11. Number Patterns II

1. In each case, start with the given number and do the operation in brackets at least 10 times.
   Then do the same pattern with corresponding common fractions. Check that your answers are the same.
   E.g. 0,8 (+ 0,2) → 1,0 + 0,2 → 1,2 + 0,2 → 1,4 + 0,2 → ….
   $\frac{8}{10}$ (+ $\frac{2}{10}$) → 1 + $\frac{2}{10}$ → $1\frac{2}{10}$ + $\frac{2}{10}$ → $1\frac{4}{10}$ + $\frac{2}{10}$ → ….

❶ 6,4 (+ 0,3) → *6,7 → 7 → 7,3 → 7,6 → 7,9 → 8,2 → 8,5 → 8,8 → 9,1 → 9,4 → 9,7 → 10*

❶ $6\frac{4}{10}$ (+ $\frac{3}{10}$) → $6\frac{7}{10}$ → *7* → $7\frac{3}{10}$ → $7\frac{6}{10}$ → $7\frac{9}{10}$ → $8\frac{2}{10}$ → $8\frac{5}{10}$ → $8\frac{8}{10}$ → $9\frac{1}{10}$ → $9\frac{4}{10}$ → $9\frac{7}{10}$ → *10*

❷ 4,42 (+ 0,1) → *4,52 → 4,62 → 4,72 → 4,82 → 4,92 → 5,02 → 5,12 → 5,22 → 5,32 → 5,42*

❷ $4\frac{42}{100}$ (+ $\frac{1}{10}$) → $4\frac{52}{100}$ → $4\frac{62}{100}$ → $4\frac{72}{100}$ → $4\frac{82}{100}$ → $4\frac{92}{100}$ → $5\frac{2}{100}$ → $5\frac{12}{100}$ → $5\frac{22}{100}$ → $5\frac{32}{100}$ → $5\frac{42}{100}$

❸ 8,4 (– 0,3) → *8,1 → 7,8 → 7,5 → 7,2 → 6,9 → 6,6 → 6,3 → 6 → 5,7 → 5,4*

❸ $8\frac{4}{10}$ (– $\frac{3}{10}$) → $8\frac{1}{10}$ → $7\frac{8}{10}$ → $7\frac{5}{10}$ → $7\frac{2}{10}$ → $6\frac{9}{10}$ → $6\frac{6}{10}$ → $6\frac{3}{10}$ → *6* → $5\frac{7}{10}$ → $5\frac{4}{10}$

❹ 0,3 (+0,4) → *0,7 → 1,1 → 1,5 → 1,9 → 2,3 → 2,7 → 3,1 → 3,5 → 3,9 → 4,3*

❹ $\frac{3}{10}$ (+ $\frac{4}{10}$) → $\frac{7}{10}$ → $1\frac{1}{10}$ → $1\frac{5}{10}$ → $1\frac{9}{10}$ → $2\frac{3}{10}$ → $2\frac{7}{10}$ → $3\frac{1}{10}$ → $3\frac{5}{10}$ → $3\frac{9}{10}$ → $4\frac{3}{10}$

❺ 1,37 (-0,1) → *1,27 → 1,17 → 1,07 → 0,97 → 0,87 → 0,77 → 0,67 → 0,57 → 0,47 → 0,37*

❺ $1\frac{37}{100}$ (- $\frac{1}{10}$) → $1\frac{27}{100}$ → $1\frac{17}{100}$ → $1\frac{7}{100}$ → $\frac{97}{100}$ → $\frac{87}{100}$ → $\frac{77}{100}$ → $\frac{67}{100}$ → $\frac{57}{100}$ → $\frac{47}{100}$ → $\frac{37}{100}$

❻ 11,6 (-0,4) → *11,2 → 10,8 → 10,4 → 10 → 9,6 → 9,2 → 8,8 → 8,4 → 8 → 7,6*

❻ $11\frac{6}{10}$ (- $\frac{4}{10}$) → $11\frac{2}{10}$ → $10\frac{8}{10}$ → $10\frac{4}{10}$ → *10* → $9\frac{6}{10}$ → $9\frac{2}{10}$ → $8\frac{8}{10}$ → $8\frac{4}{10}$ → *8* → $7\frac{6}{10}$

❼ 12,67 (-0,9) → *11,77 → 10,87 → 9,97 → 9,07 → 8,17 → 7,27 → 6,37 → 5,47 → 4,57 → 3,67*

❼ 12,67 (- $\frac{9}{10}$) → $11\frac{77}{100}$ → $10\frac{87}{100}$ → $9\frac{97}{100}$ → $9\frac{7}{100}$ → $8\frac{17}{100}$ → $7\frac{27}{100}$ → $6\frac{37}{100}$ → $5\frac{47}{100}$ → $4\frac{57}{100}$ → $3\frac{67}{100}$

❽ 25,6 (halve) → 12,8 → → *6,4 → 3,2 → 1,6 → 0,8 → 0,4 → 0,2 → 0,1 → 0,05 → 0,025*

❽ $25\frac{6}{10}$ (halve) → $12\frac{8}{10}$ → $6\frac{4}{10}$ → $3\frac{2}{10}$ → $1\frac{6}{10}$ → $\frac{8}{10}$ → $\frac{4}{10}$ → $\frac{2}{10}$ → $\frac{1}{10}$ → $\frac{5}{100}$ → $\frac{25}{1000}$

2. Use your knowledge of addition and subtraction of common fractions to calculate the following:
   (a) 0,5 + 0,3 *(0,8)*
   (b) 0,9 + 0,2 *(1,1)*
   (c) 0,28 – 0,15 *(0,13)*
   (d) 0,58 – 0,4 *(0,18)*

Teacher Notes:

What learners may do:

- Learners may want to do all the decimals sequences first, and later come back to the common fractions sequences. They should be encouraged to complete the sequences in the order in which they appear, so that they are constantly reflecting and comparing their decimal and common fractions answers.
- In order to complete common fractions sequences like the second one beginning with $4\frac{42}{100}$ (+ $\frac{1}{10}$), learners will have to remember that $\frac{1}{10}$ is the same as $\frac{10}{100}$.
- They might use equal signs in Sequencing II. This is **wrong** and must be pointed out to them. They should rather use the arrow notation.

What learners may learn:

- Number concept of common fractions as well as decimal fractions.
- The connection between adding and subtracting decimal fractions and common fractions.

12. Which One is Bigger?

1. In each case, say which decimal fraction you think is biggest and *why.*

   (a) 0,03 or 0,3

   (b) 5,31 or 5,13

   (c) 3,5 or 3,412

   (d) 4,09 or 4,1

   (e) 0,76 or 0,760

   (f) 0,89 or 0,089

2. Sometimes we can take the zero away and it does not change the size of the number. Other times, we cannot take the zero away or the number will change.

   In each case say whether or not we can take the zero away without changing the size of the number, and *why*.

   (a) 1,04

   (b) 3,480

   (c) 0,42

   (d) 2,055

   (e) 8,80

Teacher Notes:

This activity has been designed to challenge common misconceptions among learners regarding decimal fractions. **It is essential that teachers encourage discussion among learners about their answers.**

In cases where all the learners agree on the incorrect answer, the teacher should ask the learners what the decimal fractions mean in terms of common fractions.

What learners may do:

- Learners may make the error of assuming that the number that looks longer is bigger. For example, they may say that 0,03 is bigger than 0,3 and that 4,09 is bigger than 4,1.
- Learners may forget or may not know that a zero at the end of a decimal fraction is extraneous, in other words that 0,76 is the same as 0,760.
- Learners may make the error of assuming that ALL zeros are unnecessary, for example that 0,89 is the same as 0,089. The second question addresses the issue of zero as a place holder. It is less important that learners know this terminology than that they understand the important role played by the zero after the comma.

What learners may learn:

- Comparison of the size of decimal fractions
- That the length of the number does not have a direct connection to its size
- The role of zero after the comma and at the end of a decimal fraction
- Consolidation of the meaning of decimal fractions.

13. Paper

How thick do you think one sheet of paper is?
Can you measure it with your ruler?

Dumisani has a bright idea. He measures 100 sheets of paper. The stack is 14 mm thick.

1. Calculate how thick each sheet of paper is. *$\frac{14}{100}$ mm*
2. How thick will a document of 7 pages be?
   7 x $\frac{14}{100}$ = $\frac{98}{100}$ mm (very close to 1 mm)
3. If 245 copies of this document are printed and stacked on top of another, how high will the stack be?
   ($\frac{98}{100}$ x 245 = 240,1 mm)
4. Complete the diagram:

3,5 — + 0,01 → *3,51* — + 0,01 → *3,52* — + 0,01 → *3,53*

3,53 — + 0,01 ↓ *3,54*

3,57 ← + 0,01 — *3,56* ← + 0,01 — *3,55* ← + 0,01 — *3,54*

3,57 — + 0,01 ↓ *3,58*

3,58 — + 0,01 → *3,59* — + 0,01 → *3,6* — + 0,01 → *3,61*

3,61 — + 0,01 ↓ *3,62*

3,65 ← + 0,01 — *3,64* ← + 0,01 — *3,63* ← + 0,01 — *3,62*

Teacher Notes:

This task mainly concerns hundredths. This concept (along with tenths, thousandths and later ten thousandths, etc.) is needed for a stable number concept. The learners must be given time to make sense of this on their own.
It is possible that the learners have not met the millimetre as a unit before. Teachers should clarify this for the learners, explaining that it is a measuring unit and showing it to them on a ruler. The symbol for a millimetre (mm) must also be given to them.

What learners may do:

- They will clearly not be able to measure the one sheet of paper with their rulers. Some of the learners might however try. Allow them to explore.
- In question (a) the learners might divide 14 into 100 equal parts. The answer $\frac{14}{100}$ is quite acceptable. Learners should not be forced to write the answer as 0,14 or to simplify the fraction.
- In questions (b) and (c) the learners must understand the situation. This is a practical situation where a whole number is multiplied by a fraction.
- Question 3 can be given as homework if the learners know that $\frac{1}{100}$ can be written as 0,01.

What learners may learn:

- Develop a concept of hundredths.
- Multiplication of fractions by whole numbers.

After 'Paper' the following activity can be done orally with the children:

For this activity, learners need to know how to programme their calculators to count using a certain interval. Most calculators can be programmed to do that and can thus be turned into a "counting machine". Different calculators have different procedures, so learners should play with their own calculators to find out if they can be programmed and if so, how this can be done.
See the next page for two ways for programming a calculator to count in 3's:
(This is a more complete explanation of the one that can be found in 'Snakes with Decimals')

Programming your calculator:

- Press $\boxed{3}\,\boxed{+}\,\boxed{=}$ If you keep on pressing $\boxed{=}$, the calculator will go on counting in 3's. However, if you press any of the operation functions (+; –; ×; ÷) or clear the screen, you have to start the process from the beginning again.

  You can press any number (without clearing the screen) and the calculator will count in 3's from that number onwards. For example:

  Press $\boxed{3}\,\boxed{+}\,\boxed{=}$.

  Now press $\boxed{4}\,\boxed{1}$ and $\boxed{=}\,\boxed{=}\,\boxed{=}$ …

  Your calculator should give 44; 47; 50; 53; …
- Press $\boxed{3}\,\boxed{+}\,\boxed{+}\,\boxed{=}$ and follow the same procedures as for the first method.

1. Programme your calculator to count in 0,1's. Press the $\boxed{=}$ key several times and count aloud with the calculator. Count up to 2,5.

2. Programme your calculator to count in 0,01's.
   Now enter 0,9 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key and count aloud with the calculator. Count up to 1,2.

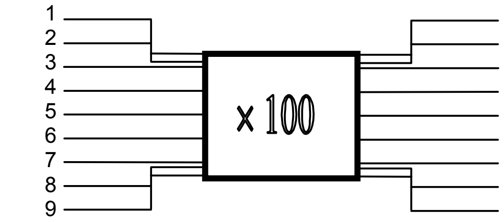
3. Programme your calculator to count in 0,1's. Now enter 111,11111 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key. What do you notice?

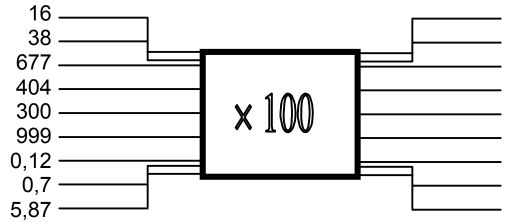
4. Programme your calculator to count in 0,01's. Now enter 111,11111 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key. What do you notice?

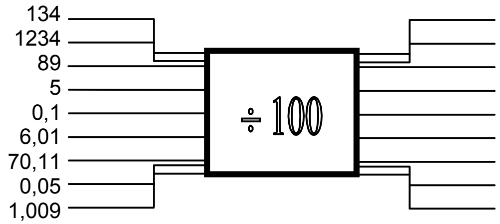
5. Programme your calculator to count in 0,001's. Now enter 111,11111 and press $\boxed{=}$. Keep on pressing the $\boxed{=}$ key. What do you notice?

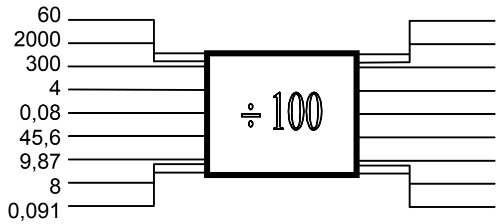
14. The Wonderful Number 100

1. Complete the following:









Teacher Notes:

This is a very important place value activity. The teacher can also design more of these if they are needed.

Also see the teacher notes for 'The Wonderful Number 10'.

15. Decimal Invaders

43,598

Procedures to play the game:

1. Two players need one calculator
2. Player 1 enters any decimal number e.g. 43,598. This number must be 'shot down' (replaced by 0 by subtracting).
3. Players take turns to 'shoot down' a digit. (One at a time.)
4. The player that ends with 0 wins.
5. If a player changes the number on the screen but does not shoot down a digit, the other player gets two turns.

Example:

	Press	Number on screen
Player 1:	43.598	43.598
Player 2:	$\boxed{-}$ 0.5 $\boxed{=}$	43.098 - The '5 has been shot down.
Player 1:	$\boxed{-}$ 40 $\boxed{=}$	3.098 - The '4' has been shot down.

Repeat this with different numbers!

Teacher Notes:

This activity can also be used as a diagnostic activity to see which learners still need help and which learners have mastered decimal place value.

What learners might do:

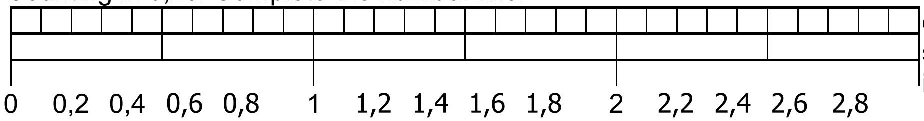
- Subtract, for example, 1 instead of 100 to "shoot down" the '1' or 5 instead of 0,05 to "shoot down" the '5'. This is a valuable learning moment. Give learners enough time to resolve this.

What learners might learn:

- Place value of decimal numbers.

16. Decimal Fractions and the Number Line

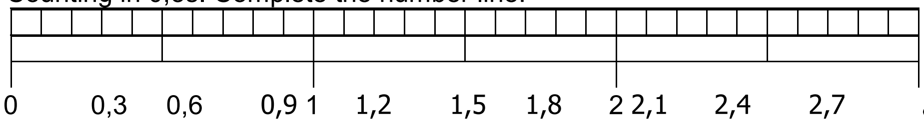
1. Counting in 0,2s. Complete the number line:



   (a) How many 0,2s in one whole? 5

   (b) What common fraction is 0,2 therefore? $\frac{1}{5}$

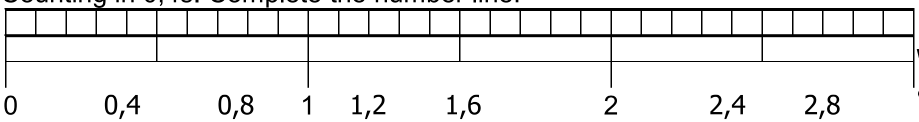
2. Counting in 0,3s. Complete the number line:



   (a) How many 0,3s in 3? 10

   (b) What common fraction is 0,3? $\frac{3}{10}$

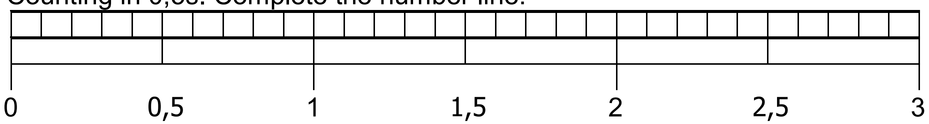
3. Counting in 0,4s. Complete the number line:



   (a) How many 0,4s in 2? 5

   (b) What common fraction is 0,4? $\frac{2}{5}$

4. Counting in 0,5s. Complete the number line:



   (a) How many 0,5s in one whole? 2

   (b) What common fraction is 0,5? $\frac{1}{2}$

Teacher Notes:

This later introduction of the number line is a whole new approach to decimal fractions. The teacher must remember that the number line is not a spontaneous idea that learners develop. They might not understand it immediately.

The teacher should also ensure that all the learners know what a common fraction is. This is terminology that the teacher can introduce.

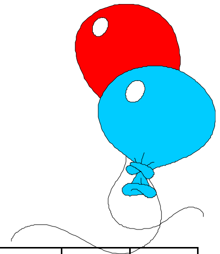
What learners might do:

- They might not see the connection between the numbers the arrows and the actual number line, therefore pointing their arrows between lines and not towards a specific line. Especially if this is their first experience of number lines.
- When they have to say how many of a certain decimal fraction there is, they might count wrong. Either by counting one to few or by counting the 1 and the 2 as one of the decimal fractions.
- They might see the link between question 1.1 and 1.2 and generalize that for the other questions. E.g. seeing there are 10 0,3's in 3, saying that is $\frac{1}{10}$.

What learners might learn:

- Grouping. This will help them to give meaning to division with decimal fractions (e.g. how many 0,2's in 1), because sharing is not sensible.
- That the answer doesn't get smaller when you divide by a decimal fraction.
- The relationship between common fractions and decimal fractions.

17. Balloons



A party box of 100 balloons weighs 250 g and costs R 6,29.

1. What is the mass of one balloon?
2. Complete the table:

Number of balloons	1	2	3	4	5	10	15	25	50
Mass (g)	2,5	5	7,5	10	12,5	25	37,5	62,5	125

3. What is the price of each balloon?
   The calculator gives 6.79 $\boxed{\div}$ 100 $\boxed{=}$ **0.0679** which is R0,0679 or 6,79c.
   Is this a sensible answer?

   Sometimes we have to *round* decimal fractions *off* in order to get sensible answers. For example, when we are working with money, it does not make sense to talk about R0,5214 so we would *round* this *off* (or *round* it *down*) to R0,52 or 52c. In the same way, it does not make sense to talk about R0,5281, so we would *round* this *off* (or *round* it *up*) to R0,53 or 53c.

   How would you round off R0,5551?

4. Complete the table:

Number of balloons	1	2	3	4	5	10	15	25	50
Cost (c)	6	13	19	25	31	63	94	157	315

5. What will 5 boxes of balloons cost?
6. What is the mass of 5 boxes of balloons?

Teacher Notes:

This task provides experience in the use of decimals to describe fractions of a *unit* (grams and cents).

It would be of great value if a real balloon could be weighed in class, so that the children can develop a feeling for a unit such as one gram.

What learners might do:

- Some children will get the answer $2\frac{1}{2}$ when dividing 250 g by 100. Social interaction during discussion, can help to clarify the concept that $2\frac{1}{2}$ is the same as 2,5 (the calculator answer).
- Questions (c) and (d) provide a practical situation where a realistic amount must be found by rounding off the decimals. Some learners might initially not round off the decimals. The teacher (or other pupils) can challenge them by asking whether it is sensible to pay 6,29c.
  This raises another mathematical issue. We never round off results that we are still going to use in calculations. Only the final answer may be rounded off. This is social knowledge and we can't expect the child to know this. The teacher has to **tell** this to the learners. This should however be done **after** different learners have obtained different answers, because this shows the need for such a convention.
  The terminology and conventions associated with rounding off are introduced as social knowledge. Once again, this should only be discussed **after** different learners have obtained different answers to Question (c). It is assumed that learners have some knowledge of rounding off whole numbers, and the conventions associated with the number '5'. If this is not the case, the teacher should tell the learners about the conventions.

What learners might learn:

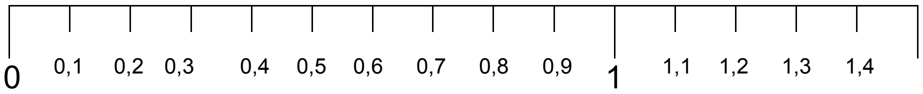
- How to round off correctly and the convention that results are not rounded off if they are still going to be used in further calculations.
- How to use decimals to describe fractions of a unit.
- That a decimal fraction is just another way of writing a common fraction, e.g. $2\frac{1}{2}=2,5$
- A feeling for the 'muchness' of one gram.

- That mathematics must be realistic in our practical world. The answers that the learners obtain must be sensible for the situation in which they are to be used.

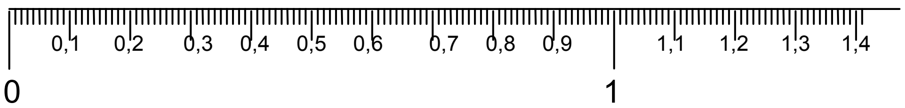
18. Measuring

Here are two rulers with which we are going to measure in the following activity. Cut them out neatly and keep them safe:

RULER 1



RULER 2



For each of the lines below (SEE RIGHT):
Use ruler 1 to measure the line (estimate the length to at least two decimal places). Write your answer in column 2 of the table.
Now measure the line with ruler 2. Write your answer in the given column. (How close was your estimate?)

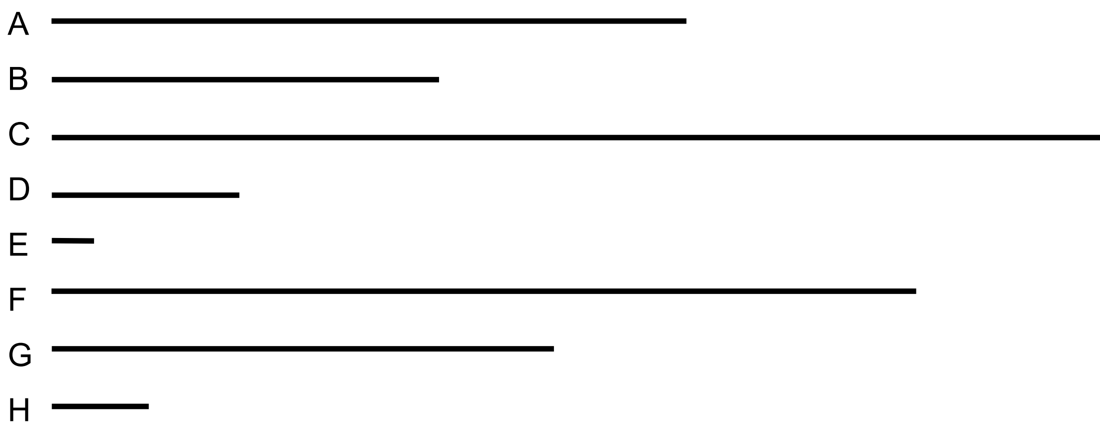
Line	Reading on ruler 1	Reading on ruler 2
A		
B		
C		
D		
E		
F		
G		
H		

Which ruler would you rather use? Why?

Teacher Notes:

This activity is a good indicator of the learners' understanding. The teacher can see whether they have a feeling for decimal fractions. It reinforces the concept of decimal fractions.

Also see the teacher notes for 'Estimates'.



19. Marking Homework

The following worksheet was given to Zanele for homework. Mark the work, correcting all the mistakes.

Decimals: Name: Zanele

1. Write 0,2 as a common fraction $\frac{1}{2}$ ($\frac{2}{10}$ or $\frac{1}{5}$)
2. Write 3,5 as a common fraction: $\frac{3}{5}$ ($3\frac{1}{2}$)
3. 3,6 + 0,3 = 3,9 (3,9)
4. 4,8 + 4,3 = 8,11 (9,1)
5. 0,7 - 0,1 = 0,6 (0,6)
6. 0,27 - 0,1 = 0,26 (0,17)

Write down the next three terms in each sequence:

7. 0,2 ; 0,4 ; 0,6 ; 0,8 ; 0,10; 0,12 (Adding 0,2's)
8. 1,2 ; 0,9 ; 0,6 ; 0,3 ; 0 (Subtracting 0,3's)
9. 0,34 ; 0,36 ; 0,38; 0,40; 0,42 (Adding 0,02's)
10. 0,5 ; 0,10; 0,15; 0,20 (Adding 0,05's)
11. 0,25 ; 0,50 ; 0,100 ; 0,200 (Doubling)
12. 0,8 ; 0,4 ; 0,2 ; 0,1 ; $0,\frac{1}{2}$; $0,\frac{1}{4}$ (Halving)

Teacher Notes:

This is a very good activity for the children to challenge their own beliefs about decimal fractions. It can elicit a lot of discussion and the teacher should allow the learners to talk the issues through, until they are comfortable with it.

What learners might do:

- They might not see some of the mistakes. Therefore enough discussion time should be allowed.

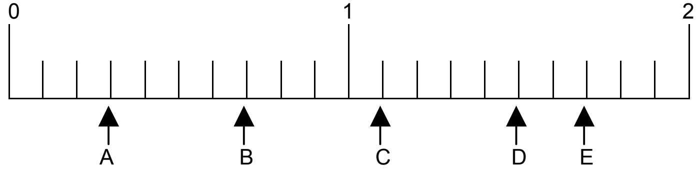
What learners may learn:

- Reflection on their own work while they are doing this can lead to a more stable concept of decimal fractions.
- To be aware of potential 'potholes' where one can easily make mistakes.

20. Scale Readings

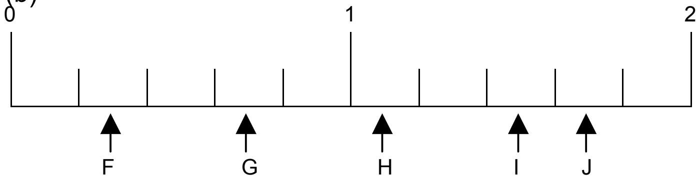
1. Give the readings on the scales. Fill your answers into the box next to the scale:

(a)



A	=	0,4
B	=	0,8
C	=	1,2
D	=	1,6
E	=	1,8

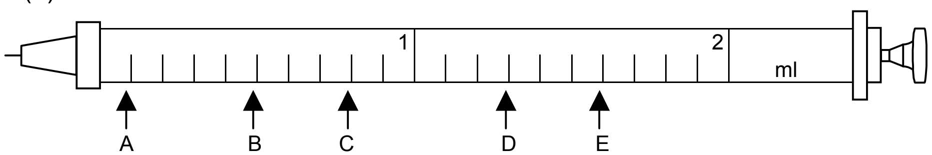
(b)



F	=	0,4
G	=	0,8
H	=	1,2
I	=	1,6
J	=	1,8

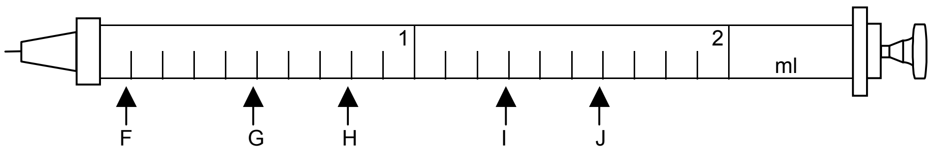
2. Pretend that you are a doctor or a nurse. Read each of the following doses as accurately as possible. Write your readings on the lines below.

(a)



0,1 0,5 0,8 1,3 1,6

(b)



0,1 0,5 0,8 1,3 1,6

Teacher Notes:

Estimating a reading between two calibrations is a valuable activity, which requires understanding of the relative size of decimals and the meaning and use of zero in decimal numbers. Besides being an important skill in its own right, scale reading discriminates very clearly between learners who have a deep understanding of decimal numbers and those who do not. Doing scale reading at this stage can therefore be a diagnostic activity where possible misconceptions can be exposed.

There is a huge variation in the degree of difficulty in scale reading tasks in general. For a start we include two pairs of scales. Each pair has one set of questions with a scale marked in tenths, which is the easiest scale for learners to read (see question 1(a)).
Question 1(a) is calibrated in tenths and 1(b) in fifths. The points A, B, C, D and E correspond to points F, G, H, I and J. If a child has read point F in (b) as 0,2 the teacher can refer him to point A in (a) which is the same distance from the zero point. Learners normally do not have problems in reading scales in tenths (like (a)) correctly. Comparing the two readings may enable the learner to correct his/her own errors.

What learners might do:

- They might have problems in interpreting the scale readings in 1(b) and 2(b). The teacher can refer the learners to 1(a) and 2(a) to help overcome this.

What learners might learn:

- Equivalent decimal fractions.
- How to interpret a scale that is marked in fifths only.

TO PREVIOUS

TO NEXT