Malati

Mathematics learning and teaching initiative.

Introductory Calculus

Module 2

Grade 10 and 11

TEACHER DOCUMENT

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Activity 1

A. The Broken Tachometer

Mrs Brown and her husband are on a very long journey by car. They are travelling on a highway that bypasses all towns, and Mr Brown had extra fuel tanks fitted to his car to extend the range to about 1500km.

Mrs Brown likes to keep track of the distance she has covered, but discovers soon after departing from home that the tachometer (distance meter) in the car is broken, and Mr Brown forgot to bring the road map. The speedometer still works and Mrs Brown is wearing a watch.

1. Describe how they could determine the distance they have travelled.

2. Mrs Brown decides she will record the speedometer reading from time to time, and that she will use these readings to estimate the distance covered from this record. The record for the first two hours of the journey is given below. (They left home at 8:30 am). The speeds are given in km/h.

Time	8:35	8:49	9:01	9:09	9:21	9:25	9:29	9:47
Speed	84	126	122	93	75	81	84	118

9:53	9:59	10:07	10:13	10:17	10:21	10:25	10:29
112	122	134	131	124	114	109	116

3. Estimate, as well as you can, what distance they have covered from 8:30 till 10:30. Take time and trouble to really make the best estimate you can.

4. Provide a written account of how you obtained your estimate.

5. If you undertook a similar journey and encountered the same problems would you take similar readings to Mrs Brown? Is there anyway of using the same method as Mrs Brown but obtaining a better estimate of the distance travelled?

6. Can you predict how far they will have travelled when they stop at midday for lunch?

B. The broken speedometer

Gowell's transport company has to make a delivery from Bloemfontein to Cape Town. The distance between the two cities is 1041 km. The driver sets off at 5:30 and must arrive in Cape Town before 17:00, every hour that he is late the company has to pay a penalty fine. For safety reasons the company insists that the driver take 3 half-hour stops to rest and refuel if necessary. Shortly after leaving Bloemfontein he notices that his speedometer is not working. The tachometer still works and the driver is wearing a watch.

1. Describe how the driver could determine when he will arrive in Cape Town

The driver decides to record the tachometer recordings from time to time, and he will use these readings to estimate his speed. After four hours the driver takes his first rest. The record for these four hours is given below.

Time	5:50	6:25	7:00	7:45	8:10	8:55	9:25	9:30
Distance	35	98	168	255	305	386	436	441

2. Estimate, as well as you can, the speed of the vehicle during the first four hours.

3. Provide a written account of how you obtained your estimate.

4. If the driver managed to maintain a similar speed for the rest of the journey would he reach Cape Town before 17:00? Explain your answer.

5. Due to the passes through the mountains in the Western Cape, the driver will have to reduce his average speed by 10 km/h for the last 250 km. Will the driver make it to Cape Town on time? Explain your answer.

6. During the driver’s first stop he meets you in the service station and explains his problem to you. Could you help him to devise a plan that will give him a more accurate or easier to calculate estimate of his speed? Explain your plan.

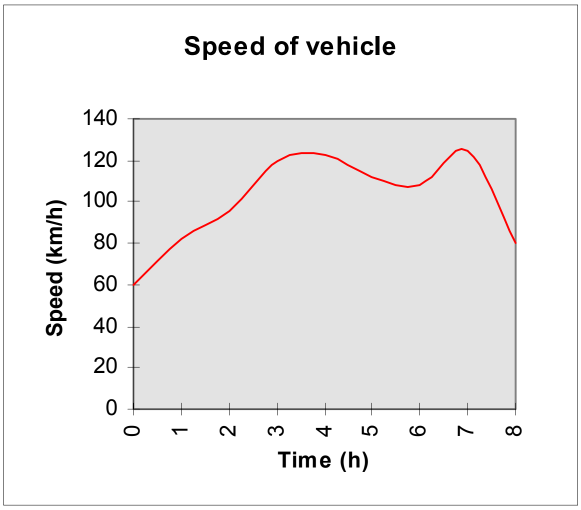
Recap
In the broken tachometer problem we knew the speed of the vehicle and from this we could estimate the distance travelled. In the broken speedometer problem we knew the distance travelled and from this we could estimate the speed.

1. Describe the relationship between distance and speed.

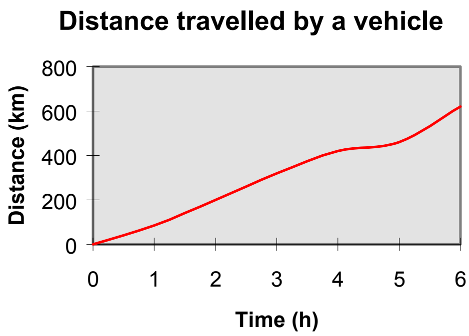
2. The units we used were kilometres per hour (km/h) for speed, kilometres (km) for distance and hours (h) for time. What does km/h mean?

3. Can you use this to explain the relationship between distance and speed?

4. Use the following graph to determine the distance travelled by a vehicle during an eight-hour journey. You may assume that the time it took to reach 60 km/h initially and the stopping time are negligible. Explain your technique clearly and discuss ways that could improve your estimate. Is there anyway of exactly determining the distance travelled?



5. Use the following graph to determine the average speed of a vehicle for each hour of a six-hour trip. Explain your technique clearly. Could you use a similar technique to determine the average speed during a half-hour interval? Could you determine the exact speed of the vehicle after 3h15 min?

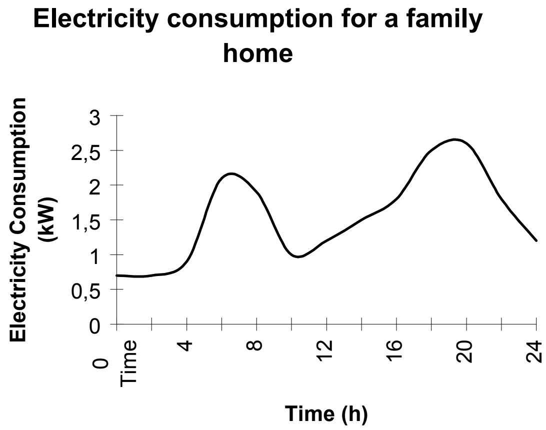


6. When do you think the driver stopped for a cup of coffee? Justify your answer.

Activity 2

A. Electricity consumption

Below is a graph of the average electricity consumption for a family home in Bellville.



1. Write a story explaining why the average electricity consumption would look like this?
2. How could you estimate the amount of electricity consumed in any hour?
3. What is the maximum and minimum amount of electricity used in any hour?
4. When is the electricity consumption increasing and when is it decreasing?
5. At what stage during the day does the electricity consumption increase most rapidly?
6. Estimate the total number of kilowatts used by the household in an average day. Explain your method clearly.

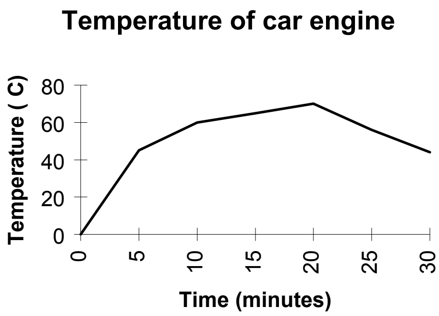
Recap
We say a **function is increasing** if the dependent variable increases as the independent variable increases. A **function is decreasing** if the dependent variable decreases as the independent variable increases.

1. Two rabbits were introduced to an island where they had no predators. A scientist kept a record of the rabbit population.

Time (years)	1	2	3	4	5	6	7	8	9
Rabbit population	2	4	8	16	32	64	128	256	512

(a) Which is the dependent and independent variable?
(b) Is the function increasing or decreasing?

2. The temperature in a motor car engine is recorded during a 20-minute journey and for 10 minutes after the engine has been switched off.



(a) What is the independent variable?

(b) When is the temperature increasing and when is it decreasing?

We say a function has a <u>maximum</u> if there exists a function value which is greater than all other function values, and a <u>minimum</u> if there exists a function value which is less than all other function values.

3. What is the minimum number of rabbits on the island during the nine years when the scientist recorded the rabbit population?

4. What is the maximum temperature of the car engine during the given half-hour?

B. Cost of electricity

The electricity department charges R40 monthly service fee then an additional 20c per kilowatt-hour. A kilowatt-hour is the amount electricity used in one hour at a constant power of one kilowatt.

1. Use your estimate for the daily electricity consumption of a family home in the previous worksheet to predict what the monthly account would be for electricity.

2. Three people live in a flat. Their monthly electricity account is approximately R180. How many kilowatts per month do they usually use?

3. In winter the average electricity consumption increases by 20%, what would the monthly bills be for the household and for the flat?

4. Determine a formula to assist the electricity department to calculate the monthly electricity bill for any household. State clearly what your variables represent and the units used.

5. After careful consideration the electricity department decide to alter their costing structure, they decide that there will no longer be a monthly service fee of R40 but now the each kilowatt-hour will cost 25c.

6. What would be the new electricity accounts for the household and the flat?

7. Do both the household and the flat benefit from this new costing structure?

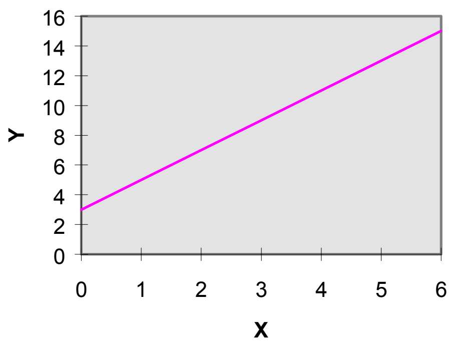
8. If people using the electricity had the option of choosing either of the two costing structures which would you recommend? Clearly explain your answer and make use of tables, graphs and formulae to support your reasoning.

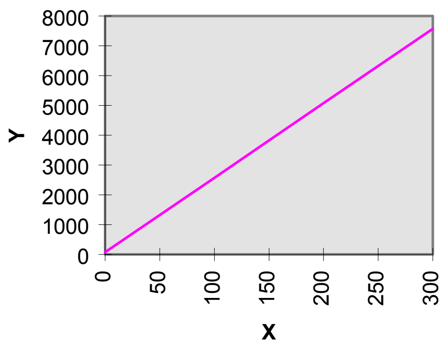
Recap

In both costing structures the client pays a fixed amount for every additional kilowatt-hour used. The difference between the two structures was that in the first structure the client paid a fixed amount before any electricity was used in the second the client did not pay anything if no electricity was used.

With straight line graphs there are two characteristics that can alter the line, namely the gradient and the starting point (or y-intercept). In each of the following graphs identify the gradient and y-intercept.

Straight line graphs





1. What is special about the gradient of a straight line graph?

2. Are there any other graphs that could have the same properties but not be a straight line?

3. In the algebraic representation, $y = mx + c$, of a straight line graph what represents the gradient?

4. Each time x-value increases by one what happens to the y-value?

5. In the two electricity problems what were the gradients and what were the x and y values?

Activity 3

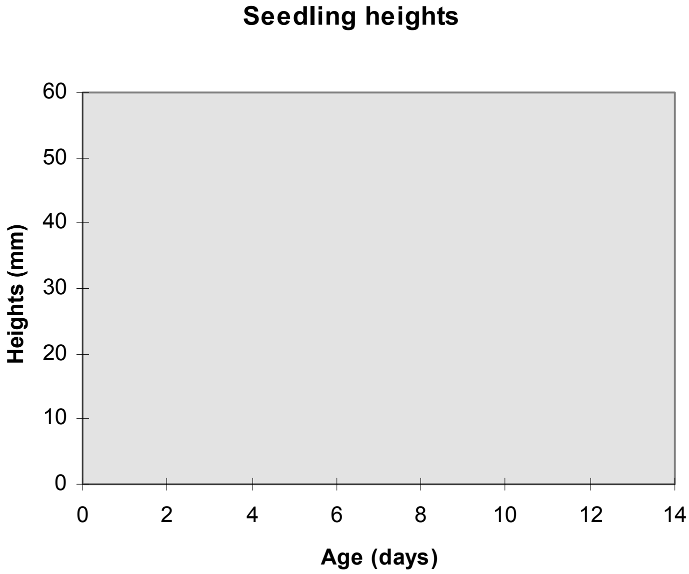
Plant growth

A Biology class measured the height of two seedlings over a two-week period. The following information was recorded.

Day	0	2	4	6	8	10	12	14
Seedling A (Height in mm)	0	6	12	18	24	30	36	42
Seedling B (Height in mm)	0	4	10	19	26	37	45	56

1. What was the daily growth of seedling A?
2. What was the daily growth of seedling B?
3. What is different about the growth of the two seedlings?
4. When was seedling A 21 mm tall? Explain your method of calculation.
5. When was seedling B 21 mm tall? Explain your method of calculation.
6. After 11 days what was the height of seedling A?
7. After 11 days what was the height of seedling B?
8. Explain how the age and the height of seedling A are related. Can you provide a formula to determine the height of the seedling at any time during the two-week period?
9. Explain how the age and the height of seedling B are related. Can you provide a formula to determine the height of the seedling at any time during the two-week period?

10. Plot the heights of both seedlings over the two-week period on the same graph.



11. If seedling A continued to grow at the same rate when would it be 100 mm high?

12. Can you predict when seedling B would have been 100 mm high?

13. Can you predict the height of the seedling after 20 days?

14. For which seedling can you more accurately predict information about the height of the seedling? What makes it easier for you to predict the height?

Project

Grow your own seedlings carefully measuring their growth over the initial two-week period. Analyse the growth of the seedling and determine whether your seedling behaved in the same way as those of the Biology class. If not, can you explain the relationship between your seedling's age and height, can you predict the height of your seedling four weeks later? (Keep your seedling to see whether your prediction is correct).

Recap
Seedling A had linear growth, in other words every day it grew by the same amount. Seedling B had erratic growth, in other words it grew by different amounts every day.

With linear functions it easy to interpolate (find a value between two existing values). For example, finding the height after 11 days for seedling A, you might have used either of the following methods:

Method 1: height after 10 days is 30 mm
daily growth is 3 mm
$\therefore$ height after 11 days = 30 mm + 3 mm = 33 mm

Method 2: height after 10 days is 30 mm
height after 12 days is 36 mm
$\therefore$ height after 11 days = (30 mm + 36 mm) $\div$ 2 = 33 mm

However with erratic growth, it is not so easy. For seedling B you could not use Method 1 as the daily growth isn't a constant value. You could use method 2 but this is still inaccurate, as you do not know that the seedling grew by the same amount on the 11^{th} and 12^{th} day. There are several ways to improve your estimate but it will always only be an estimate. (*Use the trend-line facility on a computer to study different possible ways of estimating the growth)*

With linear functions it easy to extrapolate (predicting future values).
For example, predicting the height of seedling A after 20 days you might have used the following method:

Method 1: height after 14 days is 42 mm
daily growth is 3 mm
growth for 6 days = 6 $\times$ 3 mm = 18 mm
$\therefore$ height after 20 days = 42 mm + 18 mm = 60 mm

We must assume that for seedling A, the growth continues at a constant rate. To make this assumption you need to know more about plant growth. As a mathematician, you can only say what would happen if this assumption were true.

For seedling B, it gets even more complicated, not only do we have to assume a given growth rate after a two week period, but we also have to work out what that growth rate should be. You could use the average growth rate over the two-week period or you could use the average growth rate from the 12^{th} to the 14^{th} day or any other time period.

The fact that we can easily interpolate and extrapolate linear functions (straight line graphs) because their rate of change is constant, makes linear function a very useful modelling tool. Often when we collect data, the data is not quite a linear function but we use a linear function to *approximate* the data so that we can make predictions.

1. A shoe company is staring the production of a new running shoe. The cost to design the new shoe, train the production staff and change the advertisements has been estimated at R570 000. Once the above have taken place it is estimated that each pair of shoes will cost R45 to make.

   Complete the following table of costs.

Number of pairs of shoes	0	20	350	1500	70000
Cost (Rand)					

   (a) Explain how you determined the cost in the above table.

   (b) The cost is a function of the number of pairs of shoes produced. What type of a function is it? Justify your answer mathematically.

   (c) The pairs of shoes sell for R300 each. What is the revenue (the money received by the factory) on 60 pairs of shoes?

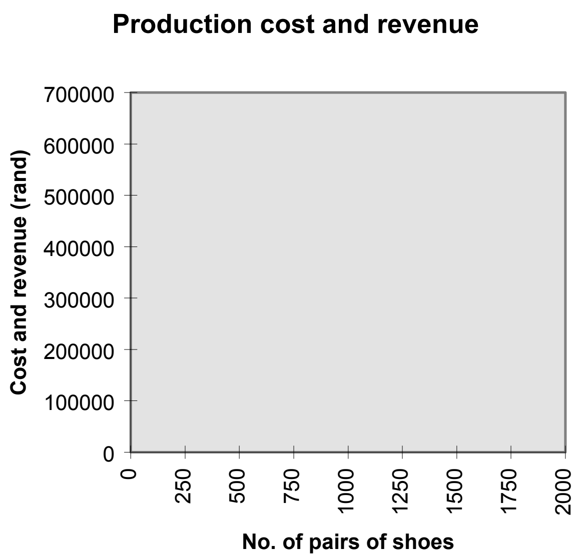
   (d) What type of a function is the revenue function? What is the independent variable?

   (e) Describe what is meant by profit.

   (f) How many pairs of shoes would the company have to make and sell in order to make a profit?

   (g) Can you provide a formula that can be used to determine the profit? If you can, describe what type of function it is.

   (h) Plot both the cost and revenue functions on the same set of axes. Describe the similarities and differences between the two graphs.



(i) Do the graphs cross at any stage? If they do what is important about that point?

(j) Plot the profit function on the above graph. Do you notice anything? Explain your observations.

2. The Grade 7 mathematics class is studying the circle. They have each drew a different a different sized circle and has measured the radius, diameter and circumference of their circle. A few of their results are shown below.

Radius (mm)	10	12	25	13	20	8
Diameter (mm)	20	24	50	26	40	16
Circumference (mm)	63	75	157	82	126	50

Are there any relationships between the measurements taken? Analyse all the pairs of measurements e.g. radius and circumference, and radius and diameter. Explain the relationships, if any, that you discover. Use graphs, tables and formulae to assist your explanation. Do you think your relationships will hold true for any circle? Motivate your answer.

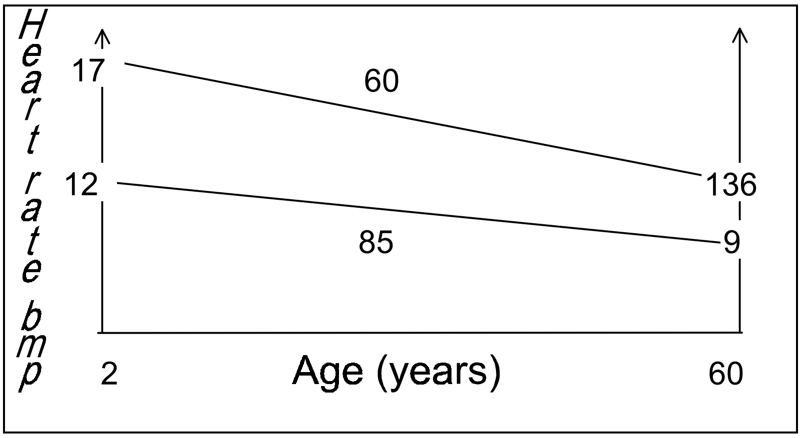
Activity 4

Heart rate

While you are exercising your heart rate increases so that more blood is pumped round your body. The blood carries oxygen to the muscles so that they can work harder. As you get older your heart rate slows down and you have to be careful that you don't over exert yourself.

1. Heart rate is given in beats per minute (bpm). Measure your heart rate.

2. Below is a table that sports people use to determine how hard they are exercising by measuring their heart rate. (The scale is not accurate.) Use this table to answer the following questions:



2.1 The following sports people want to know what their heart rate must be if the are exercising at 85 % of their maximum heart rate.

Name	**Age**	**Heart rate**
Penny Heyns	21	
Elana Meyer	30	
Chester Williams	27	
Paul ‘Gogga’ Adams	23	
Doctor ‘16V’ Khumalo	35	
Jacob ‘Baby Jake’ Matlala	43	

2.2 During the warm up session for Orlando Pirates soccer practice, the coach decides that the players' heart rate must be 60% of their maximum heart rate. Use the table below to calculate the ages of the following players.

Heart rate (at 60% of maximum)	**Age**
117	
114	
108	

2.3 This table only starts at 20 years old can you work out a way of calculating what 60% and 85% of your maximum heart rate would be? Explain your method carefully.

2.4 Run on the spot for 1 minute then measure your heart rate again. What do you notice?

2.5 Run quickly on the spot for another minute then measure your heart rate again. What percentage of your maximum heart rate have you reached? Explain how you got your answer.

2.6 Your grandfather is 80 years old. After walking to the shops his heart rate is 60% of his maximum heart rate. What is his heart rate in beats per minute?

2.7.1 What is the maximum heart rate of a 40 year-old?

2.7.2 If a person's heart rate is 50% of its maximum while they are at rest what is the resting heart rate of a 30 year-old?

2.7.3 Can you determine a formula for calculating the maximum heart rate of a person of any age?

Recap

In this activity we used a linear model to represent the relationship between heart rate and age. Do you think that everybody of the same age has exactly the same heart rate? Obviously not, we all differ physiologically, which is why some people are better athletes than others. This linear model just gives a rough idea of what the heart rate should be for a given age. Using the linear model people can calculate their maximum heart rates and can thus prevent possible heart problems by not exceeding this limit.

Project

Investigate another relationship in real-life that could be represented by a linear model. Justify why you think a linear model would be suitable and use your model to make predictions about what would happen in the future and what might have happened in the past.

Activity 5

Tides

At the entrance to a small fishing harbour there is a ruler painted onto the side of the jetty sticking into the sea. Boats can use this to check whether the water is deep enough for them to enter or whether they must wait for high tide when the water is deeper. The harbour manager keeps a record every day of the water-levels so that he can accurately tell boats how the tides are changing so the boat captains know when to come and off-load their fish catches.

Below is a table of the data the harbour manager collected over a period of slightly longer than 24 hours. (Do not complete the last column at this stage).

Time	Water-level (m)	Change in water level (m)
Wednesday:00:00	1.6	
01:02	1.9	
02:04	2.12	
03:06	2.2	
04:08	2.12	
05:10	1.9	
06:12	1.6	
07:14	1.3	
08:16	1.08	
09:18	1.0	
10:20	1.08	
11:22	1.3	
12:24	1.6	
13:26	1.9	
14:28	2.12	
15:30	2.2	
16:32	2.12	
17:34	1.9	
18:36	1.6	
19:38	1.3	
20:40	1.08	
21:42	1.0	
22:44	1.08	
23:46	1.3	
Thursday: 24:48	1.6	

The harbour manager always takes readings at intervals of 1 hour and 2 minutes. Can you give any explanation why the harbour manager has chosen this time interval?

1. When was high-tide? When was low-tide?

2. Will high-tide be at the same time on consecutive days?

3. Is there any way of predicting what time high-tide will be on the following day? Explain your method.

4. The time it takes to go from one high-tide to the next is called a tidal cycle or period. What is the tidal cycle for this harbour? If we measure the time between low-tides will it be the same?

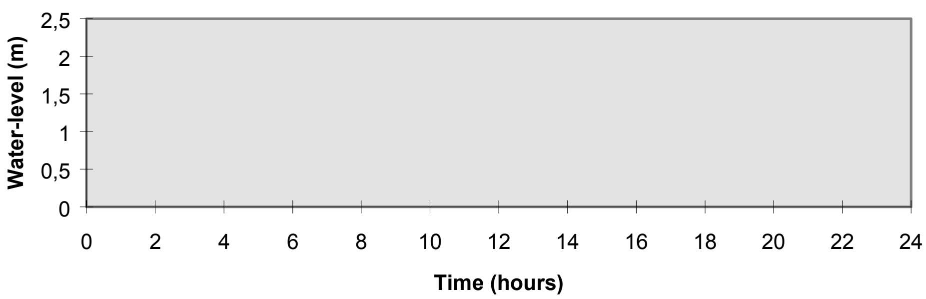
5. A fishing boat wishes to come into the harbour on the Wednesday. The boats keel is 1,8m deep. At what time(s) during the day will it be able to enter the harbour?

6. The captain of this boat needs to do repair work to the wood just below the water-line of the boat. He wishes to scrap down the wood and apply a seal to stop it from leaking. The job will take 5 hours to complete. Will the wood below the water-line be exposed for long enough to complete the job or will the captain have to do it over two tidal cycles?

7. Another fishing boat has a keel depth of 2,5m. Can it enter the harbour?

   Use the recorded data to plot a graph of the water level.

Water-level in Harbour



8. Describe the shape of the graph you have drawn.

9. Use your graph and the table to answer the following questions.

10. Does the water level change at a constant rate every hour? Justify your answer.

11. When is the water level increasing and when is it decreasing?

The speed of the seawater flowing into or out of the harbour is effected by the speed at which the water level is changing. If the water level is increasing rapidly, the water will flow quickly into the harbour. When the boats come in to moor, the captains must know the speed of the water. For this reason we are going to look at the rate of change of the water level.

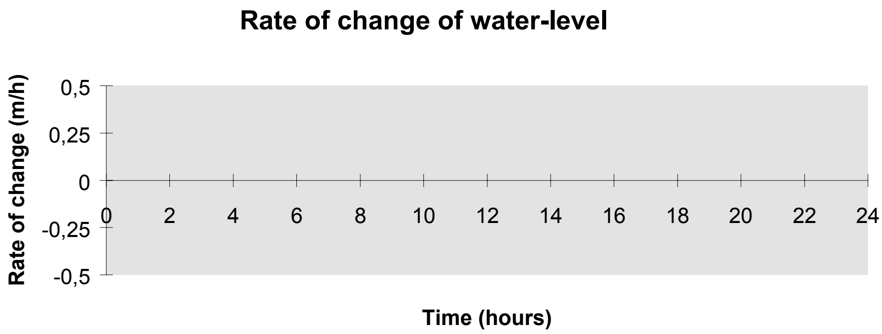
Return to the table of data and calculate the change in the water level during any given time interval by taking the current water level and subtracting the water level in the previous time interval. You will have to estimate what the water level was late on Tuesday night to be able to fill in the first entry. (You should be able to do this by now). If the water level is decreasing we make the rate of change negative, for example

19:38	1.3	
20:40	1.08	1.08 – 1.3 = –0.22

1. At what times do you think the water would neither flow into or out of the harbour? What is the rate of change of the water level at these moments?

2. At what times do you think the water would flow the fastest? What would be the rate of change of the water level at these moments?

3. Using the data you have filled in on the table, and the answers to the previous two questions, plot the graph of the rate of change of the water level on the graph below.



4. Look carefully at the two graphs. Comment on any similarities and differences between the two graphs.

5. Using the rate of change of the water level graph can you predict when the water will flow most rapidly into the harbour and when it will flow most rapidly out of the harbour? Does this agree with your previous answers?

6. Can you predict when the water will flow most rapidly into the harbour on Thursday?

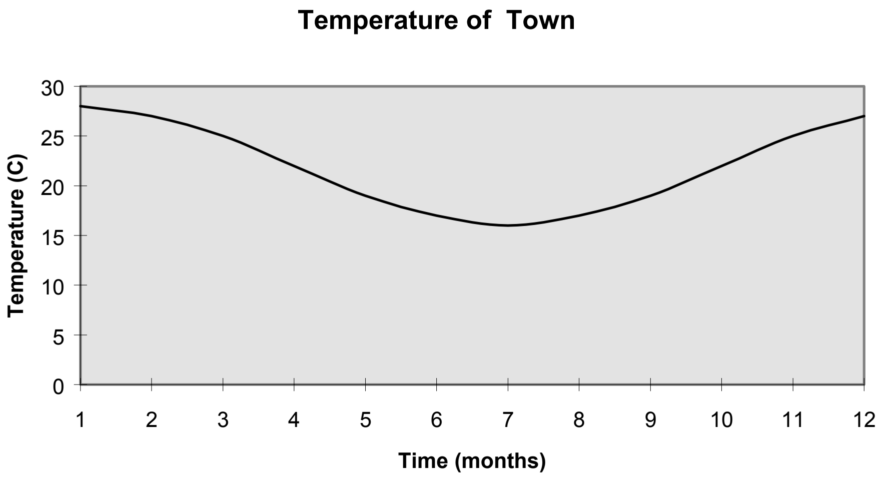
7. Is the length of the cycle the same for both the water-level graph and the rate of change of the water level graph?

8. When the water-level graph has a maximum or minimum what is happening to the rate of change of the water level graph? Can you explain your findings?

Recap
Both graphs repeat themselves after a given time (or period) thus they are called periodic. They also have a wavy shape. Mathematically we call this sinusoidal, which comes from the Latin word *sinuare,* which means bend. Thus both the water-level graph and the rate of change of the water level have graphs that are sinusoidal graphs.

The graph of the water level and the rate of change of the water level had the same period (time interval between repeats) but the maximums of the two graphs did not occur at the same time, there was one quarter of a period separating the maximums. Will this always be true for the difference in time between the maximum of a function and the maximum of the rate of change of the same function?

Sinusoidal graphs occur commonly in real-life. Below is the how the monthly average temperature in a town changes during the year.



Is this town in the northern or southern hemisphere? Explain your reasoning.

Project

Investigate another relationship in real-life that would be periodic and sinusoidal. Find the period and sketch the graph of the relationship. Analyse the rate of change of this function.

Activity 6

Growth and Decay

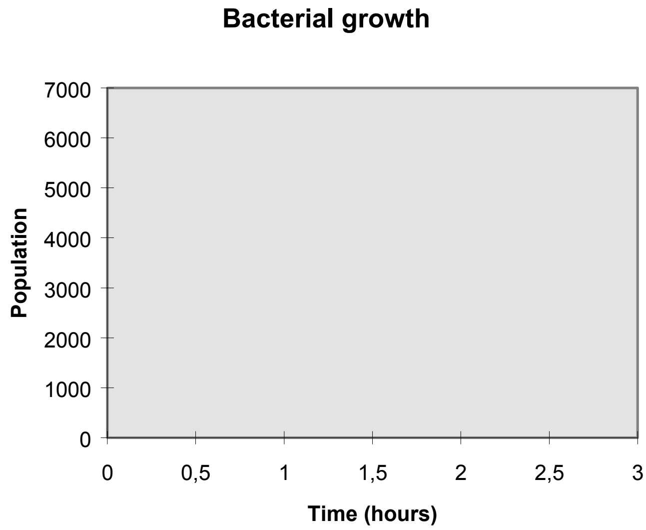
The bacteria population in a certain culture doubles every 30 minutes. The number of bacteria initially present is 100, complete the following table.

Time (hours)	0	½	1	1½	2	2½	3	3½
Bacteria population	100	200	400					
Change in population		100	200					

(The change in population is given as the difference between the population at the end of the current time-interval, and the population at the end of the previous time interval).

1. Describe the similarity between the population and the rate of change of the population.

2. Plot the graphs of both the population and the rate of change of the population on the axes below.



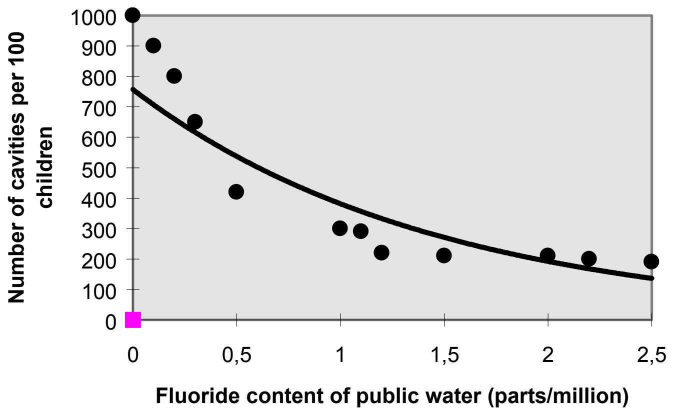
3. Can you predict how many bacteria there will be after 5 hours? Explain your method clearly.

4. Estimate the number of bacteria after 2 hours and 20 minutes. Explain your method.

5. In the last twenty years cavities in teeth have become far less common. Probably eating less sugar and better health education have helped reduce the occurrence of cavities, but could this account for the dramatic decrease? There has been one other important change in tooth care since 1971, the introduction of fluoride in the public water supplies.

As early as 1933, it was noticed that in areas where the water naturally contained high levels of fluoride, children had fewer cavities. A series of studies in America confirmed this. They also showed that though there was a marked improvement in children's teeth as the level of fluoride increased to 1 mg/l, there was very little further improvement if the concentration of fluoride was higher than this.

The relationship between fluoride in the drinking water and the incidence of cavities



Above is a graph showing the relationship between the fluoride content in the public water and the number of cavities per 100 children. The **black dots** represent the findings of several samples taken and the **solid line** is a trend-line, this gives an overall picture of what is happening.

1. Name the dependent and independent variables.

2. Estimate how many cavities you would expect in a group of 100 children where the fluoride content in the public water is 0,75 parts per million.

3. Where is the rate of change in the number of cavities relative to the fluoride content largest?

4. As the fluoride content increases what happens to the rate of change in the number of cavities?

5. On the same axis above plot what you think the rate of change graph would look like. Explain your reasoning for the graph you have drawn.

6. What type of function do you think the trend-line is? Complete the table below for the trend-line.

Fluoride content (parts per million)	0	0.5	1	1.5	2	2.5
Number of cavities per 100 children						
Rate of change in number of cavities per 100 children						

(a) Analyse this table of data. Using the trend-line can you predict what will happen if the fluoride content is raised to 5 parts per million?

(b) What do you notice about the rate of change in the number of cavities in 100 children as the fluoride content increases?

(c) Is there a formula or way of predicting the rate of change of this function? If there is show how you could use it?

Recap
Both the bacteria problem and the cavity problem could be modelled with exponential functions. Both are models, the exact bacteria population size at a given time or the exact number of cavities per 100 children for a given fluoride content are probably different than the values given by the models. The models however gives us useful trends so we can predict future bacteria populations or what will happen if we further increase the fluoride content in the water.

If we look at the bacteria model we would notice that the ratio between consecutive population sizes (at the end of ½ hour intervals) was constant, in this case the ratio was 2. The same applied to the rate of change of the population size at the end of the ½ hourly intervals.

In the cavity model you would have noticed that this ratio between consecutive counts of the number of cavities (at fluoride increases of multiples of 0,5 parts per million) was also constant (not a nice number this time!) If you analyse the rate of change of the number of cavities you will also note that the same ratio occurs, again at fluoride increases of 0,5 parts per million.

By looking at function values at discrete intervals we are actually working with a geometric series not an exponential function. An exponential function is a continuous function. However by making the discrete intervals very small and we would have a function that looks and behaves in a similar manner to the exponential function.

1. Will the rate of change of all geometric series be similar to the function itself?

2. Complete the following tables to help you answer the above question.

Value of series	2	6	18	54	162	486
Rate of change		4	12			

Value of series	48	24	12	6	3	1,5
Rate of change						

Value of series	1500	1050	735	514.5	360.15	252.105
Rate of change						

Value of series	2500	2575	2652.25	2731.82	2813.77	2898.19
Rate of change						

Tasks

1. Radiocarbon has a half-life of 5 568 years. Radiocarbon in all animals and plants is continually replaced from the atmosphere while the animal or plant is alive. However, as soon as the animal or plant dies, the radiocarbon starts to decay. Normal carbon however, is stable and thus does not start to decay once the animal or plant dies. An archaeologist can thus measure the ratio of radiocarbon to normal carbon in fossils to determine the age of the fossils.

   (a) It is found that 50% of the original radiocarbon in a wooden archaeological specimen has decomposed. When was this wooden specimen part of a living tree?

   (b) It is found that 87,5% of the original radiocarbon in a mollusc fossil uncovered in a geological survey has decomposed. When was this mollusc alive? Over the next 5 568 years, how much more of the original radiocarbon will decompose?

2. If you invest an amount in a savings account and at the end of each investment period you reinvest the interest earned we say the interest is being compounded. Similarly, if you borrow money from the bank and at the end of each time period you can not repay the loan, the bank adds the interest to the loan account and then charges you interest on this total amount, we say the bank is compounding the interest.

   (a) You invest R1 000 in an account paying 12% interest compounded annually. Calculate the amount you will have after 5 years.

   (b) Will the increase in the amount be the same during the sixth year as it was during the third year? Explain you answer carefully.

   (c) Determine the increase in the amount during the 30th year. (Do not calculate what happens each year for 30 years, there are easier ways to calculate this).

   (d) During which year will the change in the amount first exceed R2 000? Clearly show your method of determining this.

Activity 7

Rabbits and Jackals

A Karoo farmer was having a problem with jackal taking his lambs, so he set traps for the jackal. It took six months for the entire jackal on his land to be killed. What the farmer did not foresee was that the rabbit population would grow as their main natural predator, the jackal, had been removed. In ideal conditions a rabbit population will double every six months.

Answer the following questions giving clear explanations.

1. In the initial six months, while the jackal are being trapped, would the rabbit population have doubled?

2. In the following six months how would the rabbit population have changed?

3. Would the rabbit population have continued to grow indefinitely? If not, what factors would have influenced the rabbit population.

4. After two years the farmer realised that the rabbits were becoming a strain on the carrying capacity of the land, so he allowed jackal to return to the land.

5. Do you think the jackal population returned to its original size? If not, what do you think happened?

6. On the same system of axes, plot rough sketches of how you think the rabbit and jackal population would have changed over a 10 year period, starting when the jackal were being trapped. Assume the initial ratio of rabbit to jackal was 25:1 and that there were an estimated 10 jackals on the farmers land. Clearly show initial populations, growth and or decline rates and any significant turning points on your graphs. Label these points and give clear reasons for their existence.

7. What conclusions can you draw? Do you think a mathematical knowledge of how certain animal population's change under varying conditions is important to agricultural and environmental management?

Activity 8

Parachute Jumping

1. Two glasses fall from a table. The one glass falls on the top of a smaller table, and it does not break. The other glass falls right down to the floor, and breaks.

   Why does the one glass break and the other one not?

> When an object is dropped, it starts to fall quite slowly at first. However, the speed gradually increases. A small hailstone can create a lot of damage when it strikes the earth, because it started falling high up and by the time it reaches the earth it is falling very fast.
>
> This is why people use parachutes when they jump from aeroplanes. Without a parachute, a person would just fall faster and faster and will probably get killed when hitting the ground. For instance, 6 seconds after having jumped from an aeroplane, a person without a parachute would fall at a speed of about 180 metres per second (that is more than 600 km/h) and would certainly get killed when hitting the ground.
>
> With a parachute the speed increase is slowed down because of the air resistance of the parachute. A person with an open parachute falls at a gentle, almost constant speed.

2. Vincent uses a big parachute that slows him down to 5 m/s when falling. Dudu uses a smaller parachute, which slows her down to 8 m/s when falling.

   (a) How far will Vincent fall in 12 seconds with his parachute open?
   (b) How far will Dudu fall in 12 seconds with her parachute open?
   (c) How long will it take Vincent to fall through a distance of 120 m with his parachute open?
   (d) How long will it take Dudu to fall through a distance of 120 m with her parachute open?

3. Miriam and Henry, two parachutists, have jumped from an aeroplane at the same time. They did not open their parachutes immediately. The distances they have fallen after various periods of time are given in the table below.

Time (seconds)	0	2	4	6	8	10	12	14	16
Miriam's fall distance (metres)	0	20	41	49	57	65	73	81	89
Henry's fall distance (metres)	0	20	80	180	217	235	253	271	289

(a) Approximately when did Henry's parachute open?
(b) Approximately when did Miriam's parachute open?
(c) If you look carefully at the data in the table, does Henry's speed become constant at some time after he jumped from the plane?
(d) At what speed is Miriam falling, 14 seconds after she had jumped from the plane?
(e) At what speed is Henry falling, 14 seconds after he had jumped from the plane?
(f) What is Henry's effective speed over the first 6 seconds after had he jumped from the plane?
(g) At precisely 6 seconds after he jumped from the plane, is Henry's speed equal to his effective speed over the first 6 seconds after he jumped from the plane?
(h) What is Henry's effective speed over the period from 2 seconds up to 6 seconds after he jumped from the plane?
(i) What is Henry's effective speed over the period from 4 seconds up to 6 seconds after he jumped from the plane?

4. Which of the following is the best approximation of the speed at which Henry is falling at exactly 6 seconds after he had jumped from the plane?
(a) His effective speed over the first 6 seconds after he had jumped from the plane.
(b) His effective speed over the period from 2 seconds up to 6 seconds after he jumped from the plane.
(c) His effective speed over the period from 4 seconds up to 6 seconds after he jumped from the plane.

5. Determine Henry's actual speed at each of the following moments as accurately as you can. In some cases you will only be able to give an estimate of his actual speed.
(a) 16 seconds after he had jumped.
(b) 14 seconds after he had jumped.
(c) 12 seconds after he had jumped.
(d) 10 seconds after he had jumped.
(e) 8 seconds after he had jumped.
(f) 6 seconds after he had jumped.

(g) 5 seconds after he had jumped.
(h) 4 seconds after he had jumped.

6. Here is a more complete table of data concerning Miriam and Henry's jumps:

Time (seconds)	0	0,5	1	1,5	2	2,5	3	3,5	4
Miriam's fall distance (metres)	0	1,25	5	11,25	20	31,25	37	39	41
Henry's fall distance (metres)	0	1,25	5	11,25	20	31,25	45	61,25	80

Time (seconds)	4,5	5	5,5	6	6,5	7	7,5	8	8,5
Miriam's fall distance (metres)	43	45	47	49	51	53	55	57	59
Henry's fall distance (metres)	101,25	125	151,25	180	199	208	212,5	217	221,5

(a) Use the information in this table to make a better estimate of Henry's actual speed at exactly 6 seconds after he had jumped from the aeroplane.
(b) Calculate Henry's effective speed over the period 4 seconds until 4,5 seconds after he had jumped from the plane.
(c) Precisely at 4,5 seconds after he had jumped from the plane, is Henry falling at the same speed as his effective speed over the period 4 seconds until 4,5 seconds after he had jumped from the plane?

7. For the part of the fall before he has opened his parachute, the distance $\boldsymbol{S}$ which Henry has fallen in $\boldsymbol{t}$ seconds after he jumped from the plane can be calculated with the formula $\boldsymbol{S} = 5\,\boldsymbol{t}^2$. Use this formula to complete the following table:

Time (seconds)	5,4	5,5	5,6	5,7	5,8	5,9	6	6,1	6,2
Henry's fall distance (metres)		151,25					180		

Use the information in this table to make a better estimate than before, of Henry's actual speed at precisely 6 seconds after he had jumped from the plane.

8. Use the formula given in question 7 to generate data which will make it possible for you to obtain an even better estimate of Henry's speed at precisely 6 seconds after he had jumped from the plane.

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