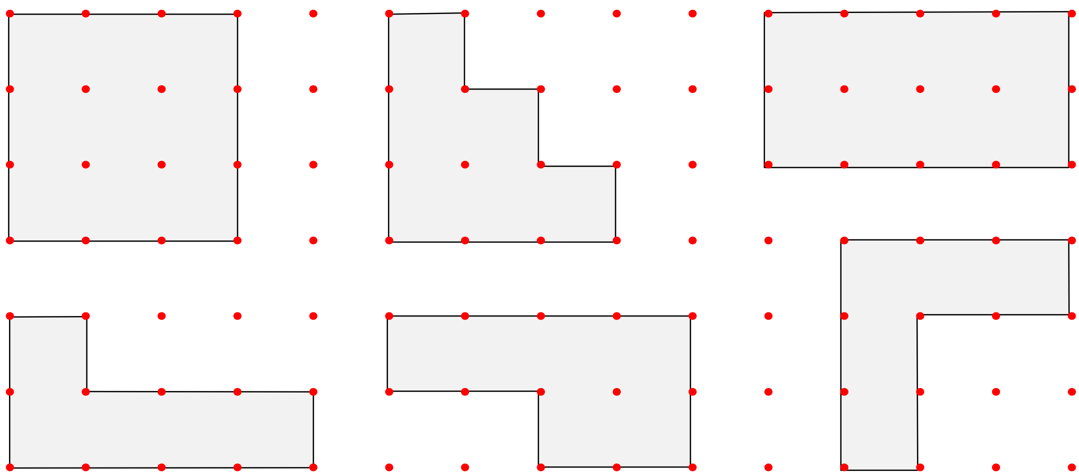


4. Pule finds it difficult to build a rondavel in the city. He says that he will build a *square* house as shown above.

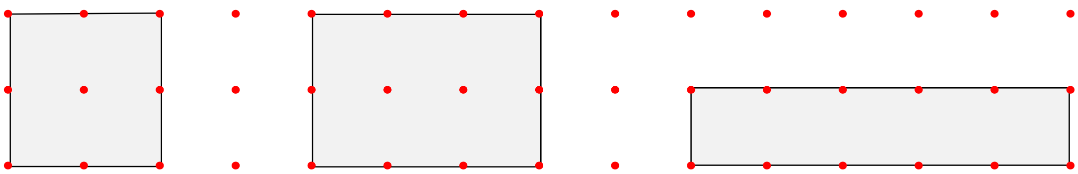
Pule says the square house is 16 steps around. This is more steps than around the rondavel. So the house is bigger than the rondavel.
Is this reasoning correct?

Is measuring *around* the house a good way to say how big a house is?

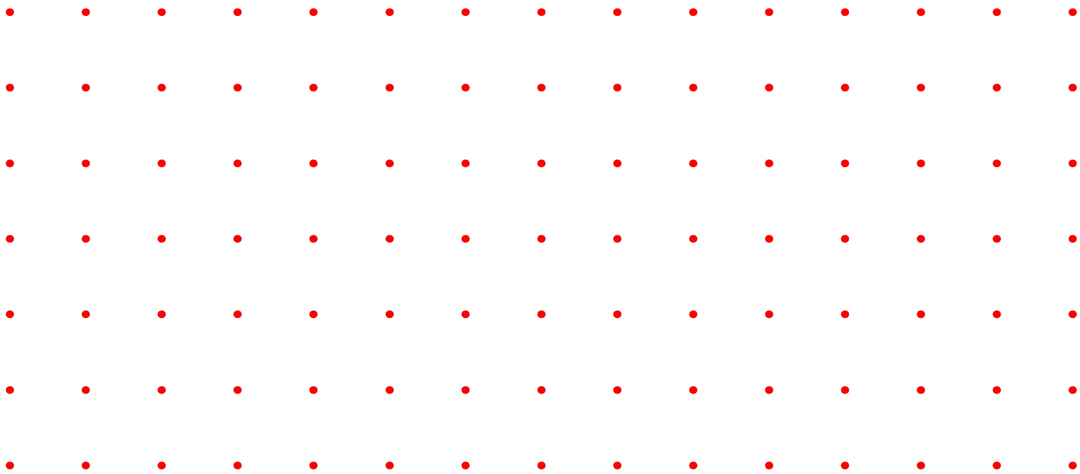
5. Do you agree that these "houses" are all 12 steps around?
   But are they equally big?
   How big is each house? Which house is the biggest?



6. Look at these "houses".
   How many steps is it around each house?
   Which house is the biggest? How big is it?



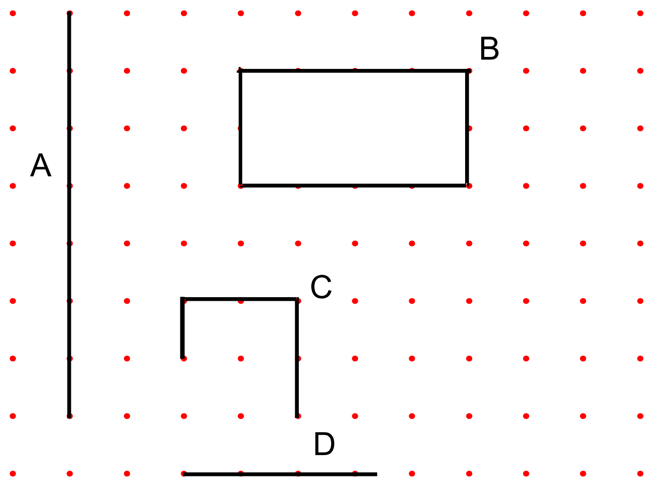
7. Draw several different houses that are 14 steps around.
   How big is each house? Which house is the biggest?



Activity 2

PERIMETER 1

Determine the perimeter of the following figures if possible.



Teacher Notes: Activity 2

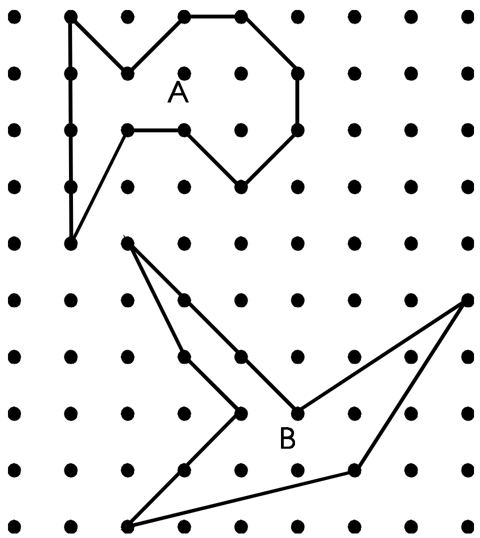
The aim of this activity is to present the learner with situations in which it is not possible to consider perimeter. It needs to be emphasised that the notion of perimeter only refers to figures that are closed. (Figure B)

Activity 3

PERIMETER 2

Which of the figures below have the larger perimeter?

Explain how you decided.



Teacher Note: Activity 3

In this activity the learner need not determine the actual perimeter of the figures. The perimeter can, for example, be approximated in terms of the sides and the diagonals of the square or rectangular figures;

Figure A	***Figure B***
7 side units of 1 by 1 square	
5 diagonal units of 1 by 1 square	*6 diagonal units of 1 by 1 square*
1 diagonal unit of 1 by 2 rectangle	*1 diagonal unit of a 1 by 2 rectangle*
	2 diagonal units of 2 by 3 rectangle *> 3 units of 1 by 1 square*
	1 diagonal unit of 1 by 4 rectangle *> 4 units of 1 by 1 square*

Figure B therefore has a larger perimeter than Figure A.

Activity 4

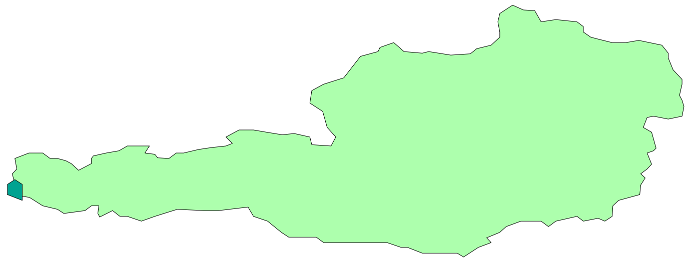
ISLANDS

Which island below has the largest area?

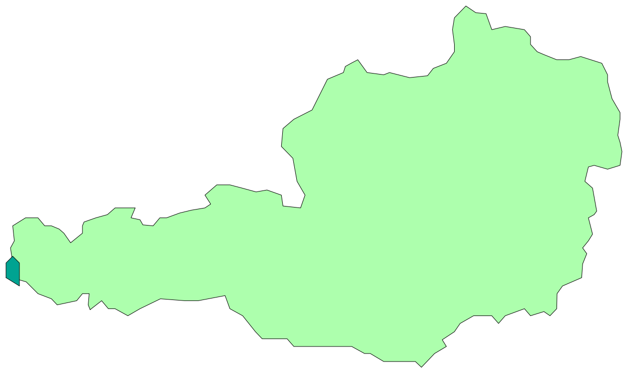
Which island below has the largest perimeter?

Discuss how you made your decisions.

Island A



Island B



Teacher Notes: Activity 4

The aim of this activity is to give the learners experiences in finding ways of estimating the area and the perimeter of irregular figures.

<u>Estimating the area</u>
If the learners suggest the use of grids, ask them how their estimate can be improved with this method. Thereafter provide the learners with different sized grid paper.
The learners can also be asked to calculate the following:

- *total area of the squares that are inside the island (this area is called the "inner sum")*
- *the total area of the squares that are either inside or touch the island (this area is called the outer sum)*

These sums can be investigated for different grid sizes.

This activity can be revisited to focus on the notion of a limit.......the inner and outer sums get closer to a single number, which is the area of the region.

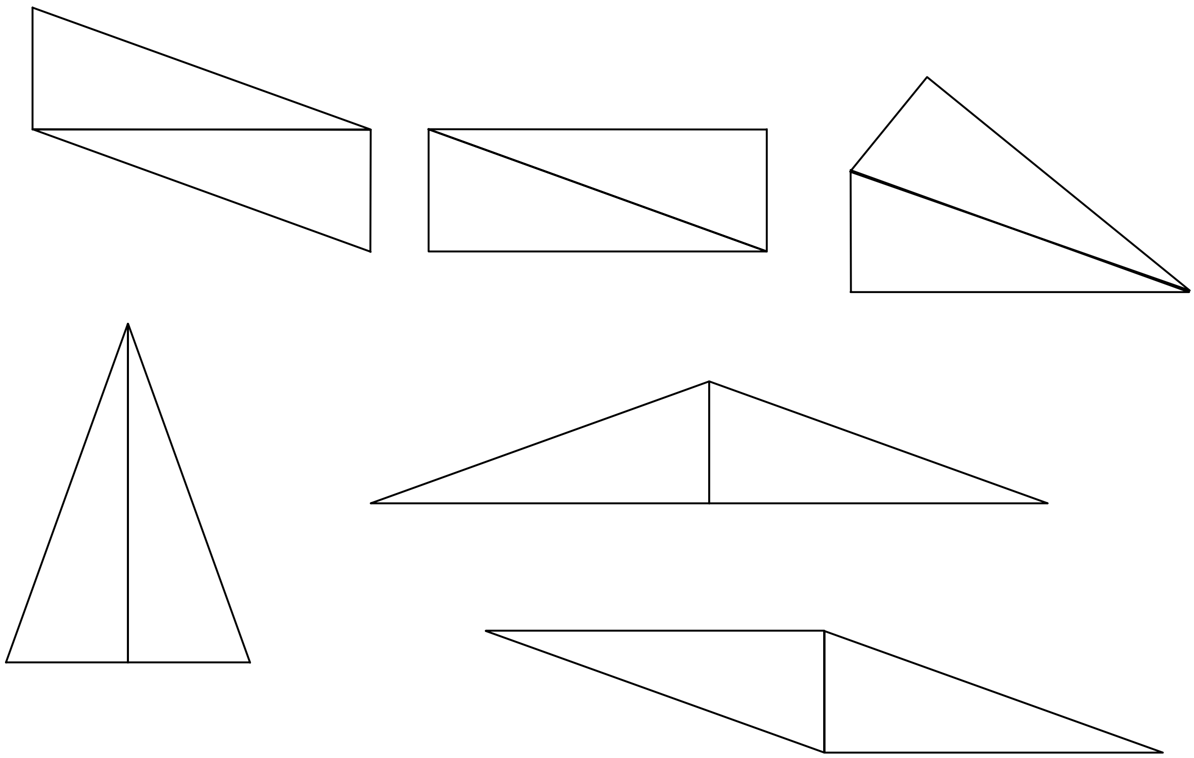
<u>Estimating the perimeter</u>
The marking off of line segments, also known as linear approximation, can be discussed. The learners can also be asked to draw circles and to approximate the perimeter of their circles using this method.

Activity 5

BUILDING FIGURES 1

Andrew builds 6 different figures with the following two congruent triangles:

Discuss the area and the perimeter of the figures below.



Teacher Notes**: **Activity 5

In this activity the learners are introduced to the idea that rearranging figures does not change area.
Ask the learners if they can identify the line inside the figures on the original triangles.

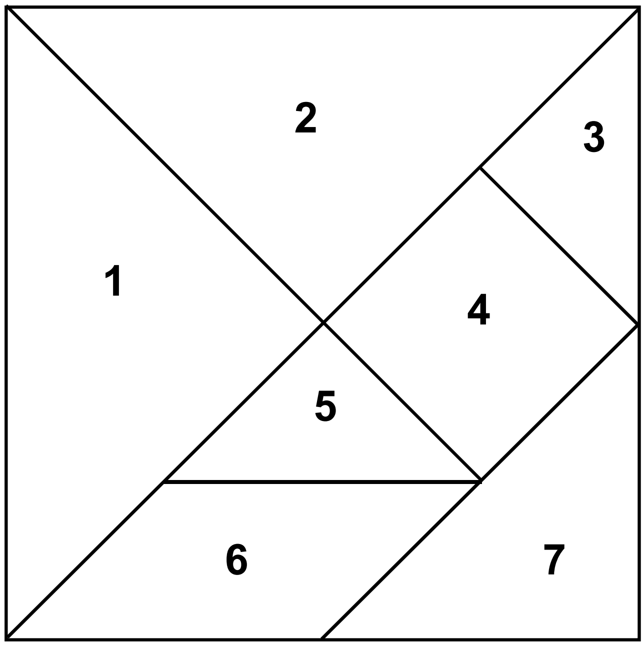
What does this line do to the new figure?

- *In all the figures it divides the figure in half.*
- *In some of the figures it is a line of symmetry.*

ACTIVITY 6

BUILDING FIGURES 2

A tangram is a geometric puzzle in which a square is cut into seven specific pieces as shown below.



1. Use the seven tangram pieces to build a rectangle.

   Does the square have a larger area, a smaller area, or the same area as the rectangle? Explain your answer.

2. Use the two small triangles, 3 and 5, to build three different figures that are the same as three of the other tangram pieces.

   Compare the areas of the three figures that you have made.
   Compare the area of one of the small triangles with the area of each of the figures that you have made.

3. Use the two small triangles, 3 and 5, and the medium triangle, 7, to build the large triangle, 1 or 2.

   Compare the area of the small triangle and the medium triangle.
   Compare the area of the small triangle and the large triangle.
   Compare the area of the medium triangle and the large triangle.

Teacher Notes: Activities 6 and 7

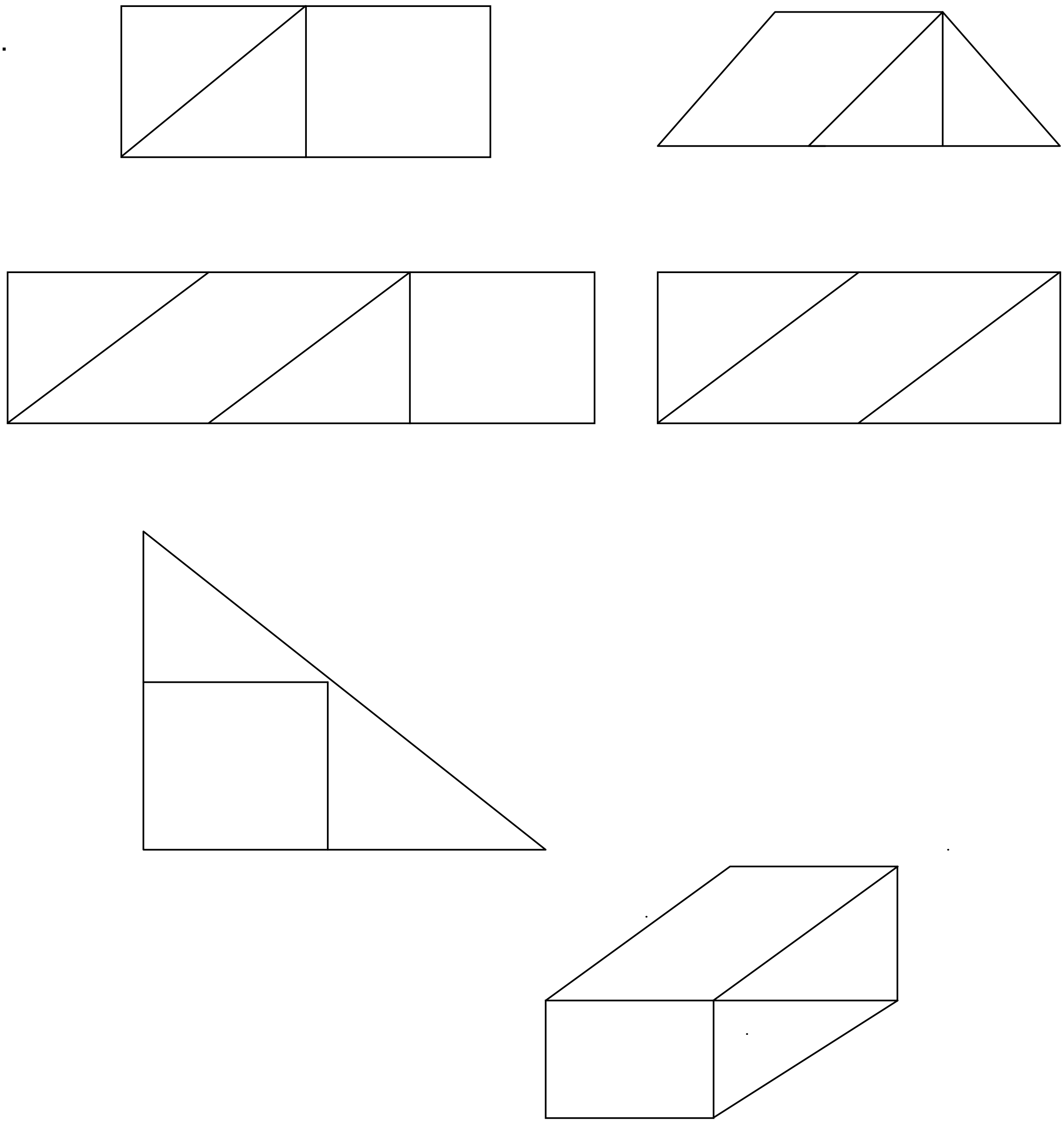
These activities encourage the discussion of ways to tell whether figures have the same area and also reinforce the notion of area as a covering of space.

Activity 7

BUILDING FIGURES 3

Donald builds the following figures with his tangram pieces, 3, 4, 5 and 6.

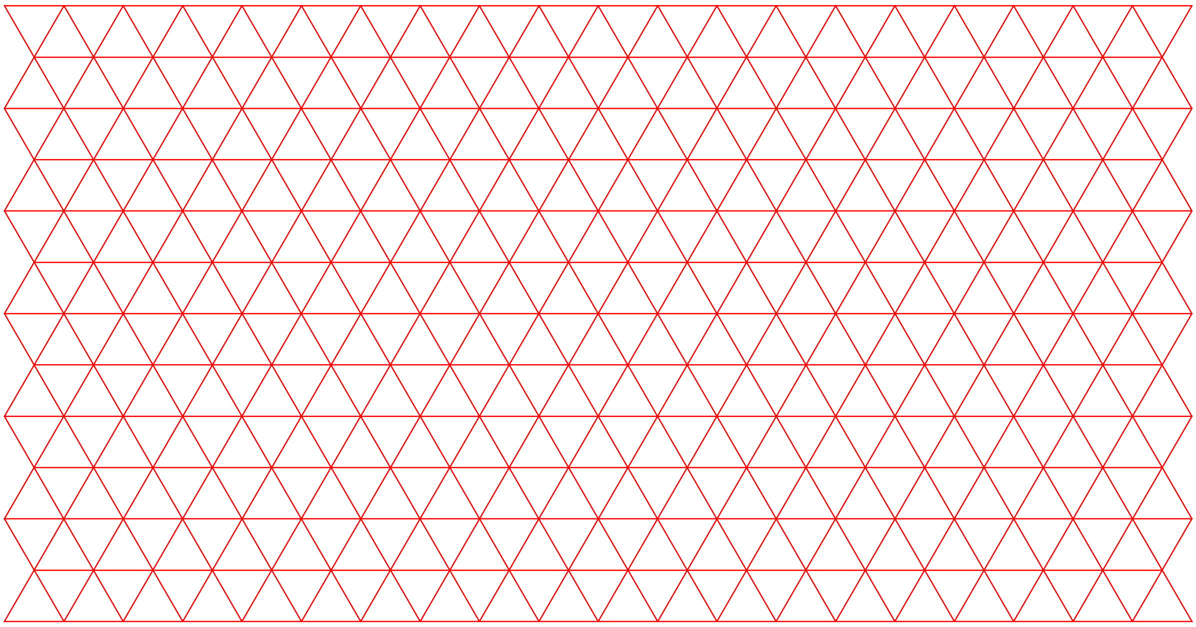
Which figures have the same area? How do you know?



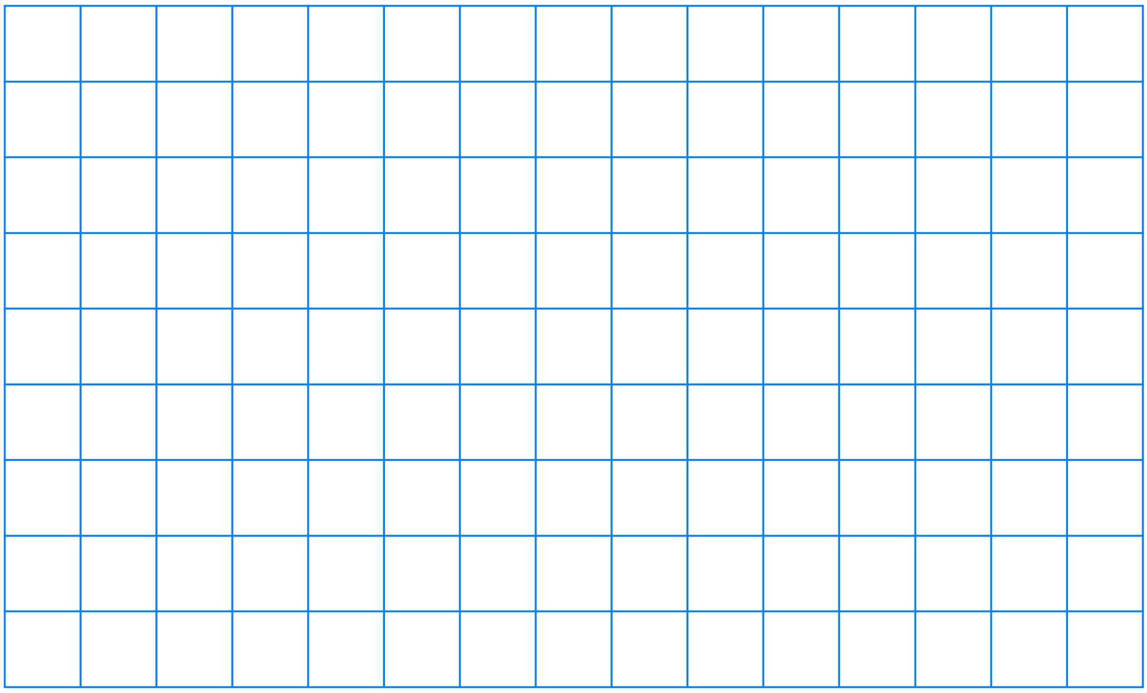
Activity 8

BUILDING FIGURES 4

Draw two different figures that have the same area on the triangular grid below:



Draw two different figures that have the same area on the square grid below:

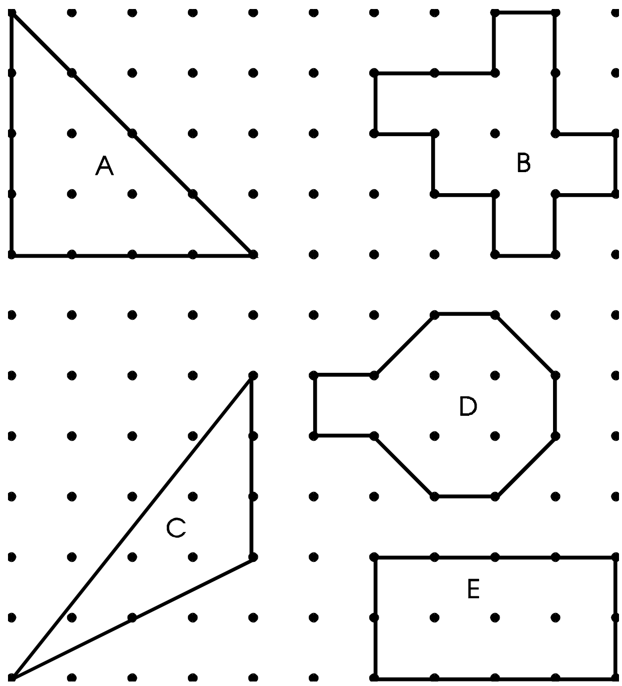


Teacher Notes: Activities 8 and 9

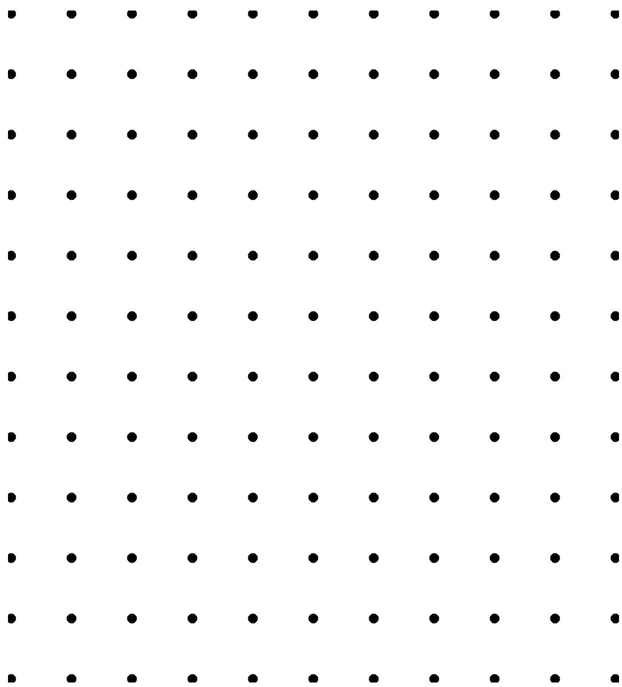
These activities reinforce the idea of area as a covering of surface. Learners are given opportunities to determine the area in terms of units and to create figures that have the same area.

Activity 9

AREA 1



1. What is the area of figures A, B, C, D and E?
2. Draw other figures with the same area as these figures.

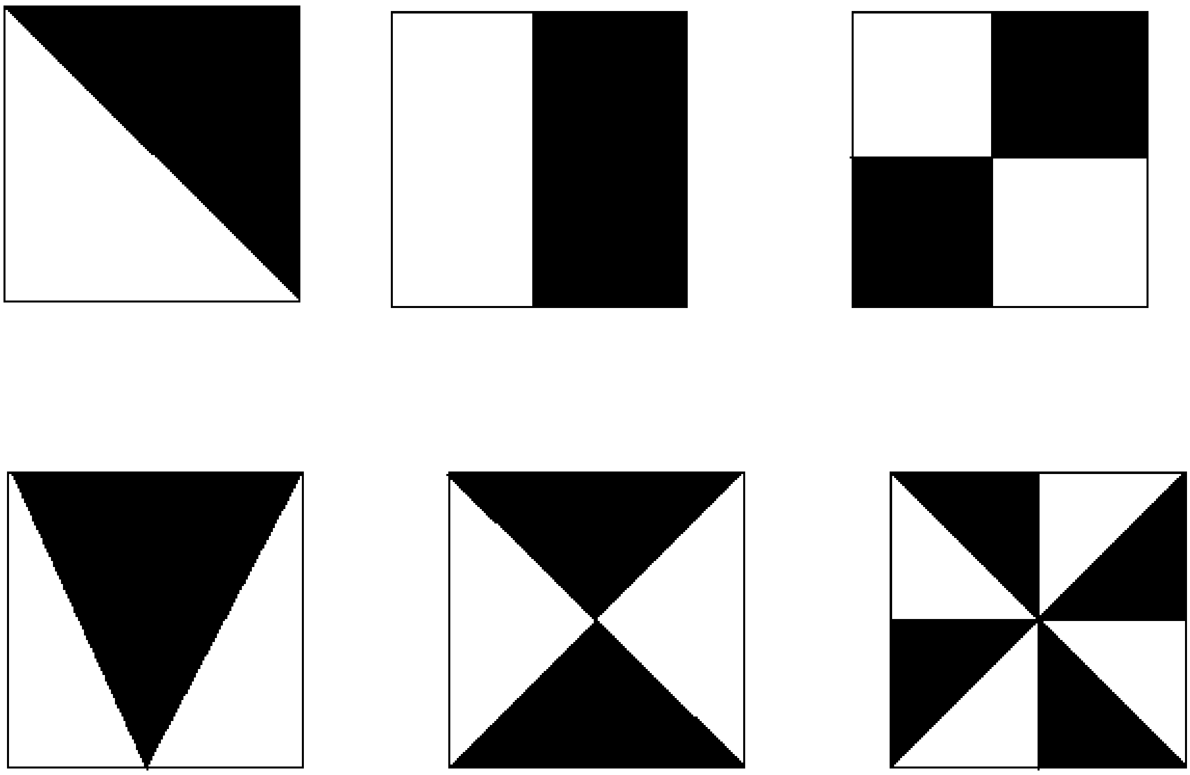


Activity 10

TILES 1

Which of the square tiles below have more black than white?

How do you *know*?



Teacher Notes: Activity 10

Ask the learners if they are able to cut the black section of the tiles so that they all look like the first tile.

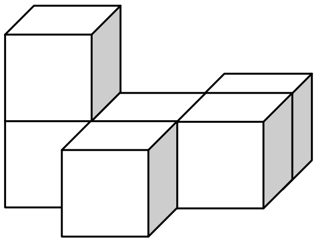
The learners can also be asked what kind of transformations are necessary to get the black pieces in the position so that all the tiles look like the first tile.

Activity 11

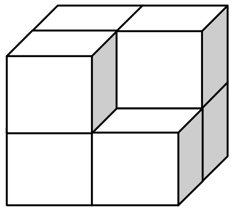
BLOCK BUILDINGS 1

These block buildings are made up of 1 cm by 1 cm by 1 cm blocks. The buildings are placed on a table.
For each building write down its surface area.
Explain your answer.

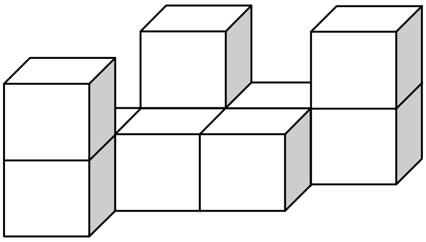
1.



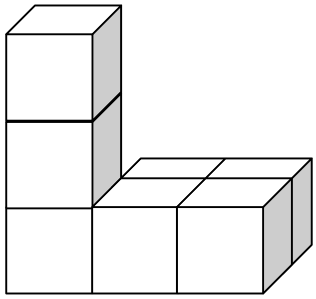
2.



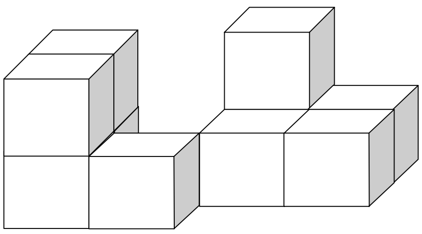
3.



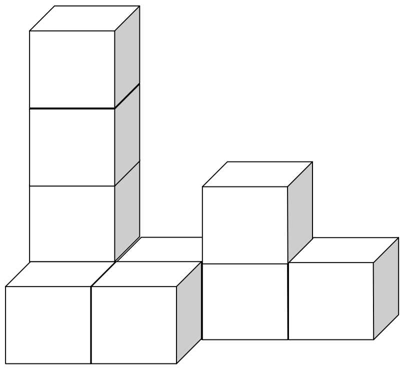
4.



5.



6.



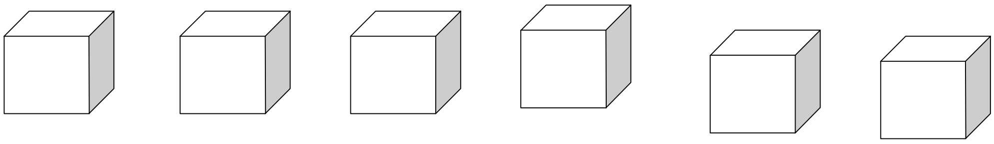
Teacher Notes: Activities 11 and 12

In these activities the learners are given opportunities to consider the area of 3D figures. The notion of surface area is introduced.

Activity 12

BLOCK BUILDINGS 2

Lindy is given six, 1 cm by 1 cm by 1 cm blocks. She is asked to build as many different kinds of block buildings, using all 6 blocks.



1. Draw floor plans of all the possible block buildings and write down the area of the floor that is covered by each building.

2. Determine the surface area of the block buildings.

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