Malati

Mathematics learning and teaching initiative

Statistics

Probability 1

Grades 4, 5 and 6

Teacher document

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We acknowledge the valuable comments of Heleen Verhage and Donald Katz.

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Introduction and overview of Module 1

The activities in this Module have been designed for Grades 4 to 6. We provide our suggestions on the use of these activities in the different grades, but this will depend on different learners' prior experiences of activities of this nature. The teacher should thus select activities appropriate for different grades and different learners within a grade.

This Module has two focuses:

1. **To encourage the use of systematic representation in solving of problems:** We regard this is an important skill for problem solving in general. This is reflected in the Curriculum 2005 document which emphasises the use of logical processes in problem solving (SO2 and SO10) and knowledge of "ways of counting" (SO6).

   Further, we are also using this form of thinking as a cognitive tool to help learners to solve probability problems by helping them to easily visualize the various outcomes. This is a skill that, according to the research on probability, is lacking in many people. Initially learners are allowed to use their own methods to solve simple counting problems, for example, determining how many different outfits can be compiled with certain items of clothing. Some learners are likely to start by listing the possibilities randomly. By increasing the number of possibilities and requiring that learners be sure they have included all the possibilities, we challenge learners to use more systematic representations. Once learners have recognised the need for systematic listing, the tree diagram can be introduced as a mathematical way of representing possibilities in a systematic way. If the teacher feels at any point that some/all learners are ready for it, simple probability questions can be posed, for example, questions beginning with 'What do you think the chances are of …?'

   The following performance indicators are relevant for the intermediate phase:
   - Recognises the need for systematic representation in the solving of problems
   - Constructs and uses a tree diagram to solve counting problems
   - Generalises to find a mathematical formula for solving counting problems.

   Our experience suggests that some learners might not initially recognise the need for a systematic representation and will continue to use random methods in spite of intervention from the teacher or fellow learners. A learner who does not see this need will struggle to construct an accurate tree diagram. The teacher should thus use the discussion of the initial activities ('Toy Cars' and 'Choosing Clothes 1') to identify those learners who are listing randomly. For those learners who do not see the need for systematic representation, it is recommended that the teacher use the same context (for example, clothes), but change the numbers, for example, by increasing the number of shirts and/or jeans. It is hoped that when faced with the greater number of possibilities, the learner will realise that random listing is not an efficient method. In cases where these additional strategies do not assist the learner, he should be given opportunities at a later date to revisit similar problems. It is important that the teacher does not attempt to introduce the tree diagram until the learner has recognised the need for systematic representation.

The activities in this section are sequenced as follows:

Core:	Toy Cars Choosing Clothes 1 Choosing Clothes 2 Choosing Clothes 3 Ice-Cream Choices 1	Informal **assessmen**t during these activities. The teacher can provide additional practice for learners using these or different contexts, for example, sandwiches and pizzas.
Enrichment / Consolidation:	Holiday in Zimbabwe Counting Shapes	

2. To introduce learners to the ideas of chance:

In Curriculum 2005 both "evidence of ways of counting" and "understanding of the concept of probability" are included along with data handling in SO6. As discussed in our rationale document, the concept of probability is recognised as being particularly difficult. We have identified the following performance indicators as relevant for the intermediate phase:

- Understands the notion of chance: Learners should be aware that the chance / probability of an event can be expressed on a scale of likelihood
- Describes the likelihood of given events using descriptive words (impossible, unlikely, 50% chance, likely, certain) and understands the distinction between the everyday and mathematical uses of these words
- Chooses a suitable event given a description of its likelihood
- Understands that not all events have an equal likelihood of happening, for example, the outcomes of tossing a drawing pin
- Understanding of the notion of "fairness" in relation to equal likelihood.

Some of the difficulties associated with teaching probability stem, firstly, from the fact that the mathematical use of words such as "likely" and "unlikely" conflicts with the everyday use of these words. For example, the word "likely" used in everyday language means that something "could happen", whereas it is used in the context of probability to describe the likelihood of events on a scale, that is, it describes an event that has more than 50% chance of happening.

Secondly, the topic often contradicts learners' everyday experiences and intuitions, for example, learners might say that a slice of bread buttered on one side will land butter-side down when dropped because "it always happens this way", or that a soccer team will throw "heads" in their next game because the team has thrown "tails" in the last six games. Research by Hawkins and Kapadia (1984) suggests that some learners can give the correct theoretical predictions, but tend to revert to their original "hunches" when the results of experiments do not confirm the prediction. In this Module, therefore, we are acknowledging that learners' experiences cannot be ignored. We emphasise the use of discussion, for it is in this atmosphere of sharing that the differences between everyday experiences and the mathematical probability can be made explicit and in which learners can convince one another.

It is important that the teacher diagnose problems in these two areas. "Diagnostic Activity 1" is provided for this purpose. Consistent with our use of the "subjectivist" approach in our wider theoretical framework (see Malati probability rationale document), we have found that discussion amongst learners can assist in addressing this problem. By sharing their ideas with one another, learners can reflect on their own ideas and possibly re-evaluate these if necessary. In the case where some learners who have a good understanding of chance and have displayed the ability to justify their responses in the class discussion, it is recommended that the teacher place these learners with those requiring remediation. The group can work through the assessment questions or sections of the "Likelihood Scale 1" together. In the case where the problem is a general one in the class, the teacher should provide additional challenges, for example, in a problem in which three balls are placed in a bag, the teacher could increase the number of balls.

The activities for this section are as follows:

Core:	Likelihood Scale Soccer A Choir Competition	Diagnostic Activity 1

We suggest that the following activities are appropriate for use in the different grades in the intermediate phase. The teacher should, however, select according to the needs of the learners. If required, the teacher can also select activities from Grade 7 in the senior phase Module.

GRADE 4	**GRADE 5**	**GRADE 6**
Toy Cars *More simple counting activities – see teacher* notes for 'Toy Cars' Soccer	Choosing Clothes 1 Choosing Clothes 2 Wrapping Presents Choosing Clothes 3 Soccer	Revisit Choosing Clothes Revisit Wrapping Presents Ice Cream Choices 1 Holiday in Zimbabwe Counting Shapes Likelihood Scale A Choir Competition

Toy Cars

A toy factory makes toy cars. They make big cars and small cars. They paint some of them blue, some of them red and some of them yellow.

The factory then sells these cars. For example, you could buy a big red car, or a small yellow car.

1. What would you buy?

2. How many DIFFERENT kinds of cars does the factory make?

3. The toy factory decides to make their cars in different colours. They still make big cars and small cars, but they use more colours:

   Blue
   Red
   Yellow
   Green
   Black

   How many DIFFERENT kinds of cars do they make now?

Teacher Notes: Toy Cars

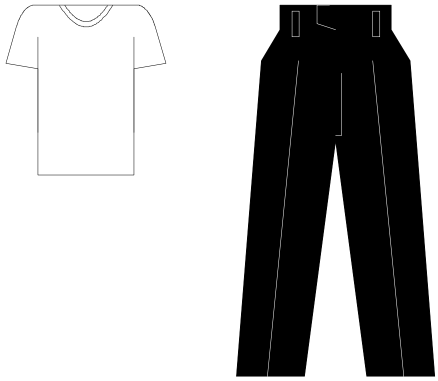
This activity is intended as an introduction to systematic ways of counting. Learners should be encouraged to draw or list their solutions, using colours if possible. Although this may seem tedious, it forces the learners to understand the problem correctly, and they are more likely to come up with all the possibilities than if they simply try and write down an answer.

Some learners might respond to question 2 by saying there are 5 different kinds of cars. In this case, they have simply added 'big', 'small', 'red', 'blue' and 'yellow' and have not understood the question. It is therefore important that learners draw or list all the possibilities as these representation provide access to their thinking.

In the Junior Primary Interim Core Syllabus, this kind of problem was referred to as Multiplication (Combination). However, the teacher should NOT point out to the learners that this activity requires multiplication. Rather, learners should be encouraged to compare their solutions with each other, and to motivate how they know that they have ALL the possible cars. However, if some learners have realised the multiplication rule by the time they answer question 3, they should be encouraged to share their reasoning with the rest of the class.

<u>Further Activities:</u> The teacher should provide additional simple counting activities. The car context can be extended by varying the number of colours or sizes (as in question 3) or by introducing a new context. In the latter case the teacher should choose contexts that are suitable for drawing.

Choosing Clothes 1



Siyanda has a white, a yellow and a red shirt and she has black, green and navy jeans.

Siyanda wants to dress differently every day of the week.

On Monday she wears the white shirt with the black jeans. Can you help her to make combinations for the rest of the week?

Are there enough combinations to dress differently every day of the week?

Teacher Notes: Choosing Clothes 1

This activity deliberately avoids suggesting any form of systematic representation. ***It is important that learners are only given the first activity to start with and that they decide on their own form of representation before seeing the tree diagram.*** *Learners can simply give possible combinations for the different days of the week. They should find that there are more combinations than days of the week.*

In the discussion that follows, it will become evident that different learners used different combinations. Learners might experience a need to list ALL the possible different combinations. For example, learners may simply write the combinations as follows:

> *white shirt, black jeans*
> *white shirt, green jeans*
> *white shirt, navy jeans*   *etc.*

Learners might want to write out all the possibilities in full each time – this is time-consuming and they should be encouraged to abbreviate, for example, use WB, WG etc.

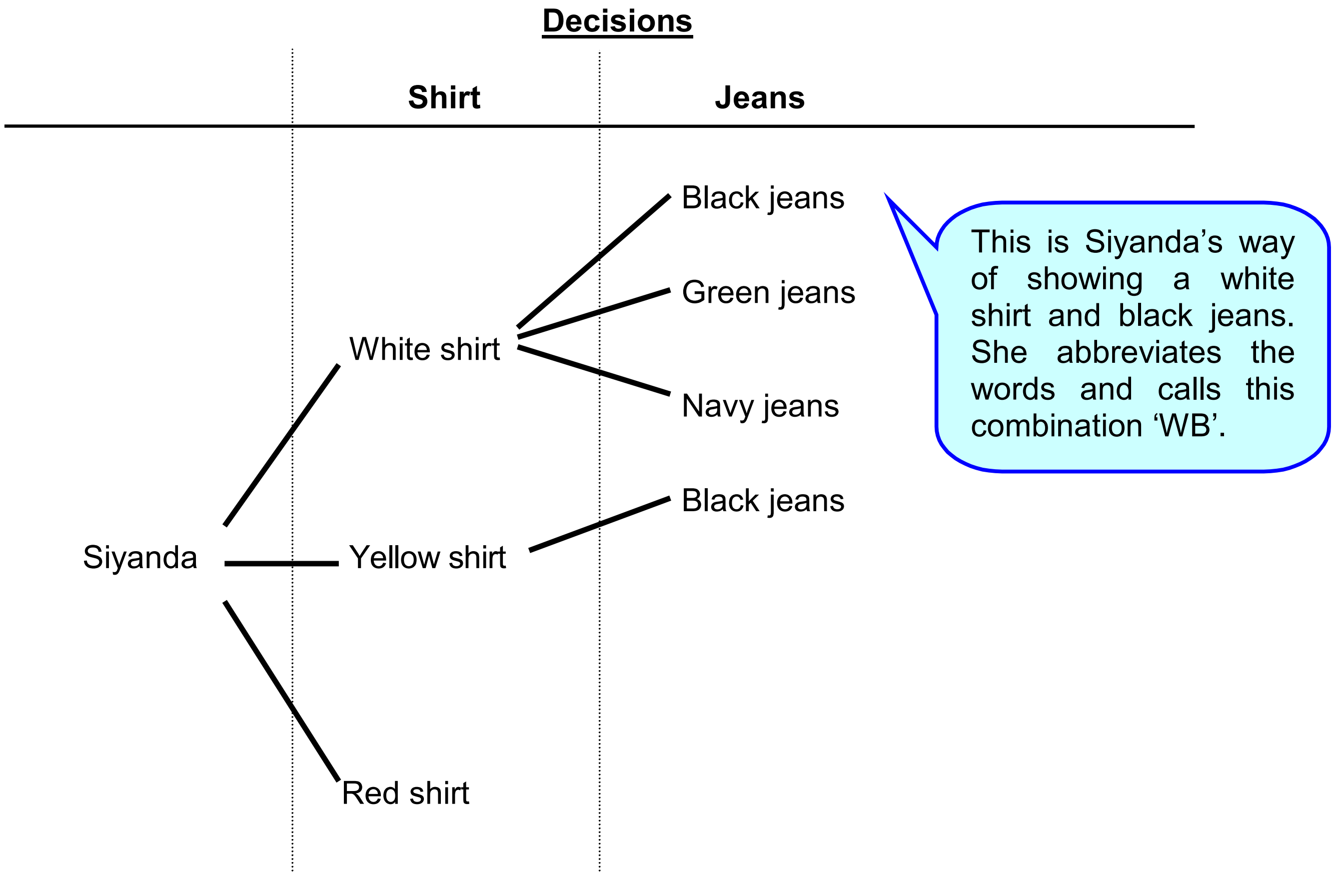
Younger learners are inclined to 'match' the words (or even pictures) with lines, for example:

Siyanda has a white, a yellow and a red shirt and she has black, green and navy jeans.

A whole-class discussion might result in a systematic way of listing these combinations. All methods are acceptable during the whole-class discussion, as long as the learner shows some indication of understanding that there are several combinations, and that there is a ***need to be systematic*** *about indicating these combinations. If learners do not show any tendency towards the need for systematic representation, they can be challenged by asking them how they know that all the possibilities have been mentioned and by being given larger numbers in the same context. If learners still do not see the need for the systematic representation they should not go on to Choosing Clothes 2, but should be given more counting activities at a later stage.*

Choosing Clothes 2

1. Siyanda uses this diagram to help her work out how many ways she can dress:



(a) On Tuesday, Siyanda wears a yellow shirt and black jeans. Colour the branches in the diagram which show this combination.

(b) Complete the diagram for Siyanda. According to this diagram, in how many different ways can Siyanda dress?

(c) Using your diagram, make a list of all the combinations using the two-letter combinations. For example, WB, WG, ...

(d) Siyanda's diagram is called a '*tree diagram*'. Discuss why you think it is given this name.

2. Siyanda buys a new shirt – a pink shirt. Extend the tree diagram. In how many different ways can she dress now?

3. Siyanda buys two hats to wear with her shirts and jeans. One hat has stripes and the other hat has dots.

   (a) In how many different ways can she dress now?

   (b) Use a tree diagram and make a list of all the possible combinations, using three letter combinations, e.g. WBS means white shirt - black jeans - striped hat.

Teacher Notes: Choosing Clothes 2

It is unlikely that learners, especially younger learners, will use the tree diagram spontaneously. In this case the teacher can introduce it as a mathematical way of representing all the possible outcomes. Question 1 encourages learners to ensure that they understand how the diagram works. Then the learners are encouraged to complete the diagram and count the 9 ways of dressing. The listing using the initials of the words provides practice in reading off the tree diagram. It is important that learners understand the word 'tree diagram' and how the number of ways/combinations is arrived at: each combination is a complete set of branches across the different levels, for example, white shirt and black jeans, not just the last branch (in this case, black jeans).

Question 3 provides more practice in using the tree diagram within the same context. Some learners might calculate 12 different combinations by looking at their new tree diagram, while others will immediately know that it is '3 more' because the new shirt can be worn with three different jeans. Similarly, different methods might be used to calculate that there are 24 different combinations including the hats (Question 3), if one includes the new shirt. The teacher should focus attention on the different tree diagrams produced by, for example, starting with the hats or starting with the shirts. The learners could be asked to discuss whether these tree diagrams produce the same results, and which one is more 'efficient' in terms of recording.

If learners have finished and are waiting for their peers, the teacher can ask them to add/take away an item of clothing at any of the levels and work out how many combinations are now possible.

Wrapping Presents

Dali makes extra pocket money by running a present-wrapping service for his friends.

Dali wraps each present in paper and adds a coloured ribbon.
Dali has three different types of wrapping paper – plain, dotty and striped. He has five different coloured ribbons – blue, red, green, yellow and white.

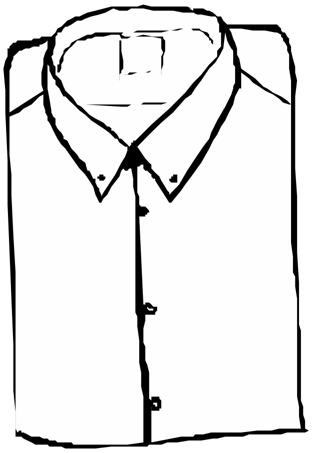
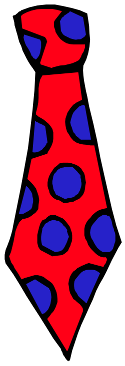
Draw a tree diagram to work out how many different ways Dali can wrap a present.

Teacher Notes: Wrapping Presents

This activity provides learners with an opportunity to practise drawing a tree diagram. The teacher can extend the activity by changing the number of different papers and/or coloured ribbons or by extending the service to, for example, adding gift tags.

The teacher can use different contexts to provide additional consolidation, for example, white or brown sandwiches with different fillings; different pizza bases and toppings.

Choosing Clothes 3

Mzwake has five different shirts and three different ties to wear to work.
His wife says she has an easy way of working out how many different combinations of a shirt and tie he can wear. She says it is:

"the number of shirts" × "the number of ties"

But Mzwake disagrees. He says the number of different combinations is:

"the number of shirts" + "the number of ties"

Without making a list or drawing a tree diagram, explain which of these rules you think is correct and **why**.

Does you rule work in Wrapping Presents?

Teacher Notes: Choosing Clothes 3

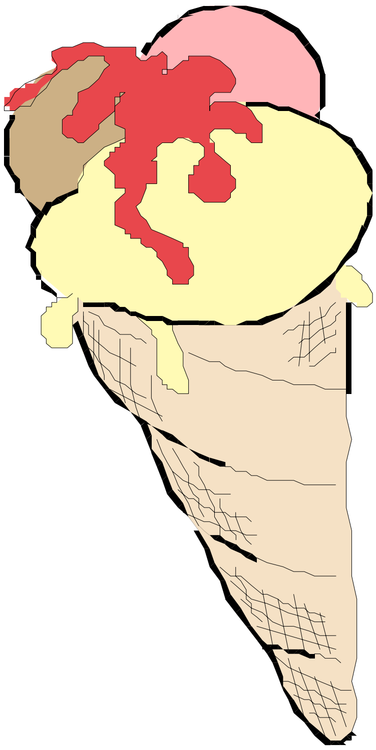
In this activity, learners are encouraged to reflect on the mathematics behind the problem of "how many different combinations", and should be able to explain clearly why multiplication and not addition produces the correct number of combinations. Saying that "multiplication is correct because it gives the right answer" is NOT a sufficient response...the teacher should ask WHY is this the case.

For example, learners might say that there are 15 combinations because for each shirt (and there are 5), there are 3 different choices of tie.

Learners may be encouraged to use tree diagrams or other systematic representations to PROVE their explanations.

The activity can be extended by changing the number of shirts and/or ties.

Ice Cream Choices 1!



MENU

Cones

Plain Chocolate

Ice-Cream Flavours

Vanilla Strawberry Fudge

Toppings

Nuts Jelly tots Banana

This is the menu in an ice-cream shop. The shop sells “combos” which consist of one cone, one scoop of ice-cream and one topping. For example, you can order a “combo” with a plain cone, strawberry-flavoured ice-cream and nuts on top.

What is another possible “combo” you can order?

1. How many different “combos” can you order at this ice-cream shop? Draw a tree diagram to show all the different “combos” (**with a cone, one flavour and one topping**).

   Use the tree diagram to answer the following questions:

   (a) How many different ice-creams can you order which have banana on top?

   (b) Andile does not like chocolate. How many different ice-creams can she order?

   (c) Denise does not like strawberry or fudge ice-cream. How many different ice-creams can she order?

(d) Phumeza does not like nuts. How many different ice-creams can she order?

(e) Lungi does not like chocolate OR nuts. How many different ice-creams can she order?

2. During the holiday season the owner of the ice-cream shop decides to add to the menu: She adds lemon-flavoured ice-cream to the ice-creams and Flake to the toppings.

   Without listing the possible "combos" or drawing a tree diagram, explain how you can find the number of "combos" you can now order from the ice-cream shop.

Teacher Notes: Ice-cream Choices 1:

This activity provides more practice in drawing and interpreting tree diagrams. Learners should be encouraged to reflect on their 'clothes' tree diagrams in order to help them construct this new tree diagram. ***It is essential that the learners complete Choosing Clothes 1, 2 and 3 before attempting this activity.***

It is important that learners realise that each 'level' on the tree diagram represents a different decision – for EACH choice of cone there are three choices of ice-cream, and for EACH of these there are three choices of topping. Thus there are two groups of three groups of three ice-creams (2 x 3 x 3) = 18. It is important that the teacher points out that the branches at each level do not represent separate ice-creams, but that one ice-cream (e.g. plain cone, vanilla ice-cream and nuts) is represented across all three levels – the teacher should follow the branches across the various levels to demonstrate this. Learners can then be asked to come and use colours to indicate on a tree diagram on the blackboard where, for example, the ice-cream with chocolate cone, strawberry flavour and banana topping, is represented. They must show the ***whole*** *series of branches and not just the last one (topping).*

Teachers should ensure that learners can interpret the tree diagram before continuing.

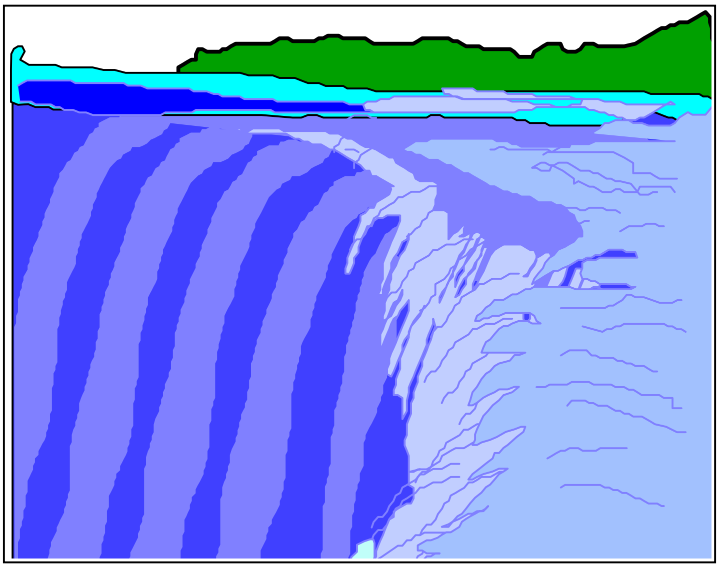
The effect of cancelling out different 'branches' of the tree diagram is also investigated in this activity… not having a chocolate cone cancels out one half of the tree diagram while not having banana on top cancels only a third.
By counting the ice-creams on the tree diagram, learners should complete the questions and validate their answers in the groups:

- *1(a) 6 with banana on top*
- *1(b) excluding the ice-creams with chocolate cone, there are 9 possible ice-creams*
- *1(c) two groups of 3, or 18 – 12 gives 6 ice-creams*
- *1(d) six groups of 2, or 18 - 6 gives 12 ice-creams*
- *1(e) only the 'plain cone' half of the tree diagram can be considered, and only the ice-creams without nuts, in other words (18 - 9) - 3 or 3 x 2 gives 6.*

Question 2 requires that learners reflect on the mathematics behind the problem, namely that there are 2 x 4 x 4 combinations. The number of items on the menu has been changed deliberately to prevent learners using a known answer in their explanation.

Teachers can easily extend this problem by asking learners who are finished to add or subtract another choice at any of the levels.

Holiday in Zimbabwe



The Lukhele family is going on holiday to Zimbabwe. The travel agent can offer the following options:

Length of Stay	Transport	Accommodation
S1 - 3 weeks	T1 - bus	A1 - camp
S2 - 4 weeks	T2 - train	A2 - hotel
	T3 - aeroplane	

1. How many different options does the Lukhele family have for their holiday?

2. Mr Lukhele hears of some very good apartments which can be hired by tourists who do not want to camp or stay in a hotel. How many different holiday options does the Lukhele family have now?

3. Mr Lukhele has a limited budget. He will not be able to afford to travel by aeroplane AND to stay in a hotel. How many different holiday options does the Lukhele family have?

4. The travel agent is promoting tours to Italy and Spain. It is too far to travel by bus or train, so if the Lukhele family decides to choose one of these two countries for their holiday, they will have to fly. It is winter in both these countries, so the family will not be able to camp. How many different holiday options does the Lukhele family have?

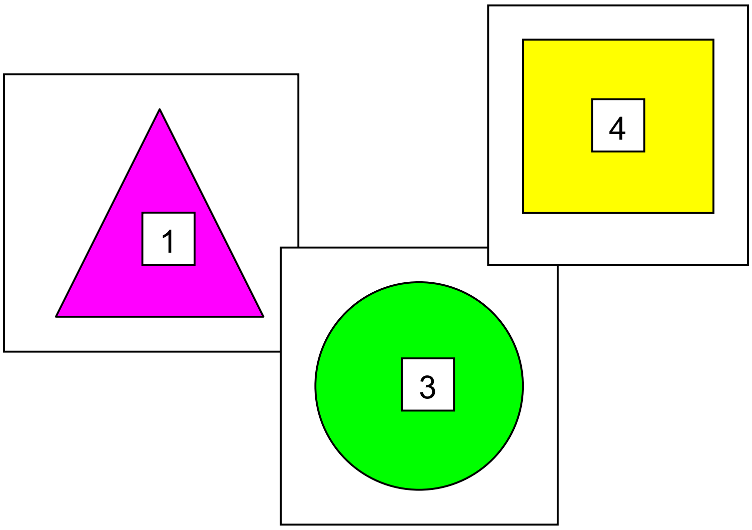
Length of Stay	**Transport**	**Accommodation**	**Country**
S1 - 3 weeks	~~T1 - bus~~	~~A1 - camp~~	C1 - Zimbabwe
S2 - 4 weeks	~~T2 - train~~	A2 - hotel	C2 - Italy
	T3 - aeroplane	A3 - apartment	C3 - Spain

Teacher Notes: Holiday in Zimbabwe

This activity also requires a systematic way of counting and recording results. Learners will find the tree diagram useful. It is suggested as an Enrichment/Consolidation activity for learners who have already grasped the concept of tree diagrams and/or systematic listing, or those who need more practice.

Additional activities can be designed using, for example, combinations of items on the menu (starters, main courses, desserts) or pizza combinations (base – thin or thick, wholewheat or plain; toppings).

Counting Shapes



Geoshop sells packs of cards. There are 15 cards in each pack – triangles numbered from 1 to 5, circles numbered from 1 to 5 and squares numbered from 1 to 5.

Andile and Khululwa play a game with these cards. They draw cards one at a time from the pack and put them back straight away. Andile wins if the card he takes is any triangle. Khululwa wins if the card he draws has an odd number (except if it is a triangle). Andile has started to show his ‘winning cards’ on this grid.

	Square	Triangle	Circle
1		**A**	
2		**A**	
3		**A**	
4			
5			

1. Complete the grid showing Andile’s and Khululwa’s ‘winning cards’.
2. Who do you think has more chance of winning this game? Why?
3. You are invited to join Andile and Khululwa in playing this game. You will win if the card drawn has an even number, except if it is a triangle. Would you accept the invitation to play? Why?

Teacher Notes: Counting Shapes

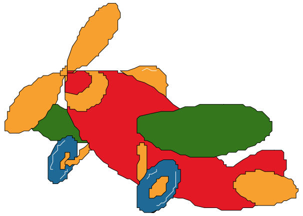
This activity suggests an alternative way of representing results, namely a grid in which the chances of 'winning' are clear at first glance. ***It is usually not necessary for the teacher to explain the grid – time for reflection and discussion with peers should enable learners to successfully complete the grid.***

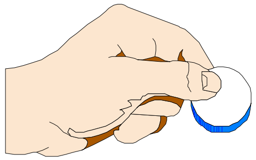
Initial discussions on chance and fairness can also follow. Learners should reach the conclusion that Khululwa has a better chance of winning than Andile (6 blocks of the grid marked as opposed to only 5), and that the player in Question 3 has even less of a chance of winning (4 blocks).

The teacher can also ask why the number of blocks adds up to 9.

The Likelihood Scale 1

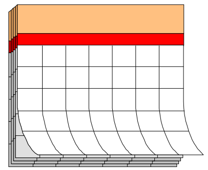
Sometimes we know that an event cannot happen, for example, we cannot fly to the sun. We say the event is **impossible**.





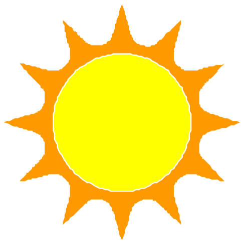
Some events have a **50% chance** of happening or not happening. For example, when we toss a coin there is an equal chance of getting 'heads' or 'tails'. So we say that there is a **50% chance** that a coin will land on 'heads' when we toss the coin.

Sometimes we are sure that an event will happen. For example, Wednesday will come after Tuesday. We say that the event is **certain**.



Impossible	Unlikely	50% chance	Likely	Certain

In summer, it doesn't rain much in Cape Town, so on a chosen day in December, it is unlikely that it will rain.



If we choose a day in June, we cannot say that it is impossible that it will rain on that day in Cape Town. We cannot say that it is certain either!
But June is in winter and it rains in Cape Town in winter so we say that it is likely that it will rain in Cape Town in June.

Impossible	Likely	**50% chance**	Unlikely	Certain

1. Choose words from the scale above to help you describe the likelihood of each of these events:

   (a) Ben has two marbles of the same size in his pocket, a green one and a red one. He puts his hand into his pocket and, without looking, takes out a **red** marble.

   (b) Cindy has three marbles of the same size in her pocket, a green, a blue and a red marble. She puts her hand into her pocket and, without looking, takes out a **red** marble.

   (c) Leroy has six red marbles of the same size in his pocket. He puts his hand into his pocket and, without looking, takes out a **blue** marble.

2. In each row of this table, an **event** is described. Put a tick in the column which best describes the **likelihood** of each event.

Event	**Impossible**	**Unlikely**	**50% chance**	**Likely**	**Certain**
(a) You will turn 2 years old on your next birthday.					
(b) Nosipho has four marbles of the same size in her pocket, a red, a blue, a green and a white marble. She takes out a white marble when she puts her hand in her pocket without looking.					
(c) A coin is tossed to decide who has the kick-off in a soccer match. Bafana Bafana wins the toss at the start of their next match.					
(d) A slice of bread with butter and jam spread on one side is dropped on the floor. It lands with the jam side facing up.					
(e) A dice is thrown and the number '3' lands on top.					
(f) A drawing pin is tossed and it lands with the pin facing downwards.					

Sometimes the words we use in mathematics are the same as those we use every day. But we have to be careful because the **meanings** of the words can be different. Look at this example:

Nosipho has five marbles of the same size in her pocket, a red, a blue, a green, a yellow and a white marble. She takes out a white marble when she puts her hand in her pocket without looking.

It is possible that Nosipho takes out a white marble, but because this is only one of the five marbles in Nosipho's pocket, there is less than 50% chance that this will happen. In mathematics we use the word "unlikely" to describe this.

3. In this table the likelihoods are marked by ✔s. In each case, think of an **event involving marbles in a bag** which would have this likelihood. Write your answer in the column called 'Event'.

Event	**Impossible**	**Unlikely**	**50% chance**	**Likely**	**Certain**
(a)	✔				
(b)			✔		
(c)				✔	
(d)		✔			
(e)					✔

Teacher Notes: Likelihood Scale 1

This activity is an introduction to chance and likelihood. Learners are required to classify certain happenings (events) according to the likelihood of each event occurring. At this stage the classification is mainly ***descriptive*** *and not numerical.*

One of the difficulties of teaching probability is that the mathematical use of words such as "likely" and "unlikely" conflicts with the everyday use of these words. For example, the word "likely" used in everyday language means that something "could happen" or it is possible, whereas it is used in the context of probability to describe the likelihood of events on a scale, that is, it describes an event that has more than 50% chance of happening. By including this activity in this Module we are acknowledging that these everyday meanings cannot be ignored, but that the difference between these and the mathematical use of the words needs to be made explicit to learners.

It is important the learners be encouraged to read through the introduction before proceeding with question 1. In order to ensure that they have read and understood this, the teacher can ask for other examples of events that are "impossible", "certain", "likely" or "unlikely" or have "50% chance" of happening .

It is essential that a whole-class discussion is held after the learners have completed 1(a) to 1(c) and discussed these in their groups

1(a) Ben has two marbles in his pocket, one of which is red, thus there is a "50% chance" of drawing either colour (both outcomes are equally likely).

1(b) Cindy has three marbles in her pocket, one of which is red. There is thus more chance of her drawing a marble of another colour (blue or green), than of drawing a blue marble. The likelihood is thus "unlikely". Another explanation is that there is LESS chance of drawing the red marble now than there was in 1(a) – because there are now 3 marbles. Thus if one looks at the scale, the likelihood must be on the left side of "50% chance".

If learners need to be challenged, the teacher can add more marbles (e.g. 8 marbles, only one of which is red), or ask the learners whether they would bet money on the red marble.

1(c) There are only red marbles in Leroy's pocket, so he cannot remove a blue marble. The event is thus "impossible".

The Likelihood Scale activity refers to a number of contexts and introduces important ideas about probability which will be revisited in later activities, namely.

- *the notion that not all events have an equal likelihood of happening*
- *in some cases the likelihood can be determined by reasoning, that is, by considering all the possible outcomes and the chance of each occurring. For example, in question 1(a) there is an equal chance of drawing a green or a red marble so there is a "50% chance" of drawing a red marble.*
- *in other cases it is only possible to predict the likelihood based on experimentation, that is, on what* ***usually*** *happens. For example, if one tosses a drawing pin, there are two possible ways the pin can land, but the likelihood of*

each happening is not equal. This example also reinforces the idea that although there might be two possible results / events, these are not always equally likely. These ideas should be emphasised in discussion.

Some learners might base their decisions on personal experience, for example, in question 1(a) a learner might feel that the green marble will be drawn because green is his/her favourite colour or in question 2(d) a learner might think that a six has a greater chance of being on the top of the dice because six is his/her lucky number or because it is the biggest number. Such responses are well-documented in the literature. It is hoped that if the learners are able to discuss their responses, they might convince one another that such responses are not mathematically correct. Some of the issues are tackled in more detail in later activities.

Some learners may argue that there is a "50% chance" of an event happening because it 'might or might not happen'. However, in the mathematical context, a "50% chance" means that an event has an equal chance of happening or not happening.

Answers:

2(a) There will not be any one-year-old children in the class, so no-one will turn 2 on his/her next birthday. Thus the event is "impossible".

2(b) There are six white balls and four red balls in Nosipho's pocket, therefore there is more than "50% chance" that a white will be drawn. So the answer is "likely".

2(c) When tossing an unbiased coin there is a "50% chance" that it will be heads and a "50% chance" that it will be tails. So there is a "50% chance" that Bafana Bafana will win the toss in their next game. Learners might base their answers on what they know has happened in previous games, but it should be emphasised that the coin cannot remember what happened in previous games! It does not matter if Bafana Bafana have won the toss 3 times in a row: there is still a "50% chance" as each coin is tossed independent of previous tosses.

2(d) Learners might base their decisions on their personal experience, that is, the slice always lands with the jam side down! Although theoretically there are only two sides on which the slice could land, the weight of the spread might affect the way it falls. It is possible that there is a greater chance of the slice landing on the jam side than on the non-jam side. The estimated difference can only be determined experimentally, but "unlikely" would be appropriate here. An answer of "50% chance" should also be accepted here.

2(e) There is an equal chance of a dice landing with any of the six numbers on top. There is less chance of the '3' being on top than of any of the other five numbers being on top, so the answer is "unlikely". Learners might be influenced by their personal experience, for example, their favourite/ lucky numbers or on the size of the numbers.

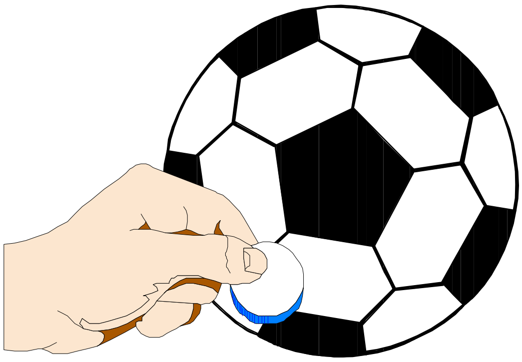
2(f) It is more likely that the pin will land with head up than with the pin facing upwards. An answer of "unlikely" is sufficient here as the estimated likelihood can only be determined experimentally.

In question 3 learners are required to think of their own events to match the given likelihoods. They should be encouraged to compare and discuss one another's suggestions. The teacher should check that the sentences used are actual 'events'. For example, "tossing a coin" is ***not*** *an event, but "tossing a coin and getting heads" is an event. Also, "7 blue marbles and 3 red marbles" is not an event, but "drawing a red marble from a bag containing 7 blue and 3 red marbles" is acceptable.*
Learners should give sound reasons for the events that they provide.

General Comments:
Some learners might be uneasy that the events in both 2(e). and 2(f) be classified as "unlikely" as, when considering the scale of likelihood, the event in 2(e) is clearly more unlikely than that in 2(f). The descriptive words used in this activity are thus not always adequate enough to distinguish between the likelihood of some events. In the activity 'Likelihood Scale 2' we introduce learners to the terms ""very unlikely" and "very likely" to make the descriptions more precise. These terms are, however, also not adequate and for this reason the use of rational numbers to describe probability is introduced at a later stage ('The Probability Scale').

If teachers feel that learners need to have the Likelihood Scale expressed in terms of percentages (e.g. "unlikely" between 25% and 50% and "likely" between 50% and 75%), they should proceed cautiously. The use of rational numbers may hinder some learners, and it is suggested that teachers consider delaying this until the learners have a sound basic concept of chance.

Soccer



In a game of soccer the two teams toss a coin to decide who has the kick-off. One team chooses “heads” and the other team chooses ”tails”. The team whose choice lands on top of the coin gets to start the game.

1. What do we mean when we speak about the “heads” and “tails” of a coin?

2. Of course both teams want to win the kick-off. Why do you think they choose to toss a coin to decide who starts the game? Explain your answer.

3. Could the teams use a dice to decide who starts the game? Explain.

Teacher Notes: Soccer

The tossing of a coin was referred to briefly in the "Likelihood Scale 1" activity. Some learners will use their personal experience to argue that a there is a greater chance of a tossed coin landing on either heads or tails. In an attempt to avoid this type of response, the context of a game of soccer has been chosen for this activity as, in our experience, learners seem happy to accept that the use of the coin in such a case is "fair".

In the case of a coin, the likelihood of getting "heads" or "tails" can be determined theoretically. Some learners might argue that the coin could land on its "edge". It should be stressed, however, that the chance of such an occurrence is so slight that we can disregard it.

Learners will be aware that the South African coins no longer have "heads". One side of the coin is simply chosen as "heads" and the other as "tails". The origin of these words can also be explored.

Use of the terms "equal chance", "equal likelihood" and "50% chance" should be encouraged in this activity. The fact that the tossing of the coin is "fair" to both teams might also arise. The teacher should note, however, that "equal likelihood / chance" is not necessarily the same as "50% chance". For example, if one has a bag containing a red, blue and green marble, there is an equal likelihood / chance of drawing any one of the three marbles from the bag, but there is not a "50% chance" of drawing each marble. There is in fact a $33\frac{1}{3}$ *% chance of drawing each marble.*

Question 3 is intended to challenge learners to consider what 'tools' can be used to make a "fair" decision amongst a number of possibilities. In the case of a dice the likelihood of the different events happening, that is, of different numbers landing on top, can be determined theoretically. Learners could suggest that one team chooses the even numbers on the dice, and the other team chooses the odd numbers, or one team could choose the numbers 1 to 3, and the other team the numbers 4 to 6. The teacher should encourage the learners to assess the validity of one another's ideas.

The notion of fairness is explored further in the activity 'A Choir Competition'.

Diagnostic Activity 1

Impossible	Unlikely	50% chance	Likely	Certain

Vusi has a bag containing marbles of the same size. Each time he closes his eyes and removes one marble from the bag.

Choose words from the scale above to describe the likelihood of each of these events:

1. There are two marbles in the bag, a black marble and a white marble. Vusi removes a black marble.

2. There are three marbles in the bag, a black one, a white one and a yellow one. Vusi removes a black marble.

3. There are five marbles in the bag, a black one, a white one, a red one, a blue one and a yellow one. Vusi removes a white marble.

4. There are five marbles in the bag, three blue marbles and two yellow marbles. Vusi removes a blue marble.

5. There are six red marbles in the bag. Vusi removes a red marble.

6. There are six marbles in the bag, three red marbles and three blue marbles. Vusi removes a white marble.

Teacher Notes: Diagnostic Activity 1

As noted in the introduction to this Module, in order for a learner to proceed after the activity "The Likelihood Scale", s/he needs to have an understanding of the notion of chance. It is thus suggested that the teacher perform this short diagnostic assessment to identify any problems in this regard. (The class discussion during the completion of "The Likelihood Scale" will also provide an opportunity for the teacher to identify these problems.)

Remediation:

It is recommended that learners with different perspectives be given the opportunity to discuss their answers. By sharing their ideas with one another, learners can reflect on their own ideas and possibly re-evaluate these if necessary. It is thus suggested that the teacher identify some learners who have a good understanding of chance and have displayed the ability to justify their responses in the class discussion. These learners should be placed with those requiring remediation. The group can work through the assessment questions or sections of the "Likelihood Scale" together. The teacher has an important role to play here, too, in providing the correct challenges and clarifying the use of the terminology for mathematical purposes. For example, in a problem in which three balls are placed in a bag, the teacher could increase the number of balls.

A Choir Competition

A choir competition is being held at the Good Hope Centre in Cape Town. None of the choirs wants to sing first.

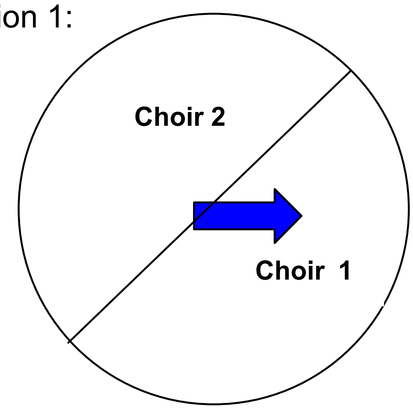
1. If there are **two** choirs in the competition, how can we decide who sings first? Remember that the method we choose must be fair to all the choirs taking part, that is, each choir must have an *equal chance* of being chosen.

2. What if there are **three** choirs in the competition?

3. What if there are **six** choirs?

4. What if there are **fourteen** choirs in the competition?

5. Marie says that she will use this 'spinning wheel' to make a fair choice between the two choirs in question 1:



She spins the wheel and looks where the arrow is pointing when the wheel stops.
If the arrow is pointing to Choir 1's semicircle, then Choir 1 must sing first.
If the arrow points to Choir 2's semicircle, Choir 2 must sing first.

(a) Is this a fair method? Explain.

(b) Now design a spinner that could be used if there are **three** choirs.

6. Godfrey sings in Choir 1 and he does **not** want to sing first.

(a) Help Godfrey to design a spinning wheel that will give his choir less chance of having to sing first than Choirs 2 and 3 will have. (Remember that it must not be too obvious that Godfrey is cheating, or the spinning wheel will not be used.)

(b) Godfrey has another other idea: he says that if there are **two** choirs, they should toss a drawing pin to decide who sings first.

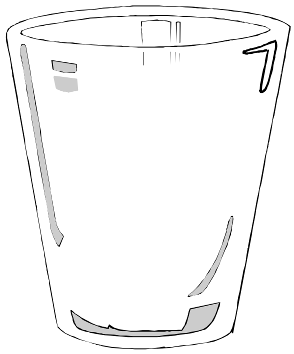
Godfrey says his choir (Choir 1) will sing first if the drawing pin lands with the pin facing up. Choir 2 must sing if the pin is facing down.

Why has Godfrey chosen this rule?

(c) Godfrey says that if there are **three** choirs he would choose to toss a plastic cup like this.

Why has he chosen to use this cup?

What rules should he choose if he does not want his choir to sing first?



Teacher Notes: A Choir Competition

This activity develops ideas introduced in the "Soccer" activity. Learners are required to decide how a 'fair' choice can be made so that each choir has an ***equal chance/ likelihood*** *of being chosen.*

Learners might suggest "taking a vote" or choosing the choir that "arrived last". They should be challenged to consider whether these methods are 'fair'. Would they be happy to accept the decision if they were singing in one of the choirs?

Possible answers:

Question 1: Tossing a coin; concealing two pieces of paper behind your back; drawing names from a hat; selecting certain numbers on a dice, eg odd and even.

Question 2: Drawing names form a hat; selecting certain numbers on a dice, eg 1 and 2, 3 and 4, 5 and 6. Note that using a coin and the process of elimination is ***not*** *a fair method.*

Question 3: Drawing names from a hat, tossing a dice.

Question 4: Drawing names from a hat.

Question 5: The 'fairness' of the spinning wheel depends on the ***angle*** *in the sector. A fair spinning wheel would have three angles of 120° at the centre. Learners might not be able to calculate the size of each angle, but should show that each sector has the same angle.*

Question 6: The notion of 'bias' can be discussed here. In 6(b) it is more likely that the drawing pin will land with the 'head' of the pin up. The actual likelihood cannot be calculated, but can be estimated using experiments. This differs from the use of the coin in which there is an equal chance of either outcome ("heads" and "tails"). In 6(c) there are three possible ways the cup can land, but it is most likely that the cup will land on its side.