Mathematics learning and teaching initiative

## Geometry

## Module 2

## Representations

## Grades 4 to 7

## Teacher Document

Malati staff involved in developing these materials:
Kate Bennie
Zonia Jooste
Dumisani Mdlalose
Rolene Liebenberg
Piet Human
Sarie Smit

We acknowledge the assistance of Zain Davis, Shaheeda Jaffer, Mthunzi Nxawe and Raymond Smith in shaping our perspectives.

## COPYRIGHT

All the materials developed by MALATI are in the public domain. They may be freely used and adapted, with acknowledgement to MALATI and the Open Society Foundation for South Africa.

## Representations of three-dimensional objects in two dimensions (Photographs Theme)

## The previous activities required that learners

1. Interact with actual objects in space.
2. Interact informally with the representations of these objects in 2-dimensional space (using perspective drawings).

It is important to remember that in the case of representations of 3-dimensional objects in 2 dimensions (in the form of plans, perspective drawing, nonperspective drawing and nets), there are certain conventions associated with these representations. A knowledge of these conventions is necessary for the understanding of given visual representations as well as for communicating visual images in the form of drawings. Such knowledge is social knowledge that needs to be provided by the teacher.

These activities are thus designed to explore some of the conventions associated with the different forms of representation:

- the relative position and size of objects and an observer in photographs
- the notions of "aerial view" / plan, "side view" and "front view"
- the difference between perspective and non-perspective drawing.

Assistance is also provided for learners in making their own drawings (perspective and non-perspective drawings).

It is felt that once some understanding of the conventions is established, these visual representations (rather than the actual objects) can then be used more extensively in the exploration of space. The activities that follow this module make use of the visual representations and require the use of some drawing skills.

An effort has been made in the design of these activities to provide learners with experiences in various sized spaces:

- "Small-Scale Space": For example, visualising what a representation of a cereal box will look like from different perspectives.
- "Large-Scale Space": This is space which surrounds the individual, for example, the classroom or the school grounds. Some such space can be known to the learner, for example, the neighbourhood, or might be unknown in the case of a town the learner has not visited.

We believe that the skills developed in this Module are vital if we wish learners to "describe and represent experiences with shape, space, time and motion" as required in Specific Outcome 7 of Curriculum 2005. The development of these skills of representation and description will equip learners to explore the position of an object in space, changes in shape of an object, the orientation of an object, and the interconnectedness of shape, space and time as required in Curriculum 2005.

LIST OF ACTIVITIES:<br>Vusi's Holiday Photos*<br>In Front or Behind?*<br>Near and Far*<br>Practice Photos*<br>Draw What You See*<br>Which Camera?<br>Different Photos*<br>Block Buildings 1*<br>Block Buildings 2*<br>Block Buildings 3*<br>A Box<br>Drawing a Box<br>Strange Pictures<br>Sorting 2*<br>What Can Vusi See?*<br>Hide-and-Seek*

Note that the activities marked with the symbol * require that the learners work with the visual representations without the use of the actual objects. In order to be able to do this the learners have to have had sufficient experience interacting with the real objects, as was the case in the activity Where are the Objects? in Spatial Module 1. If the teacher feels that some learners require more experiences with concrete objects s/he could use activities such as "Draw What You See" and "Different Photos", but using the concrete objects.

The marked activities should only be used by learners when they have had rich experiences with the concrete objects. These learners should be permitted to use the concrete objects as a checking mechanism for the solutions obtained using visual means. The concrete objects can also be used by learners to explain their reasoning to one another.

## Vusi's Holiday Photos



Vusi was given a camera before his holiday. It is a simple camera and does not have a zoom lens. We are going to look at some of the photos he took on his holiday.

These are some of the photos Vusi took in the Malati Game Park:

1. He took two photos of a zebra resting under this tree.


Photo 1


Photo 2
(a) How are these photos different?
(b) Can you explain how Vusi could take two different photos of the same scene?
2. While Vusi was taking these photos he saw a springbok and he took this photo.


Vusi is very confused. He says he knows that the tree is much bigger than the springbok, but it does not look like this in the photo.
Can you help him?

Photo 3
3. Vusi took this photo of the hut they stayed in. Can you help Vusi so that he will take a better photo next time?


## Teacher Notes: Vusi's Holiday Photos

This activity explores the relative position of objects as depicted in 2-dimensional representations (photographs) and the relative position of the photographer and the objects.

Question 1: Learners should note that the tree and the zebra is bigger in photo 1 than in photo 2. Vusi was in a different position relative to the objects when he took the photos: He was closer to the tree and the zebra for photo 1 than for photo 2.

Question 2: Learners should note that the springbok looks bigger because it is closer to Vusi.

Question 3: Vusi should move back so that the hut will look smaller and he will be able to fit more of it into the photo.

## In Front or Behind?

1. Is the tree behind the house, or in front of the house?

2. Is the tree behind the house, or in front of the house?

3. Shade with your pencil or pen so that the tree is behind the house.

4. Shade with your pencil or pen so that the tree is in front of the house.


## Teacher Notes: In Front or Behind?

This activity explores how objects that are in different positions in relation to the viewer are drawn in 2-dimensions. In question 1 we cannot see part of the tree, so it is behind the house. In questions 3 and 4 learners must shade the pictures to show the relative positions of the tree and the house.

## Near and Far

1. Meet Gertrude, Sam and Benny. Here they are standing in a row next to each other:


In the picture below, Benny is standing far away from us. Sam is standing close to us. Gertrude is standing further away than Sam, but not as far away as Benny.

(a) Who is the tallest and who is the shortest?
(b) Make a drawing in which Benny is standing close to you, Gertrude is standing far away, and Sam is standing somewhere in between.
(c) Make a drawing in which Sam is standing close to you, Getrude is standing far away, and Benny is standing somewhere in between, with Benny on the left, Sam in the middle and Gertrude on the right.
2. Meet Thandi, Thulani and Saul. Here are three photographs of them:

(a) Who is the tallest and who is the shortest?
(b) Explain why the people have different heights in the three photographs.
(c) Make a drawing in which Thandi is standing close to you, Thulani is standing far away, and Saul is standing somewhere in between.
3. In each of the two drawings below, you can see two identical rondavels. There is a tree close to each rondavel.

In each case, which tree is the taller? Explain how you know.

4. In the previous three questions, you have worked with drawing techniques that can be used to show how far different objects are from you.

Talk about these techniques with some classmates. Then write a short letter to a friend to explain how we can make one object look further or closer than another in a drawing.

## Teacher Notes: Near and Far

This activity develops the ideas in the activity "Vusi's Holiday Photos". It explores how objects that are in different positions in relation to the viewer are drawn in 2dimensions. So an object that is further away from the viewer is drawn smaller than an object closer to the viewer.

In question 1(a) learners should note that, although the people appear different sizes in the second picture, they are actually all the same height (as suggested by the first picture). Learners are then required to draw pictures with the figures in different positions.

In question 2 the first picture indicates that Thulani is the tallest, then Thandi and Saul is the shortest. The people have different heights in the different pictures because they are at different distances from the person taking the photograph.
In question 3 learners should note than in the first picture, the rondavels can be used as a reference point. Since they are the same size and the same distance from the viewer, one can see the that the tree on the left is bigger than that on the right. But in the second picture the rondavels are at different distances from the viewer, so one cannot compare the size of the trees.

Question 4 requires that learners summarise in words what they have learnt about drawing objects in different positions. This can be used as an assessment activity.

## Practice Photos

1. Vusi wanted to practice taking "close up" photos so he took these photos of his suitcase. Where was Vusi standing when he took each of these photographs?



Photo 5


Photo 6


Photo 7

2. Vusi was asked to draw the rear view of this block:

This is what he drew:
Is Vusi's drawing correct? Explain.


## Teacher Notes: Practice Photos

This activity focuses on a particular form of representation, that is, a plan or orthogonal projection. This is obtained by viewing an object "straight on". The terminology for the different views is introduced.
Note that in this form of representation, the size of the objects in relation to the viewer is not taken into account. Consider for example this block building:


When one looks directly from above the block on the right is closer to you so it will appear larger than the one on the left:


But this difference in size is not reflected in a plan:


The second page of the activity in which the terminology is introduced, should only be given to learners on completion of and after discussion of the learner responses.

Learners could be required to identify Vusi's position on the drawing of the suitcase but should also attempt to describe the position.
Photo 5: Vusi took the photo from above.
Photo 6: Vusi took this from the side (two possibilities here).
Photo 7: Vusi took this from the front or behind.
The terminology for these different views is introduced. The teacher should emphasise that the objects are being viewed "directly on" in each case. So when one draws the aerial view of the suitcase one does not draw any sides. Question 2 is designed to reinforce this notion.

The terminologoy is reinforced in the activities "Take Your Own Photos" and "Draw What You See".

## Draw What You See

1. Look carefully at this drawing of a box:

(a) Draw the front view of the box.
(b) Draw the rear view of the box.
(c) Draw the aerial view of the box.
(d) Draw the side view of the box.

Now answer these questions for each of the objects drawn below:
(a) Draw the front view of the object.
(b) Draw the rear view of the object.
(c) Draw the aerial view of the object.
(d) Draw the side view of the object.
2.

3.

4.

5.

6. This is a photo of two boxes:

(a) Draw the aerial view of the two boxes.
(b) Draw the side view of the two boxes (from point $A$ on the left).
(c) Draw the side view of the two boxes (from point B on the right).
(d) Draw the rear view of the two boxes.

## Teacher Notes: Draw What You See

## Teacher Aids:

Teachers should have some of the objects in the classroom for learners to check their drawings. Learners should only be permitted to check once they have completed the activity.

## Notes:

In this activity learners are required to visualise what these 2-dimensional representations of 3-dimensional objects will look like when viewed from different positions.
Again, the details in the drawings are not important, but the shape and the relative positions of the different parts of an object should be emphasised. For example, the handle of the cup in Question 5 should be in the correct position (on the right) when viewed from behind. This ability to orient oneself in relation to other objects is an important skill in map reading (see "Which Direction?", "Giving Directions" and "Bafana Bafana").

Encourage learners to consider which figures have the same shape when viewed from different viewpoints.
Ask learners to describe their drawings so that they have to develop a vocabulary for talking about the shapes.

Question 7 requires that learners consider the relative position of the boxes as well as their own position relative to the boxes. In (b) the drawing should indicate that only the large box can be seen, whereas in (c) the top of the large box will be seen behind the small box. In the latter case learners can be challenged to consider at what point one would not be able to see the larger box. Learners should note that the large box will be on the right when viewed from behind in (d).

## Which Camera?

Which camera sees which picture?


Write your answer like this:
1 sees ....
2 sees ..
3 sees ....
4 sees ....

In each case, explain why you chose each picture.

## Different Photos

1. Look at this picture of these objects on a round table.


Our photographer Vusi has taken photos of these objects from different places.
Where did he take photos $A, B$ and $C$ from? Uses the letters $A$ to $C$ to show on the picture above where he stood when he took the photographs.
In each case explain how you got your answer.


Photo A


Photo B


Photo C
2. Look at this picture of these objects on a round table.


Use the letters $A$ to $C$ to show on the picture above where Vusi stood when he took these photos. In each case explain how you got your answer.


Photo A


Photo B


Photo C

## Teacher Notes: Which Camera? and Different Photos

These activities require that learners visualise what a group of objects looks like from different perspectives. The activities differ from "Draw What You See" as learners do not have to draw the objects, but have been given possible viewpoints. Learners are thus provided with an opportunity to respond to the task in a medium other than drawing.

It is important that learners explain how they got their answers as this requires that they describe the relative positions and sizes of the objects.

## Block Buildings 1

1. Look at this building of blocks:

front

On the dotty paper below, draw each of the following:
(a) The front view of the building
(b) The aerial view of the building
(c) The side view (from the right) of the building
(d) The side view (from the left) of the building
(e) The rear view of the building.

The first one has been done for you:


Now do the same for each of these block buildings:
2.

(a)
(b)
(c)

(d)
(e)
3.


## Block Buildings 2

The diagram on the right shows the aerial view of this block building. This is also called a floorplan.


We can use numbers to show how many blocks there are in each stack:


How many blocks are there in this block building?

1. Now draw the floor plans for each of these buildings. In each case write down how many blocks there are in the building.
(a)

(b)

(c)

(d)


2 Draw the floor plan for each of these block buildings:
(a)

(b)

(c)

3. Make a block building with each of these floor plans:
(a)

(b)


## Teacher Notes: Building Blocks 2

Learners should note that in question 2(c )there is more than one possible solution as one cannot see what is "behind" the given cubes.

## Further Activity:

Give learners a fixed number of blocks, say five. How many different block buildings can be made with five blocks. Then increase the number of blocks.

## Block Buildings 3

These block buildings are made up of 1 cm by 1 cm by 1 cm blocks. The buildings are placed on a table.

1. For each building write down:
(a) the area of the table covered by the building
(b) the number of blocks in the building.
2. Which building takes up the most space?

Which building takes up the least space?
Explain your answer.


## Teacher Notes: Block Buildings 3

This activity requires that learners have a knowledge of area. The activity provides an informal introduction to the concept of volume, that is, the amount of space occupied by the solid. Learners might have different conceptions of what it means to take up space, for example, they might consider the shape of the block building.

## A Box

Look at this picture of a box:


1. How many faces of the box can you see in the picture?
2. How many corners (vertices) of the box can you see in the picture?
3. How many edges of the box can you see in the picture?

Now look at a box and count the number of faces, vertices and edges. Can you explain this?

When we look at a picture of an object like a box some parts of the object will be hidden:


We could draw the box like this to show that some parts are hidden:

4. Look at this picture of a cube:


How many faces are hidden?
Draw in the missing edges (use dotted lines).
5. Look at this picture of a triangular prism:


How many faces are hidden?
Draw in the missing edges (use dotted lines).

Teacher Notes: A Box
Learners should not be given page 2 of the activity until page 1 has been completed and discussed.

Terminology such as faces, vertices and edges is used - The teacher might have to remind learners about the use of this terminology:


The aim of this activity is to familiarise learners with the conventions used to make 2-dimensional representations of 3-dimensional objects. Learners should note that not all faces are drawn and that these can be indicated using dotted lines.

## Drawing a Box

## Teacher Notes: Drawing a Box

Consider the following example: We know that the top of the table is a rectangle, but when we look at it, it might not appear so. The top could look like a trapezium:


The figure that one sees will depend on where one is looking from. Although we might know that the actual shape of part of an object might be a rectangle, it looks different from different perspectives.
This is used in perspective drawing so that the representation of a 3dimensional object looks realistic.

The aim of this activity is to alert learners to the fact that, although we know that an object might be a particular shape in real life, it might look different, depending on where one is viewing it from. This is used to explore perspective drawing in the activity "Strange Pictures".

## Getting Started:

For this activity learners need to be able to look into a classroom through the windows.
A large box should be placed inside the classroom - learners should be able to see this box through the window. Each learner or group of learners should have a piece of transparency which is pasted onto the outside of the window. The learner must look at the box through the transparency and draw what s/he sees from that particular position.
For example, consider a box like this:


This will look different from different perspectives.
A learner viewing the box from "straight" on will see this:


But a learner viewing the box from an angle might see this:


The different drawings generated should be shared with the class - learners should discuss why the pictures are different.

A Further Activity: The teacher should collect the different responses and make copies. A set of all the drawing should then be distributed to each group in the class. Learners should use these drawings of the box to identify the position from which each was drawn.

## Strange Pictures!



Vusi has looked at these pictures and says he is very confused. He says that he knows that railway lines should not meet, that a brick is made up of rectangles and that the shape of a tennis court is a rectangle. But it does not look like this in the pictures!

Can you explain?

Now look at the drawings of a box:


Which drawing do you think is the best? Explain.

When looking at Vusi's holiday pictures we noted that objects that are closer to the photographer appear larger than other objects in a photograph, for example, the springbok looked bigger than the tree.
So if we want to draw pictures that look realistic we must remember this when we draw!

When we look at a box, the front of the box is closer than the back, so we must draw the back part smaller than the front part:


Then we can draw the rest of the box:


Then we can rub out the lines we do not need and we have a drawing of a box that looks realistic:


This is called a perspective drawing.

Now practice drawing realistic pictures of these 3-dimensional objects yourself. One face has been given for you each time:
1.


3.

4.


Sometimes in mathematics we want to draw pictures that show what the objects really look like.
For example, we want to show that the opposite faces of a rectangular prism are the same shape and size (congruent). So we draw two congruent rectangles:


Then we join up the rectangles:


Then we rub out the lines we do not want:


Now we have a picture of a box that shows the equal edges and congruent faces. Measure the lines to test this.

Now practice drawing pictures of these 3-dimensional objects like this. One face has been given for you each time:
1.

2.

3.

4.


## Teacher Notes: Strange Pictures!

This activity explores the representation of 3-dimensional objects in 2-dimensional drawings.

Learners should note that, although we know that railway lines do not meet in real life, we draw them as if they will meet so that they look realistic. Learners can be referred to photographs to look at the representations of a variety of objects.

Learners in the higher grades should note that, if one extends the parallel lines in a perspective drawing, the lines will meet in a point (called the "vanishing point").

Learners are introduced to conventions for drawing. Two possibilities are discussed here:

- Perspective drawings in which the objects are drawn as they are seen so that they look realistic. So those parts of an object which are behind others should be drawn smaller than those in the front.

- Non-perspective drawings which show some of the mathematical properties of object, for example, congruent faces and sides of an object are reflected in the picture.


At this level learners should be made aware of the differences between the two representations and should be able to use both in drawings. Drawings in the rest of this Module are not drawn in perspective.

## Sorting 2

Which of these figures and objects belong together?
Why did you group them in this way?
Is there another way to group them?
A


C

D

E

F

G

H

K

M

N

P


T

$T$


## Teacher Notes: Sorting 2

This activity is similar to the sorting activities used in the first Module, but requires that learners work with 2-dimensional figures as well as 2-dimensional representations of 3-dimensional objects. The use of this activity should be delayed until learners are familiar with the conventions for the representations, as is addressed in this Module.

There is likely to be a range of responses:

- Some learners will be able to distinguish between the representations of the 3and 2-dimensional figures, whereas others might look at the individual figures, for example, figures $C$ and $V$ both have circles.
- Some will regard figures $B$ and $H$ as different because the orientation is different, whereas other will see them both as squares. Some learners will include the cube (figure $P$ ) with the squares.
- Some learners will group figures $M$ and $N$ together, whereas others will indicate that the "corners" in figure $N$ are rounded.

After the learners have worked through the activity, a class discussion should be held so that the varying responses can be explored. This is a useful opportunity for the teacher to focus the learners on different properties of the figures and to introduce or reinforce the appropriate terminology.

## What Can Vusi See?

Vusi and his family visit a farm during their holiday. This is what Vusi sees as they drive along the flat road towards the farm buildings.


1. What will happen to the size of the buildings and the windmill as the car gets closer to them?
2. Vusi notices that, as he gets closer to the buildings he can see less and less of the windmill. Then he cannot see the windmill at all! Can you explain this?

## Teacher Notes: What Can Vusi See?

The aim of this activity, like the activity "What Can I See?", is for learners to explore the relationships between themselves and objects in space and between these objects themselves. But this activity requires that the learners work with visual representations, rather than the objects themselves. It also requires that learners reflect not only on what they see, but also on how they see.

Some learners will not be able work with the visual representations and should be given more opportunities to explore with the real objects.

Question 1 revisits the idea that, when one moves closer to an object, it becomes larger.

In question 2 the learner should note that, although the windmill is getting larger as Vusi moves towards it, one can see less of it. A diagram can be used to show the sight lines:


The dotted lines show that one can see the top of the windmill from points $A$ and $B$, but not from point $C$.

Source of Ideas: Realistic Geometry Education, The Freudenthal Institute

## Hide-and-Seek

Vusi and his brother, Jacob, are playing hide-and seek in between the farm buildings. This is an aerial view of the buildings:


- V

1. Jacob is hiding next to the house as shown. Can Vusi see him from where he is standing?
2. Show on the diagram where Vusi must be so that he can see Jacob. And where he must be if he cannot see Jacob.
3. Vusi is now standing still in the position shown below. Jacob has to hide from him. Can you suggest where he should hide? Show a number of different places. Is it possible for Jacob to hide between the farmhouse and the office as he did before?


Source of Ideas: Realistic Geometry Education, The Freudenthal Institute

## Teacher Notes: Hide-and-Seek

This activity deals with sight lines, but this time in an aerial view. Learners should be encouraged to draw on the diagram.

In the first case Vusi will not be able to see Jacob: The sight line drawn from Vusi to Jacob goes "through" the farmhouse!


The shaded part shows where Vusi should stand to see Jacob. Learners could also answer this question placing Vusi on the other side of the building.

Question 3 requires that learners work from Vusi's position which is now fixed. The sight lines for Vusi are shown and can be used to find the sections in which Jacob can hide.


