Malati

Mathematics learning and teaching initiative

Fractions

Phase 3

Grades 6 and 7

Teacher document

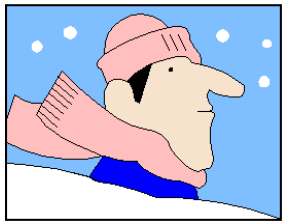
Malati staff involved in developing these materials:

Therine van Niekerk
Amanda le Roux
Karen Newstead
Bingo Lukhele
Agatha Lebethe

We acknowledge the valuable comments of Hanlie Murray and Sarie Smit.

December 1999

1. Scarves and Socks



1. Mrs. Pieterse uses 15 balls of red wool to knit 9 red scarves, all of them the same length. How many balls of wool does she use to make one scarf?

2. She also uses 35 balls of yellow wool to knit 7 pairs of socks (14 socks). How many balls of wool does she use to make one sock?

Teacher Notes:

What learners may do:

- In question 1, learners may distribute one whole ball of wool to each scarf, and then cut the remaining six into 9 pieces each, obtaining an answer of $1\frac{6}{9}$.
- Or they may cut the remaining six balls of wool into thirds, and obtain an answer of $1\frac{2}{3}$ balls of wool per scarf. Learners should be encouraged to compare these different answers and convince each other that they are the same.
- Similarly, in question 2, learners may obtain answers of $2\frac{7}{14}$ or $2\frac{1}{2}$.

What learners may learn:

- Consolidation of sharing situations in which the remainders can be further shared.
- Teachers should ensure that learners are able to use the correct symbols to write the fractions.
- Equivalent fractions.

2. Picnic

1. For a special treat a teacher decides to have a picnic with her class and to buy them Tex-bars (chocolate). She buys 15 bars, but only 9 of the pupils show up for the picnic. They decide to share the bars equally between them. Can you help them to do this?

2. She also buys 35 vienna sausages. If the sausages are shared equally among the 9 children, how many sausages will each one get?

3. Marius eats $1\frac{2}{3}$ of the 12 slices of cake that the teacher buys. Is this more or less than his fair share? What would his fair share be?

Teacher Notes:

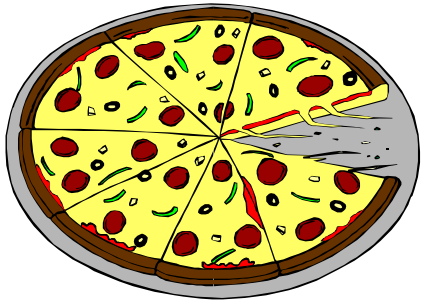
What learners may do:

- Learners may realise that the first problem has the same structure (and answer) as question 1 of 'Scarves and Socks'.
- Learners may think that it is not practical to give each child $3\frac{8}{9}$ vienna sausages, and may suggest that one child should have three while the others all have four. Such suggestions are sensible and should be taken seriously.
- Learners may conclude that a fair share of the 12 slices of cake would be $1\frac{3}{9}$ or $1\frac{1}{3}$, depending on how they share the remainder after allocating one whole slice per child. These answers should be compared and learners should convince each other that they are the same.

What learners may learn:

- Consolidation of sharing situations in which the remainders can be further shared.
- Correct writing of the fractions symbol.
- Equivalent fractions.
- Comparison of fractions ($1\frac{2}{3}$ and $1\frac{1}{3}$).

3. Pizza Party



1. At a pizza party, Jane ate $\frac{1}{6}$ of a pizza, Thobeka ate $\frac{1}{4}$ of the pizza and Hector ate $\frac{1}{3}$ of the pizza.

   (a) Who ate the most pizza?
   (b) How much of the pizza was eaten?

2. At another party, Zonia ate $1\frac{1}{3}$ of a pizza, Karen ate $1\frac{1}{4}$ of a pizza and Dumisani ate $2\frac{3}{8}$ of a pizza. How much pizza did these three eat altogether?

Teacher Notes:

What learners may do:

- If learners conclude incorrectly that $\frac{1}{6}$ is bigger than $\frac{1}{4}$ or $\frac{1}{3}$, they should be referred to the fraction walls given in the Phase 1 fractions package.
- If learners conclude that $\frac{1}{3}$ is bigger than $\frac{1}{4}$ or $\frac{1}{6}$, but give their reason as "the smaller the denominator, the bigger the fraction", they should be challenged. For example they can be asked to compare $\frac{1}{3}$ and $\frac{3}{4}$. The rule "the smaller the denominator, the bigger the fraction" is only true if the numerators of the two fractions are the same.
- Learners who add fractions by adding the numerators and the denominators ($\frac{1}{6}+\frac{1}{4}+\frac{1}{3}=\frac{3}{13}$) can be challenged to reflect on the reasonableness of this answer. They should also be referred back to a fraction wall (Phase 1) which includes sixths, quarters, thirds and twelfths (or twenty-fourths).

What learners may learn:

- Comparison of unit fractions (with denominator of 1).
- Addition of fractions and mixed numbers.

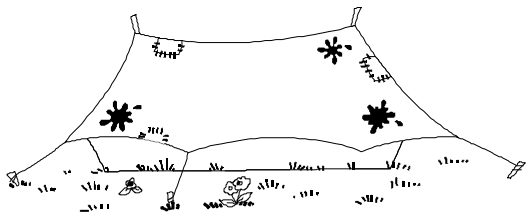
4. Camping Trip

1. Phillip is going on a camping trip for six days. He has some energy bars that he wants to make up into six small packs (one for each day). He can break up the energy bars.

   (a) How many energy bars will be in each pack if there are 25 bars in total?
   (b) How many energy bars will be in each pack if there are 26 bars in total?
   (c) How many energy bars will be in each pack if there are 27 bars in total?
   (d) How many energy bars will be in each pack if there are 28 bars in total?
   (e) How many energy bars will be in each pack if there are 29 bars in total?
   (f) How many energy bars will be in each pack if there are 30 bars in total?

2. There are 25 energy bars that must be made up into equal sized packs for each day of the trip. How many bars will be in each pack if:

   (a) He goes hiking for 4 days?
   (b) He goes hiking for 5 days?
   (c) He goes hiking for 6 days?
   (d) He goes hiking for 7 days?
   (e) He goes hiking for 8 days?
   (f) He goes hiking for 9 days?
   (g) He goes hiking for 10 days?
   (h) He goes hiking for 11 days?



Teacher Notes:

What learners may do:

- Again, several opportunities will arise to discuss equivalent fractions. Learners may, for example, obtain answers of $4\frac{2}{6}$ or $4\frac{1}{3}$ for question 1(b).

What learners may learn:

- Consolidation of sharing situations in which a remainder is obtained which can be further shared.
- Correct use of fraction symbols.
- Equivalent fractions.

5. Designing Chocolate II

1. Design a chocolate slab that can easily be shared equally by 10 people.

2. Design two more chocolate slabs (not the same as the one in Question 1) that can easily be shared equally by 10 people.

3. What fraction of the slab will each person get in each of these cases?

4. Name the fraction of the slab that each person will receive, in two more different ways.

5. You want to make different slabs that can easily be shared equally among
   (a) 9 people
   (b) 12 people
   How must you do this?

6. What do we call fractions of equal size that have different names?

Teacher Notes:

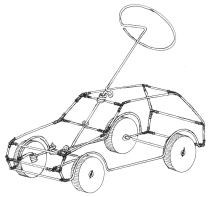
What learners may do:

- Learners may wish to draw elaborate pictures to illustrate their designs. They should be encouraged to draw neat sketches but not to take too much time.

What learners may learn:

- The meaning of the denominator and numerator. The teacher can reinforce this by asking learners how they got their answers in questions 3 and 4.
- Equivalent fractions.

6. More Wire Toys



Mr. Wilson makes toys from wire. He bought a piece of wire from the hardware store.

1. He decides to make a car and he uses a quarter of the wire. What fraction of the wire is left over?

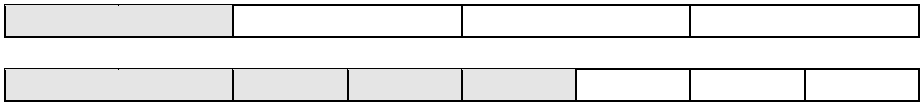
2. Then he decides to make a motorcycle and for that he uses half of what was left over of the original piece of wire. What fraction of the wire is now left over?

3. What fraction of the piece of wire that he bought did Mr. Wilson use to make the car and the motorcycle?

Teacher Notes:

What learners may do:

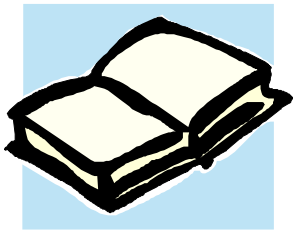
- Learners may want to know how long the piece of wire is. They should be encouraged to reflect on whether or not they need this information in order to answer the questions.
- If learners understand the meaning of a quarter (one of four equal pieces of a whole), they should have no difficulty in seeing that $\frac{3}{4}$ of the piece of wire is left over once the car has been made.
- Learners may need to draw in order to calculate what a half of $\frac{3}{4}$ is. They may also wish to refer back to the fraction wall (Phase 1).
- Other learners may realise that in order to calculate a half of $\frac{3}{4}$, one can just cut each of the quarters into two, thus obtaining eighths.
- Learners will need to think clearly in order to keep track of what answers are required. Questions 1 and 2 require the fractional part which is *left over* ($\frac{3}{4}$ and $\frac{3}{8}$ respectively) while question 3 requires the sum of the two fractional parts *used*, in other words $\frac{1}{4}$ and $\frac{3}{8}$.



What learners may learn:

- Calculating a fraction of a fraction.
- Simple subtraction of fractions.
- Addition of fractions.

7. Reading



The Grade 5's are reading library books as part of a book month.

1. Jack is reading a book with 200 pages, Sally's book has 150 pages and Rose is reading a book with 120 pages.
   "I'm exactly halfway through my book," says Jack.
   "I have read two-fifths of my book," says Sally.
   "I have also read two-fifths of my book," says Rose.
   (a) Who has read the most pages?
   (b) Who has read the least pages?

2. Steve, Sarah and Busisiwe are reading the same book with 120 pages
   On Monday, Steve read $\frac{1}{3}$ of the book, Sarah read $\frac{2}{5}$ of the book and Busisiwe $\frac{1}{2}$ of the book.

   (a) Who read the most?
   (b) Who read the least?

3. By Friday evening, Steve had finished reading $\frac{4}{5}$ of the book, Sarah had read $\frac{7}{10}$ and Busisiwe had read $\frac{3}{4}$ of the book. Who was the closest to the end of the book?

4. What fraction of the book was read by each of the children from Tuesday to Friday?

Teacher Notes:

What learners may do:

- In order to respond to question 1, learners will have to calculate how many pages each of the children has read.
- However, in order to answer question 2, learners do not have to (but may) calculate how many pages each of the children has read. The fact that the whole is the same in this case, means that the fractions can just be compared by using fraction walls or own diagrams or by calculating equivalent fractions.
- Similarly, learners do not have to calculate how many pages are left for each of the children in question 3, but they may choose to. They may also choose to calculate the difference in the number of pages before giving the fractions in question 4.

What learners may learn:

- The concept of a fraction as part of a whole where the whole is a collection of objects (in this case pages).
- Comparison of fractions where the whole is not the same (question 1).
- Comparison of fractions where the whole is the same (question 2).
- Subtraction of fractions.

8. Fractions in all Shapes and Sizes

1. (a) How much is one fifth ($\frac{1}{5}$) of a quarter ($\frac{1}{4}$) of a sausage?
   (b) How much is a quarter of a fifth of a sausage?
   (c) How much is one fifth of four sausages?
   (d) How much is a quarter of five sausages?

2. (a) How much is half of a tenth of a piece of wire?
   (b) How much is a tenth of a half of a piece of wire?
   (c) How much wire is half of 10 metres of wire?
   (d) How much wire is a tenth of two metres of wire?

3. (a) How much is a sixth of a third of a sack of flour?
   (b) How much is a third of a sixth of a sack of flour?
   (c) How many grams is a sixth of three kilograms of flour?
   (d) How many grams is a third of six kilograms of flour?

4. We can also write $\frac{1}{5}$ **of** $\frac{1}{4}$ as $\frac{1}{5} \times \frac{1}{4}$. Now calculate the following:

   (a) $\frac{1}{5} \times \frac{1}{4}$ (b) $\frac{1}{4} \times \frac{1}{5}$

   (c) $\frac{1}{5} \times 4$ (d) $\frac{1}{4} \times 5$

   (e) $\frac{1}{2} \times \frac{1}{10}$ (f) $\frac{1}{10} \times \frac{1}{2}$

   (g) $\frac{1}{2} \times 10$ (h) $\frac{1}{10} \times 2$

   (i) $\frac{1}{6} \times \frac{1}{3}$ (j) $\frac{1}{3} \times \frac{1}{6}$

   (k) $\frac{1}{6} \times 3$ (l) $\frac{1}{3} \times 6$

Teacher Notes:

What learners may do:

- Learners may initially need to draw to solve these problems. This is acceptable, but if other solutions arise, these should also be discussed.
- It is important that different answers are discussed. For example, in question 1(d), learners who reach the answer $1\frac{1}{4}$ should compare their answer with learners who reach the answer $\frac{5}{4}$. This is important so that learners can begin to formulate conjectures about the multiplication of fractions.
- Learners may need to be reminded that there are 1000 g in 1 kg. This is social knowledge which the teacher should pass on if they do not know it already.
- Learners will probably formulate conjectures about the multiplication of fractions and use them to answer question 4. In the case of multiplication of fractions by fractions, they may say that one can 'multiply the tops and the bottoms' or 'multiply the numerators and multiply the denominators'. The use of such rules is acceptable if it is clear that they have solved several problems in a way which shows understanding of the meaning of this multiplication.
- In the case of multiplication of fractions by whole numbers, learners may formulate a different rule or use the same rule as above, taking 4 as $\frac{4}{1}$. They should be encouraged to share and validate each others' conjectures.

What learners may learn:

- Calculating a fraction of a fraction.
- The notation $\frac{1}{5} \times \frac{1}{4}$ for $\frac{1}{5}$ of $\frac{1}{4}$.
- That 'of' (multiplication) is commutative.
- Calculating a fraction of a whole number.
- Multiplication of fractions (context-free).

9. More Biscuits

Mrs Williams wants to use this recipe to make a large batch of biscuits:

COCONUT BISCUITS

2 cups of flour	$\frac{3}{4}$ cups of coconut
$\frac{2}{5}$ cups butter	1 teaspoon vanilla essence
$\frac{3}{4}$ cups of sugar	$\frac{1}{5}$ cup milk
1 egg	$\frac{1}{4}$ teaspoon salt
2 teaspoons baking powder	

This recipe will be enough for about 40 biscuits

Mrs Williams has only 3 cups of butter.

1. How many biscuits can she bake?
2. How much of each ingredient will she need?

Teacher Notes:

What learners may do:

- In order to answer question 1, learners will probably add $\frac{2}{5}$'s to find out how many are in 3. They may query the fact that $\frac{2}{5}$ does not divide exactly into 3, and ask the teacher whether they should use 7 or $7\frac{1}{2}$ when calculating the rest of the ingredients. If learners suggest that one *can* bake half a batch of biscuits, they can calculate $7\frac{1}{2}$ times each ingredient. However, other learners may insist that this is not practical in the kitchen situation, and that each ingredient should be multiplied by 7.
- To calculate the rest of the ingredients, learners may use repeated addition or multiplication. They should compare answers to check their solutions.

What learners may learn:

- Division by a fraction (a grouping problem).
- Repeated addition/multiplication of fractions.

10. Newspaper Advertisements

Last week's issue of the Tonight section of the Cape Argus contained 6 pages.

Page 1 had no advertisements;

$\frac{1}{3}$ of page 2 had advertisements;

$\frac{1}{2}$ of page 3 had advertisements;

$\frac{1}{4}$ of page 4 had advertisements;

$\frac{1}{3}$ of page 5 had music advertisements and

$\frac{1}{4}$ of that page had clothes advertisements; and

$\frac{1}{6}$ of page 6 had advertisements.

How many pages of the whole Tonight section were **not** advertisements?

Teacher Notes:

What learners may do:

- In order to calculate the total number of pages covered by advertisements, learners may convert all the fractions given to suitable equivalent fractions (twelfths or twenty-fourths). However, some learners may find shortcuts, for example they may see that $\frac{1}{2}$ and $\frac{1}{4}$ and $\frac{1}{4}$ gives one whole. They should be encouraged to share their methods with each other.
- In order to calculate the total number of pages NOT covered by advertisements, learners may first calculate the total number of pages covered by advertisements and then subtract this from 6: $6 - (\frac{1}{3}+\frac{1}{2}+\frac{1}{4}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}) = 6 - \frac{11}{6} = 4\frac{1}{6}$
- Other learners may calculate how much of each page is NOT covered by advertisements and then add these fractions together: $1+\frac{2}{3}+\frac{1}{2}+\frac{3}{4}+\frac{5}{12}+\frac{5}{6}=\frac{50}{12}=4\frac{1}{6}$. The $\frac{5}{12}$ would probably be calculated by subtracting the $\frac{1}{3}$ and $\frac{1}{4}$ covered in different advertisements on page 5, from 1 whole.

What learners may learn:

- Addition and subtraction of fractions.

TO NEXT