Malati

Mathematics learning and teaching initiative

Fractions

Phase 2

Grades 5, 6 and 7

Teacher document

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We acknowledge the valuable comments of Hanlie Murray and Sarie Smit

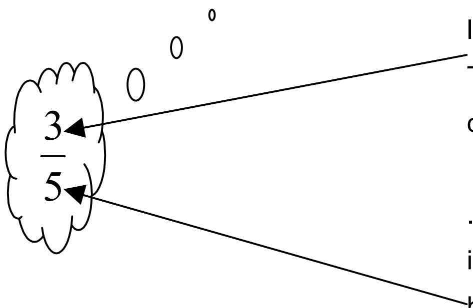
December 1999

1. Fractions

A fraction looks like two numbers but is actually one number!

For example, the fraction $\frac{3}{5}$ is a number somewhere between a half and one whole.

Imagine I could have $\frac{3}{5}$ of a chocolate bar...



The number on top of the line is called the *numerator*. This would mean that I could have **THREE**...

...of the **FIVE** equal parts into which the chocolate bar is divided.

The number below the line (showing the number of equal parts into which the whole has been divided) is called the *denominator*.

What does $1\frac{3}{8}$ mean?

Teacher Notes:

This is the learners' first introduction to the numerator and denominator in formal terms. This is social knowledge, and is thus introduced by way of telling the learners the meaning of these terms. These are difficult words, and the learners should be allowed to become accustomed to them gradually as they are used in the classroom.

What learners may do

- Learners may struggle to articulate the meaning of the 3 and the 8 in the number $1\frac{3}{8}$. They should be encouraged to discuss this and try to negotiate a way to describe these meanings.

What learners may learn

- the terms numerator and denominator
- reflection of the meanings of the numbers above and below the line.

2. Chocolate Bars:

Mrs Hermanus gives a prize to the group in her class that has behaved the best during the week. The prize is a box with 10 chocolate bars.

1. This week Ismail's group wins the prize. There are 4 people in Ismail's group. They all want the same amount of chocolate. How much chocolate does each child get?
2. Last week it was Rasheed's group that won the prize. There were 6 people in Rasheed's group. How much chocolate did each child get?

Teacher Notes:

This task and the next one (More Chocolate Bars) must be done on consecutive days. The idea of sharing is revisited and another opportunity is created for pupils to get involved in a discussion that may lead to an awareness of equivalent fractions.
It may still be necessary to encourage some pupils to draw the situation before they can progress.

What learners may do:

- A very popular strategy is first to share out the number of 'whole' chocolate bars and then to share out the remainder. In the first question, for example, the 10 bars can be shared between the 4 children so that each child gets 2 'whole' bars. Then the left over two bars can be shared equally. (This forms the base of the distributive property: 10 ÷ 4 = 8 ÷ 4 + 2 ÷ 4, although most of the learners will simply handle it concretely and **not** as division).
- Now there are two possible ways to solve 2 ÷ 4: They can divide both chocolates in 4 equal parts, and each child gets $\frac{2}{4}$ (in total $2\frac{2}{4}$) OR they can divide each chocolate in 2 equal parts, and each child gets $\frac{1}{2}$ (in total $2\frac{1}{2}$).
- Some pupils may also share each chocolate into the number of pupils in each group, giving each $\frac{10}{4}$ (or ten quarters).

What learners may learn:

- If the discussion of answers and methods leads the children to think about the situation of two names for the same fraction, they must be left alone to sort out the problem and reach consensus by reflecting and discussing.
- The concept of equivalent fractions can be formed.

3. More Chocolate Bars:

1. Roxanne and five friends share 15 chocolate bars. How much chocolate does each child get?
2. Jade and two friends share 8 chocolate bars. How much chocolate does each child get?
3. Who got the most chocolate, Roxanne or Jade? Why do you say that? Explain your thinking.
4. What is more: one-seventh of a chocolate bar or one-eighth of a chocolate bar? Explain your thinking (You may use a diagram).

Teacher Notes:

As with the previous task, this is again an equal sharing problem with a fractional answer. This task however goes a little further to comparing fractions.

What learners may do:

- Pupils may not realise that Roxanne and Jade also belong to the groups, therefore the sharing is between 6 and 3 respectively. If this happens, try to get discussions going for pupils to sort that out themselves.
- For question 1, each child will get 2 whole bars and the three remaining ones have to be shared between 6 children. Each could get either $\frac{3}{6}$ or $\frac{1}{2}$, depending how they shared.
- For question 2, each child will get 2 whole bars and the two remaining ones have to be shared between 3 children. Each could get $\frac{2}{3}$.
- In the third problem the learners are forced to compare fractions.

What learners may learn:

- Comparing fractions with different denominators.
- Getting used to the idea that a fraction with a bigger denominator is smaller than a fraction with a small denominator. (This should however not be told to them at this stage).

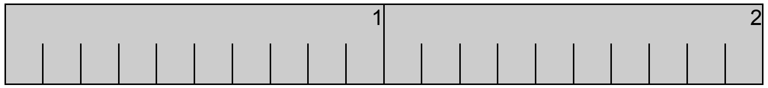
4. Let Us Measure!

When we measure lines with rulers, we give the length of the lines in centimetres and/or millimetres. (Many years ago we used “inches” and “feet” to measure lengths in South Africa.) We can also measure in other units.

1. Spread your fingers as wide as possible (see sketch). Use the span of your hand to measure the width of your desk or the table where you are working. Compare your measurement to that of your friend.

Why do we not use our own units when we measure the lengths of carpets, material, walls, etc? Why do we always use centimetres and millimetres?

2. Let us measure the lines given below with a new unit, the bina. This is your ruler that you are going to use to measure the lines:



Your ruler is two bina’s long. Your teacher will give you a similar ruler to cut out and to use for measuring the lines below. Compare your measurements with that of your friends.

<u>LINES:</u>

A

B

C

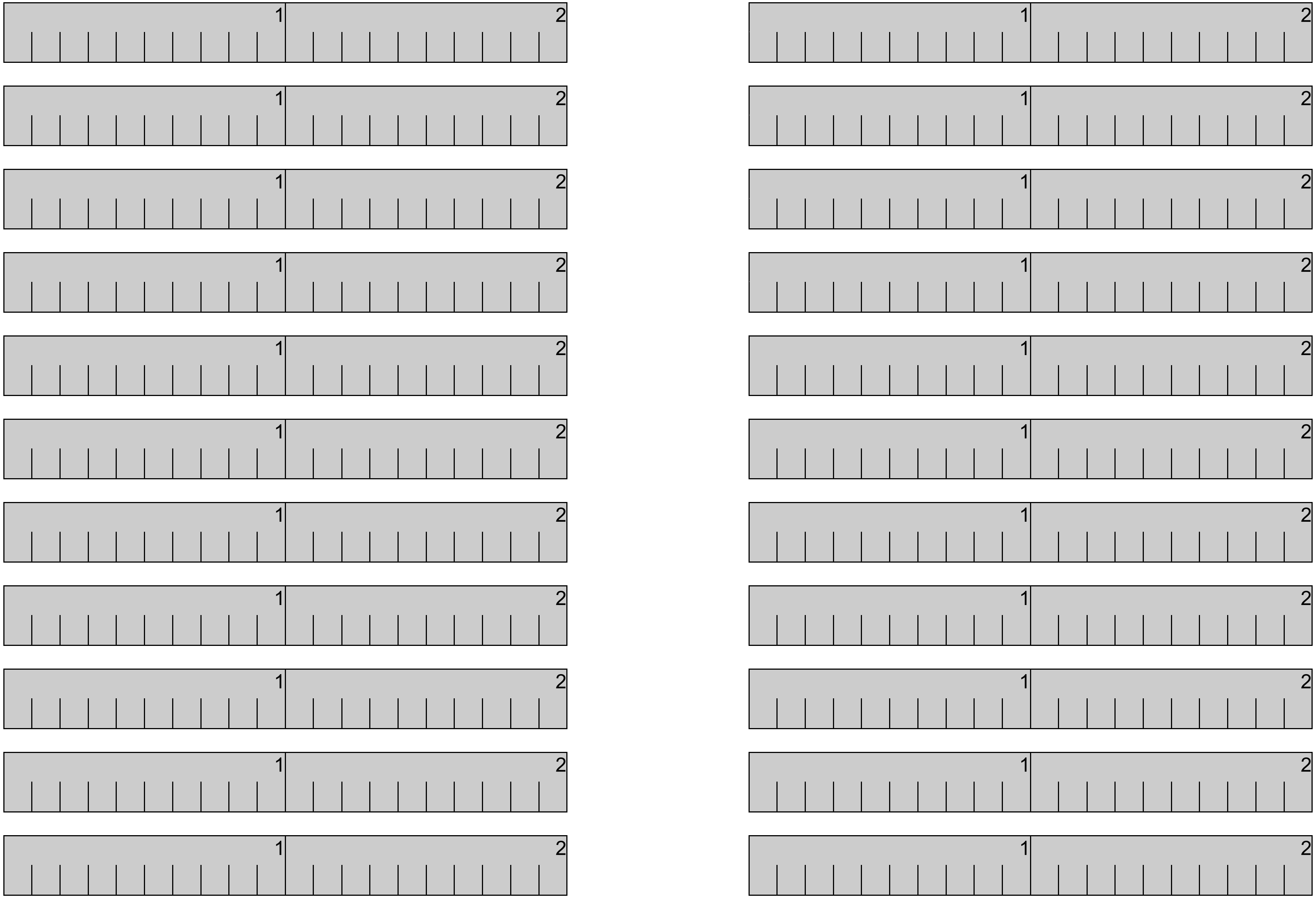
D

E

Teacher Notes:

On the next page you will find the rulers for the children.

Make sure that each learner experiences the use of a unit with which to measure and that the meaning of what a measuring unit is, is clarified.
These issues (use of measuring unit) should be discussed in the class.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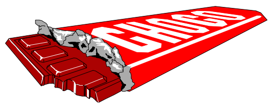
5. Designing Chocolate I

Think of various chocolate bars and chocolate slabs. Some of them are designed with divisions that make it easy to break off one or more “blocks”.



1. Design a chocolate slab that can easily be shared equally by 5 people.
2. Design two more chocolate slabs (not the same as the one in Question 1) that can easily be shared equally by 5 people.
3. What fraction of the slab will each person get in each of these cases?
4. Compare your designs and your answers to Question 3 with those of your friends.
5. Design a chocolate slab so that five people that share will each get 3 small (same-size) blocks.
   Name the fraction of the slab that each person will receive, in two different ways.
6. You want to make different slabs that can easily be shared equally among
   (a) 5 people
   (b) 7 people
   What must you do?
   What can you say about fractions of equal size that have more than one name?

Teacher Notes:

Children may stick to designs with different shapes (eg. squares, triangles, etc.) and not necessarily more smaller divisions. Do not interfere. Question 5 forces them to design a slab with 15 divisions. Give them enough time to make the breakthrough themselves.

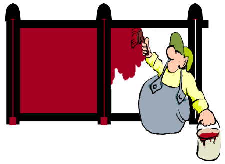
What learners may do:

- Design slab with 5 (or multiple of 5) divisions
- Design more slabs with a different shape
  or with number of divisions a multiple of 5
- Name the part that each person gets
  - in fifths
  - by using number of “small blocks”
- Compare answers to others in group, etc
  - notice different designs, smaller blocks?
  - possibility of 2×5 etc divisions
- Find designs in which each receive 3 small blocks - name fraction in 2 different ways
- Consider slabs easily shared by 5 people
- Reflect on fractions with more than one name

What learners may learn:

- To divide equally by 5, it is the easiest to have 5 (or a multiple of 5) divisions
- A number of divisions which is a multiple of 5 can also be easily shared by 5 people
- Notation of fractions
- Becoming aware of equivalent fractions
- Fractions can be expressed in more than one way. Eg. $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = ..$
- One-fifth is the same quantity than three-15ths
- Number of small blocks must be multiples of five
- Different names of same fraction will be multiples of unit

6. A Painting Job



Mr. Bengu asked Ben to paint his garden wall for him. The wall consists of 36 panels or divisions of equal size.

1. After one week, Ben painted $\frac{1}{6}$ of the wall. How many panels did he paint?
2. The next week he painted another $\frac{1}{4}$ of the wall. What fraction of the wall was painted then?
3. The third week, Ben asked Nic to help him. Ben painted $\frac{1}{3}$ of the wall and Nic painted $\frac{1}{4}$ of the wall. What fraction of the wall still has to be painted? How many panels are still not painted?

Teacher Notes:

This activity forms part of the sequence of activities to develop the concept of addition of fractions with unlike denominators

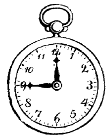
What learners may do:

- In question 2 the learners are required to calculate the sum of $\frac{1}{6}$ and $\frac{1}{4}$. Fast learners may immediately realise that 12 is a convenient common denominator to use and interpret $\frac{1}{6}$ as $\frac{2}{12}$ and $\frac{1}{4}$ as $\frac{3}{12}$. They can now add them to get $\frac{5}{12}$.
- Others might see $\frac{1}{6}$ as $\frac{6}{36}$ and $\frac{1}{4}$ as $\frac{9}{36}$. It is now easy to add them to get $\frac{15}{36}$.
- Those who are not ready to do it in this way, can calculate the number of panels painted. (This was asked in question 1 for $\frac{1}{6}$.) Ben painted $\frac{1}{6}$ of 36 = 6 panels the first week, and $\frac{1}{4}$ of 36 = 9 panels the second week. By the end of the second week 15 panels have been painted. This is $\frac{15}{36}$, which could also be $\frac{5}{12}$. (It is even possible that some learners express the answer as $\frac{5}{12}$.
- It is left to the learners to understand or sort out these two possible answers through discussion.

What learners may learn:

- At this stage the concept of a fraction as a part of a whole should be stable.
- This is another opportunity to develop the concept of equivalent fractions. It is however possible that some learners have already constructed the notion of equivalent fractions (a fraction can have more than one name).
- Addition of fractions with unlike denominators can also be developed in this activity. The number of panels provides a “bridge” to addition of fractions with unlike denominators. Question 1 asks for the number of panels to give the suggestion to learners.

7. Day Time:



1. Cyril spends $\frac{1}{3}$ of his day sleeping and $\frac{1}{4}$ of his day at school.
   (a) Does he spend more time sleeping or at school?
   (b) How much time does he have left for other activities?
2. What fraction is $\frac{1}{3}$ + $\frac{1}{4}$? Explain how you got your answer.
3. What fraction is:
   (a) $\frac{1}{3}+\frac{1}{2}$? Why?
   (b) $\frac{1}{3}+\frac{1}{8}$? Why?
   (c) $\frac{1}{3}+\frac{3}{8}$? Why?

Teacher Notes:

This activity requires that pupils compare fractions and carry out addition and subtraction with fractions.

The context is time, and two different meanings of fractions can be addressed:

- Fraction as part of a whole (e.g. $\frac{1}{3}$ of a day)
- Fraction as part of a collection (e.g. $\frac{1}{3}$ of 24 hours)

What pupils may do:

1(a),2 (a)	a) compare the number of hours (Level 1): $\frac{1}{3}$ of 24 hours = 8 hours; $\frac{1}{4}$ of 24 hours = 6 hours, 8 > 6. b) compare the fractions (Level 2): $\frac{1}{3} > \frac{1}{4}$ If different pupils come up with these solutions, discussion and comparison of them should be encouraged, but pupils should BY NO MEANS be expected to use the Level 2 method.
1(a),2(b), 2(c)	add the fractions and subtract these from 1, or add the number of hours and subtract these from 24 – if both these methods are used, pupils should share and be aware of both of them

What learners may learn:

- that larger denominators mean smaller fractions
- that the same fractions can have different names
- that the 'whole' can be a collection of objects (in this case hours)
- consolidation: addition and subtraction of fractions
- to work out fractions OF a whole number

It is however important to remember that the teacher **should not** suggest any of the above-mentioned.

8. Pirates' Pizza Parlour

Pirates' Pizza Parlour make big pizzas. They cut them into 24 slices.
The customers have a choice. They can order:

- a Single Slice
- a Big Slice ($\frac{1}{4}$ of the whole pizza)
- a Super Slice ($\frac{1}{6}$ of the whole pizza)

1. What fraction of a whole pizza is a Single Slice?
2. Which of the three choices is the smallest? Which is the biggest? How do you know?
3. Why do you think Pirates' Pizza Parlour slice their pizzas into 24 pieces?
4. Nadia is greedy – she wants TWO Super Slices! What fraction of a whole pizza does she want?
5. Jerome is also greedy – he wants a Big Slice AND a Super Slice! What fraction of a whole pizza does he want?
6. Choose your own combination of pizza slices from Pirates' Pizza Parlour and work out what fraction of the original pizza you have chosen. Check your friends' answers.

Teacher Notes:

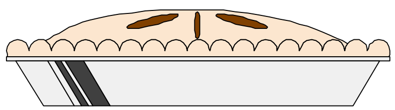
What learners may do:

- Refer to the fractions wall if necessary
- $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$ (or $\frac{1}{3}$)
  OR $\frac{1}{6} + \frac{1}{6}$ is 4 slices of pizza + 4 slices of pizza = 8 slices of pizza, which is $\frac{8}{24}$ $\frac{4}{24} + \frac{4}{24} = \frac{8}{24}$ of a pizza (or $\frac{1}{3}$)
- Similarly, $\frac{1}{4} + \frac{1}{6}$ is 6 slices of pizza + 4 slices of pizza = 10 slices of pizza, which is $\frac{6}{24} + \frac{4}{24} = \frac{10}{24}$ of a pizza (or $\frac{5}{12}$)
- In Question 6, learners should be encouraged to explore various combinations in order to become familiar with adding fractions with different denominators. The teacher should encourage them to write their answers as mathematically as possible.

What learners may learn:

- Comparing fractions with different denominators
- Revisiting adding fractions with same and different denominators
- Becoming aware of the need for equivalent fractions

9. Mrs. Daku's Apple Tarts:



Mrs. Daku bakes small apple tarts. She uses $\frac{3}{4}$ of an apple for one apple tart She has 20 apples. How many tarts can she bake?

Teacher Notes:

Encourage learners to make drawings after they had discussed the problem. Ask them to give feedback of their different responses. Encourage them to reflect on the reasonableness of their answers.

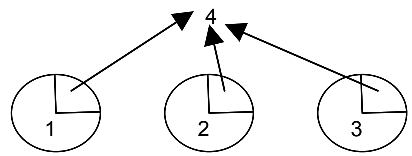
Children who zoom in on an operation or number sentence often make the wrong choice. They should be encouraged to keep the problem in mind and refer to the context to ensure that they are still on the right track.
The structure of the problem is actually division by a fraction ($20 \div \frac{3}{4}$). DO NOT TELL THE LEARNERS THAT!

What learners may do:

1. Choosing an operation (the wrong one)

   $20 \times \frac{3}{4} = 15$

   Some children will reflect on the reasonableness of the answer, and realize that it is impossible; most children will simply accept the 15 and forget about the original problem.
2.

   Every three apples give 4 tarts, so 6 groups of 3 give 24 tarts. Another 2 apples give another 2 tarts; that is 24 + 2 = 26 tarts in all.
3. 20 apples give 20 tarts; 20 quarters remain, that is 5 apples. 5 apples give 5 tarts; 5 quarters remain, that gives another tart. That is 20 + 5 + 1 = 26 tarts.
4. 20 apples give 80 quarters. You need 3 quarters per tart; there are 26 groups of three in 80, so you can make 26 tarts.
5. Ten $\frac{3}{4}$'s will give ten tarts. Another ten $\frac{3}{4}$'s will give another 10 tarts. Ten $\frac{1}{4}$'s will give 3 tarts with $\frac{1}{4}$ remaining. Another ten $\frac{1}{4}$'s will give 3 tarts with $\frac{1}{4}$ remaining.

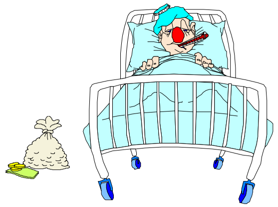
   That is 10 + 10 + 3 + 3 = 26 tarts.
6. Most children will draw the apples to help them understand the problem.

What learners may learn:

- That they should not just choose an operation, but keep the problem in mind all the time.
- The structure of the problem is division with a fraction. Most children will however simply handle it as a real situation and will not formulate the number sentence.

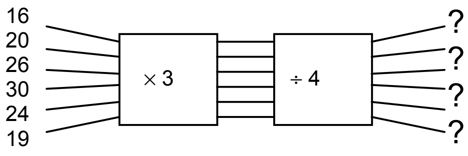
10. Earning Money for Hospital

A few friends agreed to give $\frac{3}{4}$ of what they earned during the holiday to the children's hospital.



Charlie earned R16
Bongi earned R20
Ayanda earned R26
Thandi earned R30
Daphne earned R24
Ernie earned R19

1. How much did they give to the hospital?
2. How much did each of the children give to the hospital?
3. Complete the following flow diagram:



Teacher Notes:

This activity is to develop the concept of a fraction of "a collection".

What learners may do:

- The order of the questions is deliberate so that learners can solve the question either by first adding the money and then calculating $\frac{3}{4}$ of the total amount, OR by finding $\frac{3}{4}$ of each amount and then adding them all together.
- Those who found $\frac{3}{4}$ of the total then has to find $\frac{3}{4}$ of each amount in question 2.

What learners may learn:

- Developing further the idea that a fraction can also be part of a collection of things (money in this case).
- The flow diagram is intended to draw their attention to the fact that to find $\frac{3}{4}$ of an amount, one has to multiply by 3 and divide by 4 (or the other way around). Children should however **not** be forced to try and see this. If they do see this, it must be because they observed it themselves.

FORWARD TO NEXT