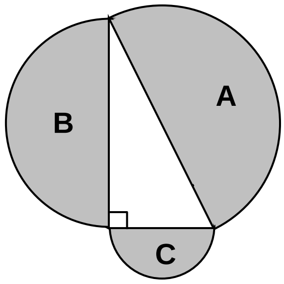
GENERALISING PYTHAGORAS

1. On the worksheet on the next page, different *similar* figures are drawn on the sides of a right-angled triangle. Find the areas of these similar figures (you can *count* the number of units, or you can *calculate* the areas using known formulae, or use Pick's theorem below). Organise your data in the table below, then look for a pattern and *generalise*. Formulate a *conjecture*. Try to *prove* your conjecture.

   In what sense is your conjecture a *generalisation* of the Theorem of Pythagoras (why is it *more general*)?

Figure	Area of figure on shortest side	Area of figure on other side	Area of figure on hypotenuse
A			
B			
C			
D			
E			
F			

   Prove for each of the cases A-F on the worksheet that the three figures shown on the sides of the triangle are *similar*. (But you only have to submit any three.)
   Formulate an argument to prove the following statements:
   - Any two squares are similar
   - Any two circles are similar

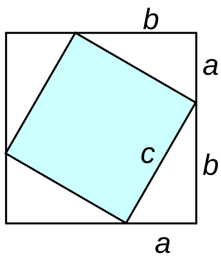


2. A, B and C are semi-circles drawn on the sides of a right-angled triangle.

   Prove that:

   Area A = Area B + Area C

   How is this result related to the Theorem of Pythagoras?



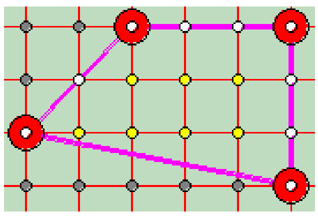
3. In the figure, four congruent triangles are inscribed in the corners of a square. Use the figure to prove the Theorem of Pythagoras, it is

$$c^2 = a^2 + b^2$$

4. Skryf 'n kort refleksie oor die **W**iskunde wat jy gedoen het, veral die wiskundige *prosesse*, asook jou *belewing* van jou voltooiing van die taak.

Pick's theorem gives the relationship between the area of a closed figure and the number of points on square dotty paper:

$$\text{Area} = \frac{1}{2}b + i - 1$$



b is the number of points on the *boundary* of the figure, and
i is the number of points *inside* the figure.

For example, in the figure alongside $b = 9$, $i = 7$, so the Area is 10,5 cm^2.

Is this correct? Convince yourself that Pick's theorem is true … How?

- Try induction: *Specialise* by using Pick to calculate the areas of the specific figures in the worksheet and compare it to answers *using a different method*, e.g. counting squares or using formulae.
- Visit this webpage for an interactive demonstration and a general proof: http://www.cut-the-knot.org/ctk/Pick.shtml

WORKSHEET

On this dotty paper, the unit area is the area of the 1×1 square X. The area of other figures can be found by dissecting or surrounding the figure and adding or subtracting known areas (the whole is equal to the sum of its parts). For example:

Area of Y = $2 \times 2 - 4 \times \frac{1}{2} = 2$ or $4 \times \frac{1}{2} = 2$

Area of Z = $2 \times 4 - (1 + 1 + \frac{1}{2} + 1\frac{1}{2}) = 4$