The Van Hiele Theory

Introduction

Teachers will be familiar with some of these incidents:

- A learner recognises a rectangle and defines it as a "long shape".
- A learner claims that a square is not a rectangle.
- A learner does not the see the need for a proof, claiming that the result is "obvious".

Two mathematics educators in the Netherlands, Pierre Van Hiele and Dina Van Hiele-Geldorf, developed a pedagogical theory to explain this phenomenon. They identified relatively stable, qualitatively different levels of understanding through which an individual passes when learning.

As noted by Schoenfeld (1986), this theory does not give a deterministic view of a fixed progression, but is an *empirical* description of relatively stable stages and provides guidance on structuring learners' experiences. Furthermore, since the 1950's this theory has been developed by Van Hiele and has been the topic of many research projects world-wide. Much of the research (Usiskin, 1982; Senk, 1983; Burger & Shaughnessy, 1986; De Villiers & Njisane, 1987) has confirmed that the Van Hiele theory can be used to describe the geometric thinking of school learners, but questions have been raised about certain aspects of the theory. ***MALATI has used the theory, together with the results of the subsequent research, as a guide, and not as a rigid framework, in designing learner materials and in working with teachers to assess learner responses and to plan appropriate instruction.***

In this document we briefly describe the theory (as well as related research) and provide suggestions on how teachers can select activities to encourage the development of geometric understanding.

The theory

The Van Hiele theory describes the way in which the understanding of a new topic may develop. MALATI has chosen to use three levels appropriate for school mathematics, namely the visual, analysis and ordering levels.

We begin by describing these three levels and typical learner responses for each level. We use the descriptors of Fuys, Geddes, Lovett and Tischler (1998).

The Visual Level

Fuys, Geddes, Lovett, & Tischler (1988) describe this as the level on which *"a learner identifies, names, compares and operates on geometric figures, for example, triangles, angles, parallel lines, according to their appearance".* They have identified the following descriptors for this level:

Descriptor **The Learner:**	**Sample Learner Response** **(where necessary)**
1. Identifies instances of a shape by its appearance as a whole (a) in a simple drawing or set of cut-outs (b) in different positions (c) in more complex shapes	
2. Constructs, draws or copies a shape	Using matches, geoboard, tiles, can copy a shape/pattern on paper
3. Names or labels geometric configurations using standard or non-standard names and labels them appropriately	Refers to angles as 'corners', or by colour/letter symbols in a diagram
4. Compares/sorts shapes on basis of appearance as a whole	Sorts because "they look alike/different" "A rectangle is wider than a square"
5. Verbally describes shapes by their appearance as a whole	A rectangle "looks like a square" A parallelogram is a "slanty rectangle" Angles are "arms of a clock"
6. Solves routine problems by operating on the shapes rather than referring to the properties in general	Trial and error in tangram puzzles Places tiles on a rectangle to determine the area of the shape
7. Identifies parts of a figure but **does not** (a) Analyse figure in terms of these parts (b) Think of properties as characterising a class of figures (c) Make generalisations about shapes or related language	

Argument at this level is based on a statement of belief, and not on logical conclusions. This can only be resolved by repeated statement or by claiming authority ("because I say so") (Murray, 1997).

<u>The Analysis Level</u>

At this level a learner analyses figures in terms of their parts and the relationships between these parts, establishes the properties of a class of figures empirically, and uses properties to solve problems (Fuys *et al.*, 1988).

Fuys *et al.* (ibid.) give the following descriptors for this level:

Descriptors **The Learner:**	**Sample Learner Response** **(where necessary)**
1. Identifies and tests relationships between parts of figures, for example, congruence of sides	Learner notes that a square has four congruent sides and four right angles.
2. Recalls and uses appropriate vocabulary for parts and relationships, for example, opposite sides, diagonals bisect each other	
3 (a) compares two shapes according to relationships between their parts (b) sorts shapes in different ways according to properties	Comparing square and rectangle by referring to similarities/differences in sides and angles Makes up rule for sorting quads according to number of right angles
4 (a) interprets and uses verbal descriptions of a figure in terms of its properties, draws a figure from this description (b) interprets verbal or symbolic statement of rules and can apply them	Can explain area rule and recognises when it does/does not apply

5. Discovers properties of figures empirically and generalises these for that class of figures	After colouring congruent angles in a triangular grid the learner notes the sum of angles in each triangle is 180° and then tries to find out whether this is the case with all triangles
6 (a) Describes a class of figures in terms of properties (b) Tells what shape a figure is, given certain properties	Learners describes a square over the telephone: "it has 4 sides, 4 right angles, all sides equal, and opposite sides parallel" Given certain clues about the shape, the learner can tell what shape it is on the basis of the properties
7. Identifies which properties used to categorise one class of figures also apply to another class of figures, compares classes according to their properties	Knowing that a parallelogram has opposite side parallel, a learner will note that this is also the case with rectangles and squares
8. Discovers properties of an unfamiliar class of figures	
9. Solves geometric problems by using known properties of figures or by insightful approaches	A learner works out how to find the area of a new shape by dividing it up into shapes of which the area s/he can already determine
10. Formulates and uses generalisations about properties of figures and uses related language, for example, "all", "every" and "none", **but does not**: (a) Explain how certain properties are interrelated (b) Formulate and use formal definitions. (c) Explain subclass relations beyond checking specific instances against a given list of properties (d) See a need for proof or logical explanations for generalisations discovered empirically, or use related language, for example, "if…then" and "because".	 A learner cannot explain how, in a parallelogram, the idea "opposite angles are equal" follows from "opposite sides are parallel" A definition of a figure consists of a list of properties, some of which are redundant A learner can list the properties of all the quads but cannot explain why "all rectangles are parallelograms" After discovering the angle sum of a triangle by colouring angles on a grid, a learner does not see the need to provide a deductive argument to show why this is valid

Murray (1996, 1997) notes that at this level the concepts can exist for the learners separate from the situations in which they were developed. These concepts exist in a network of related concepts. Terminology and symbols are exact and meaningful to learners and they can formulate their own definitions. These definitions are accepted as binding for logical arguments and discussions. Arguments can be resolved by referring to the definition, for example, "this shape must be a square because it has four sides equal and four right angles". Note that the definitions are not precise and often include redundancies.

The Ordering/ Informal Deduction Level:

According to Murray (1997) the network of related concepts developed in the analysis level becomes complete and stable at this level. Precise definitions are understood and accepted. These definitions are referred to when learners talk about the shapes. Learners understand the relations within and between figures, for example, "the opposite sides of a parallelogram are parallel therefore the opposite angles are equal", and "a square has all the properties of a rectangle therefore it is also a rectangle". Learners are capable of 'if… then' thinking (but not formal proofs) at this level, so logical reasoning can be developed.

Fuys *et al.* (1988) provide these descriptors:

Descriptor **A learner:**	**Sample Learner Response (where necessary)**
1 (a) Identifies different sets of properties that characterise a class of figures and tests that these are sufficient	Selects properties that characterise a certain class of shapes and tests by construction etc whether these are sufficient Explains that two different sets of properties can characterise the same shape
(b) Identifies minimum sets of properties that can characterise a shape	In describing a shape to a friend in the shortest possible way, the learner selects the least number of properties to ensure that the shape is a square.
(c) Formulates and uses a definition for a class of figures	Formulates a definition of a kite and uses it to explain why a figure is/is not a kite
2. Gives informal arguments (using diagrams, cut-out shapes that are folded etc)	
(a) Having drawn a conclusion, justifies the conclusion using logical relations.	Learner concludes that "if $\angle a=\angle b$ and $\angle b=\angle c$, then $\angle a=\angle c$, because they are both equal to $\angle b$"
(b) Orders classes of shapes	"A rectangle is a parallelogram because it has all the properties of a parallelogram as well as the special properties of right angles."
(c) Orders two properties	Given a list of properties of a square, the learner says, " opposite sides are equal is not needed because it already says that all four sides are equal".
(d) Discovers new properties by deduction	A learner explains that two acute angles in right-angled triangle add up to 90° because the sum of the angles in a triangle is 180°
(e) Interrelates several properties in a family tree	
3. Gives informal deductive arguments	
(a) Follows a deductive argument and can supply parts of the argument	A learner can give reasons for steps in a proof when guided through the proof
(b) Gives a summary or variation of a deductive argument	Can give own explanation for "opposite angles of a parallelogram are equal"
(c) Gives deductive arguments on own	
4. Gives more than one explanation to prove something and justifies these explanations using family trees	Learner explains the angle sum of a pentagon equals 540° by dividing it into three triangles or by dividing it into a quad and a triangle, and shows each method using a family tree
5. Informally recognises the difference between a statement and its converse	A learner can recognise that the following are different: • If the corresponding angles are equal, then the lines a parallel • If the lines are parallel, then the corresponding angles are equal
6. Identifies and uses strategies or insightful reasoning to solve problems.	
7. Recognises the role of deductive argument and approaches arguments in a deductive manner, **but does not** (a) Grasp the meaning of deduction in an axiomatic sense, for example, does not see the need for definitions and basic assumptions (b) Formally distinguish between a statement and its converse (c) Establish interrelationships between a network of theorems	A learner recognises the role of logical explanations or deductive arguments in establishing facts (versus inductive or empirical approach): "I know that the angle sum of a pentagon is 540° and I don't have to measure it" A learner has not yet experienced 'proof' in an axiomatic sense (i.e. using axioms, postulates and definitions)

The features of the theory

The hierarchical nature of the levels: According to the Van Hieles, the theory is hierarchical in that a learner cannot operate with understanding on one level without having been through the previous levels. This has been confirmed in research (Burger and Shaughnessy, 1986; Fuys *et al.*, 1988; De Villiers & Njisane, 1987).

Burger and Shaughnessy (1986) have suggested, however, that the levels are not as discrete as suggested by the descriptions. Rather, it appears that learners can be in transition between levels and that they will oscillate between them during the transition period. There is also evidence that a learner's level of thinking might vary across topics and according to how recently a topic was studied (Mayberry, 1983; Fuys *et al*., 1988);

Language: Each level is regarded as having its own language and learners on different levels cannot understand one another. For example, a rectangle might have different meanings on different levels. A learner on the ordering level might regard a rectangle as a special kind of parallelogram, but this is not understood by learners on lower Van Hiele levels. Problems can be encountered when a teacher uses language on a higher level to that of the learners. This has important implications for assessment. Can we penalise a learner who does not regard a rectangle as a special kind of parallelogram? Rather our assessment should alert us to the kind of teaching required by such learners to move them through the levels.

The role of the teacher: The Van Hieles place an emphasis on pedagogy and the importance of the teacher structuring the learners' experiences to facilitate transition through the levels. Development is thus not developmental, but the result of learners having the correct experiences. A number of researchers have indicated activities appropriate for learners on different levels.

For example, Murray (1996) suggests learners on the visual level should be given tasks in which

- the situations are 'authentic' and the new topic/concept should form a natural part of this situation. She suggests that teachers find out about the situations in which a concept was actually developed historically, rather than trying to make up experiences
- the situations need not be concrete or real world
- the stories/models used are not more complicated than the actual concepts involved
- the learners are allowed freedom of interpretation, method and representation.

Murray(*ibid*.) suggests that the teacher can assist the learners in moving through this level by encouraging discussion and argument and by introducing more formal definitions and exact terminology as required.

Holmes (1995) suggests that learners on the visual level be given activities which require them to:

- manipulate and identify geometric shapes
- sort and arrange shapes
- draw and construct shapes
- describe shapes in their own words
- solve problems involving shapes.

Holmes (1995) suggests that activities for learners on the analysis level should:

- be similar to those used for the recognition level but in which the focus is on the properties of the shapes
- involve the classifying of shapes according to properties
- require learners to deriving generalisations inductively, that is, by studying examples
- require the use of appropriate vocabulary.

The intrinsic / extrinsic nature of change:
Van Hiele indicates that the levels are characterised by differences in the objects of thought. For example, on the visual level the objects of thought are the geometric figures. These figures are, in fact, determined by the properties, but someone thinking on this level is not aware of the properties. These properties become explicit on the analysis level where the learners work with classes of figures and the properties are the objects of study. On the ordering level these classes of figures become the objects of study.

Phases within a level: According to Van Hiele, progress from one level to the next involves five phases. Each phase involves a higher level of thinking. These phases are useful in designing activities:

- Information: The learner gets acquainted with the working domain/ field of exploration by using the material presented to him/her, for example, examines examples and non-examples. This process causes him to 'discover' a certain structure.
- Guided/Directed Orientation: the learner explores the field of investigation using the material, for example, by folding, measuring, looking for symmetry.
- Explicitation/Explanation: A learner becomes conscious of the network of relations, tries to express them in words and learns the required technical language for the subject matter, for example, expresses ideas about the properties of figures.
- Free Orientation: The field of investigation/network of relations is still largely unknown at this stage, but the learner is given more complex tasks to find his/her way round this field, for example, a learner might know about the properties of one kind of shape but is required to investigate the properties for a new shape, for example, a kite. The tasks should be designed so that they can be carried out in different ways.
- Integration: A learner summarises all that s/he has learned about the subject, reflects on his/her actions and thus obtains an overview of the whole network/field that has been explored, for example, summarises properties of a figure.

(adapted from Fuys *et al.* (1988) and Presmeg (1991))

Questions from the research

De Villiers and Njisane (1987) as well as Smith (1987) have indicated that the use of hierarchical classification might not be necessary for formal deductive thinking. They have also suggested that the Van Hiele theory needs refinement with regards to the levels at which deduction can occur, and propose that simpler intuitive deductive reasoning might be possible at levels lower than the ordering level.

De Villiers and Njisane (*ibid.)* note that there is some confusion within the writings about the Van Hiele theory as to where class inclusion is supposed to occur.

De Villiers (1987) has also suggested that the Van Hiele uses a limited notion to proof – learners who cannot see the meaning in terms of logical systematisation do not see the meaning of proof. He suggest that, if other meanings of proof were to be used, then this could possibly done at lower Van Hiele levels.

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