Malati

Mathematics learning and teaching initiative

ALGEBRA

MODULE 3

Towards the notion of function

Grades 8 and 9

TEACHER DOCUMENT

MALATI staff involved in developing these materials:

Marlene Sasman
Rolene Liebenberg
Alwyn Olivier
Liora Linchevski
Richard Bingo Lukhele
Jozua Lambrechts

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Overview of Module 3

This Module is designed to develop concepts gradually. Learners are not expected to master each concept and procedure when they first encounter it , but rather to develop continually their mathematical understandings through encounters with mathematical models of realistic situations as well as activities which could be termed as 'decontexualised' (i.e. just numbers). We believe that the rich background of ideas and experiences that learners have developed around numbers could serve as a context and could foster mathematics learning by relating learners' resources and intuitions to broader ideas and to encourage them to learn mathematics in the analysis of the complex.

We have chosen a wide variety of functions in the activities so as to develop a broader notion of function. For example, pupils will deal with functions for which there are rules and for which there are no rules, functions that are discrete (rules and no rules) and continuous (rules and no rules). We also want to develop and give examples of the power of modeling as a descriptive tool to describe situations between two variables and as an analytic tool to gain additional information about the situation.

The activities in this Module focus on input and output relationships given in situations, tables and algebraic rules. Three problem-types, i.e. finding function values, finding input values and analysing the behaviour of function values (the rate of change) are studied at an informal level.

The Module also attempts to develop understanding and use of algebraic expressions (function rules) as models.

The Module introduces a new representation for functions of two variables – co-ordinate graphs (for Grades 8 and 9, *not* for Grade 7) and then makes links between the different representations (words, tables, formulae and graphs).

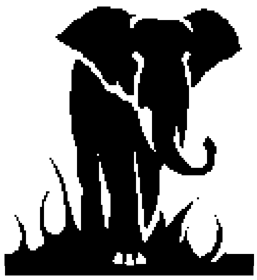
Note:
The reader should consult the MALATI rationale for school algebra, as well as the following research papers which form the backdrop for the design of the materials and the teaching approach:

Linchevski, L., Olivier, A., Sasman, M.C. &. & Liebenberg, R. (1998). Moments of conflict and moments of conviction in generalising. In A. Olivier & K. Newstead (Eds.), **Proceedings of the Twenty-second International Conference for the Psychology of Mathematics Education: Vol. 3**. (pp. 215-222). Stellenbosch, South Africa.

Sasman, M.C., Linchevski, L., & Olivier, A. (1999). The influence of different representations on children's generalisation thinking processes. In J. Kuiper (Ed.), **Proceedings of the Seventh Annual Conference of the Southern African Association for Research in Mathematics and Science Education** (pp. 406-415). Harare, Zimbabwe.

ACTIVITY 1:

MALATI Game Park




For the summer vacation Lester is working as a game ranger at the ***MALATI GAME PARK.***

1. The first group of tourists that he takes around is very keen to see the Africa's wildlife. As they drive around, they see

   lions in Timbavati Valley, leopards at Numbi Hills, rhino drinking at Skukuza Pools, a herd of buffalo grazing near Numbi Hills, elephants near Numbi Hills

   Lester would like to keep a record of these animals and the places where they were seen. Illustrate how he can summarise and organise this information .

2. When he takes the second group of visitors around they see

   lions at Trapezi Hills, elephants at Skukuza Pools, leopards near Numbi Hills elephants at Geopari Park, some rhino next to Timbavati Valley.

   Illustrate how he can organise and summarise this information.

3. Compare your illustrations in 1 and 2.

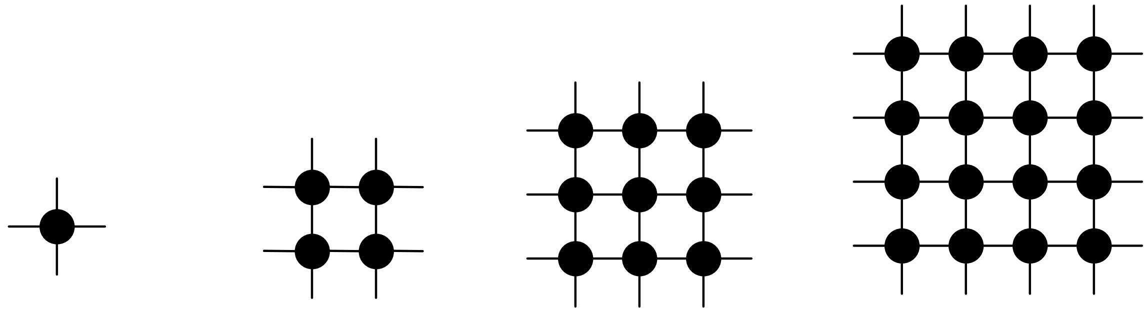
Teacher Note: Activity 1

Activity 1 is designed to test students' intuitive responses to data organisation. Here students are required to interpret contextual information about the relationship between the variables (non-numerical). It is also an attempt at developing a very necessary skill, i.e. systematically organising information – this should also build towards good classroom culture.

If children do not know how to organise the information the teacher should show them, e.g. the drawing of tables is *strategic knowledge* and if no students represent the data in tables, it must discussed with them.

ACTIVITY 2: Balls and Rods

Thabo links balls with rods in arrangements like this:



Arrangement 1 Arrangement 2 Arrangement 3 Arrangement 4

The table shows how many balls are needed for different arrangements:

Arrangement number	1	2	3	4	5	6	7	8		20			30		60
Number of balls in arrangement	1	4	9	16	25	36						625			

1. Complete the table. Show all your work.

2. Write a rule to calculate the number of balls in any arrangement.

3. How can you convince others that you are correct?

Teacher Note: Activity 2

Discussion: Whole-class setting:

It is useful to distinguish two different underlying thinking strategies (processes):

- numerical pattern recognition (induction), and
- structural analysis (deduction).

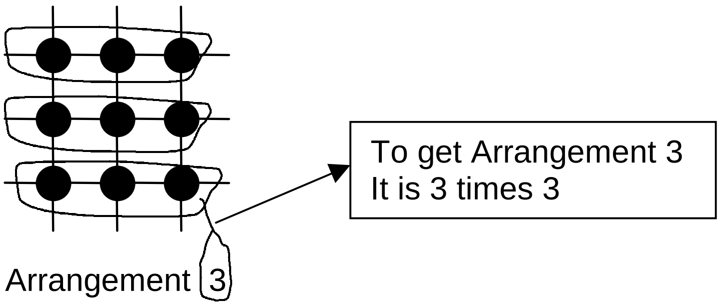
Please see the extensive notes in Activity 4.

The processes are not necessarily distinct – there is often an interplay from one to the other. Both processes are important in doing and learning mathematics.
Our research experience with *this particular activity* has shown that many students spontaneously see the functional relationship between the input and output variables. We hope that they will learn from this and also look for a functional relationship in other activities.
They may now however assume that the multiplication strategy is valid. Let them make the multiplication error and analyse
(1) the data base (induction), and
(2) the structure of the situation (deduction).

For example, is it sensible to say that the first arrangement has four times the number of balls as the fourth arrangement? Make explicit, not so much that you may do it for $y = kx$ but NOT for others, but the general attitude and strategy to see any "method" as a conjecture that should be checked against the database and the structure of the situation.

However, some children may not organise their data but use the structure of the picture in a *recursive* way, e.g. to arrangement 1 you add 3 balls to get the number of balls in arrangement 2, then you add 5, 7, 9 etc. for the successive arrangements.

Few children may intentionally take a specific input number and try to see this number in the structure of the picture, as illustrated in the following diagram:



They must be encouraged to also use the database (i.e. the given pictorial representations) in this way.

In the table the shaded columns represent the pyramid numbers not written in the table. This notation is not necessarily accessible to students and must be explained *by the teacher.*

ACTIVITY 3:

DOUGHNUTS

Yusuf sells doughnuts at a flea-market. He does not want to make calculations every time so he started preparing the following table to help him.

Number of doughnuts	1	2	3	4	5	6	7	8	9	10	…	100
Total cost (in cents)	29	58	87	116								

1. Complete Yusuf's table.

2. How much would 25 doughnuts cost?

3. Write a rule for a calculating the cost of any number of doughnuts.

4. How can you convince others that you are correct?

5. Yusuf usually balances his money after lunch, and then again in the evening.

   On Monday Yusuf sold 12 doughnuts in the morning and 18 in the afternoon.
   How much money did he receive on Monday?

   On Tuesday Yusuf sold 19 doughnuts in the morning and 25 in the afternoon.
   How much money did he receive on Tuesday?

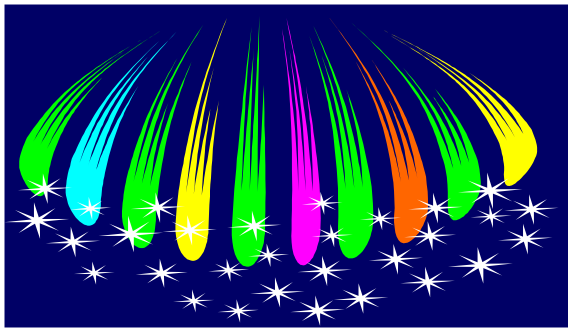
   Write a *general* rule, in words, for a calculating how much he receives on a day.
   Now write a *different* rule!

Teacher Note: Activity 3

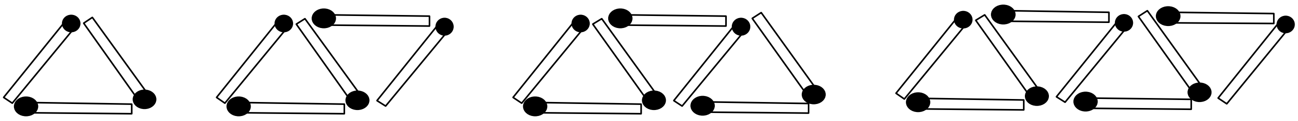
The model or function for this context is a direct proportional relationship and is thus of a type that children encounter regularly in their daily life. We think they will use functional rules, but also they MAY/CAN use multiplication as a strategy. This gives them the opportunity to use multiplication as a sensible strategy, also in contextual problems.

They may then study the relationships
1x100, 29x100
2x50, 58x50
20x5, ... 100 doughnuts cost *5 times as much as* 20 doughnuts
100 doughnuts cost *50 times as much as 2 doughnuts*, etc.

ACTIVITY 4:



1. Sipho uses matches to build pictures like this:



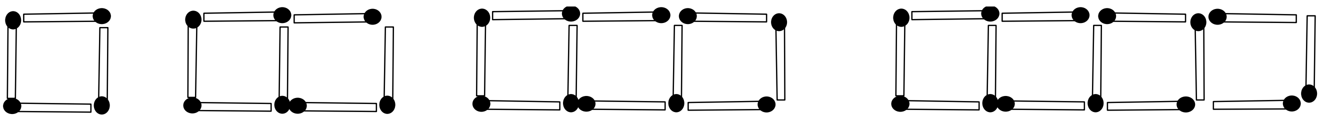
Picture 1 Picture 2 Picture 3 Picture 4

The table shows how many matches are used for the different pictures.

Picture number	1	2	3	4	5	6	7	8		19		37		59		100
Number of matches in picture	3	5	7	9	11	13										

Complete the table. Show all your work.

2. On another day Sipho builds squares, like this:



Picture 1 Picture 2 Picture 3 Picture 4

(a) How many matches does he use to form

(i) 5 squares (ii) 100 squares?

(b) Sipho has 455 matches. How many squares can he form in this way?

3. What is the same and what is different in the situations in 1 and 2?

Teacher Note: Activity 4

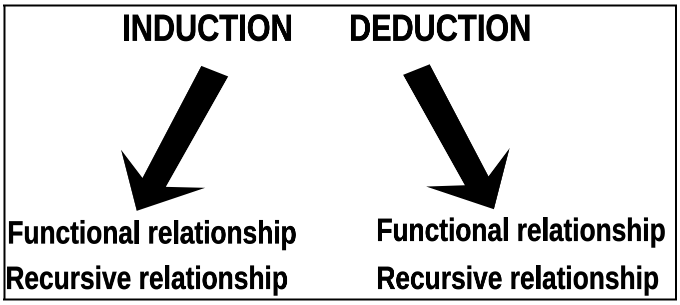
The mathematical relationship between two variables can always be described in terms of either

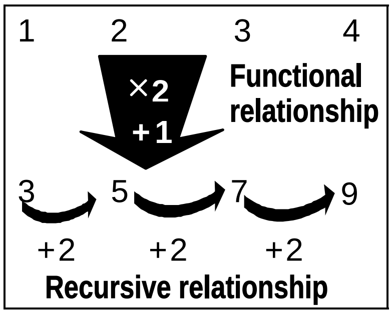
- the functional relationship between the variables which can lead to a formula such as, here, $m = f(t) = 2t + 1$, or
- the recursive relationship between (successive) function values leading to a formula, as here, $f(t + 1) = f(t) + 2$.

Both types of relationships are important in (the learning of) mathematics. They emphasise different aspects of the relationship:

- a functional formula such as $m = 2t + 1$ makes it easy to find function values, solve equations, ...
- a recursive relationship such as $f(t + 1) = f(t) + 2$ underlies the study of sequences and series, and the important concepts of change, rate of change, gradient and derivative.

Both induction and deduction can lead to a functional or a recursive relationship.



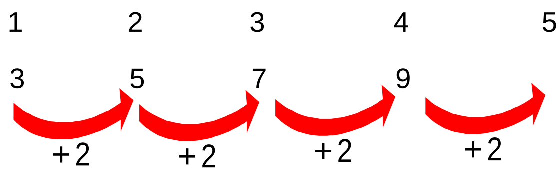


Numerical pattern recognition (induction)
The process of induction consists of two sub-processes:

1. pattern recognition in a finite set of data (abstraction)
2. pattern extension to cases not in the present set (generalisation)

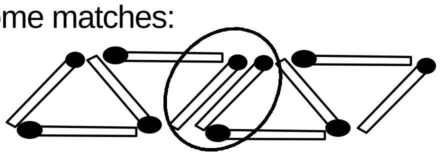
One can focus on the *numbers* given in the table (we call this the *database)* and recognise a vertical (functional) relationship $m = 2t + 1$ which easily yields all the solutions. Or one can recognise a horizontal (recursive) pattern, which can serve as a model to generate additional information about the situation.

The nature of a model is that it *simulates* the physical situation, so that one can generate information by manipulating the mathematical model instead of the practical situation. Therefore, even the simple recursive pattern $f(t + 1) = f(t) + 2$



is a useful model that allows us to determine the number of matches needed for 5, 6, 7, . . . triangles *without having to physically pack or draw the triangles and count the matches*. (This is what we mean with *predict:* to use a mathematical model, not the physical situation or a physical model of the situation, to generate additional, unknown information about the situation.) However, it is not initially feasible to continue this horisontal pattern until $t = 100$. A shorter method is needed!

Most learners recognise the need for a shorter method. However, few learners seem to use the functional relationship – most learners try to adapt the recursive relationship, but this often lead to errors, e.g f(100) = 10 $\times$ f(10). However, this is not a valid property of f. It can be *disproved* by a simple special case, e.g. f(4) $\neq$ 2 $\times$ f(2), or analysing the physical situation and realising that we will be repeating some matches:



2 × f(2) = 2 × 5 = 10.
But f(4) = 9 in the database

The property that f(kx) = k $\times$ f(x) is a property only of the model f(x) = mx (compare Activity 10 and Activity 12).

When children make this mistake here, it is very important that the teacher should help them to compare this situation with situations like in Activity 12, and analyse how they are the same and how they are different, i.e. they should analyse the *properties* of the two functions.

A valid "shortcut" can be found by looking at the horisontal pattern with different eyes, i.e. not working with the function *values*, but with the *structure* of the values:

$$
\begin{aligned}
f(1) &= 3 && = 3 \\
f(2) &= 5 && = 3 + 2 && = 3 + 1\times 2 \\
f(3) &= 7 && = 3 + 2 + 2 && = 3 + 2\times 2 \\
f(4) &= 9 && = && \ \ 3 + 3\times 2 \\
f(5) &= 11 && = && \ \ 3 + 4\times 2
\end{aligned}
$$

We can recognise the pattern in this *structure* and generalise it to

$$f(t) = 3 + (t - 1)\times 2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

from which it follows that

$$f(100) = 3 + 99\times 2 = 201$$

Structural analysis (deduction)

While the above came directly from looking at the *numbers* and ignoring the *matches*, some learners focus on the *process* of packing the *matches* and ignore the numbers. They easily formulate "you start off with 3 matches and then add another 2 matches for every additional triangle that you build". This is a model in the form of *words* (the equivalent to (1) in *symbols*) and also easily yields

$$f(100) = 3 + 99 \times 2 = 201$$

Syntactic meaning of algebraic expressions

It is important to note that the models all describe *a computational procedure*, e.g.

- in words: take the *number* of triangles, multiply it by 2 and then add one
- as a flow-diagram: $\longrightarrow \boxed{\times 2} \longrightarrow \boxed{+1} \longrightarrow$
- as an algebraic expression using symbols: $2 \times t + 1$

Equivalent transformations

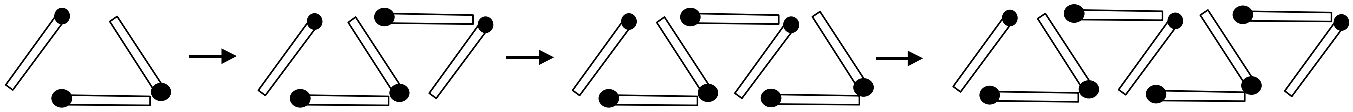
It is interesting to compare the recursive formula $3 + (t - 1) \times 2$ with the functional formula $2 \times t + 1$. Of course they yield the same *value* for the same value of t. *So they are simply* *different computational procedures!* Different *methods!* If we now convince ourselves that they are "the same" at the formal/deductive level, when we say

$$\begin{aligned} &3 + (t-1)\times 2 \\ &= 3 + 2t - 2 \\ &= 2t + 1 \end{aligned}$$

we are <u>*not*</u> *saying that 2t + 1 is the "answer" of 3 + 2(t – 1)!! We are merely saying that they are different procedures (methods) to calculate the same thing, therefore they give the same numerical result for the same value of the variable.* That is what equivalent transformation (i.e. algebraic manipulation) in this context means!

Semantic interpretation of algebraic expressions

The recursers know exactly what their formula $m = 3 + 2\times(t - 1)$ means: you start with 3 matches for the first triangle, and then you add an extra 2 matches for every extra triangle that you make. But what does the functional relationship $m = 2t + 1$ mean in the physical situation? Well, it is as simple as this:



Is a picture worth a thousand words?

ACTIVITY 5: Average Temperatures

The following table gives the monthly average temperature in degrees Celsius for the year 2000 in *ABCTown.*

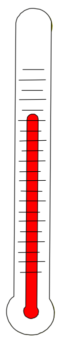
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Ave. temp.	26	28	22	21	17	13	12	15	20	21	22	25

Predict the monthly average temperature in *ABCTown* for:

(a) August 2001

(b) February 2002

Remember to **justify** your predictions.



100°C: water boils

45°C: a very hot day

22°C: tap water on a nice day

0°C: water freezes to ice

ACTIVITY 6:

TEST RESULTS

The table below shows the scores of some students at Greenside High in a mathematics test.

Student's name	**Score (in %)**
Lauren	83
Sipho	76
Peter	41
Mary	54
Mpho	62
Sheena	32
Zita	95
Oscar	
Waheeda	

The teacher forgot to write down the scores of the last two students (they were not absent).

What do you think their scores were?

Remember: in the mathematics class you must **justify** your answers.

Teacher Note: Activity 6

Teachers should realise that children often cannot make a distinction between context and mathematical knowledge. They then often push the discussion too much about non-mathematical issues. This is where the role of the teacher is very important – we should help them to divorce the mathematics from the context. The crucial aspect of these activities is the fact that the given data is insufficient to make a clear prediction. Children may however explain that if it is in the Cape it will be winter in August and on the basis of that make a prediction that is acceptable. Another child may argue that because of the effect of global warming it is possible to have an unusually high temperature in August even though it is suppose to be winter.

Teachers must insist on justification at all times – we must be clear on what counts as an explanation and what not – we should not be satisfied with any answer. It has to be clear that the predictions must be based on the database as well as on our contextualised knowledge. However we have to be careful since we are aiming at activities in which generalisation is based solely on the database.

We want to develop a classroom culture which accepts that mathematics is not just about answers. The activities on its own cannot do this – the teacher's role here is very important.

ACTIVITY 7:

THE TALLEST MAN

In American, Robert Waldow, was the tallest man who ever lived, according to the Guiness World Book of Records. His height was measured almost every year of his life, until he died when he was 21 and a half years old, and 272 cm tall. Here is a table containing his measurements.

Robert's height is a function of his age.

AGE (in years)	**HEIGHT (in cm)**
0	54
1	77
2	90
3	115
8	183
9	
10	196
12	210
13	218
17	
18	253
19	
21	
21,5	272

1. What was Robert's height when he was born?
2. What was his greatest height?
3. How old was he when he was 183 cm tall?
4. How tall do you think he was when he was 7 years old? Explain
5. Is it possible that his height when he was 17 years old was 215 cm? Explain.
6. Complete the table.

Teacher Note: Activity 7

In order to give students a broad notion of function, this activity is included so that students realise not all functional relationships can be described in terms of an algebraic expression or formula. The idea of continuity is also dripped.

ACTIVITY 8:

Shoe Size

. . . .

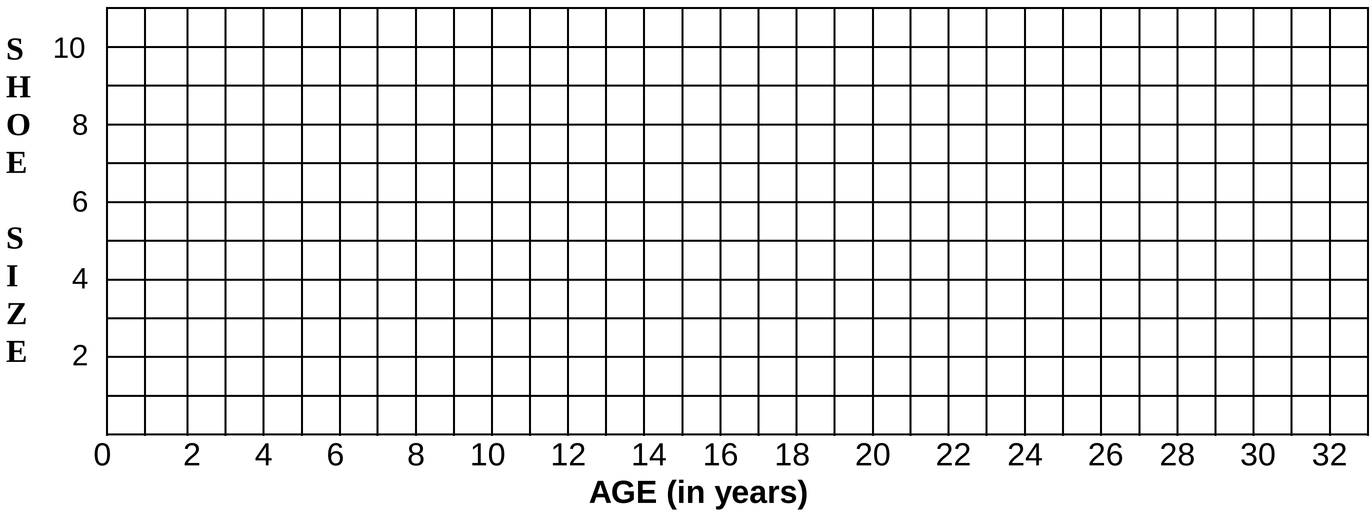
Jill's shoe size from age 12 to age 20 is recorded in the table below.

Age (in years)	12	13	14	15	16	17	18	19	20
Shoe size	2	$2\frac{1}{2}$	3	4	$5\frac{1}{2}$	6	6	6	6

1. What will the size of Jill's shoe be when she is:

   (a) 25 years old
   (b) 13 years and 6 months old
   (c) 54 years old
   (d) 19 years and 6 months old?

2. Represent the above data on the grid below:



Teacher Note: Activity 8

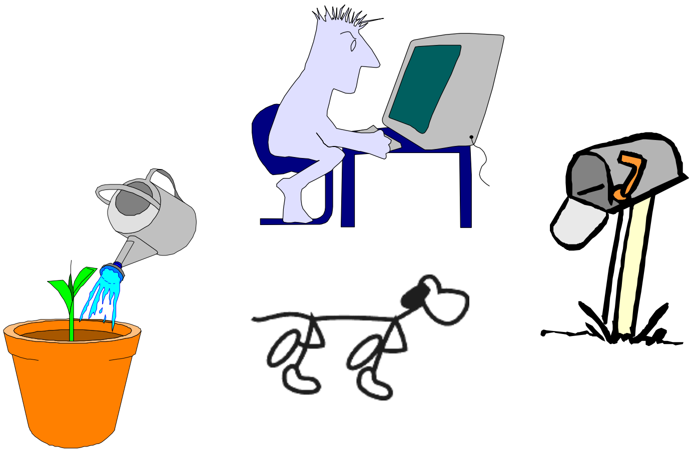
In this activity the idea of a constant function is being dripped since after some time the shoe size becomes constant.

The grid has been included to see how learners will intuitively represent data in this mode. The teacher must ensure that learners see each point on the grid as representing both the age and the shoe-size.

A discussion on continuous versus discrete functions may be dripped. Whilst shoe-size is discrete, actual foot-size is continuous.

ACTIVITY 9:

JOE'S VACATION JOB



Joe decided to run his own business during his summer vacations. When neighbours went on vacation, Joe would water their plants, take in their mail, walk their dog, etc. He charged a fee of R10 per household, plus R2 for each hour of work.

1. Joe filled in this table so that he could tell how much his customers owed him. Calculate how much money each customer paid him.

Name of customer	Time spent working (in hours)	Cost to customer (in Rands)
	T	**R**
Mr Harris	1	
Jenny	2,5	
The Ramsey's	7	
Mrs Ames	3,25	
The Heath's	15,2	
Rachael	11	

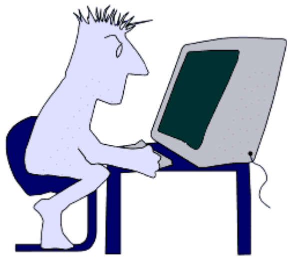
2. Write a rule that would show how much any customer owed Joe, based on the total number of hours he has worked for them.

3. How can you convince your classmates and teacher that your rule is correct?

4. Calculate Joe's *total* income for the vacation.
   Can you use two different methods to calculate the total?
   Describe your methods in words.

ACTIVITY 10:

COMPUTERS AND NUMBERS



When given the following input numbers on two different occasions, a computer produces the output numbers shown in the two tables below:

Table 1

Input number	1	2	3	4	5	6	…	17	…	60	…	n
Output number	4	8	12	16								

Table 2

Input number	1	2	3	4	5	6	…	17	…	60	…	n
Output number	5	9	13	17								

1. In each case complete the table.
2. Explain how you calculated output number 17 and output number 60 in each table.
3. How can you use the given number pairs (we call it the *database*) to verify your answers in question 2?
4. Make a conjecture about output number n in each table.
5. How can you *prove* your conjecture?
6. How are these tables the same and how are they different?

Teacher Note: Activity 10

Children will see different patterns or methods (of which some will be wrong) for the same activities. It is absolutely crucial that they realise which are not acceptable and why they are not acceptable. For example, they should realise that the multiplicative rule works in some situations but not in others, i.e. it is a property of some functions but not of others (actually the multiplicative rule is a property only of the direct proportion function, i.e. $f(x) = mx$). See Activity 4.

Proportional multiplication is a property of Table 1, but not of Table 2:
In Table 1, $f(6) = 24$, so $f(60) = 10 \times f(6) = 10 \times 24 = 240$ is correct.
But in Table 2, $f(6) = 25$, so $f(60) = 10 \times f(6) = 10 \times 25 = 250$ is *not correct.* It can be disproved by a simple special case (we call this a *counter example*), e.g. $f(4) \neq 2 \times f(2)$.
They should also be able to explain and demonstrate why their *methods* are acceptable in terms of the database and the structure of the situation.

The issue is again addressed in Activity 12.

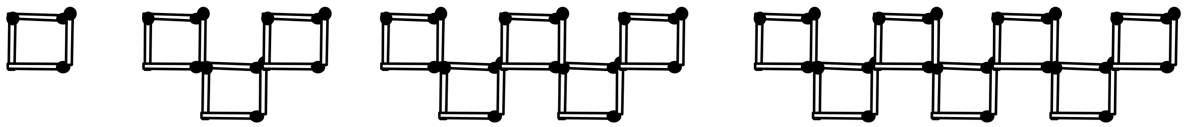
Activity 10 was designed to help learners to focus on the advantages of using *function rules* rather than *recursive patterns* in the tables. Learners should easily recognise the function rule $y = 4n$ in Table 1, and then recognising patterns between Table 1 and Table 2, to see the rule $y = 4n + 1$ in Table 2.

Note that although tables 1 and 2 are described by different functions, and each function has unique properties, both functions have the same property of a common difference between consecutive function values. This concept is later generalised to "arithmetical sequence" and the gradient of a straight line.

ACTIVITY 11:

MATCHES 1

Thandi uses matches to build shapes in the following way:



Shape 1 Shape 2 Shape 3 Shape 4

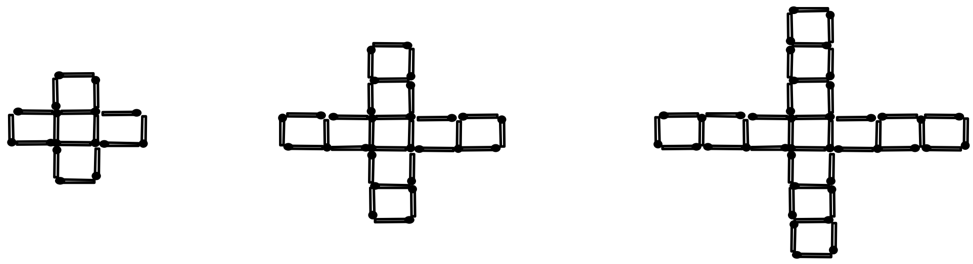
The table shows the number of matches used to build the different shapes.
Complete the table. Show all your work.

Shape number	1	2	3	4	5	6	7	8		20		30		60		*n*
Number of matches in shape	4	12	20	28	36	44										

ACTIVITY 12:

MATCHES 2

1. Thandi builds crosses with matches like this:



Picture 1 Picture 2 Picture 3

The table shows the number of matches she uses for different pictures:

Picture number	1	2	3	4	5	6	…	19	…	60	….	100
Number of matches	16	28	40	52								

(a) Complete the table.

(b) Explain to some of your classmates how you got to your answer for Picture 60.

(c) John, Siketla, Mandy and Vuyo use different *methods* (plans) to calculate the number of matches in Picture 60.
Who will get the right answer? *Explain why.*
If someone will *not* get the right answer, *explain why not.*
Which of these plans do you prefer? Explain *why*.

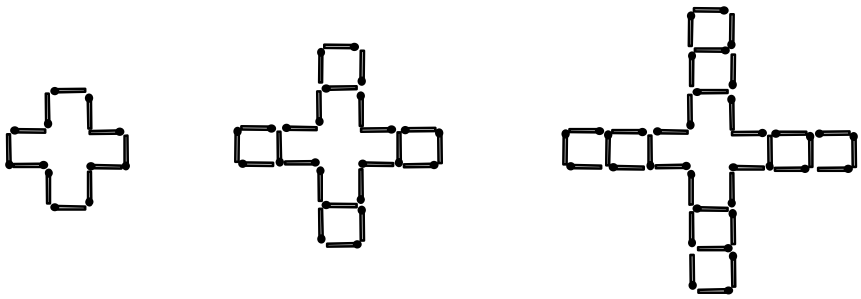
<u>John's plan:</u>
I see from 16 to 28 is +12, and from 28 to 40 is +12, and from 40 to 52 is +12. So I continue to add 12 until I reach Picture 60.

<u>Siketla's plan:</u>
I know $6 \times 10 = 60$. So I take the number of matches I got for Picture 6 and I multiply it by 10 to get the number of matches in Picture 60.

<u>Mandy's plan:</u>
I multiply the Picture number by 12, and then I add 4. So in Picture 60 there are $60 \times 12 + 4$ matches.

<u>Vuyo's plan:</u>
I see from 16 to 28 is +12, and from 28 to 40 is +12, and from 40 to 52 is +12. So Picture 6 has 76 matches. Now from 6 to 60 means I must add 54×12 which is 648 and then I must still add the number of matches for Picture 6. So it is 648 +76.

2. On another day Thandi builds crosses with matches like this:



Picture 1 Picture 2 Picture 3

The table shows the number of matches she uses for different pictures:

Picture number	1	2	3	4	5	6	…	19	…	60	….	100
Number of matches	12	24	36	48								

(a) Complete the table. Explain your method to calculate the number of matches for Picture 60.

(b) John, Siketla, Mandy and Vuyo use different *methods* (plans) to calculate the number of matches in Picture 60. Who will get the right answer? Explain *why* or *why not.*

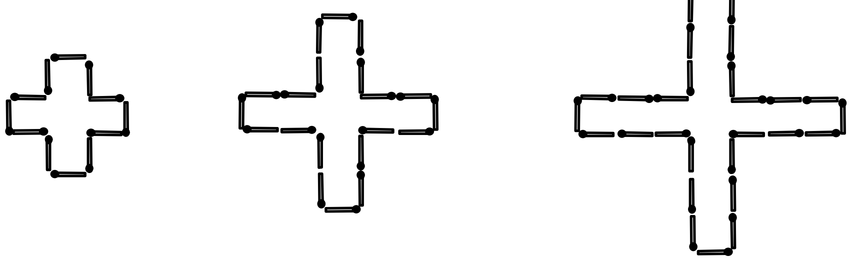
<u>John's plan:</u>
I see from 12 to 24 is +12, and from 24 to 36 is +12, and from 36 to 48 is also +12. So I continue to add 12 until I reach Picture 60.

<u>Siketla's plan:</u>
I know $6 \times 10 = 60$. So I take the number of matches I got for Picture 6 and I multiply it by 10 to get the number of matches in Picture 60.

<u>Mandy's plan:</u>
I multiply the Picture number by 12. So in Picture 60 there are 60×12 matches.

<u>Vuyo's plan:</u>
In Picture 1 there are 12 matches. Then you add another 12 matches for every picture you make. For Picture 60 I make another 59 pictures, so I must add another 59×12 matches. So in Picture 60 there are $12 + 59 \times 12$ matches.

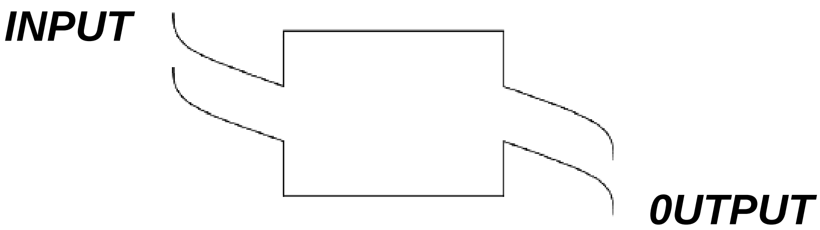
3. On another day Thandi builds crosses with matches like this:



Picture 1 Picture 2 Picture 3

Describe John, Siketla, Mandy and Vuyo's metod for these crosses.
Whose method gives the right answer for Picture 60? Explain *why* or *why not.*

ACTIVITY 13:



Here are more input-output tables that were generated by the computer.

1. Complete each table.
2. In each case write a rule for the *n*th output number.
3. In each case explain how you would convince others that your rule is correct.

Table 1

Input Value	…	3	…	7	…	12	…	61	…	113	…
Output Value		11		23		38					

Table 2

Input Value	…	5	…	9	…	13	…	73	…	204	…
Output Value		25		81		169					

Table 3

Input Value	…	5	…	9	10	…	13	…	91	…	117
Output Value				82	101		170				

Table 4

Input Value	…	7	…	29	…	67	…	119	…		…
Output Value		71		291		671				1321	

Table 5

Input Value	…	5	…	10	…	50	…	73	…		…
Output Value		13		28		148				498	

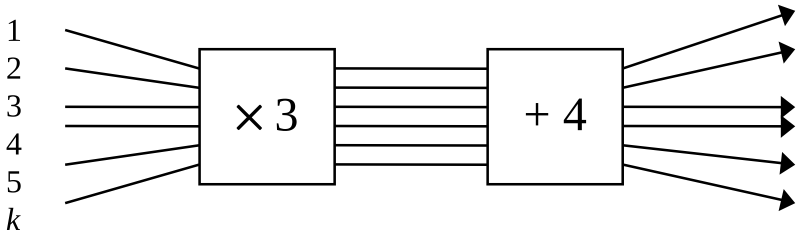
Teacher Note: Activity 13

In these tables input – output pairs are given and they are not necessarily consecutive since the main purpose is to get the children to focus on the functional (vertical) relationship between them. We want the learners to consider the input and output numbers simultaneously in their search for a relationship. This is to hopefully discouraging them from remaining with a recursive approach when looking for a pattern. We also want to make explicit the fact that a functional relationship can have a combination of operators – children often give up when looking for a “short-cut” to describe a rule since they are not used to the idea that a functional relationship can consist of several operators.

ACTIVITY 14:

Flow diagrams and tables

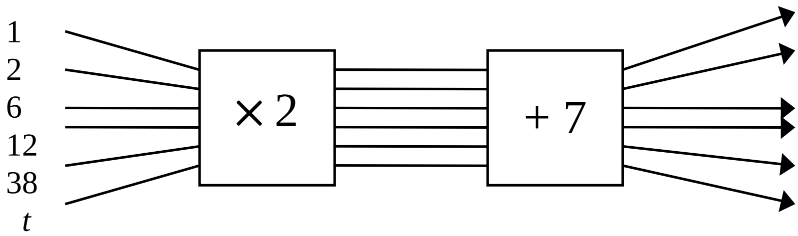
1. (a) Complete the flow diagram.



(b) Use the values of the flow diagram to complete the table.

Input values	1	2	3	4	5	k
Output values						

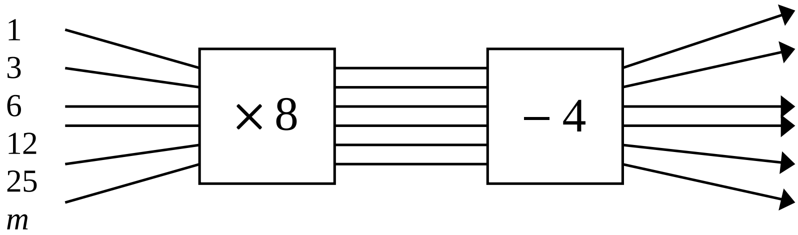
2. (a) Complete the flow diagram.



(b) Use the values of the flow diagram to complete the table.

Input values	1	2	6	12	38	t
Output values						

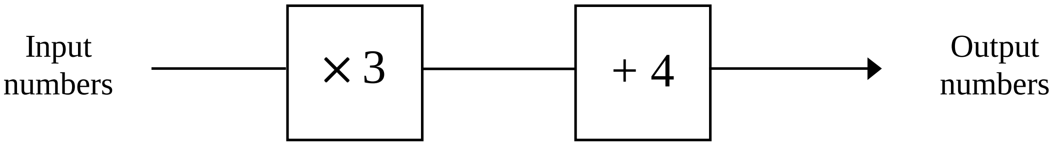
3. (a) Complete the flow diagram.



(b) Use the values of the flow diagram to complete the table.

Input values	1	3	6	12	25	m
Output values						

4. Flow diagrams can also be written in the following way. Use the flow diagram to complete the table.



Input values						
Output values						

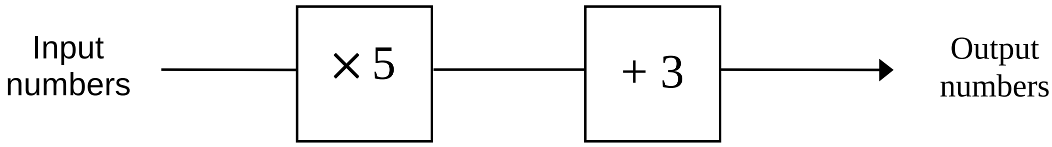
Teacher Note: Activity 14

In this exercise the students have to use the function rule to complete the table.
The main purpose of this activity is to focus learners on the functional relationship between the input and output values. It s purpose is also to make explicit the fact that some functional relationships consist of more than one operators. Many learners come unstuck when looking for a functional rule since they only focus on one operation.
Flow diagrams need to be explained to them.

ACTIVITY 15:

Flow diagrams, tables and words

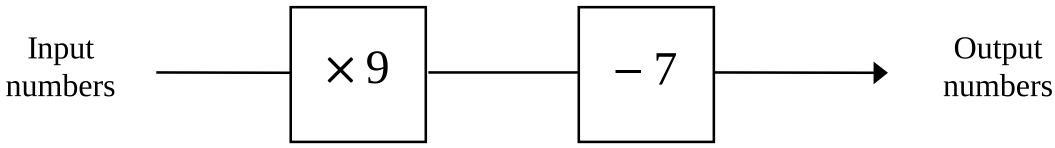
1. (a) Use the flow diagram to complete the table.



Input numbers	1	2	3	4	5	k
Output numbers						

(b) Rewrite the flow diagram in words.

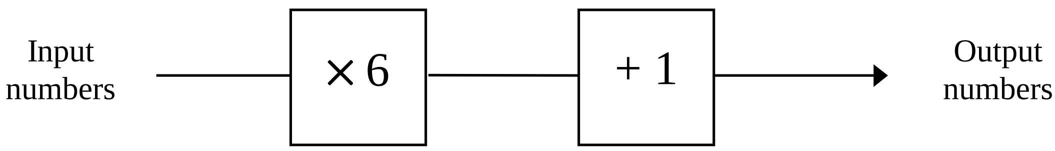
2. (a) Use the flow diagram to complete the table.



Input numbers	1	2	3	4	5	m
Output numbers						

(b) Rewrite the flow diagram in words.

3. (a) Use the flow diagram to complete the table.



Input values	1	2	3	4	5	n
Output values						

(b) Rewrite the flow diagram in words.

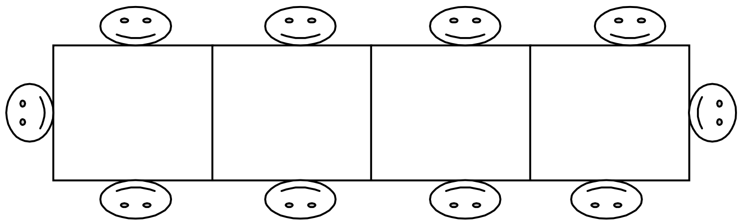
Teacher Note: Activity 15

Students have to use the function rule to complete the table and get experience in writing the function rule in words. Here they can also reflect back on previous activities and prepare flow-diagrams for some of the previous activities.

ACTIVITY 16:

THE PARTY

At a party Lester arranges small square tables next to each other in a straight row, so that one person sits on each of the available sides of the table. So for example with 4 tables, he can seat 10 people, as shown in the sketch below.



1. If Lester arranges 25 tables as shown above, how many people can be seated?
2. How many tables must he pack next to each other if he wants to seat 74 people? Explain!
3. How many tables must he pack next to each other if he wants to seat 47 people? Explain!
4. Write in words how Lester would calculate the number of tables required to seat n people. How can you convince yourself, your group members and your teacher that you are correct?

ACTIVITY 17:

STATEMENTS ABOUT NUMBERS

1. Study A and B carefully and in each case make a conjecture about the given data. Justify your conjecture in each case.

A

$3 + 5 = 8$
$47 + 5 = 52$
$11 + 9 = 20$
$13 + 27 = 40$
$51 + 85 = 136$
$141 + 85 = 226$

Conjecture:

…………………………………..………..

………………………………………….

………………………………………….

B

$24 + 57 = 81$
$12 + 17 = 29$
$10 + 9 = 19$
$147 + 534 = 681$
$4 + 9 = 13$
$84 + 93 = 177$

Conjecture:

………………………………………….

………………………………………….

…………………………………………

2. Evan makes the following conjecture:

   "An even number multiplied by an odd number is always odd"

   Do you agree or disagree with Evan? Explain why you agree or disagree.

3.

$1 \times 9 = 9$
$2 \times 9 = 18$
$3 \times 9 = 27$
$4 \times 9 = 36$
$5 \times 9 = 45$
$6 \times 9 = 54$

Investigate the digits of the multiples of 9 and make a conjecture:

………………………………………………………………………………………

……………………………………………………………………………………..

Find a way to check your conjecture:

……………………………………………………………………………………..

………………………………………………………………………………………

Teacher Note: Activity 17

Teachers should ensure that the pupils understand that a conjecture is an unproven statement and as such there should always be the need for justification, validation and convincing.

Note:
As mathematicians study, they often run across results that they are sure are true, but that they can not (yet) prove. These results are called **conjectures**. Some conjectures stand for hundreds of years before they are finally proven (or disproven). For example, Fermat's Last Theorem stood as a conjecture for 350 years before Wiles proved it recently.

ACTIVITY 18:

Mr Bean

Mr Bean buys a plant at the nursery when it is 12 cm high. He measures the height of the plant regularly and finds that it grows at a rate of 5 mm per week.

1. If the plant continues to grow at this rate, how high will it be after 1 month?
   And after 4 months?

2. How long will it take the plant to grow to a height of 30 cm?

3. What will the height of the plant be after *n* weeks?

Teacher Note: Activities 18, 19, 20, 21, 22

In Activity 18:
The class discussion should address what learners understand by the term “rate”. Reflect on previous activities – identify the rate in these activities.

Learners will also argue about whether a month consists of 4 weeks, $4\frac{1}{2}$ weeks, etc. Allow them to decide but then their solution must be based on their assumption. So different learners may have different solutions but they must be able to justify their solutions.

Activities 18, 19, 20, 21 and 22 further develop the modelling concept , encourages learners to see the advantages of linear models, but to be aware of possible problems, to extrapolate and interpolate.

Note that in Activity 18 the model for the growth of the plant is $0,5n + 12$ and that proportional multiplication does not apply, i.e. the plant is not 4 times as high after 4 weeks as after 1 week! In Activity 19 the model is **H** = 1,5**D**, and proportional multiplication is a valid property, therefore at 28 days the plant is twice as high as at 14 days.

Students that have problems with decimals must be identified and given support since they may lose track of functions because of these difficulties.

The real life contexts in these activities must be part of the class discussion, e.g. where distance and the time of day at which you are phoning also affects cost, the seedling will not grow forever, the spring cannot stretch indefinitely, etc.

Domain and range is also being dripped.

ACTIVITY 19:

Plant growth

The science class measured the growth of a seedling over a two-week period, starting from the day they planted the seedling (Day 0) as shown in the table.

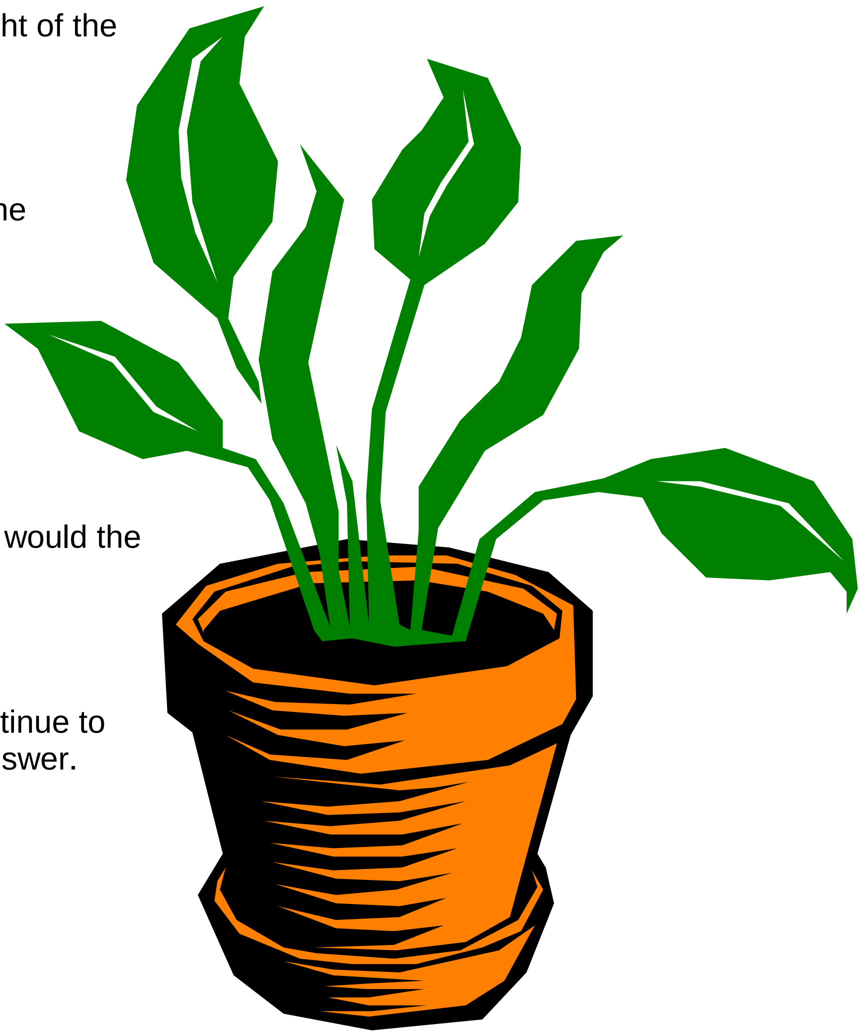
Day number	0	2	4	6	8	10	12	14
Height (mm)	0	3	6	9	12	15	18	21

1. What was the daily growth of the seedling?

2. When was the seedling 10,5 mm tall?

3. After 11 days, what was the height of the seedling?

4. Explain how the age and the height of the seedling are related.



5. If the plant continues to grow at the same rate,

   (a) How high will it be after 28 days?

   (b) When would it have been 60 mm high?

6. If the seedling is **D** days old what would the height, **H**, be?

7. Do you think the seedling will continue to grow at this rate? Explain your answer.

ACTIVITY 20:

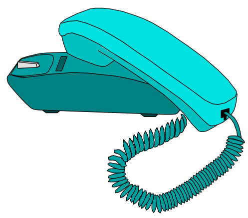
Telephones

A telephone bill includes a monthly rental and a certain cost per unit. The time you spent on the phone is measured in units.

1. Complete the following table.

Number of units	50	100	150	200	250	350		
Total cost in Rands	48,50	55,00	61,50	68,00				135,6

2. How much is the monthly rental?



3. What is the cost per unit?

4. You received a bill of R118,69 for 513 units. Are you billed correctly?

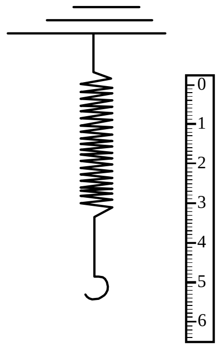
5. What will your bill be for phoning 637 units?

6. Write in words a formula to work out your bills efficiently.

7. Which other factors influences your telephone bill?

ACTIVITY 21:

SPRINGS



1. The following readings of the mass hung from a spring and the corresponding length of the spring were taken in a scientific experiment:

Mass (kg)	0	1	2	3	4	5	6,6	10	
Length (cm)	15	17	19	21	23				32,4

(a) Complete the table.

(b) By how much will the spring stretch if at any stage an additional 2 kg is hung from the spring?

(c) Write in words a formula to work out the length of the spring for any mass hung from it.

2. The following readings of the mass hung from a *different* spring and the corresponding length of the spring were taken in a scientific experiment:

Mass (kg)	0	1	2	3	4	5	6,6	10	
Length (cm)	10	13	16	19	22				32,4

(a) Complete the table.

(b) By how much will the spring stretch if at any stage an additional 2 kg is hung from the spring?

(c) Write in words a formula to work out the length of the spring for any mass hung from it.

3. How are the springs in 1 and 2 the same and how do they differ?

ACTIVITY 22:

Candles

The science class did the following experiment on new types of candles which the manufacturers claim will burn very long. Three different candles were lit and their length measured every hour. This information was written down in the following tables.

Candle 1

Time in hours	0	1	2	3	4	5	6	9	15
Length in cm	33	31	29	27	25	23			

Candle 2

Time in hours	1	2	3	4	5	6	11	19	30
Length in cm	25,6	25,2	24,8	24,4	24,0				

Candle 3

Time in hours	0	1	2	3	4	5	6	13	23
Length in cm	38,5	37	35,5	34	32,5	31			

1. Complete the tables.

2. They forgot to measure the initial length of Candle 2. What was the initial length?

3. Which candle was burning the slowest? Explain your answer.

4. How long will Candle 1 burn?

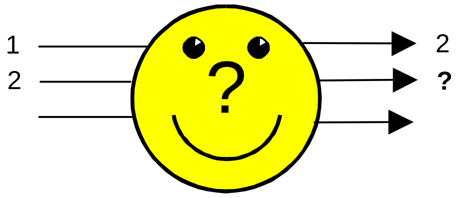
5. Which candle will burn the longest? Explain.

ACTIVITY 23:

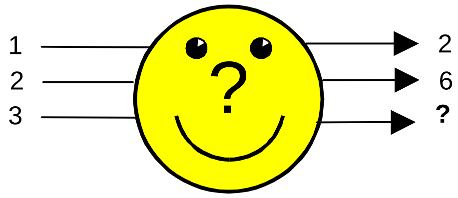
GUESS MY RULE

Smiley is thinking of a secret function rule. He will not tell us what the function rule in his head is, but he will tell us that if the input number is 1, his rule gives the output number 2.

You must guess his secret function rule, by *predicting* his output number if the input is 2. Smiley will tell you if you are correct and give you the right answer if you are wrong.



Smiley says that if the input number is 2, his output is 6. Now *predict* the output number for 3 and see at the bottom of the page if Smiley agrees . . .



Now play in pairs. Think of a secret function rule, give your pair mate some input-output numbers and see if he/she can find your rule. (You and your pair mate must reverse roles.)

1.

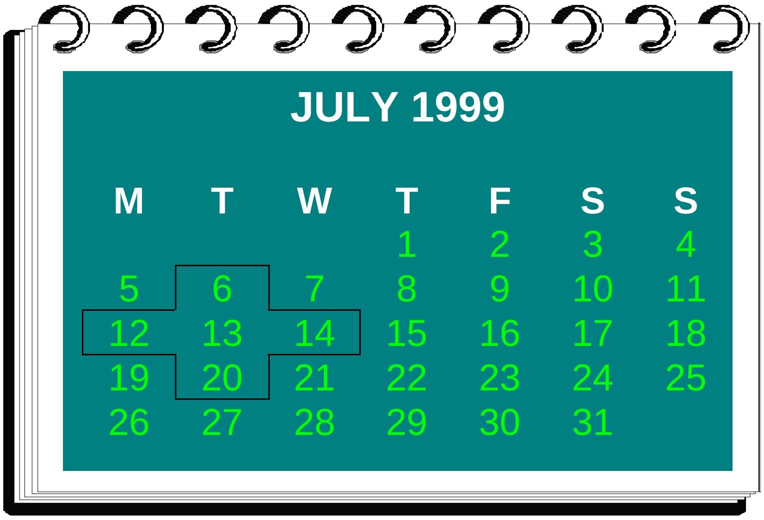
2.

3.

4.

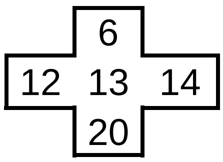
Smiley says that for 3 his output number is 12

ACTIVITY 24: THE CALENDAR

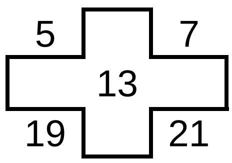


1. Study the calendar page for the month of July 1999 above and describe as many number patterns as you can find.

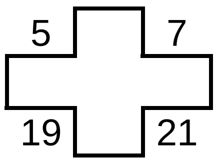
2. Lester selected the following pattern of numbers from the calendar. He says that that *anywhere* in the calendar, for numbers arranged in this pattern, the sum of the outside four numbers will be four times the number in the middle. Do you think that Lester is correct? How can you check? Explain!

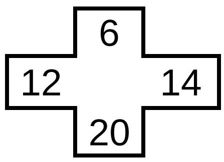


3. Is there a similar pattern between the middle number and the four corner numbers?



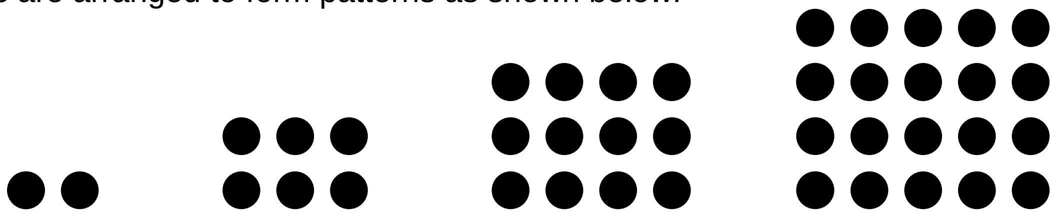
4. Lester also says that the sum of the "four numbers" in questions 2 and 3 are always the same. Do you agree? Can you explain *why*?





ACTIVITY 25: DOTS

1. Dots are arranged to form patterns as shown below:



Pattern 1 Pattern 2 Pattern 3 Pattern 4

(a) How many dots are in:

- Pattern 5?
- Pattern 11?
- Pattern 200?

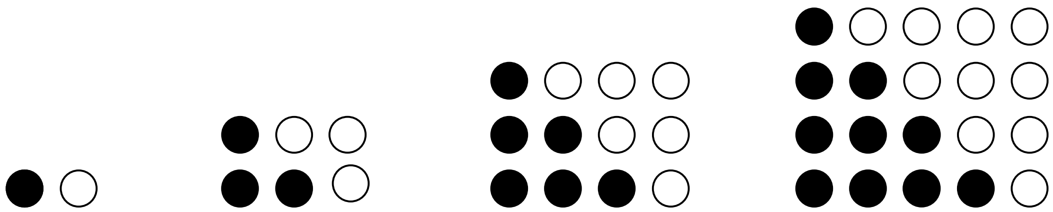
(b) Describe how you found your answers.

(c) Make a conjecture about the number of dots in Pattern *n*.

(d) Explain how you arrived at your conjecture.

(e) *Prove* your conjecture.

2. Black and white dots are arranged to form patterns as shown below:



Pattern 1 Pattern 2 Pattern 3 Pattern 4

Complete the table and explain the plan you used.

Pattern number	1	2	3	4	5	6	7	8	…	53	…	n
Total number of dots	2	6	12	20					…		…	
Total number of black dots	1	3	6	10					…		…	

3.

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10$$
$$1 + 2 + 3 + 4 + 5 = 15$$
$$1 + 2 + 3 + 4 + 5 + 6 = 21$$
$$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = ?$$
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = ?$$

$$1 + 2 + 3 + \ldots + 98 + 99 + 100 = ?$$

Conjecture: The sum of the first *n* consecutive, positive integers is ……………………

……………………………………………………………………………………………………

Explain how you arrived at your conjecture.

ACTIVITY 26: BLOCK OF FLATS

1. There is a block of flats that is 9 storeys high and on each floor there are six flats. Think of a way to number the flats so that each flat has a number which describes its *position*. Discuss your method in your group. Are your methods the same?

2. From your numbering system, can you determine:
   - on what floor a flat is located ?
   - whether it is at the end of the passage or in the middle?

   If not, could you devise another numbering system that would be able to help you answer these questions?

3. Use your (improved) numbering system to number all the flats in a building that is 4 storeys high and has 8 flats on each floor.

4. If a new block of flats was built that is much larger, say 30 storeys high and 20 flats on each floor would your numbering system still work?
   If not, could you change your numbering system to make it work?

Teacher Note: Activity 26

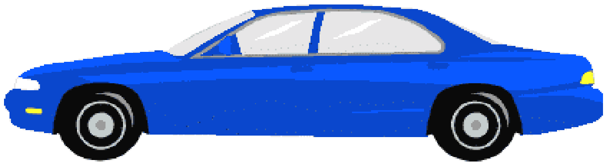
To see the need and to develop an ordering system (introduce notation)
Students must motivate the need for *two* numbers in order to find the location.

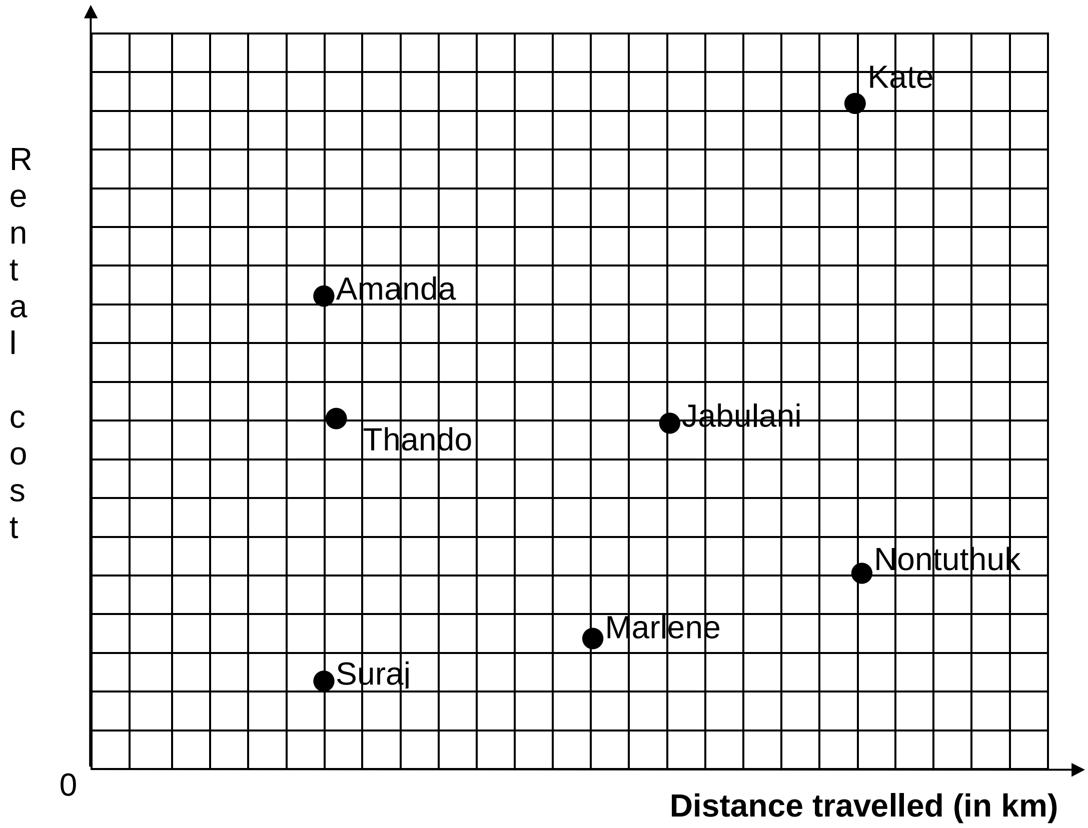
NOTE
As an introduction to co-ordinates we advise that the following MALATI materials would be a good introduction:

- In Geometry: Primary module 4 on Position and location
- In Statistics: Probability 2

ACTIVITY 27:

Renting a car



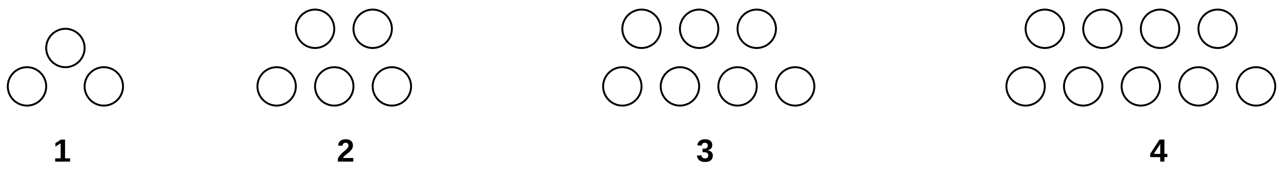


*Use the **graph** above to answer the following questions:*

1. Who rented the most expensive car? Give a reason for your answer.
2. Who rented the least expensive car? Give a reason for your answer.
3. Consider Thando and Jabulani: How do their rental costs and distances they travelled differ?
4. Compare rental costs and distance travelled by Amanda and Suraj.
   Write down your comparison.
5. Which people do you think paid the same cost per kilometre? Explain your answer.
6. Who paid the most and who paid the least?
7. Who travelled the furthest?
8. Who travelled the least distance?

ACTIVITY 28:

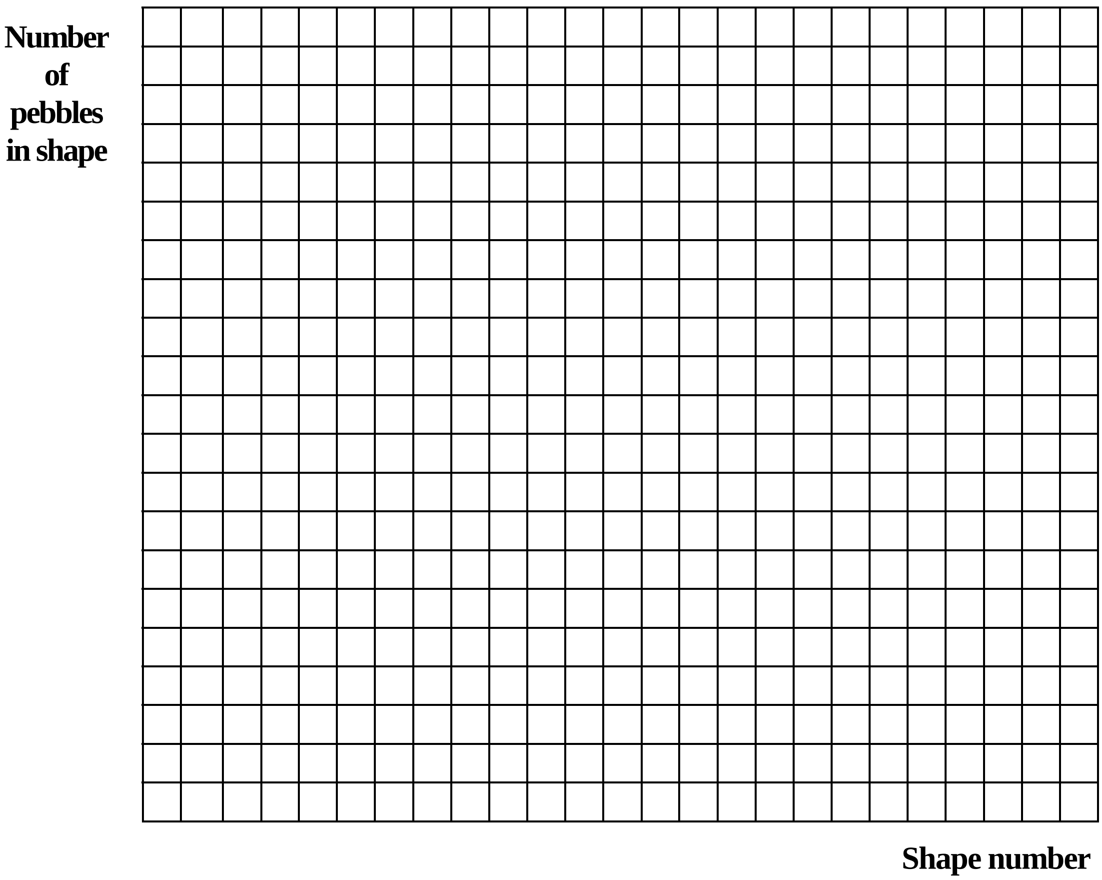
Pebbles ...



1. Complete the table.

Shape number	1	2	3	4	5	6	7		10		30		60		*n*
Number of pebbles in shape	3	5	7	9											

2. Make a graph of your findings.



ACTIVITY 29:

Expressions and Words

1. Write suitable algebraic expressions for each of the following:

   (a) A natural number is multiplied by 5 and 13 is added to the result.

   (b) 7 is added to a number and the result is multiplied by 4.

   (c) A whole number increases by 10 and the result is divided by 5.

   (d) 3 is subtracted from a number and the result is multiplied by 8.

   (e) A number is multiplied by 15 and the result is subtracted from 200.

   (f) A number is multiplied by itself.

   (g) An even number is multiplied by itself and 5 is added to the result.

   (h) 10 is added to a number, the result is multiplied by 3, and then 7 is subtracted.

2. Discuss in your group and in the whole class with your teacher how you wrote the expressions in 1. Discuss the conventions of algebraic language and notations.

3. Write, in words, statements which correspond with each of the following algebraic expressions. What do these algebraic expressions *mean?*

   (a) $x + 12$

   (b) $7 \times x - 25$

   (c) $45 - x$

   (d) $2(x + 9) - 10$

   (e) $x^2 + 4$

   (f) $100 - x^2$

   (g) $(x + 5)^2$

   (h) $5(2x - 3)^2$

Teacher Note: Activities 29 & 30

The main focus of Activities 29 and 30 is to give practice and consolidation in symbolic representation. Learners need to develop flexibility in describing functional rules and seeing the usefulness of using symbolic expressions. If need be they may revisit previous activities to ‘refine’ their functional rules.

ACTIVITY 30:

Expressions and Flow-diagrams

1. Write suitable algebraic expressions for each of the following flow diagrams:

(a) $\longrightarrow \boxed{\times 3} \longrightarrow \boxed{-4} \longrightarrow$

(b) $\longrightarrow \boxed{\times 5} \longrightarrow \boxed{-8} \longrightarrow$

(c) $\longrightarrow \boxed{+2} \longrightarrow \boxed{\times 7} \longrightarrow$

(d) $\longrightarrow \boxed{+4} \longrightarrow \boxed{\div 3} \longrightarrow$

(e) $\longrightarrow \boxed{\div 2} \longrightarrow \boxed{-8} \longrightarrow$

(f) $\longrightarrow \boxed{\times 12} \longrightarrow \boxed{+7} \longrightarrow \boxed{\div 4} \longrightarrow$

(g) $\longrightarrow \boxed{\times 12} \longrightarrow \boxed{-7} \longrightarrow \boxed{\div 4} \longrightarrow \boxed{+3} \longrightarrow$

2. Design a flow diagram that will correspond to each of the following expressions:

(a) $7x + 10$

(b) $\frac{8x-5}{3}$

(c) $9(x - 5)$

(d) $5(3x + 5) - 2$

(e) $5(\frac{x}{3} + 7)$

ACTIVITY 31:

Different but the same !

In each of the following exercises you are given one representation (words, formula, table or graph) of how two variables are related and you will need to give this same relationship in another representation. In order to make the formulas easier to write call the input "x" and the output "y".

1. *Words:* The output is the sum of twice the input and five

   Table:

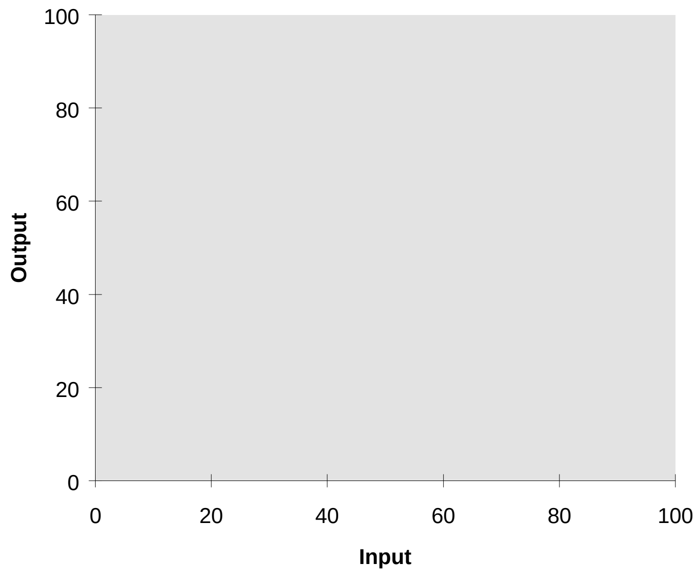
Input	0	1	3	8	10	25	100
Output							

2. *Words*: The output is the cube of the input

   Formula:

3. *Words:* The output is the result when 100 is divided by the input.

   Graph:



4. *Formula:* (Remember input is "x" and output is "y")

$$y = 2(x + 1)$$

Words:

5. *Formula:*

$$y = x^2$$

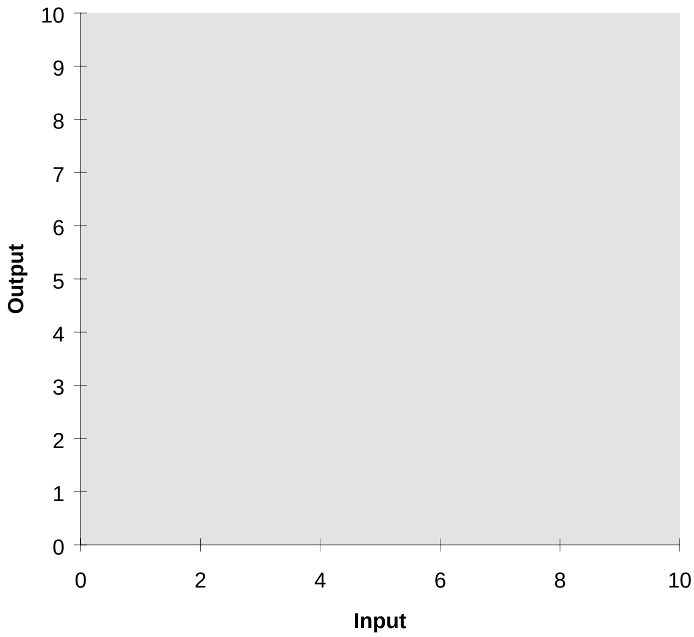
Table:

Input	0	1	3	8	10	25	100
Output							

6. *Formula:*

$$y = 10 - x$$

Graph:



7. *Table:*

Input	0	1	2	4	10	20	100
Output	3	4	5	7	13	23	103

Words:

8. *Table:*

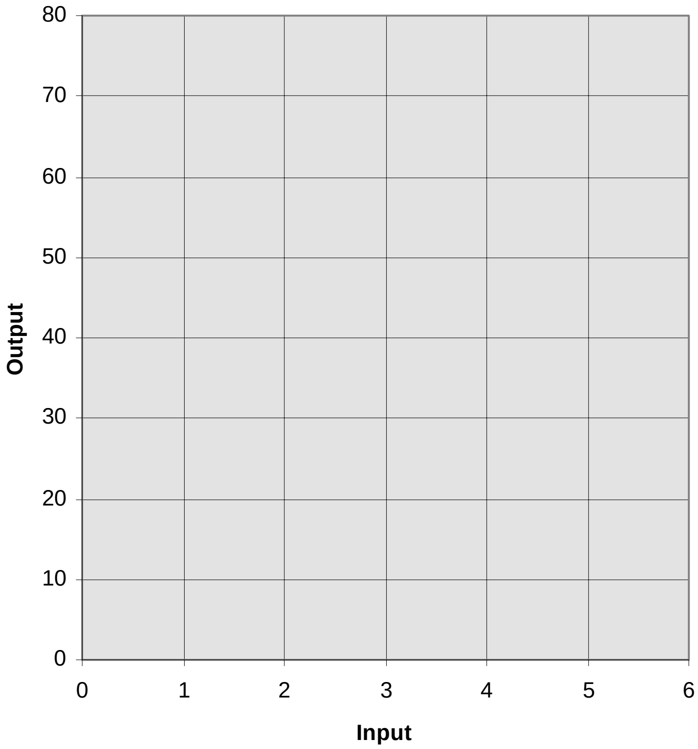
Input	0	1	2	3	5	10	100
Output	0	3	6	9	15	30	300

Formula:

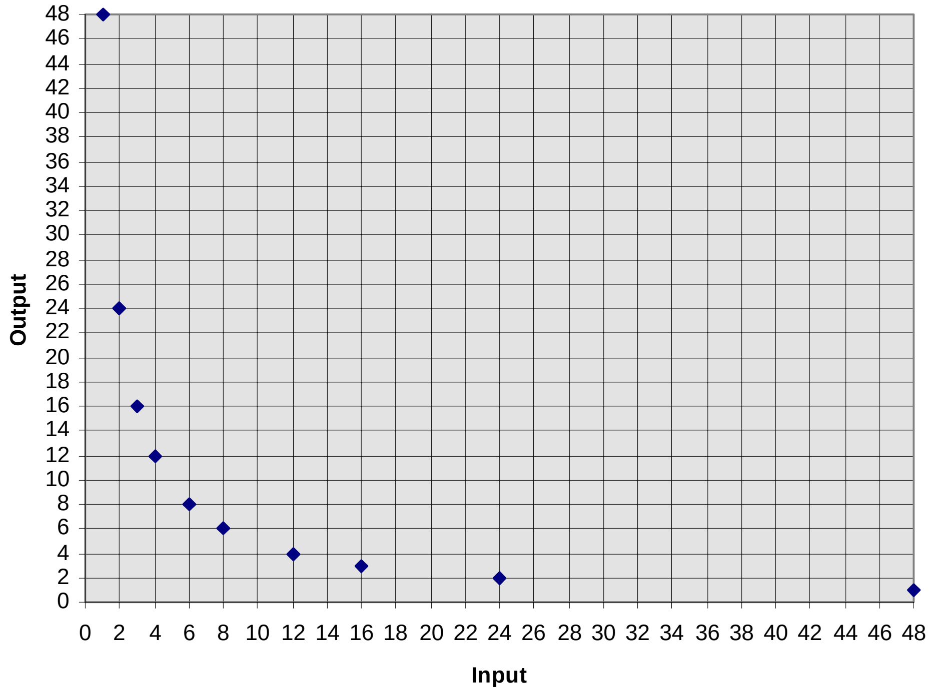
9. *Table:*

Input	0	1	2	3	4	5	6
Output	1	2	4	8	16	32	64

Graph:

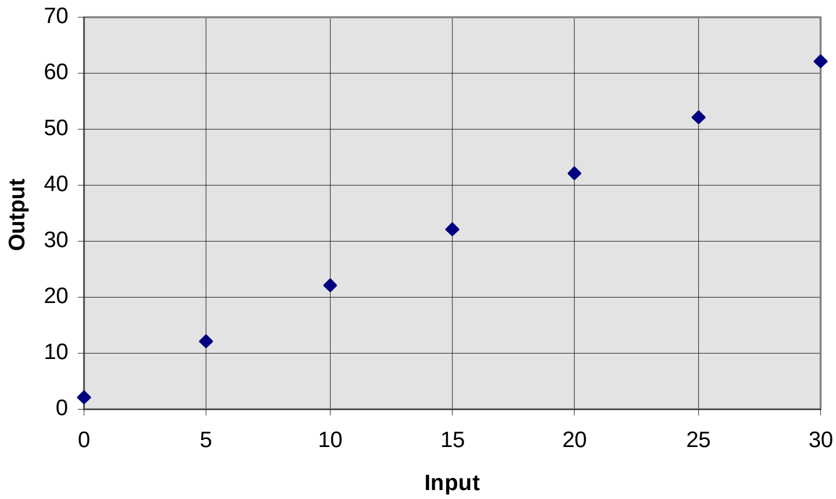


10. *Graph:*



Words:

11. *Graph:*



Formula:

12. *Graph:*

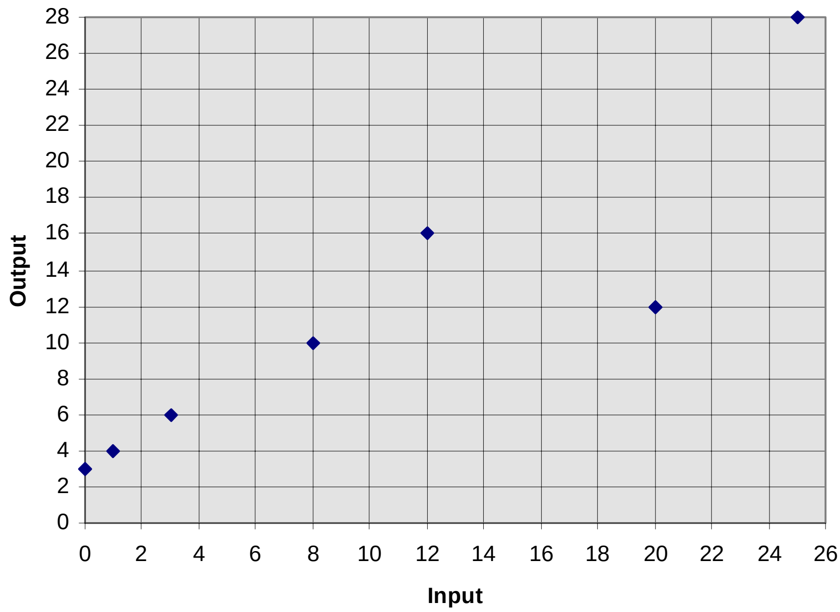


Table:

Input							
Output							