# **GEOMETRY, MODULE 1: SIMILARITY**

#### LIST OF ACTIVITIES:

The following three activities are in the Sec 01a file:

#### Visual Level:

Communication Under the Magnifying Glass Vusi's Photos

The activities below are in this Sec 01b file: Cycles Rectangles 1 Traffic signs Triangles1 Triangles 2 Polygons

#### Analysis Level:

**Drawing Houses** Lucas's Similar Figures **Bingo's Figures** Mark's Enlargement Mark's Reduction Hexagons Well-scaled Fishes Parts Of The Elephant **Scaled Pictures** Scaled Figures How To Shrink It Horses **Rectangles 2** Wayne's Triangles Renata's Arrow Scaling Polygons Thabo and Tembi's Triangles Can You See the Triangles? Terry's Problem **Problems At The Press** Mike's Triangles

#### Project / Extension:

Our Flag: the Right shape 1 Our Flag: the Right shape 2 Slicing Off Similar Triangles

#### CYCLES

Which of the following figures are similar to figure A? Explain.



# *Teacher Notes: Cycles; Rectangles 1; Traffic Signs; Triangle 1;Triangles 2; Polygons*

These activities provide the learners with some more experience in providing a visual justification for why the figures are similar or not. The learners often provide many different descriptions of the specific transformation undergone, for example, in the activity "Cycles":

Figure A was described as "stretched in length "; "compressed from the sides" Figure B was described as "expanded"; "stretched by width"; "enlarged and squashed from the top".

The learners find it more difficult however to describe how the figures that are similar have been transformed, except for the congruent shape, for example:

Figure D was described as "the same but enlarged in size"; "The whole circle expanded in all directions"

Figure E was described as "shrunk"; "shrunk in all directions"

The responses of the learners suggest that most learners do not intuitively focus on the relationships between the corresponding sides of the figures or even the nature of the angles in the figures.

In the next set of activities we provide the learners with experiences in which these relationships become the focus.

Note: In the activities "Triangles 1" and "Triangles 2" the learners can be encouraged to point out which properties of the triangles change as its shape changes. Most learners will recognise the changes in the angles. At this point they do not however see equiangularity as a sufficient condition for similarity in triangles. According to the van Hiele theory this will be at the deductive level of reasoning and since the pupils are still engaged in visual level activities at this stage we need not enter into this kind of discussion now. These activities can be revisited again once the learners have had experiences with the van Hiele analysis level activities.

In the activity "Polygons" the learners will recognise that the figures F, G and H are not similar as the angles are not congruent. If the learners feel the need to check whether the sizes of the acute angles in the figures are equal to that in figure A, they can make a traced copy of the angle and see whether it can be superimposed on the other angles. We have specifically chosen figures in which all the angles are congruent but the corresponding sides are not proportional (Figures C and J). so that the learners realise that not all polygons that are equiangular are necessarily similar. Note, since most learners do not intuitively focus on the relationships between the corresponding sides encourage them to provide alternate visual explanations to show why the figure are not similar. For example, Figures A, C and J are equiangular but it is visually clear that the lengths of the arms of the acute angle in the different figures are not in the same in relation to each other. In figure J the horizontal arm of the acute angle is much shorter than in the other two figures that are created if we complete the figures to make a rectangle (please turn the page):







#### **RECTANGLES 1**

Which of the following figures are similar to figure A? Explain.



### **TRAFFIC SIGNS**

Which of the following pairs of traffic signs are similar? Explain *why* you say that they are similar or not.







Which of the following triangles are similar? Explain!

#### **TRIANGLES 2**

Which of the triangles are similar to the shaded triangle? Explain!



#### POLYGONS

Which of the following figures are similar to figure A? Explain.



#### **DRAWING HOUSES**

Draw three similar houses.

The three houses that you draw must have different sizes:

- (a) one house must be the same size as the house drawn below
- (b) one house must be smaller
- (c) one house must be larger



What can you deduce about the angles and the sides of similar figures?

#### Teacher Notes: Drawing Houses; Lucas's Figures; Bingo's Figures

In these activities we provide the learners with experiences in drawing figures so that they may focus explicitly on the relationships between the corresponding angles and sides in figures that are similar.

For example, in the "Drawing Houses" activity the learners had difficulty in drawing the roof of the house and this made them focus explicitly on the angle of the roof that had to be the same for the houses to be similar. In the whole-class discussion it needs to be pointed out that all the corresponding angles must be congruent. Ask the learners to mark or identify the corresponding angles.

In the activity "Bingo's Figures", the learners are asked to draw a figure three times the size of the original figure so that they focus how each of the sides is created. Ask the learners to examine the angles in the new figure and to explain whether the corresponding angles are congruent. The use of the square grid allows the learners to explain why the angles are equal without the need to measure the angles directly, for example: the angles can be explained to be equal in terms of the arms of the angles, a horizontal and a diagonal of a small square.



We recommend these drawing experiences so that idea of the congruent angles and the proportionality of the corresponding sides in similar figures are re-enforced. Changing the orientation of the original figure provides the learners with challenging drawing experiences as well as showing that orientation is not a necessary condition for two figures to be similar.

The proportionality concept is a difficult concept for the learners as it connected with other concepts such as fractions and ratio. The formal introduction to the notion of the proportionality of the corresponding sides can be suspended in the discussion for now. The focus can simply be on the procedures to be carried out to make the figures similar. In the discussion the learners need to say that in creating the new figure **all the sides** were, for example, increased by three times that of the original lengths. In Hart's (1981) research it was shown that learners tend to use **additive** operations to describe the changes in creating the new figures. We observed that some learners produced the correct drawings of similar figures but when asked the question: "By how much did you enlarge or reduce the new figure?", they often used additive operations to describe the changes of each of the new sides. These learners do not yet see the significance of the transformation factor as describing the common change. In the next set of activities we focus on this aspect in developing the notion of the **scale factor**.

Reference:

Hart, K. M. (1981). Children's Understanding of Mathematics: 11-16.

#### LUCAS'S SIMILAR FIGURES

Lucas draws the figure on the square grid below. He decides to draw two figures that are similar to this figure.

He only draws one of the line segments in each of the similar figures. The line segment A'B' in the one figure is drawn and the line segment B'C' in the other figure is drawn. Complete the figures so that they are all similar figures.



What can you deduce about the angles and the sides of similar figures?

# **BINGO'S FIGURES**

Bingo draws the figures A, B and C on the square grid below and shows them to his friend, Agatha.

Agatha says that only figures A and B are similar.

Do you agree with Agatha? Explain why you agree with her or not.

Now draw a similar figure that is three times the size of figure B.



#### MARK'S ENLARGEMENT

An enlargement refers to a similar figure that is bigger than the original figure.

Mark wants to draw an enlargement of the figure below.

He starts by drawing an enlargement of the side marked AB.

Complete the drawing.

By how much did he enlarge the figure?



#### Teacher Notes: Mark's Enlargement; Mark's Reduction; Hexagons

The aim of these activities is to develop the notion of scale factor. In this process we need to re-enforce the idea that a similarity transformation creates a constant change to the properties of the figure, namely:

- 1. All the angles remain the same
- 2. All the sides of the figure are either enlarged or reduced by multiplying by a constant called the scale factor.

In the whole-class discussions the learners can be asked to draw conclusions about the nature of scale factor for enlargements and reductions. (Enlargements have a scale factor greater than 1 and reductions have scale factors between 0 and 1)

In the activity "Hexagons" the special case of a congruent figure is considered. The domain of the numbers that are scale factors is extended to include the number 1. Make it explicit in the discussions that in this case the shape and size is invariant.

Note: The learners need to realise that a scale factor can be any positive number.

#### MARK'S REDUCTION

A reduction refers to a similar figure that is *smaller* than the original figure.

Mark wants to draw a reduction of the figure below.

He starts by drawing a reduction of the side marked AB.

Complete the drawing.

By how much Mark did he reduce the figure?



The *scale factor* is the term used to describe by how many times the lengths of the original figure has been **multiplied** to be as long as the corresponding lengths of the new figure.

In the drawing above we say that Mark's figure has been scaled by a factor of \_\_\_\_\_.

Note: The terms enlargements, reductions and scaled copies refer to figures that are *similar*.

#### **HEXAGONS**

Which of the figures B, C and D are scaled copies of figure A? Explain.

Determine the scale factor of the scaled copies.



#### WELL-SCALED FISHES



The original picture of fish

Decide which of the pictures are enlargements or reductions of the original picture. How did you decide?

By how much have the fishes been enlarged or reduced?



#### Teacher Notes: Well-scaled Fishes

The aim of this activity is to determine the scale factor in non-rectilinear similar figures. In the whole-class discussion we can again focus on the conditions for similarity. Visually the learners will recognise how the "curvature" changes in those figures that are not enlargements or reductions. We can suggest to the learners that a rectilinear frame be drawn around the figures to see the "angular" changes, as shown below:



In determining the scale factor of the enlargements and the reductions, emphasise that **corresponding** parts of the fish must be measured to see by how many times the original part must be multiplied to give the measure of the enlargement or the reduction.

#### PARTS OF THE ELEPHANT



Original drawing

New drawing

The new drawing of the elephant above is a reduction of the original drawing of the elephant.

Let us measure some of the corresponding parts of the elephant on the two drawings.

Complete the table by measuring other body parts of the elephant.

Body part	Original drawing	New drawing	Difference in size of the body parts	How many times as small is the body part of the new drawing?
Height				
Length of the ear				
Length of tusk				
Length of trunk				

What patterns do you find in the measurements? Discuss!

By what factor has the new drawing been scaled?

#### SCALED PICTURES

What scale factor will transform the picture on the left into its scaled copy on the right?



#### **Teacher Notes: Scaled Pictures**

In this activity the learners are given further experiences in determining the scale factor of different kinds of objects.

Remind the learner that a scaled copy is a similar figure of the original figure. In order to find the scale factor, corresponding parts in the figure need to be measured first.

Revisit the definition of the scale factor in the previous activities ("Mark's Reductions").

In the case of the birds the learners will recognise that they are congruent and hence the scale factor is one.

Other aspects that can be raised in the whole-class discussion:

- Ask the learners what the scale factor will be if the picture on the right is the original one.
- What is the relationship between these numbers?
- What kinds of numbers can scale factors be?
- A figure is scaled by the following values: 4; 1;0,33; 1,75;  $\frac{3}{4}$ . Will the new figure be

larger or smaller? The same size?

#### SCALED FIGURES

The pairs of figures below are representations (not the actual or accurate drawings) of *similar* figures. Look carefully at the information provided in these representations.

- 1. In each case, determine by how much the figure on the left was scaled to produce the figure on the right.
- 2. What are the lengths of the sides indicated by question marks?



#### **Teacher Notes: Scaled Figures**

In this activity the learners are not given accurate drawings of the figures so that they may not rely on a visual justification. Reasoning about the figures is now only on the analysis level of Van Hiele thinking. Hence the learners need to analyse the information given and to proceed with logical deductions, for example:

Given the fact that the figures are similar we can deduce that the corresponding angles are equal and that all the corresponding lengths are enlarged (in these cases) by the same factor.

In the whole-class discussion the learners can be asked to identify the corresponding angles in the figures also.

The formula for the scale factor can be given as:

scale factor =  $\frac{\text{length of the new figure}}{\text{length of the original figure}}$ 

Learners need to be given many more activities like this in which they identify the corresponding angles and lengths in similar figures. By getting the learners to write down the different ratios for the corresponding lengths we can eventually introduce the terminology of the corresponding lengths being proportional.

# HOW TO SHRINK IT



Amanda, Anwar and Andile love Alwyn's house, but find it a little to large for their liking. In other words, they want to shrink the house down to a smaller size while keeping it exactly the same shape.

After a long discussion, each came up with a strategy for drawing the smaller house.

Amanda's Way: Keep all the angles as they are and subtract 5 cm from each side.

Anwar's Way: Keep all the lengths as they are and divide all the angles by 2.

Andile's Way: Keep all the angles as they are and divide all the lengths by 2.

Try to shrink the house by using each of the methods above. Show what result each method produces. Explain why the method does or does not work.

#### HORSES

The pictures below shows two horses labelled with the letters A through H:



Here are the distances between some of the letters:

AB = 3 cm CD = 2 cm EF = 1,8 cm GH = 1,2 cm

How can you use these measurements to *convince* someone that the two pictures are well scaled copies of each other?

Teacher Notes: Horses; Rectangles 2; Wayne's Triangles

The aim of these activities is to consolidate the idea that in a well scaled copy (similar figures), the ratios of the corresponding parts are the same (the corresponding parts are proportional)

#### **RECTANGLES 2**

Rectangle	Length (cm)	Width (cm)
A	4	1
В	3	2
С	10	5
D	4	6
E	5	3
F	16	4
G	8	4
н	4,5	3

Here are the length and width measurements of eight rectangles:

Match the rectangles that are similar. How did you decide?



#### WAYNE'S TRIANGLES

Wayne has two triangles with different measurements.

The sides of one triangle are 4, 6 and 8 and the sides of the other are 9, 6 and 12.

He says that the triangles are not similar, because

$$\frac{4}{9} = 0,44$$
  
 $\frac{6}{6} = 1$   
 $\frac{8}{12} = 0,66$ 



Do you agree with Wayne? Explain!

#### Teacher Notes: Constructing Enlargements

In the previous activities the learners explored the characteristics of similar (scaled) figures.

By getting the learners to draw enlargements, they had the opportunity to reflect on the properties of similar figures. In the following activities the learners will explore another technique of drawing enlargements and reductions using what is called a "point-by-point" process.

Before starting these activities get the learners to think about some of the most common occurrences of scaled pictures. For example, the shadows you cast when walking in front of a bright flashlight, the images you see when you watch a slide show and the enlargements made with a photocopy machine. And enlargements of photographs.

Whole-class discussion:

#### Shadows

Use a bright flashlight, for example, to make shadows. Find out how to make the shadow the shape as the original, and how to make the shadow distorted.

Ask the pupils to make a brief set of rules that describes how to affect the shadows you make.

Describe and make a sketch of the following:

Where, in relation to the light source, would you put a  $7\frac{1}{2}$  cm  $\times$  10 cm piece of paper to

cast a 15 cm  $\times$  20 cm shadow on a wall?

#### • On how a projector works

A projector uses a very bright light bulb, plus a series of lenses to make all the light coming from the projector act like it is coming from one point:



Ask the pupils to imagine some tiny picture on the frame of the film getting enlarged by this process, and appearing on the screen.

Ask the pupils what they would do to make the image on the screen smaller or larger.

What might make the image distorted?

Reflection: Will we get enlargements if the rays are parallel?

Reflection:

To scale a figure, a projector sends a beam of light through it and catches those beams on a parallel surface. This is a "point-by-point" process. A two dimensional model will look like this:



# **RENATA'S ARROW**

Renata draws the arrow below. She wants to draw a similar arrow but she wants it *reduced in size*.

Renata starts the drawing of her similar arrow by drawing a set of lines that meet at a common point.

Complete the drawing and explain how and why Renata's method works.



#### Teacher Notes: Renata's Arrow

In this activity the pupil will use the point-by-point process to draw a reduction of a nonrectilinear figure. In the discussion point out to the learners that the common point represents the source of light and that the rays coming out of it represents the beams of light. The learners need to realise that one cannot possibly consider every point in the figure, as there are infinitely many points. The more points that are chosen however the more accurate the reduction will be. The learners can draw in a few more rays to obtain a more accurate reduction.

# **Scaling Polygons**

Draw your own polygon like ABCDE below. Pick a centre of enlargement (below it is labelled F). Draw rays from that point (F) through every vertex of the polygon:



In order to enlarge polygon ABCDE by a scale factor of  $\frac{1}{2}$ :

- 1. Find the midpoints of the segments FA, FB, FC, FD and FE.
- 2. Connect the midpoints to form a new polygon.
- 3. Is our new polygon scaled by a  $\frac{1}{2}$ ? How can you tell?
- 4. Does this method work if point F is inside the polygon?
- 5. Does it work if point F is on ABCDE itself?
- 6. What about placing F at a vertex of the polygon?
- 7. Now enlarge a new figure by any scale factor you choose.

#### Teacher Notes: Scaling Polygons

The aim of this activity is to give the learners further experiences in scaling rectilinear figures using the "point-by-point" process.

Note: In this activity we refer to the common point as the centre of enlargement. We regard a reduction as an enlargement with a scale factor between 0 and 1.

Ask the learners to look for something that all the scaled polygons had in common with the original one. The observation that the polygons are orientated in the same way and their sides are all parallel suggests a quicker way of enlarging or reducing the polygon. This method is known as the **parallel method**:

1. Draw a polygon, for example , a quadrilateral:



2. Choose a point as the centre of enlargement and draw rays from that point through the vertices:



3. Now draw line segments parallel to the original quadrilateral. Once the learners have used this technique a few times they need to reflect on why this technique results in a scaled copy. The following activities develop an intuitive understanding of why triangles with the same angles are similar in order to help the learners understand why this technique of scaling works.

# Thabo and Tembi's Triangles

Thabo and Tembi draw a triangle with angles  $40^{\circ}$ ;  $60^{\circ}$  and  $80^{\circ}$ .

After having drawn the triangle they show each other their drawings. Thabo remarks: *"It appears as if the angles of a triangle determines its shape."* 

Draw your own triangle with the angles  $40^{\circ}$ ;  $60^{\circ}$  and  $80^{\circ}$ . Compare your triangle with other members in your class.

Discuss Thabo's remark.

#### Teacher Notes: Thabo and Tembi's Triangles

In this activity the learners will explore whether the shape of a triangle is determined by its angles. The learners can experiment with another set of angles. The learners should see that there is a kind of "rigidity" if the angles of a triangle are fixed. In other words, the learners will be able to conclude from their experimentation that if the angles of one triangle have the same size as the angles of another triangle, then the triangles are similar.

The question, which can be raised, is whether this rigidity is unique to the triangles. The learners can now revisit the activity "Polygons" in which the hexagons that had congruent angles did not necessarily have the same shape.

The learners can also investigate whether the length of the sides of a triangle fixes its shape. Does this rigidity exist in other polygons?

The learners will be able to conclude from their experimentation that **if the sides of one triangle have the same lengths as the sides of another triangle, then the triangles are congruent**.

It can be pointed out that this is a special case in which the ratios of the corresponding sides are equal to 1. It can also be pointed out the learners that the distinction between triangles and other polygons, namely those triangles are rigid and polygons in general not makes the following statements true:

- 1. If two triangles have their corresponding angles equal, then the triangles are similar.
- 2. If two triangles have their corresponding sides' proportional, then the triangles are similar.

# Can You See the Triangles?

1. Look at the diagram of the scaled polygons below:



- 2. Can you see four pairs of nested triangles (a small triangle inside a bigger one)?
- 3. Below we have highlighted one pair in bold lines:



4. Can you make a conjecture about these triangles? Can you *prove* your conjecture?

#### Teacher Notes: Can You See the Triangles?

The aim of this activity is for the learners to realise that the nested triangles are similar triangles because they are **equiangular**. The line segment inside the triangle is parallel to one of the sides of the triangle and the learners can use the properties of the parallel lines to make deductions about the angles of the triangles.

The learners may conjecture that if a segment is parallel to one of the sides of a triangle, then it splits the other two sides proportionally.

Note this is only a conjecture and we may point out to the learners that this conjecture can be proved true but that it requires an argument that involves the area of the triangles. This proof can be revisited once the learners have explored the areas of figures.

Note: Since each of the pair of nested triangles shares a common side, it is means that each of the sides of the original figure has been scaled by the same factor.

#### **Terry's Problem**

Terry enlarges a  $\triangle ABC$  by a factor of 2.

He follows this procedure:

- 1. He measures the distance DA and finds it to be 1. So he moves out along ray DA until he find a point A' so that AA' = 2 (twice as long as DA).
- 2. He measures the distance DB and finds it to be 2. So he moves out along ray DB until he find a point B' so that BB' = 4 (twice as long as DB).
- 3. He measures the distance DC and finds it to be 1,5. So he moves out along ray DC until he find a point C' so that CC' = 3 (twice as long as DC).



4. He draws  $\Delta A'B'C'$ .

To Terry's surprise,  $\Delta A'B'C'$  has sides that are proportional to  $\Delta ABC$  but are not twice as long.

If they are not twice as long, how much as long are they? Can you *explain* what is going on here?

#### Teacher Notes: Terry's Problem; Problems At The Press.

In these activities the learners need to apply their understanding of the method of scaling discussed in the previous activities.

# **Problems At The Press**

Oh no! There's been a problem at the printing press!

The picture below was supposed to show  $\triangle ABC$  and its enlarged companion,  $\triangle A'B'C'$ , but an ink pot spilled onto the page.

Can you salvage the disaster by calculating by how much  $\triangle ABC$  has been scaled?







#### Teacher Notes: Mike's Triangles

In this activity the learners need to view Mike's triangle as a general triangle and not focus on the drawing in the activity. Hence the sides that are to be extended can be considered to be x and y and the new lengths x + z and y + z.

The ratios of the corresponding sides are:

$$\frac{x+z}{x}$$
 and  $\frac{y+z}{y}$ 

For the original and the new triangle to be similar the ratios of the corresponding sides must be equal. and the ratios  $\frac{x+z}{x}$  and  $\frac{y+z}{y}$  can only be equal if x = y.



The shape of our flag, but not its size, is determined by law, as in this sketch:



Which of these flags do you think have the right shape? Why do you say that? Does everyone in your group agree? How can you be sure?





1. Draw a nice big South African flag in your workbook.

Convince a classmate that it is the right shape.

2. Jane wants to draw a flag so that the Y-part is 2 cm wide.

Calculate the dimensions of all the other parts of the flag.



#### SLICING OFF SIMILAR TRIANGLES

In this activity you are going to investigate how to make small triangles inside a larger one so that the small ones are similar to the large one.

Find as many lines that can be drawn so that the triangle, which is cut off, is similar to the original one.



Write a report on your investigation.

Describe in words the lines that produce smaller triangles that are similar to the original one.

#### Teacher Notes: Slicing off similar triangles

The aim of this activity is for the learners to see that lines that are parallel to a side of a triangle create a similar triangle. Ask the learners to explain why a line parallel to one side of a triangle makes a similar triangle.

It is also important for the learners to realise that lines parallel to a side of the triangle are not the only lines that create similar triangles. For example, a triangle that is formed by drawing a parallel line:



If the triangle fromed is flippd as shown below and then rotated it can be placed back inside the original triangle.



Note, the line creating this triangle is not parallel to any of the sides of the original triangle:

