MALATI rationale for teaching *Statistics*

Statistics constitutes a substantial part of the Primary and Secondary curriculum in Curriculum 2005, as well as the curricula in many other countries such as the United States, the Netherlands and the United Kingdom. Probability and Data Handling can be characterised as the "*systematic study of uncertainty"* (Shaughnessy et al., 1996). It is our belief that it is exactly this uncertainty which makes the study of statistics difficult but also important as it encourages the use of different (or broader?) kinds of reasoning and tools which are essential in mathematical modeling.

> *Statistics plays an important intermediate role between exactness of other mathematical studies and the equivocal nature of a world dependent largely upon individual opinion'*
>
> (NCTM, 1989)

This is a content area that has traditionally been neglected or omitted, so teachers and learners may be uncomfortable with the emphasis suddenly given to it in Curriculum 2005. The concepts, skills and underlying the philosophy described in this document are all compatible with those expressed in the Curriculum 2005 documentation. We are in agreement that learners of all ages should become aware of and appreciate the role that statistics plays in society, and also the scope of statistics.

> *"Living in an age of information makes it imperative for students to develop conceptual and practical tools to make sense of that information"*
>
> (Shaughnessy, Garfield & Greer, 1996)

This document describes the MALATI approach to the teaching and learning of Data Handling and Probability and the basic principles underlying the development of the teacher and learner materials on these topics.

Probability

Probability is regarded as a particularly difficult concept as, unlike most areas of school mathematics, it deals with uncertainty. The problems arising from this can be exacerbated by a classroom culture in which learners are accustomed to producing one, correct answer to a mathematical problem. Furthermore, as in any other area of mathematics, learners come to the study of the topic with intuitions and experiences. In the case of probability, these intuitions can conflict with the formal probability taught at school. Carpenter, Kepner, Lindquist and Reys (1981) note that some intuitions actually get stronger with age, in spite of instruction at school.

Research indicates that teaching probability by definitions and rules does not allow the development of a meaningful concept. Students who have been exposed to this method will attest to the fact that, although they have memorised formulae, they have little underlying conceptual understanding. Furthermore, it seems unlikely that such an understanding will develop in an environment in which learner intuitions are ignored. Hawkins and Kapadia (1984) have used the findings of the Assessment Performance Unit (APU) in England to note that learners can sometimes give the correct theoretical predictions, but tend to revert to their original "hunches" when the results of experiments do not confirm the prediction.

Theoretical Framework

Researchers have identified four different definitions of probability and the MALATI approach would be classified as being a combination of "a priori", "frequentist" and "subjectivist" approaches. These are defined as follows:

- *"A priori / theoretical / classical probability":* The probability is obtained by making an assumption of equal likelihood and the probability of an event is simply the ratio of the number of occurrences of an event to the total number of equally likely occurrences[1]. For example, the probability of obtaining an even number when throwing a dice is theoretically 3 out of 6 or $\frac{3}{6}$.

[1] Konold (1991) notes that the "a priori" definition is actually logically flawed in that the definition of probability is circular: Probability is defined in terms of equally-likely outcomes, yet "equally-likely" outcomes actually means "equally probable".

- "*Frequentist / empirical / experimental probability":* Probability is defined in terms of the limit to which the relative frequency of the occurrence of an event tends in an infinite, or near infinite, number of trials (Konold, 1991). For example, it is not possible to calculate theoretically the probability of a drawing pin landing with the pin facing up when tossed, but we can use an experiment to determine that the frequency of this event tends to 40%.
- *"Subjectivist" approach to probability:* Probability is defined as the measure of belief in the truth of a proposition. In this approach people might assign different values to the probability of an event, but these initial probabilities are revised on the basis of new information, and the probabilities of different people will converge on the frequentist's limit. This approach is reflected in MALATI's emphasis on ensuring that learners are given the opportunity to share their experiences and ideas with one another, and on discussion in which these ideas can be evaluated.

Materials Design

Using this theoretical framework, the MALATI Statistics Group has designed materials to introduce the concepts of chance and probability to learners in Grades 4 to 9. The activities are designed to encourage the development of an "a priori" / "frequentist" approach to probability. The contexts used are those which learners will encounter in everyday life (game shows, examples from the press) and learners encounter probabilities based on equal as well as unequal likelihoods at an early stage. Formal language, terminology and mathematical rules are only introduced after the relevant concept has been developed and on the basis of the understanding of this concept.

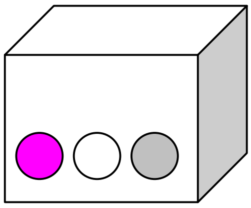
The MALATI activities and suggested teaching approach have been formulated to challenge those intuitions that conflict with formal probability and to address particular problems referred to in the literature and observed in MALATI project school classrooms. In general the suggested strategy in this regard is to provide a classroom culture in which ideas can be shared and discussed. In such an environment, learners will be able to reflect on and evaluate their ideas. Below, we use examples from our work in MALATI project schools to illustrate some of the learners' intuitions and some specific strategies that we have identified for the teacher.

Consider the following activity from the MALATI probability materials:

ZAMA ZAMA !

Mpande is a contestant in the Zama Zama Game Show.

She is told that a box contains three balls of the **same size**: a pink ball, a yellow ball and a blue ball. She cannot see the balls in the box.



In this game she must remove a ball from the box **and then put it back** before selecting another one.

1. What is the chance of Mpande removing the pink ball? And the blue ball?
   (a) Show your answers on the likelihood scale below.
   (b) Now write your answers as fractions.

Impossible   Very unlikely   Unlikely   50% chance   Likely   Very likely   Certain

- Terminology: Learners have problems distinguishing between the everyday and mathematical uses of terms. For example, the word "likely" is associated with "possible". So a learner might say that it is likely that Mpande will remove a pink ball in the Zama Zama game, as a "likely" event is interpreted as one that "could occur". A learner might, on the other hand say that there is a "50% chance" of removing the pink ball, as "50% chance" could indicate that something "might or might not happen". Hawkins and Kapadia (1984) suggest that, while this mismatch between everyday and technical interpretations arises in many areas of mathematics, the problems that arise are "particularly acute" in the area of probability. In trying to address this problem we encourage teachers using

MALATI activities to make the differences between the everyday use and mathematical use of the words explicit.

- Personal Preferences: Some learners base their decisions on, for example, their favourite colour. So a learner might indicate that a pink ball is most likely to be drawn randomly from the box in the Zama Zama game because “pink is my favourite colour”. Learners will probably have different preferences, so discussion should encourage them to reflect on this reasoning.
- The “Outcome Approach”: Learners using this approach to probability think that they are being asked whether an event (for example, removing a pink ball) will occur on the next trial, rather than quantifying **how likely** the event is. Konold (1991) indicates that in this approach, probability values are evaluated in terms of their proximity to the values of 100%, 50% and 0% - which are interpreted as “yes”, “I don’t know”, and “no” respectively. So if an event is likely, it means that it will happen. This response could also be interpreted as being related to problems with the language of probability, that is associating “likely” with “possible” as described above.
- Representativeness: In this case decisions are based on a current trend. Learners might predict that, as Mpande has removed five blue balls in a row, the next ball removed will be blue (“postive recency effect” – the trend continues). Others might suggest that this trend will not continue, and that the next ball removed must be yellow or pink (“gambler’s fallacy” / “positive recency effect”). The neglect of sample size is also classified in this category: A learner may suggest that the probability of drawing six pink balls from a collection of 50% pink and 50% blue balls is the same as drawing 60 pink balls in 100 draws.
- Availability: Learners often base their decisions on their past experiences and what comes to mind most recently. For example, a learner may claim that the pink ball will be selected in the Zama Zama game as “last night on Zama Zama the contestant removed only pink balls”. Another familiar example is that of the slice of bread spread with jam: a learner may claim it is “certain” that a slice of bread spread with jam on one side will land jam-side down, because “it always lands with the jam down”.
- The Conjunction Fallacy: In more complex probability problems, learners may regard the probability of two distinct events occurring simultaneously as greater

than the probability of the individual events occurring. For example, a learner may claim that there is less chance of being blonde, than of being female and blonde.

- Learners may also develop limiting constructions as a direct result of the way in which they have already been taught probability. For example, if learners have only been exposed to problems in which all probabilities can be calculated theoretically and in which events are equally likely, they may predict that this is true for all probabilities. In the MALATI materials we include events with equal and unequal likelihoods from the beginning.
- Often when learners are unconvinced by an argument they try an experiment to test the prediction and then base their response on the result of this very short experiment. So they might take a box with the three balls as in the Zama Zama game and remove a ball, say ten times. Then if the blue ball is removed eight out of ten times in this experiment, the answer is blue. When more than one group of learners has used this method to verify their answer, we find it useful to get the groups to compare the results of their experiments – owing to the size of trial, results are often very different, suggesting to learners that their method of testing might not be adequate.

Classroom Culture

Although conflicts can arise between learners' experiences and formal school probability, the moments of conflict that arise can be valuable in the learning process. Furthermore, Shaughnessy (1992:479) notes that some intuitions can, in fact, be useful when learning formal probability. We have, for example, found that placing the problem in the context of gambling can be useful as learners are hesitant to place a bet on the outcome of an event unless they are fairly sure of the possible result.

The classroom culture should allow discussion of learners' experiences so that learners have an opportunity to discuss these conflicts and to assess the validity of the claims of different class members. These basic principles are directly in line with the Critical Outcomes of Curriculum 2005, which include critical thinking, group work, effective communication, and problem solving.

Such an environment is likely to provide an opportunity for learners to adapt their intuitions where necessary, for as Konold (1991:152-154) has suggested, learners should be encouraged to evaluate their intuitions according to the following three criteria: Firstly, do my beliefs agree or fit with the beliefs of others? Secondly, are my beliefs internally consistent? And lastly, do my beliefs fit with empirical observations? Of course, such reflection requires the appropriate classroom culture, which once developed, could be valuable in all aspects of mathematics learning.

The Rational Number Concept

Both Green (in Shaughnessy, 1992) and Garfield and Ahlgren (1988) identify poor learner understanding of ratio as one of the main underlying causes of poor learner performance on school probability. Bramald (1994) notes that the choice of the traditional approach to probability means that learners are led too quickly to the manipulation of ratios, without providing an opportunity for reflection on reality. The MALATI Probability Materials requires, therefore, that learners begin the study of chance in a non-numerical context and, as mentioned earlier, are given the opportunity to express and discuss their own ideas. The use of fractions to express probabilities is only introduced at Grade 7 level. Support in the areas of ratio, common fractions, decimals and percentages should thus be provided where necessary from this grade onwards.

Probability and Data Handling

The Statistics Working Group that designed these materials has also designed materials for the introduction and development of the concepts associated with data handling. The issue of which concept should be studied first has not been resolved in the literature. For the initial trialling in 1998, the decision was made to trial the probability first, with a view to revising this sequence during the process. This decision was based on the following analysis of the content: The concepts of chance and probability are necessary for understanding random and representative samples, as well as the significance of the results of statistical tests. It is in the testing of hypotheses and the determining of confidence intervals, that probability and data handling come together, and learners who have a sound concept of probability will be better equipped to make judgements and decisions regarding statistical data. It is felt that learners should experience probability as useful in interpreting and evaluating statistics reported in the press, and not as a section of the syllabus which is covered by playing games and which has no relevance to the rest of mathematics or the real world.

However, reflection on learners' difficulties regarding probability, evident both in the literature and in our experiences in MALATI classrooms, has led us to conclude that the study of data handling should not be delayed until probability has been grasped. Difficulties experienced with the concept of probability should not stand in the way of learners studying important concepts in data handling. It is our opinion that the proposed data handling in the intermediate phase as well as much of the work done in the senior phase can be done *without* explicit reference to the links between probability and data handling. The teacher may, however, choose to consider covering systematic counting first, as this can be a useful tool in data handling and other areas of mathematics, as discussed below.

Systematic Counting

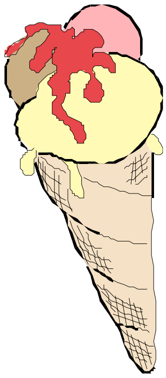
Our approach to probability begins in the lower grades with a module on 'systematic counting' (in the higher grades this content is included in other materials). The documentation for Curriculum 2005 refers to 'ways of counting' in Specific Outcome 6, and these kinds of problems are currently part of the syllabus for multiplication in the primary school. In the Netherlands, the 'systematic counting' approach has been found to provide a successful cognitive tool to help learners to solve probability problems by helping them to easily visualize the various outcomes, a skill that, according to the research on probability, is lacking in many people. Our material therefore introduces learners to tree diagrams, graphs and grids, and encourages them to reflect on which method is appropriate in a variety of situations. These methods provide learners with strategic knowledge that will facilitate the solution of problems involving relatively complicated concepts such as permutations, combinations and conditional probability.

Systematic thinking and systematic recording of results are also important skills in other areas of mathematics, and in general.

Below we provide examples of the MALATI approach to the use of systematic counting:

Initially learners are challenged to count the number of possible options or outcomes. This is an example from the MALATI materials:

Ice Cream Choices 1!



MENU

Cones

Plain Chocolate

Ice-Cream Flavours

Vanilla Strawberry Fudge

Toppings

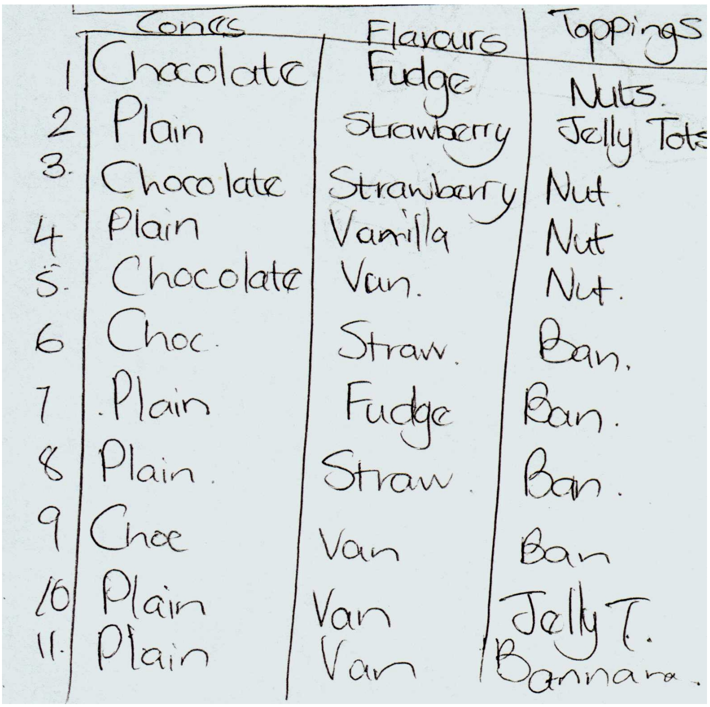
Nuts Jelly tots Banana

This is the menu in an ice-cream shop. The shop sells "combos" which consist of one cone, one scoop of ice-cream and one topping. For example, you can order a "combo" with a plain cone, strawberry-flavoured ice-cream and nuts on top.

What is another possible "combo" you can order?

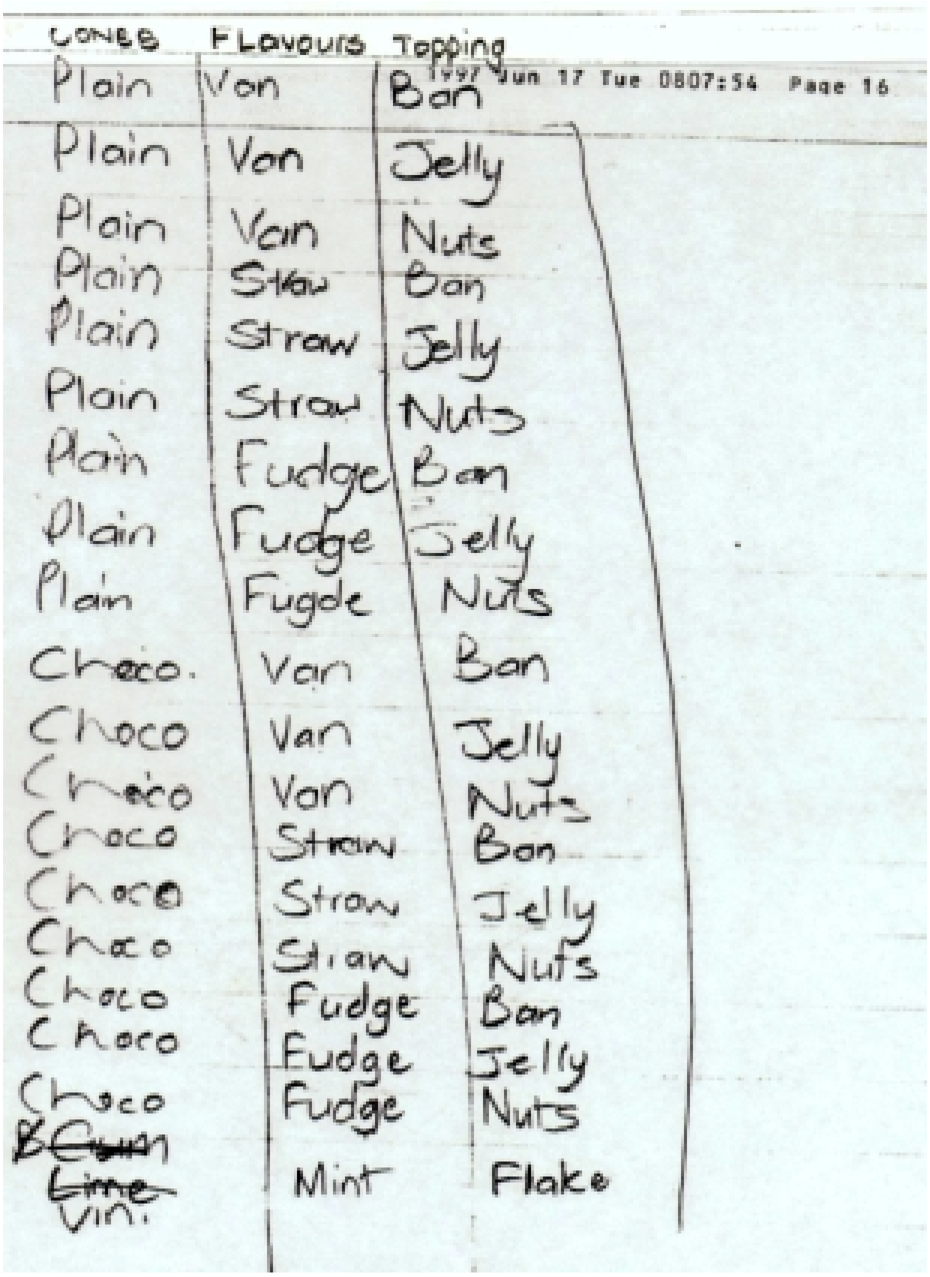
1. How many different "combos" can one order at this ice-cream shop? Show how you got your answer.

Initially most learners begin by listing the choices randomly:



	Cones	Flavours	Toppings
1	Chocolate	Fudge	Nuts.
2	Plain	Strawberry	Jelly Tots
3.	Chocolate	Strawberry	Nut.
4	Plain	Vanilla	Nut
5.	Chocolate	Van.	Nut.
6	Choc.	Straw.	Ban.
7	.Plain	Fudge	Ban.
8	Plain.	Straw.	Ban.
9	Choc	Van	Ban
10	Plain	Van	Jelly T.
11.	Plain	Van	Bannana.

But learners begin to reassess their strategy when they find they are having difficulty keeping track of which combinations have been included, or when they are challenged to explain how they know that all possible combinations have been included. This can lead to the use of a more systematic strategy:



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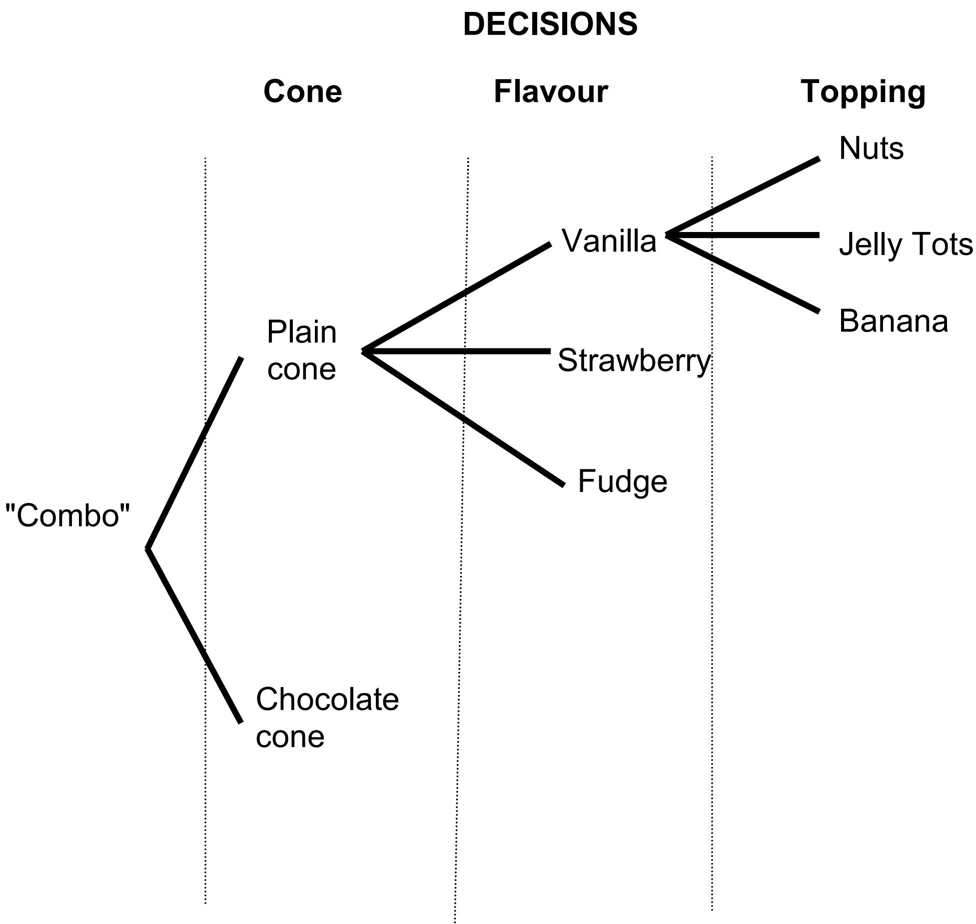
CONES	FLAVOURS	Topping
Plain	Van	Ban
Plain	Van	Jelly
Plain	Van	Nuts
Plain	Straw	Ban
Plain	Straw	Jelly
Plain	Straw	Nuts
Plain	Fudge	Ban
Plain	Fudge	Jelly
Plain	Fugde	Nuts
Choco.	Van	Ban
Choco	Van	Jelly
Choco	Van	Nuts
Choco	Straw	Ban
Choco	Straw	Jelly
Choco	Straw	Nuts
Choco	Fudge	Ban
Choco	Fudge	Jelly
Choco	Fudge	Nuts
~~B Gum~~ ~~Lime~~ Vin.	Mint	Flake

Once learners have seen the need for a systematic method of representation, we introduce the tree diagram and the grid as tools for systematic listing:



It is important that learners see the need for a systematic form of representation **before** the tree diagram and grid are introduced. Our experience in project schools suggests that these forms of representation remain meaningless if the *need* for systematic representation is not recognised.

We then use the tree diagram as a way of listing all the possible outcomes (the sample space) in order to calculate probabilities, for example, if we want to calculate the probability that a combo with a chocolate cone will be selected, we identify all the branches having a chocolate cone. 9 out of the 18 possible combos have a chocolate cone, so we say the probability is $\frac{1}{2}$.

MALATI probability and Curriculum 2005

Specific Outcome 6 of the Curriculum 2005 Learning Area 'Mathematical Literacy, Mathematics and Mathematical Sciences' (MLMMS) is defined as 'Use data from various contexts to make informed judgements'. This Specific Outcome includes two Assessment Criteria related to ways of counting, chance and probability:

Assessment Criterion 8: "Evidence of knowledge of ways of counting"
Assessment Criterion 9: "Understanding of the concept of probability"

In the Primary School Probability materials (Grades 4 to 7) we deal with systematic counting, the concepts of chance, and experimental and theoretical probabilities.
In the Secondary School materials (for Grades 8 and 9), the module Probability 1 deals explicitly with the concepts of chance and likelihood specified in Assessment Criterion 8. In Probability 2 we introduce systematic counting and systematic representations of sample space as tools to facilitate the calculation of probabilities. Predictions, experimental results and theoretical probabilities are covered in Probability modules 1, 2 and 3.

Structure of the MALATI probability modules

When deciding which activities should be done by learners in different grades, it is necessary to distinguish between those learners that have never studied the topic before, and those that will have studied the topic through each phase as proposed in Curriculum 2005. For this reason we have not specified which activities should be selected for different grades, but leave this to the discretion of the teacher. In order to assist the teacher in selecting activities according to the needs of individual learners, we provide diagnostic assessments at certain points in the materials. If the teacher finds that the necessary concepts are not developed, s/he must return to more basic activities as indicated.

Teacher notes are provided for each activity. Some assessment activities are included. Core activities which are important for all learners to complete are distinguished from those which can be used for consolidation or enrichment.

Data Handling

In the MALATI Data Handling materials, we aim to develop learners' ability to *make sense of information* and to *make decisions based on relevant information*, by exposing them to certain concepts as well as the necessary methodological process. In addition to the statistical concepts that we hope to develop, learners should also develop the skills listed below. Specific outcome 6 of the Learning Area 'Mathematical Literacy, Mathematics and Mathematical Sciences' (MLMMS) specifies similar skills. The relevant Assessment Criteria are given in brackets: (See Appendix 1 for a breakdown of the concepts and skills covered in the MALATI materials.)

- articulating a research question or problem **(AC1)**
- deciding what data is relevant for investigating this question or problem **(AC1)**
- collecting the necessary data (this includes sampling techniques) **(AC2)**
- organising the data **(AC3)**
- representing the data in a variety of ways **(AC5)**
- interpreting the data **(AC3, 4, 5 & 7)**
- summarising the data and identifying trends **(AC4)**
- manipulating the data **(AC4)**
- clearly communicating information and findings **(AC6)**
- critically analysing the data **(AC7)**.

It is essential that learners are able to *critically interpret and analyse* existing data with which they are presented, as well as their own data. Statistics are often used (and abused) in the media, and learners should be equipped with the necessary skills to recognise bias and to identify factors which influence the validity and reliability of media statements.

The MALATI philosophy and vision encompasses a classroom culture which facilitates the development of these skills. Learners learn by solving 'problems' (for which they do not immediately 'know' the solution) and by discussing and arguing with their peers. Our basic principles are thus directly in line with the Critical Outcomes of Curriculum 2005, which include critical thinking, group work, effective communication, effective use of technology, problem solving and the collection, analysis, organisation and critical evaluation of information**.**

Teachers are encouraged to find additional examples (for example, in the media) with which concepts and skills can be consolidated and further discussion facilitated.

Teaching data handling in this way provides the teacher with a challenge. Traditional approaches to data handling may have involved learners memorising the necessary concepts before being required to apply this knowledge to activities in context. The MALATI approach requires the teacher to fulfill a different role in the classroom: The teacher is discouraged from showing the learners how to solve a problem but should allow the learners to make their own decisions. The teacher introduces new terminology and conventions, facilitates discussion between the learners and moves discussion towards more formal mathematics. However, as the content may be unfamiliar to many teachers, it may be difficult for them to know *what* mathematics to highlight and consolidate, and *how* to do this. Our approach to teacher development, therefore, includes teacher notes, content-based workshops and classroom support.

The MALATI approach also provides special challenges when it comes to assessment. Traditional tests typically assess skills in isolation from a problem context. If such assessment is used, a mismatch will occur between the assessment and the learner outcomes that we aimed for. We have found it useful to use projects to assess learners' understanding of the underlying concepts and skills in a more open-ended way. Examples of possible projects are given in the primary and secondary materials. Teachers are encouraged to design their own assessments using newspaper articles or current school issues, for example fundraising efforts or water-saving drives.

Our materials have been designed with the following basic principles in mind:

- Contexts should be familiar to the learners. This does not mean that learners need to have first-hand experience of every context, but they must have an understanding of the contextual situation. In order to motivate learners, the contexts should also be interesting and relevant.

- Contexts should be varied but certain contexts should be extended for use in the development of several skills and concepts. Thus learners should not have to familiarise themselves with too many different contexts while simultaneously grappling with new content.
- The contexts in which data handling is studied provide the teacher with opportunities to make links to other Learning Areas, for example, Language, Literacy and Communication and Natural Sciences.
- Although there are concepts and tools with which we would like to equip the learners, there should also be opportunity for open-ended exploration. For example, learners can be required to complete mini research projects in which they can summarise and represent the data in any way they choose.
- Common misconceptions should be addressed by the material and challenged through discussion. The material thus encourages discussion about, for example, the size and representativeness of the sample.
- Available technology must be taken into account. In Curriculum 2005 the incorporation of technology into the different Learning Areas, including MLMMS, is encouraged. The use of technology as a Critical Outcome is also specified. One of the MALATI Secondary School modules incorporates the scientific (graphing) calculator TI-82 (see 'Using the TI82 for the teaching and learning of data handling' below). In addition, throughout all the activities, the terminology used is compatible with that used in available computer data handling and spreadsheet packages. The use of calculators to facilitate lengthy computations is encouraged throughout.

Using the TI-82 for the teaching and learning of data handling

The Statistics Working Group has produced two Data Handling modules for Grades 8 and 9, one which makes use of the TI-82 graphing calculator while the other does not. In both of these modules the study of Robben Island has been used a context. Here we discuss the use of the graphing calculator as a tool for teaching and learning data handling. In the discussion that follows we will be referring to these modules as the TI-82 module and the non-calculator module respectively.

Concepts such as regression and correlation are extremely powerful and useful in interpreting the data from many sources with which we are constantly bombarded. Many of the calculations associated with these concepts can, however, be complicated and tedious. This can be a particular problem when real data is used, as in the case of the two Robben Island modules.

The graphing calculator is a useful tool in the study of data handling as it does these calculations for the learner. The learner can thus be exposed to concepts such as regression at a lower level than would be the case if these calculators were not available.

Furthermore, when freed from focusing on performing calculations, the learner is able to focus on the actual statistical and graphing concepts involved. The MALATI TI-82 module has been designed in such a way that learners can use the calculator to perform the calculations, while engaging in other important mathematical processes such as estimating, predicting, interpreting graphs, and studying the effects of different statistical methods, for example, collapsing categories.

Other papers

Readers should also consult the following MALATI research papers, which form a backdrop for the design of the materials and the teaching approach:

Bennie, K. (1998). The "slippery" concept of probability: Reflections on possible teaching approaches. **Proceedings of the Fourth Annual Congress of the Association for Mathematics Education of South Africa** (pp. 53-67). Pietersburg: University of the North.

Bennie, K. and Newstead, K. (1999). Obstacles to implementing a new curriculum. In M.J. Smit & A.S. Jordaan (Eds.), **Proceedings of the National Subject Didactics Symposium** (pp. 150-157). Stellenbosch: University of Stellenbosch.

Appendix 1: MALATI Activities and Specific Outcome 6 of MLMMS

Assessment Criteria:	Primary school activities	Secondary school activities
1. Identification of situations for investigation	Questions for investigation:Car colours, Letters of the alphabet, Which hand do you use?, School tuckshop	Identifying trends: History, African Penguin 2 Working with real data: All activities
2. Collection of data	Collecting own data: Car colours, Letters of the alphabet, School tuckshop, How many people? Reflection on sampling and data collection procedures: Yuk or Yum Interpretation of ratings: Yuk or Yum	
3. Organisation of data	Organising own data into tables: Car colours, Letters of the alphabet, School tuckshop Tallies: Letters of the alphabet	Listing data: African Penguin, Weather, African Penguin 2 Collapsing categories: African Penguin Organising events into sensible data: Cape of Storms
4. Application of statistical tools	Mode: Sandwich Survey, How many people? Average: Averages, How many people?	Frequency: African Penguin Range: African Penguin, Weather Mode: African Penguin Average, Variance: Weather Regression, correlation: African Penguin 2
5. Display of data	Table: Car colours, Yuk or Yum, School tuckshop, How many people? Pictogram: Car colours, Sandwich Survey Bar graph: Sandwich Survey, How many people? Pie chart: Yumyums cake shop, Which hand do you use? Choice: School tuckshop	Table: African Penguin, History, Cape of Storms, Weather, African Penguin 2 Bar graph: African Penguin, History), Cape of Storms Pie chart: African Penguin, Cape of Storms Line graph: Weather Scatterplot, regression line: African Penguin 2
6. Communication of findings	All activities	All activities
7. Critical evaluation	All activities	All activities

References

Bramald, R. (1994). Teaching probability. **Teaching Statistics, 16,** 85-89.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S. Jnr, Lindquist, M.M., & Reys, R.E. (1981). What are the chances of your students knowing probability? **Mathematics Teacher, 74,** 342-344.

Garfield, J. & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. **Journal for Research in Mathematics Education, 19,** 44-63.

Hawkins, A.S. & Kapadia, R. (1984). Children's conceptions of probability – A psychological and pedagogical review. **Educational Studies in Mathematics, 15,** 349-377.

Konold, C. (1991). Understanding students' beliefs about probability. In E. von Glasersfeld (Ed.), **Radical Constructivism in Mathematics Education**, pp. 139-156. Dordrecht: Kluwer Academic Press.

National Council of Teachers of Mathematics (1989). **Curriculum and Evaluation Standards for School Mathematics**. Reston, VA: NCTM.

Shaughnessy, J.M. (1992). Research on probability and statistics: Reflections and directions. In D. Grouws (Ed.), **Handbook of Research on the Teaching and Learning of Mathematics**, pp. 465-494. New York: Macmillan.

Shaughnessy, J.M., Garfield, J. & Greer, B. (1996). Data handling. In A. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), **International Handbook of Mathematics Education**, pp. 205-237. Dordrecht: Kluwer Academic Press.