Malati

Mathematics learning and teaching initiative

Fractions

Rational Numbers

Grades 8 and 9

Teacher Document

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Rational Numbers for Grade 8 and 9

Some general remarks:

- Research in Grades 8 and 9, locally and internationally, has shown that learners have a poor conception of rational numbers e.g. they believe that the more numbers after the decimal comma, the bigger (or smaller) the number (e.g. that 0,54 is bigger than 0,7).
- This is often caused by a poor conception of common fractions.
- The more mechanical calculations pupils carry out, the further they ‘hide’ their misconceptions.
- Regular rational number *concept development*, and where necessary, *remediation* of basic rational number concepts, is important in Grades 8 and 9. The purpose with this short "intervention" package is to provide teachers with appropriate learning activities to try to address learners' problems with rational numbers.

Content of this package:

Fractions (4 sessions)

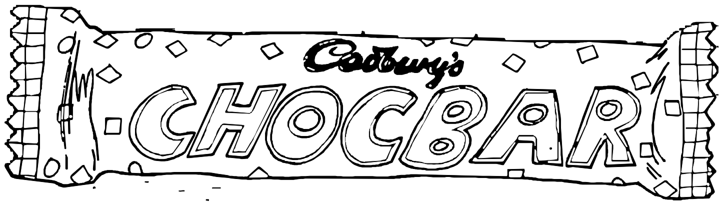
1) **Simple sharing problems** with remainders – pupils should draw the solution and express their answers as common fractions.
2) **Developing equivalent fractions** using the ‘fractions wall’ (a wider range of fractions than the traditional ‘fractions families’).
3) **A word problem** which can be solved in a variety of ways (using various operations with fractions).
4) **The log of wood** (multiplication of fractions) – a problem which provides the opportunity to reflect on how fractions are named, and what the name means.

(Homework: **Snakes/chains**)

Decimal fractions (At least 4 sessions)

5) **Introductory activities using calculators**: Comparing own solutions with calculator solutions, and reflecting on the meaning of the calculator answer.
6) **Counting** using the calculator and/or **sequencing**: Decimal place value
7) **Calculator games**: Decimal place value
8) Word problems (Choice of 2): **Operations** with decimals and **beliefs** about decimals

Sharing Chocolate



1. Three friends share four Chocbars equally. How much Chocbar does each friend get? Use a diagram to show your answer.

2. Three friends share five Chocbars equally. How much Chocbar does each friend get? Use a diagram to show your answer.

3. Two friends share eleven Chocbars equally. How much Chocbar does each friend get? Use a diagram to show your answer.

4. Three friends share eleven Chocbars equally. How much Chocbar does each friend get? Use a diagram to show your answer.

5. Ten friends share twenty-two Chocbars equally. How much Chocbar does each friend get? Use a diagram to show your answer.

6. Five friends share twenty-two Chocbars equally. How much Chocbar does each friend get? Use a diagram to show your answer.

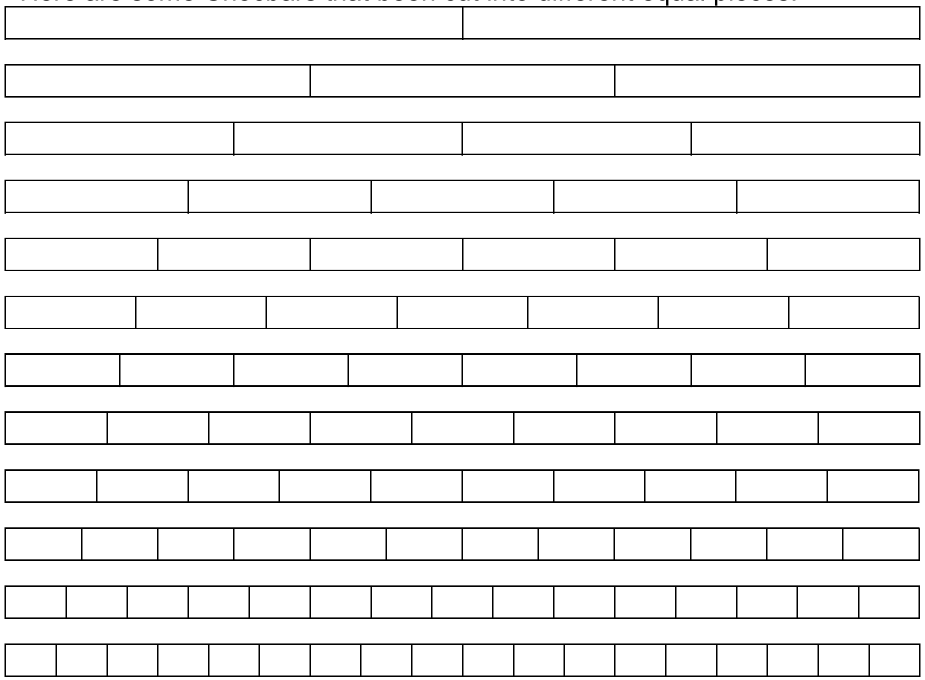
Teacher Notes:

The MALATI approach to introducing fractions to learners makes use of equal sharing situations with a remainder that can be shared out. We also use this type of problem to remediate conceptual problems with fractions, and to help learners overcome limited concepts related to learners having been exposed to halves and quarters only.

For this task learners should not express their answers as decimals. We encourage learners to *draw* their solutions. One reason for this is that the teacher can easily diagnose which learners are experiencing conceptual problems and which are simply not able to name or write the fraction. The latter is *social knowledge* which can be given by the teacher or by peers.

Which Piece is Bigger?

Here are some Chocbars that been cut into different equal pieces:



1. Fill in the appropriate fractions on each piece of Chocbar.

2. Which piece is bigger?

a) $\frac{1}{3}$ or $\frac{2}{6}$

b) $\frac{1}{2}$ or $\frac{3}{5}$

c) $\frac{2}{4}$ or $\frac{3}{6}$

d) $\frac{5}{10}$ or $\frac{2}{5}$

e) $\frac{1}{7}$ or $\frac{1}{8}$

f) $\frac{7}{15}$ or $\frac{4}{8}$

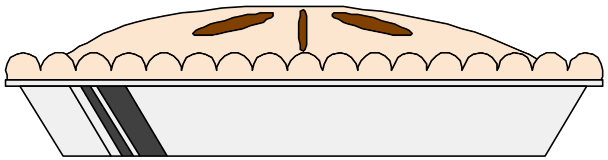
g) $\frac{9}{18}$ or $\frac{6}{10}$

Teacher Notes:

This worksheet addresses the equivalence of fractions and comparisons between sizes of different fractions. Learners may need to use direct physical comparisons of the fractions, in other words the ‘fractions wall’. Other methods (such as converting to appropriate equivalent fractions with the same denominators) are also acceptable, as long as it is clear that the learners understand why they are doing what they are doing.

The teacher should use his/her discretion as regards the ensuing discussion about the effect of the denominator on the size of the fractions. Learners should make sense of the relationship based on their observations of ‘how many pieces’ rather than memorise a rule. Many primary school learners simply remember that ‘the bigger the fraction looks the smaller it is’ which leads to problems when the numerator is not 1 and in the case of decimal fractions.

Mrs Daku Bakes Apple Tarts



Mrs. Daku bakes small apple tarts. She uses $\frac{3}{4}$ of an apple for one apple tart. She has 20 apples. How many tarts can she bake?

Teacher Notes:

For this problem, discussion of various strategies and answers is essential. Learners who zoom in on an operation or number sentence often make the wrong choice ($\frac{3}{4}$ OF 20). They should be encouraged to keep the problem in mind, and discussion with peers will help to clarify that their answer is not reasonable in terms of the problem.

Learners should be allowed to draw their solutions, and should in fact be encouraged to do so even if they used a more formal mathematical method.

The structure of the problem is actually division by a fraction ($20 \div \frac{3}{4}$). The teacher may or may not wish to point this out depending on the nature of the discussion which follows. If some learners have solved the problems by working out that there are 80 quarters, and thus $\frac{80}{3}$ groups of three-quarters, the teacher may wish to point out the resemblance to the 'invert-and-multiply' rule which some learners may remember. However, learners should not be expected to memorise this method if they do not understand it.

Some learners may find out by trial and error that 20 x $\frac{4}{3}$ gives them a reasonable answer, but not be able to explain why this works. They should be encouraged to draw the solution. Discrepancies in the remainders should be discussed: 20 x $\frac{4}{3}$ gives $\frac{80}{3}$ =26 $\frac{2}{3}$ (TARTS) which is different to the result of more diagrammatic solutions which give 26 TARTS and $\frac{2}{4}$ or $\frac{1}{2}$ APPLE. Drawings should help to clarify this discrepancy.

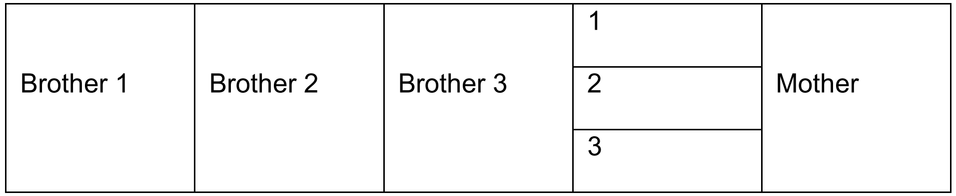
The Log of Wood

Three brothers buy a log of stinkwood. Their mother says that she will take over a fifth of the log. The brothers share the remaining wood equally. What fraction of the *original* log does each brother get?

Teacher Notes:

Learners should not have trouble solving this problem with the help of a drawing, but may have trouble *naming* each brother's share.

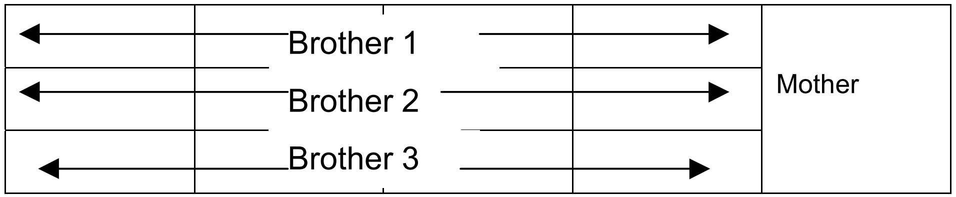
Learners may solve this in various ways, for example:



Each brother gets a big piece and a small piece: $\frac{1}{5} + \frac{1}{15}$

Each big piece is the same as three small pieces so each brother gets $\frac{3}{15} + \frac{1}{15} = \frac{4}{15}$.

Other learners may solve the problem as follows:



Each brother gets a row of small pieces: $\frac{4}{15}$.

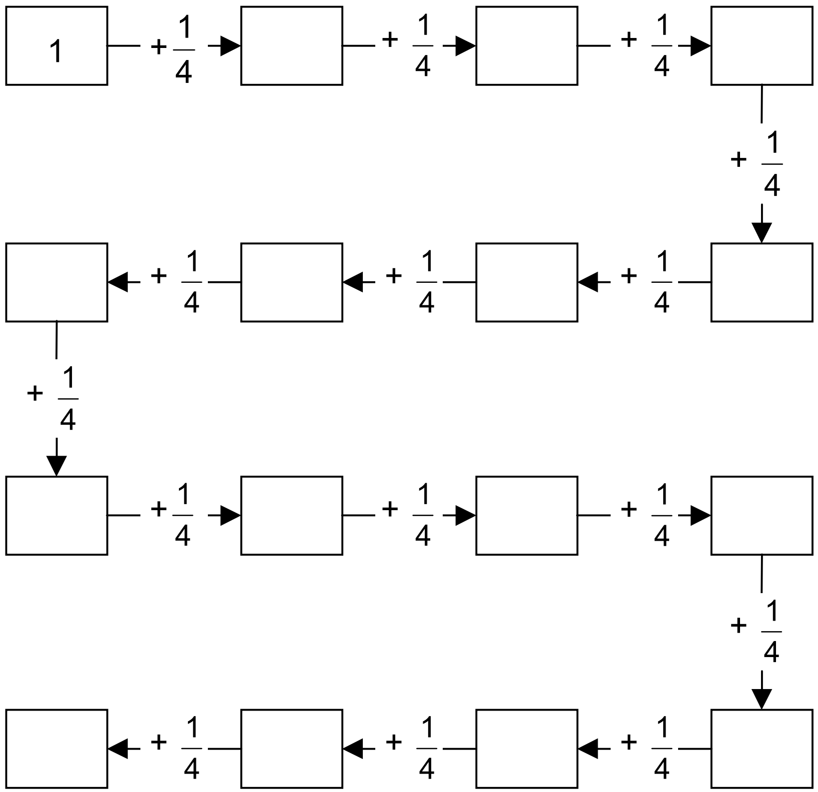
In both cases, the teacher can ask "How do we know that the small piece is a fifteenth?" or "Why fifteenths?".

Abstractly, this problem can be represented as $\frac{1}{3}\times\frac{4}{5}$ or $\frac{4}{5}\div 3=\frac{4}{5}\times\frac{1}{3}$. The diagrammatic solution illustrates how the abstract calculation actually works. Learners must therefore be encouraged to verbalise and share their solutions.

In the process of observing learners solve this problem, we have come to question our traditional method of *first* teaching the learners how to multiply and ***then*** giving them such problems. In our experience they can reflect on the meaning of multiplication and division in the process of solving this problem.

Snake

Complete the following diagram:



Sharing With and Without the Calculator

1. Share 21 sausages equally among 10 friends. Draw your answer.
   Now do this problem on your calculator. What do you think the answer on the calculator means? Explain.

2. Share 21 sausages equally among 5 friends. Draw your answer.
   Now do this problem on your calculator. What do you think the answer on the calculator means? Explain.

3. Share 21 sausages equally among 2 friends. Draw your answer.
   What answer do you think your calculator will give? Why?
   Now do it on the calculator. Were you correct?
   What does the answer on the calculator mean?

For the next four problems, first do the problem yourself, then say what answer you think the calculator will give, then do it on the calculator.

4. Share 15 chocolates equally among 10 friends.
5. Share 17 chocolates equally among 10 friends.
6. Share 18 chocolates equally among 5 friends.
7. Share 17 chocolates equally among 2 friends.

Teacher Notes:

The purpose of this task is to enable learners to assign meaning to decimals as an alternative notation for fractions. It is very important that the learners *experience* the calculator answers as simply a different notation/expression for their own common fraction answers.

The learners **should not** be told the 'logic' of the decimal notation (i.e. that the digit after the comma means tenths etc.) before they have done this activity.

Understanding of equivalent fractions is necessary for learners to find, for example, the decimal equivalent of $2\frac{2}{5}$. They should not, however, be expected to simply remember that 0.4 actually means 4 tenths – to start off with it is sufficient that they merely regard '0.4' as an alternative notation of $\frac{2}{5}$. As they progress through the activity and discuss their answers, they should make more sense of the decimal notation.

The Counting Machine

The calculator can be used as a counting machine. For example, here are two ways in which calculators can be programmed to count in 3's:

- Press $\boxed{3}$ $\boxed{+}$ $\boxed{=}$. If you keep on pressing $\boxed{=}$, the calculator will go on counting in 3's. However, if you press any of the operation functions (+; –; ×; ÷) or clear the screen, you have to start the process from the beginning again. You can press any number (without clearing the screen) and the calculator will count in 3's from that number onwards. For example: Press $\boxed{3}$ $\boxed{+}$ $\boxed{=}$. Now press $\boxed{4}$ $\boxed{1}$ and $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ …
  Your calculator should give 44; 47; 50; 53; …
- Press $\boxed{3}$ $\boxed{+}$ $\boxed{+}$ $\boxed{=}$ and follow the same procedures as above.

Now try the following:

1. Programme your calculator to count in 0,1's. Press the $\boxed{=}$ key several times and count aloud with the calculator. Count up to 2,5.
2. Programme your calculator to count in 0,01's. Now enter 0,9 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key and count aloud with the calculator. Count up to 1,2.
3. Programme your calculator to count in 0,1's. Now enter 111,11111 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key. What do you notice?
4. Programme your calculator to count in 0,01's. Now enter 111,11111 and press $\boxed{=}$ Keep on pressing the $\boxed{=}$ key. What do you notice?
5. Programme your calculator to count in 0,001's. Now enter 111,11111 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key. What do you notice?

Teacher Notes:

For this activity, learners need to know how to programme their calculators to count using a certain interval. Most calculators can be programmed to do this and can thus be turned into a "counting machine". Different calculators have different procedures, so learners should play with their own calculators to find out if they can be programmed and if so, how this can be done.

The two methods given for programming a calculator to count in 3's can be replaced by an adequate teacher explanation/whole-class discussion, before learners attempt the activities.

Sequencing

1. Complete the following sequences. Add the first number repeatedly. Check your answers with a calculator after each sequence. If you find a mistake, write down why you think it occurred.

❶ 0,2 ; 0,4 ; 0,6 ; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______

❷ 0,3 ; 0,6 ; 0,9 ; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______

❸ 0,4 ; 0,8 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______

❹ 0,5 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______

❺ 0,6 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______

2. More sequences....complete as before.

❶ 0,25 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______

❷ 0,05 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______

❸ 0,15 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______

❹ 0,09 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______

❺ 0,125 ; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______; ______; ______; ______; ______; ______; ______; ______; ______;
______; ______

3. Start with the given number and do the operation in brackets at least 10 times.

E.g. 0,8 (+ 0,2) → 1,0 + 0,2 → 1,2 + 0,2 → 1,4 + 0,2 → ….

❶ 6,4 (+ 0,3) → 6,7 →

❷ 4,42 (+ 0,1) →

❸ 8,4 (– 0,3) →

❹ 0,3 (+ 0,4) →

❺ 1,37 (– 0,1) →

❻ 11,6 (– 0,4) →

❼ 12,67 (– 0,9) →

❽ 25,6 (halve) → 12,8 →

4. Again, start with the given number and do the operation in brackets at least 10 times.

   E.g. 0,82 (+ 0,02) → 0,84 + 0,02 → 0,86 + 0,02 → 0,88 + 0,02 → ….

❶	6,43 (+ 0,03) → 6,46 →
❷	4,42 (+ 0,01) →
❸	8,44 (– 0,03) →
❹	0,3 (+ 0,15) →
❺	1,37 (– 0,04) →
❻	11,6 (– 0,03) →
❼	2,67 (– 0,09) →
❽	112,64 (halve) → 56,32 →

Teacher Notes:

This activity should first be done by hand and checked **afterwards,** with a calculator. Again the calculator can be programmed to do this (see 'Counting Machine'), giving immediate feedback to the learner, for example in the case of the common misconception that the decimal point is merely a point between whole numbers: 2,8 + 0,2 = 2,10.

By checking their answers with the calculator, learners might discover their own misconceptions and try to resolve them. The teacher should discourage them from simply copying the calculator answers. They should not be allowed do all the sequences before checking on the calculator, but should be encouraged to discuss each sequence after checking it on the calculator.

The teacher should ensure that learners do not use the '=' sign incorrectly. The arrow sign is acceptable (and preferable) in Questions 3 and 4. The teacher should also help the learners to verbalise decimals correctly, e.g. "six comma four three' not 'six comma forty-three'. In the case of 'nought point ten', the significance of the zero at the end should be discussed and learners should conclude that this the same as nought point one and therefore smaller than nought point eight.

Decimal Invaders

<u>Procedures to play the game</u>:

43,598

1. Two players need one calculator
2. Player 1 enters any decimal number e.g. 43,598. This number must be 'shot down' (replaced by 0 by subtracting).
3. Players take turns to 'shoot down' a digit. (One at a time.)
4. The player that ends with 0 wins.
5. If a player changes the number on the screen but does not shoot down a digit, the other player gets two turns.

<u>Example</u>:

	Press	Number on screen
Player 1:	43.598	43.598
Player 2:	$\boxed{-}$ 0.5 $\boxed{=}$	43.098 - The '5' has been shot down.
Player 1:	$\boxed{-}$ 40 $\boxed{=}$.	3.098 - The '4' has been shot down.

Repeat this with different numbers!

Teacher Notes: Decimal Invaders

This activity can also be used as a diagnostic activity to see which learners still need help and which learners have mastered decimal place value. Mistakes provide valuable learning opportunities, and learners should be given enough time to resolve these.

Paper

How thick do you think one sheet of paper is? Can you measure it with your ruler?

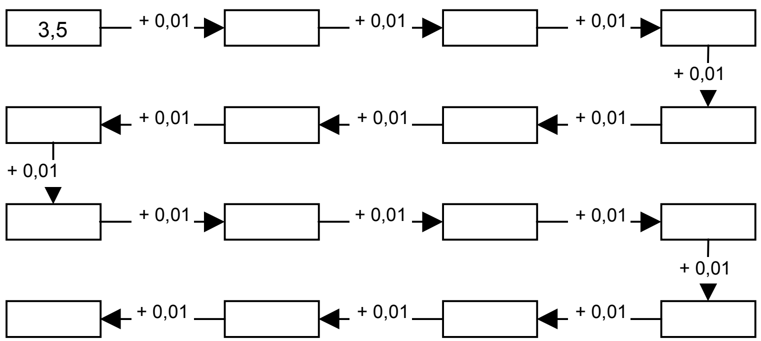
Dumisani has a bright idea. He measures 100 sheets of paper. The stack is 14 mm thick.

1. Calculate how thick each sheet of paper is.

2. How thick will a document of 7 pages be?

3. If 245 copies of this document are printed and stacked on top of one another, how high will the stack be?

4. Complete the diagram:



Teacher Notes:

This task mainly concerns hundredths. This concept (along with tenths, thousandths and later ten thousandths, etc.) is needed for a stable number concept. The learners must be given time to make sense of this on their own.

In question (a) the learners might divide 14 into 100 equal parts. The answer $\frac{14}{100}$ is quite acceptable. Learners should not be forced to write the answer as 0,14 or to simplify the fraction. However, discussion between peers should help to develop this concept.

Questions 2 and 3 present practical situations in which a whole number is multiplied by a fraction.

Question 3 can be given as homework if the class has discussed that $\frac{1}{100}$ can be written as 0,01.

Marking Homework

The following worksheet was given to Zanele for homework. Mark the work, correcting all the mistakes.

Decimals: Name: Zanele

1. Write 0,2 as a common fraction: $\frac{1}{2}$
2. Write 3,5 as a common fraction: $\frac{3}{5}$
3. 3,6 + 0,3 = 3,9
4. 4,8 + 4,3 = 8,11
5. 0,7 - 0,1 = 0,6
6. 0,27 - 0,1 = 0,26

Write down the next three terms in each sequence:

7. 0,2 ; 0,4 ; 0,6 ; 0,8 ; 0,10 ; 0,12 (Adding 0,2's)
8. 1,2 ; 0,9 ; 0,6 ; 0,3 ; 0 (Subtracting 0,3's)
9. 0,34 ; 0,36 ; 0,38 ; 0,40 ; 0,42 (Adding 0,02's)
10. 0,5 ; 0,10 ; 0,15 ; 0,20 (Adding 0,05's)
11. 0,25 ; 0,50 ; 0,100 0,200 (Doubling)
12. 0,8 ; 0,4 ; 0,2 ; 0,1 ; 0,$\frac{1}{2}$; 0,$\frac{1}{4}$ (Halving)

Teacher Notes:

This is a very good activity for the learners to challenge their own beliefs about decimal fractions. It can elicit a lot of discussion and the teacher should allow the learners to discuss the issues thoroughly. Reflection on their own concepts while learners are doing this can lead to a more stable concept of decimal fractions and make them more aware of the 'potholes' which lead to common mistakes.