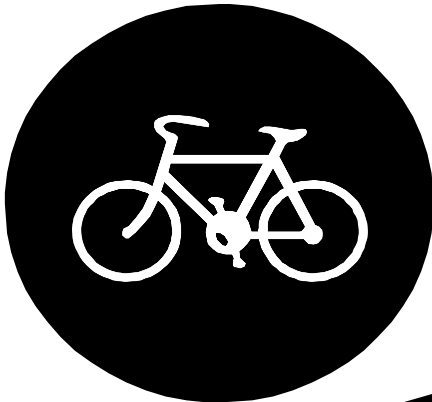
## CYCLES

Which of the following figures are similar to figure A? Explain.

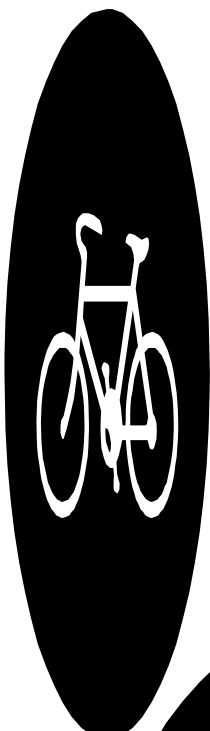
A



B

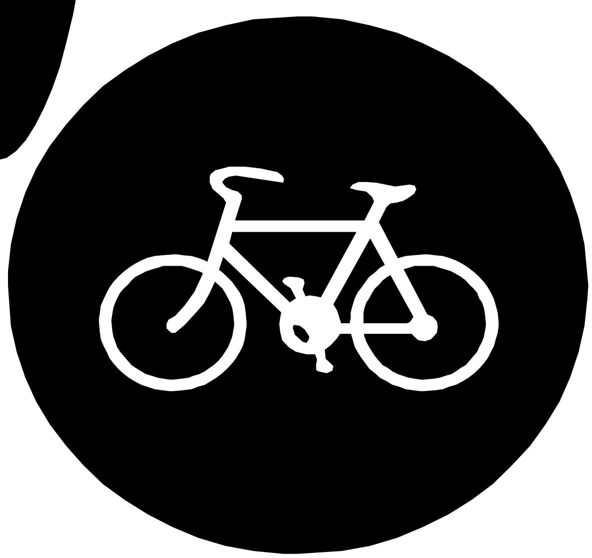
D

C



F

E



G

***Teacher Notes: Cycles; Rectangles; Traffic Signs; Triangle 1;Triangles 2; Polygons***

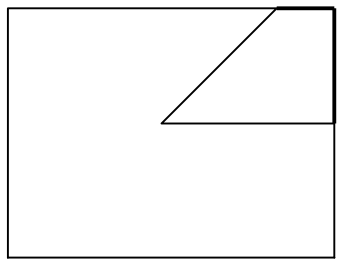
*These activities provide the learners with some more experience in providing a visual justification for why the figures are similar or not. The learners often provide many different descriptions of the specific transformation undergone, for example, in the activity "Cycles":*
*Figure A was described as "stretched in length "; "compressed from the sides"*
*Figure B was described as "expanded"; "stretched by width"; "enlarged and squashed from the top".*
*The learners find it more difficult however to describe how the figures that are similar have been transformed, except for the congruent shape, for example:*
*Figure D was described as "the same but enlarged in size"; "The whole circle expanded in all directions"*
*Figure E was described as "shrunk"; "shrunk in all directions"*
*The responses of the learners suggest that most learners do not intuitively focus on the relationships between the corresponding sides of the figures or even the nature of the angles in the figures.*

*In the next set of activities we provide the learners with experiences in which these relationships become the focus.*

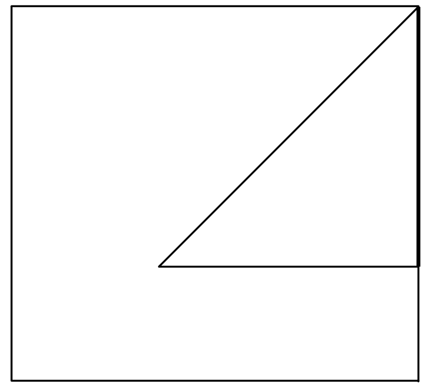
*Note: In the activities "*Triangles 1*" and "*Triangles 2*" the learners can be encouraged to point out which properties of the triangles change as its shape changes. Most learners will recognise the changes in the angles. At this point they do not however see equiangularity as a sufficient condition for similarity in triangles. According to the* van Hiele theory *this will be at the deductive level of reasoning and since the pupils are still engaged in visual level activities at this stage we need not enter into this kind of discussion now. These activities can be revisited again once the learners have had experiences with the van Hiele analysis level activities.*

*In the activity "*Polygons*" the learners will recognise that the figures F, G and H are not similar as the angles are not congruent. If the learners feel the need to check whether the sizes of the acute angles in the figures are equal to that in figure A, they can make a traced copy of the angle and see whether it can be superimposed on the other angles. We have specifically chosen figures in which all the angles are congruent but the corresponding sides are not proportional (Figures C and J). so that the learners realise that not all polygons that are equiangular are necessarily similar. Note, since most learners do not intuitively focus on the relationships between the corresponding sides encourage them to provide alternate visual explanations to show why the figure are not similar. For example, Figures A , C and J are equiangular but it is visually clear that the lengths of the arms of the acute angle in the different figures are not in the same in relation to each other. In figure J the horizontal arm of the acute angle is much shorter than in the other two figures. Another way of looking at these equiangular shapes is to look at the kind of figures that are created if we complete the figures to make a rectangle (please turn the page):*

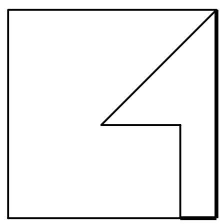
A



C



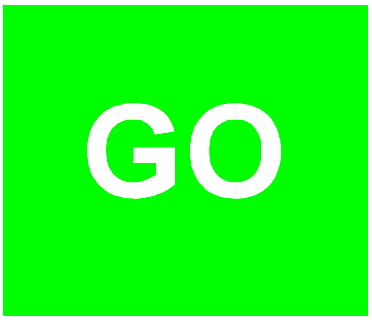
J



## RECTANGLES 1

Which of the following figures are similar to figure A? Explain.

**A**



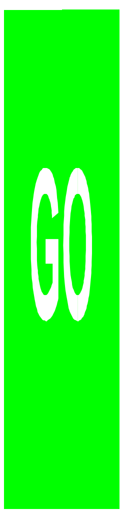

**B**


**C**


**D**


**E**


**F**

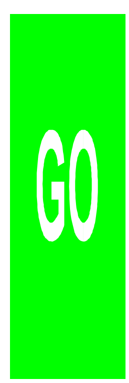



**G**


**H**


**I**

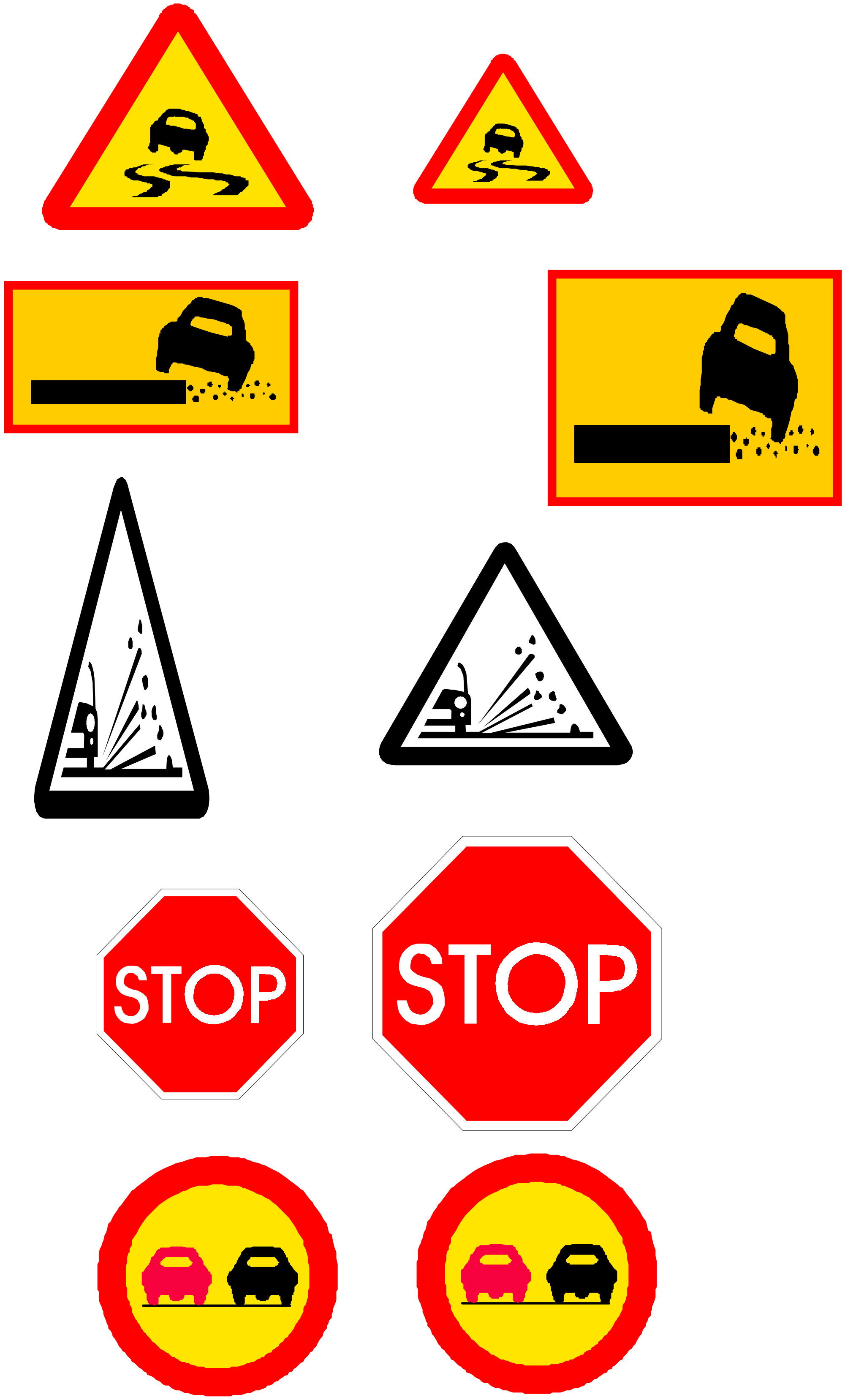

**J**



## TRAFFIC SIGNS

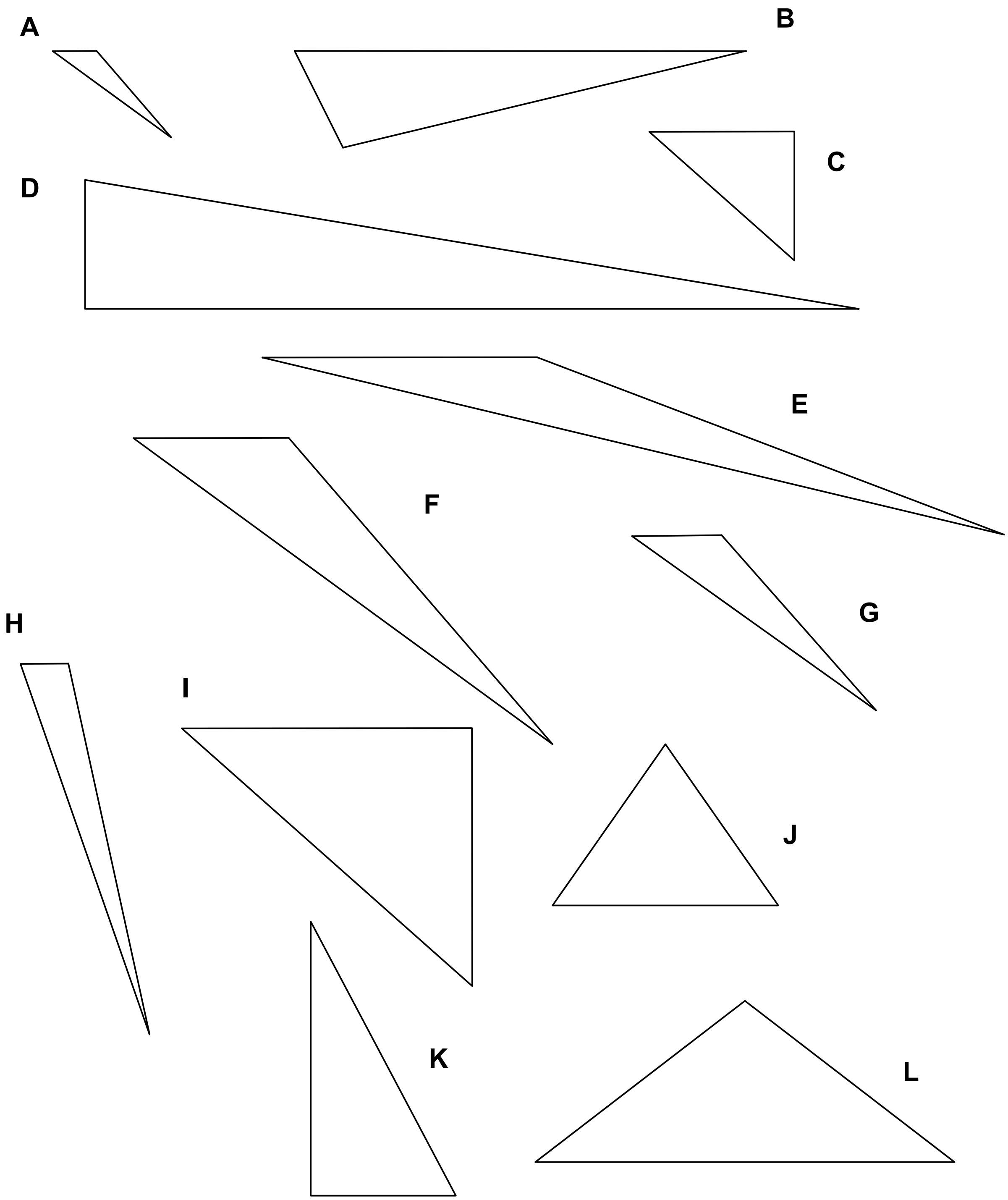
Which of the following pairs of traffic signs are similar?

Explain *why* you say that they are similar or not.



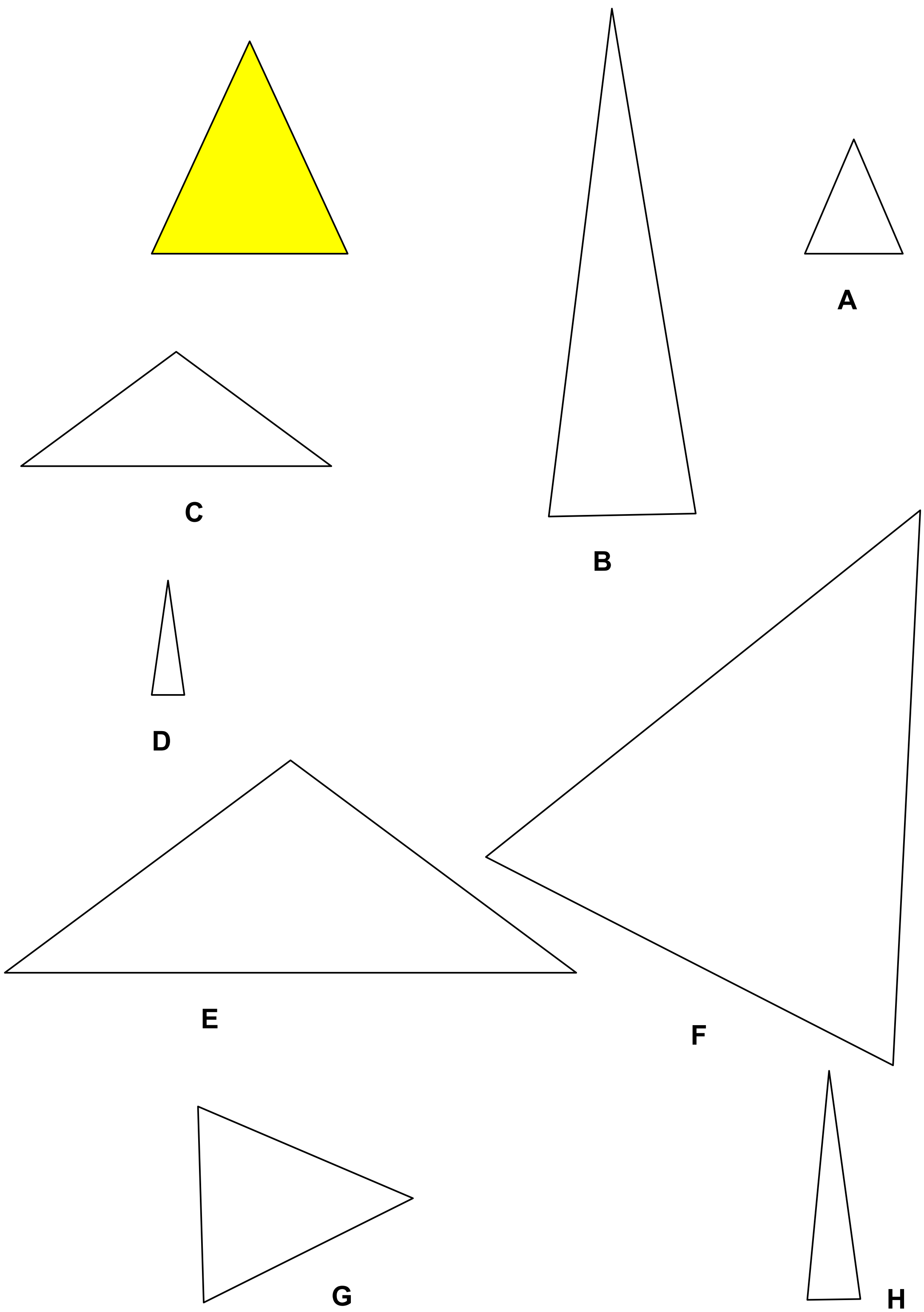
# TRIANGLES 1

Which of the following triangles are similar? Explain



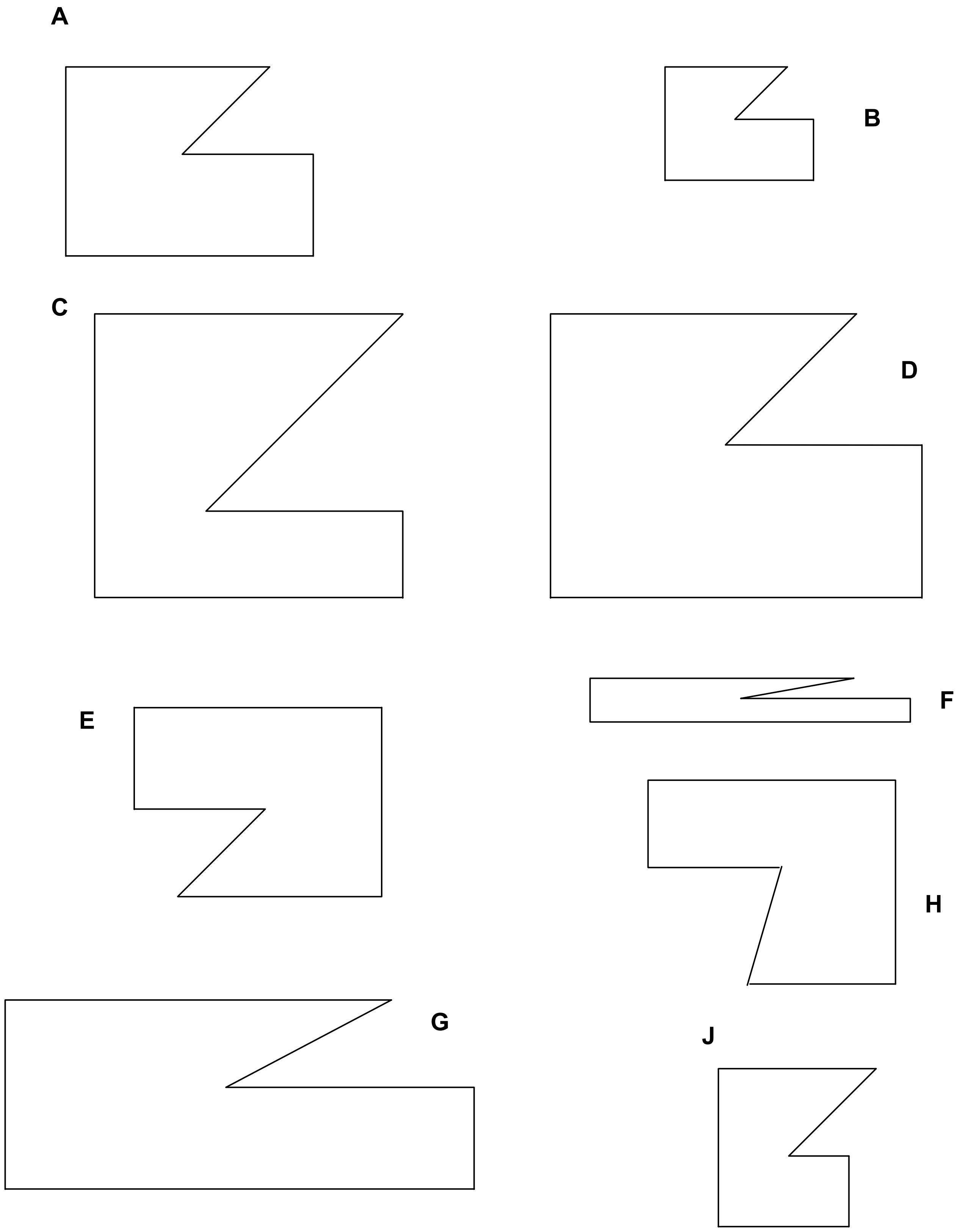
## TRIANGLES 2

Which of the triangles are similar to the shaded triangle? Explain!



**POLYGONS**

Which of the following figures are similar to figure A? Explain.



## DRAWING HOUSES

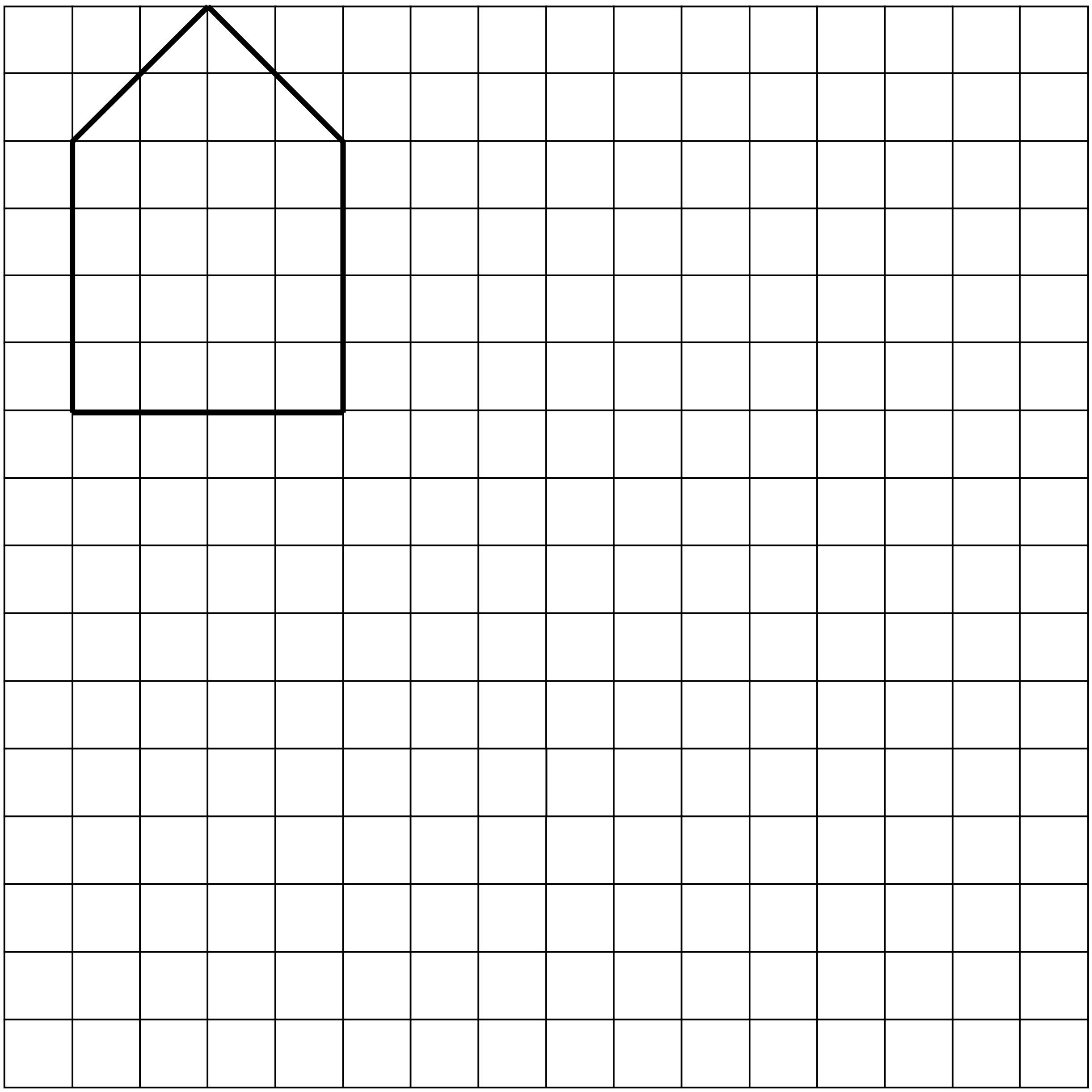
Draw three similar houses.

The three houses that you draw must have different sizes:

(a) one house must be the same size as the house drawn below

(b) one house must be smaller

(c) one house must be larger



What can you deduce about the angles and the sides of similar figures?

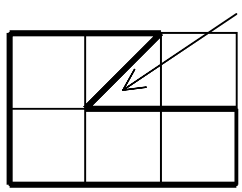
***Teacher Notes: Drawing Houses; Lucas's Figures; Bingo's Figures***

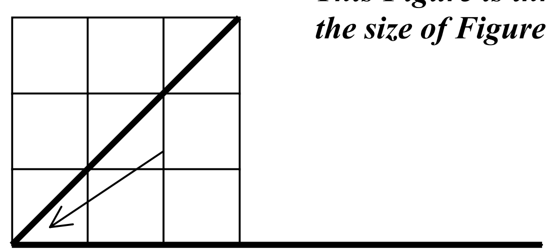
*In these activities we provide the learners with experiences in drawing figures so that they may focus explicitly on the relationships between the corresponding angles and sides in figures that are similar.*

*For example, in the "Drawing Houses" activity the learners had difficulty in drawing the roof of the house and this made them focus explicitly on the angle of the roof that had to be the same for the houses to be similar. In the whole-class discussion it needs to be pointed out that all the corresponding angles must be congruent. Ask the learners to mark or identify the corresponding angles.*

*In the activity "*Bingo's Figures*", the learners are asked to draw a figure three times the size of the original figure so that they focus how each of the sides is created. Ask the learners to examine the angles in the new figure and to explain whether the corresponding angles are congruent. The use of the square grid allows the learners to explain why the angles are equal without the need to measure the angles directly, for example: the angles can be explained to be equal in terms of the arms of the angles, a horizontal and a diagonal of a small square.*

***Figure B***





***This Figure is three times the size of Figure B.***

*We recommend these drawing experiences so that idea of the congruent angles and the proportionality of the corresponding sides in similar figures are re-enforced. Changing the orientation of the original figure provides the learners with challenging drawing experiences as well as showing that orientation is not a necessary condition for two figures to be similar.*

*The proportionality concept is a difficult concept for the learners as it connected with other concepts such as fractions and ratio. The formal introduction to the notion of the proportionality of the corresponding sides can be suspended in the discussion for now. The focus can simply be on the procedures to be carried out to make the figures similar. In the discussion the learners need to say that in creating the new figure **all the sides** were, for example, increased by three times that of the original lengths. In Hart's (1981) research it was shown that learners tend to use **additive** operations to describe the changes in creating the new figures. We observed that some learners produced the correct drawings of similar figures but when asked the question: "By how much did you enlarge or reduce the new figure?", they often used additive operations to describe the changes of each of the new sides. These learners do not yet see the significance of the transformation factor as describing the common change. In the next set of activities we focus on this aspect in developing the notion of the **scale factor**.*

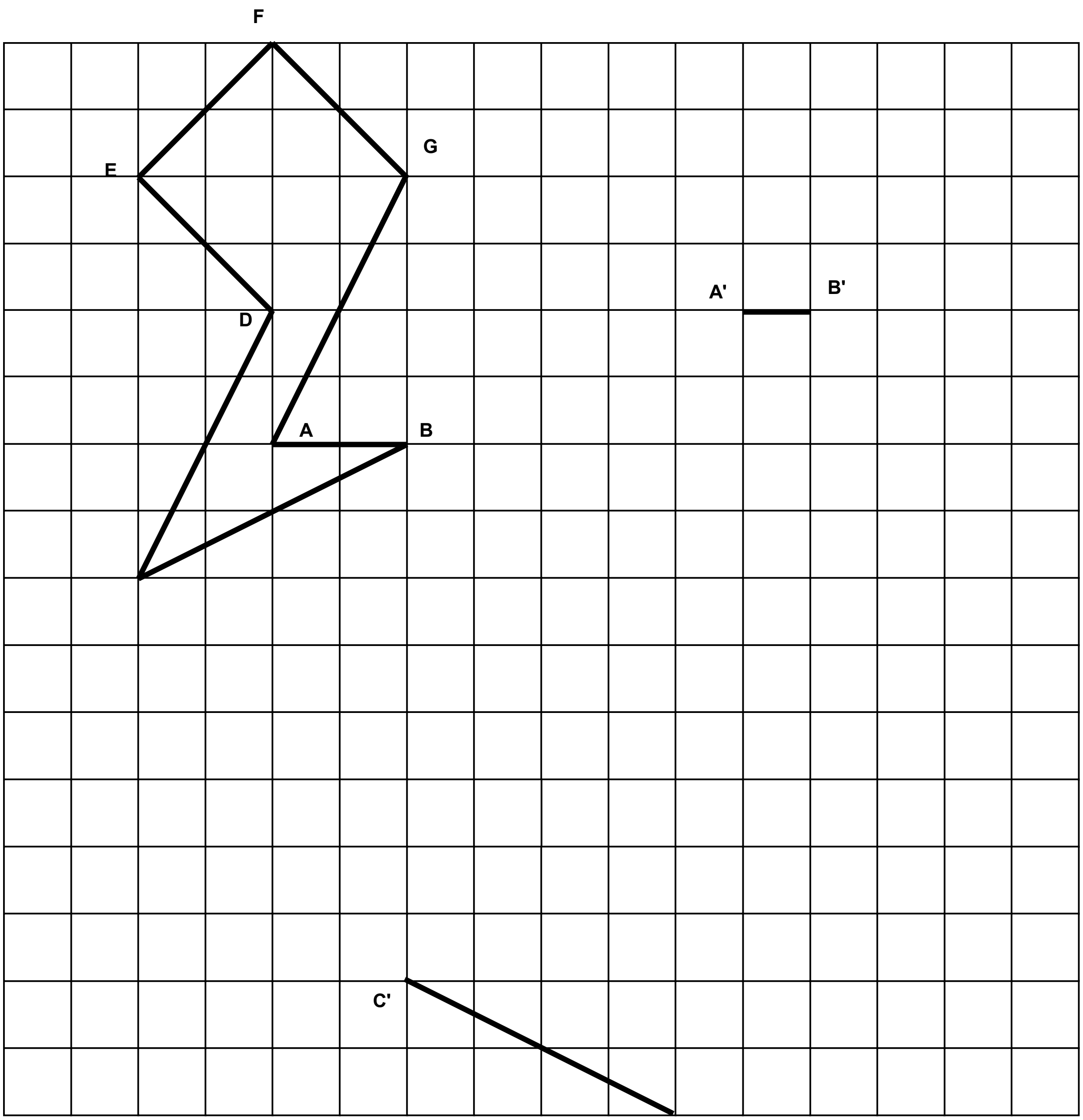
Reference:

Hart, K. M. (1981). **Children's Understanding of Mathematics: 11-16.**

## LUCAS'S SIMILAR FIGURES

Lucas draws the figure on the square grid below. He decides to draw two figures that are similar to this figure.

He only draws one of the line segments in each of the similar figures. The line segment A'B' in the one figure is drawn and the line segment B'C' in the other figure is drawn. Complete the figures so that they are all similar figures.




What can you deduce about the angles and the sides of similar figures?

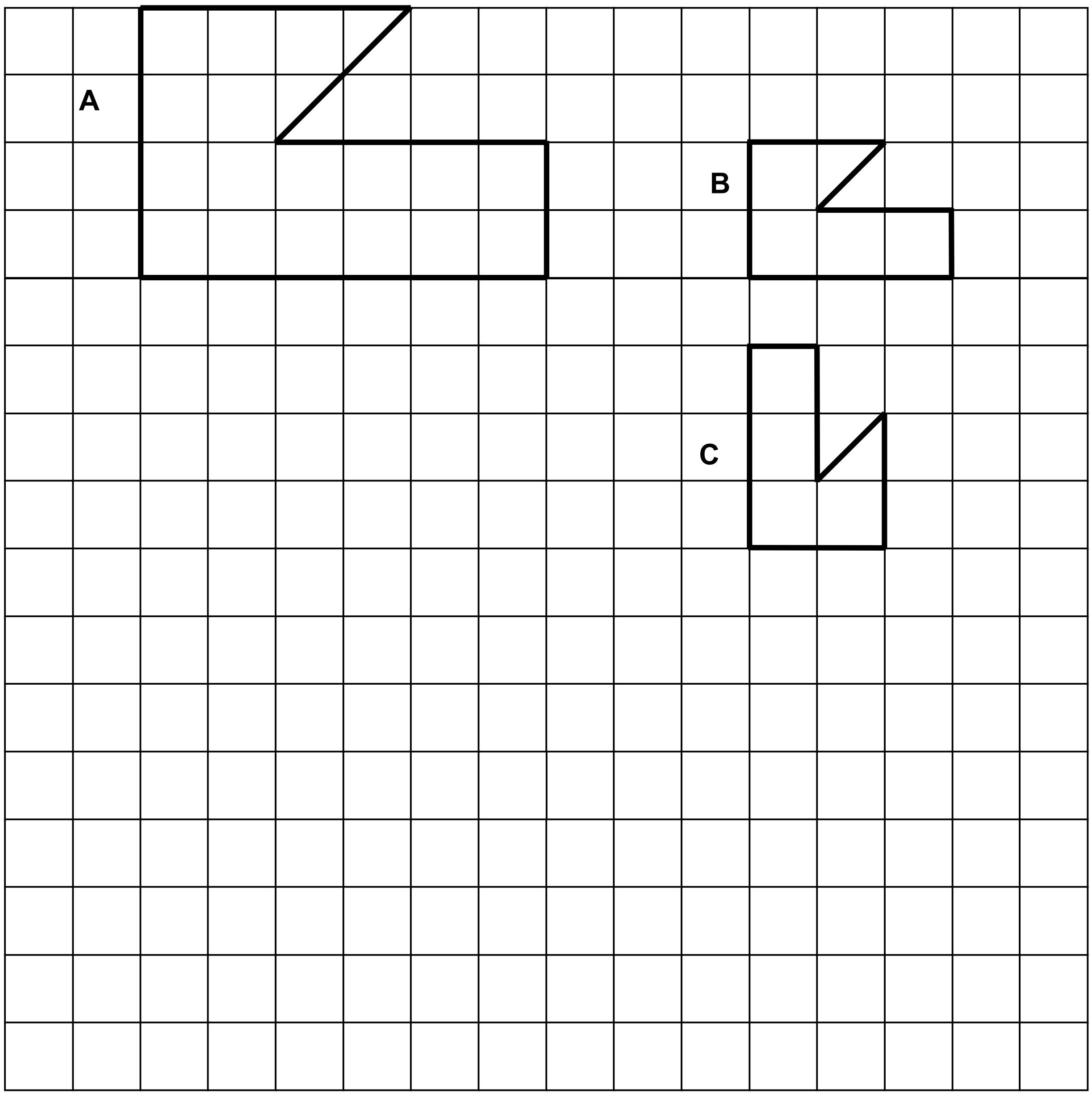
## BINGO'S FIGURES

Bingo draws the figures A, B and C on the square grid below and shows them to his friend Agatha.

Agatha says that only figures A and B are similar.

Do you agree with Agatha? Explain why you agree with her or not.

Now draw a similar figure that is three times the size of figure B.



## MARK'S ENLARGEMENT

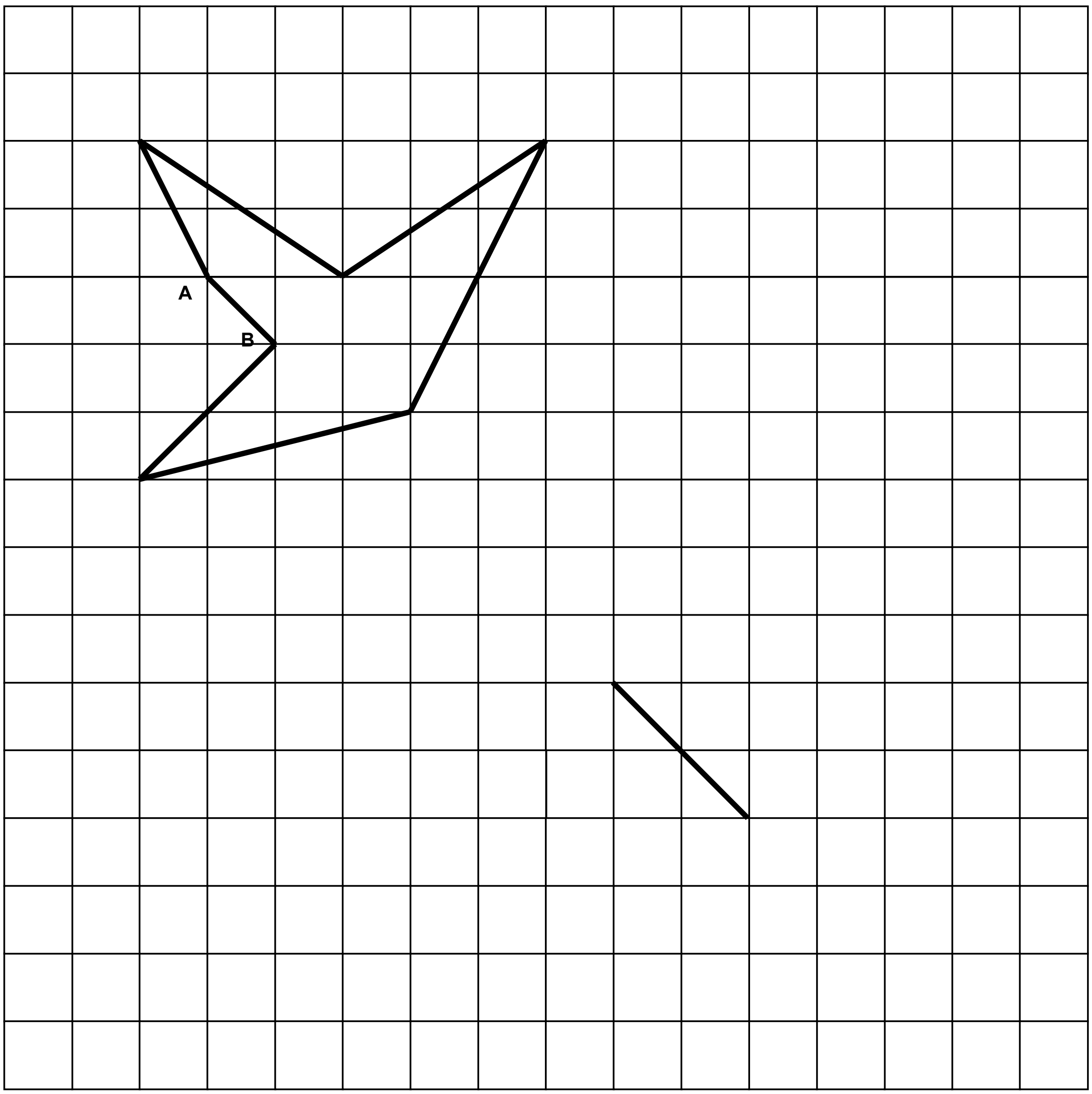
An enlargement refers to a similar figure that is **bigger** than the original figure.

Mark wants to draw an enlargement of the figure below.

He starts by drawing an enlargement of the side marked AB.

Complete the drawing.

By how much did he enlarge the figure?



***Teacher Notes: Mark's Enlargement; Mark's Reduction; Hexagons***

*The aim of these activities is to develop the notion of scale factor. In this process we need to re-enforce the idea that a similarity transformation creates a constant change to the properties of the figure, namely:*

1. *All the angles remain the same*
2. *All the sides of the figure are either enlarged or reduced by multiplying by a constant called the scale factor.*

*In the whole-class discussions the learners can be asked to draw conclusions about the nature of scale factor for enlargements and reductions. (Enlargements have a scale factor greater than 1 and reductions have scale factors between 0 and 1)*

*In the activity "*Hexagons*" the special case of a congruent figure is considered. The domain of the numbers that are scale factors is extended to include the number 1. Make it explicit in the discussions that in this case the shape and size is invariant.*

*Note: The learners need to realise that a scale factor can be any positive number.*

## MARK'S REDUCTION

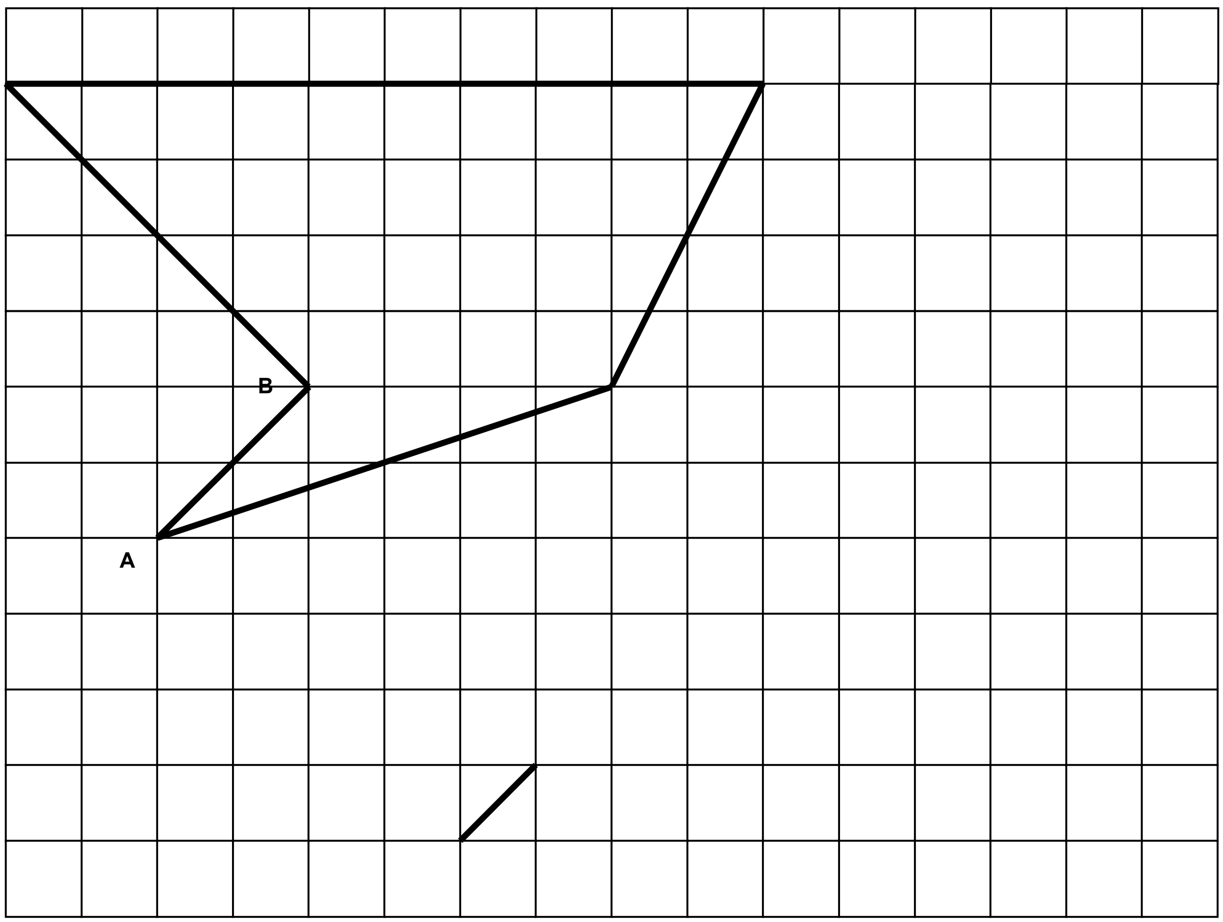
A reduction refers to a similar figure that is *smaller* than the original figure.

Mark wants to draw a reduction of the figure below.

He starts by drawing a reduction of the side marked AB.

Complete the drawing.

By how much did he a reduce the figure?




The ***scale factor*** is the term used to describe by how many times the lengths of the original figure has been **multiplied** to be as long as the corresponding lengths of the new figure.

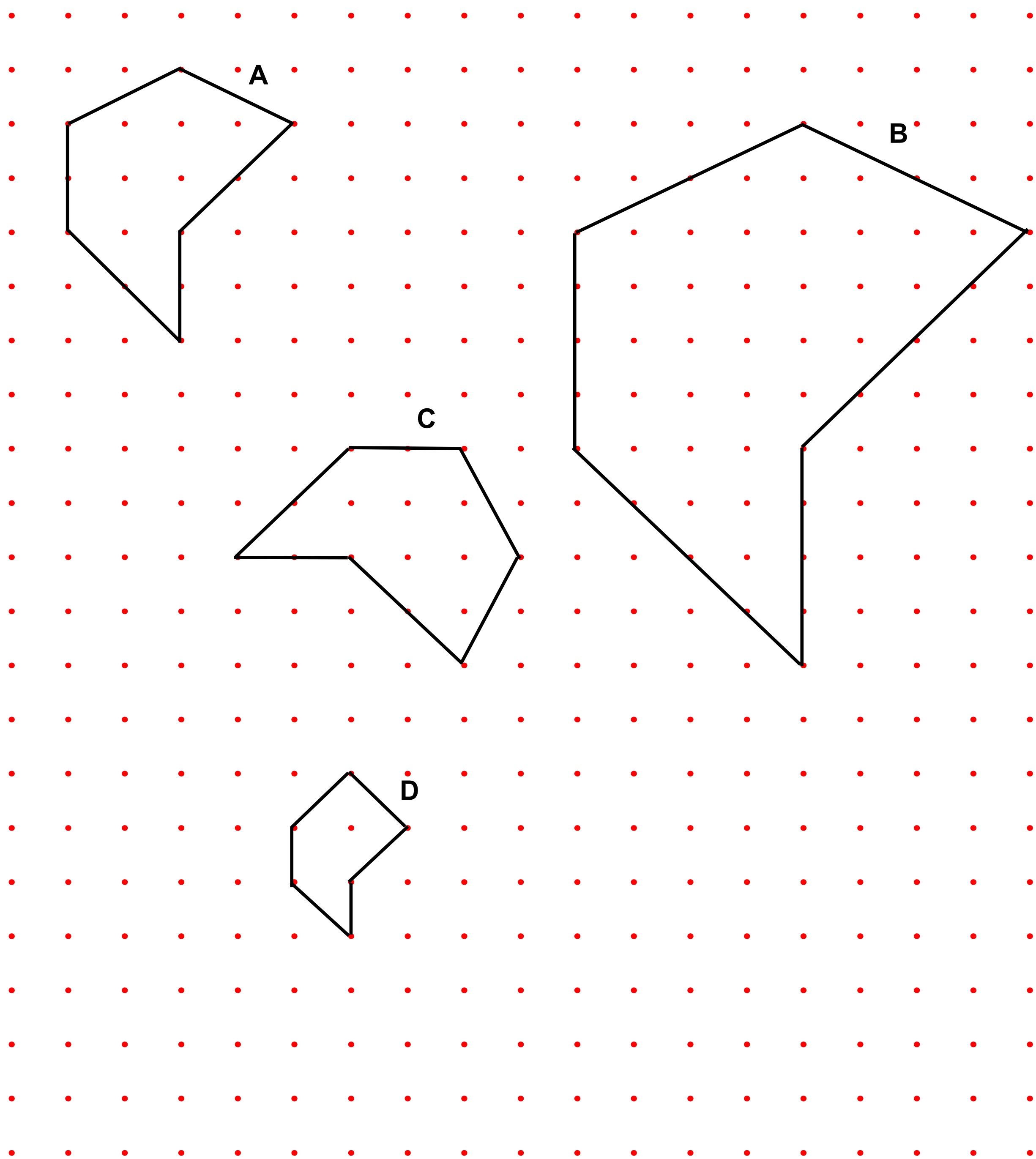
In the drawing above we say that Mark's figure has been scaled by a factor of __________.

Note: The terms enlargements, reductions and scaled copies refer to figures that are **similar**.

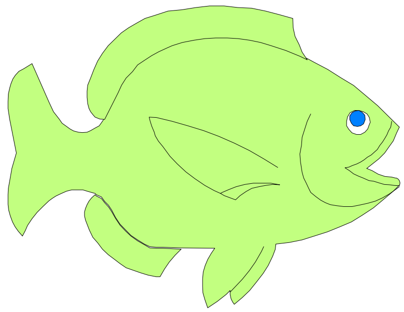
## HEXAGONS

Which of the figures B, C and D are scaled copies of figure A? Explain.

Determine the scale factor of the scaled copies.

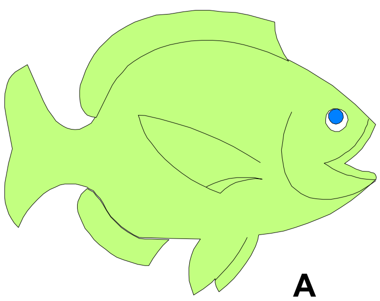


## WELL-SCALED FISHES

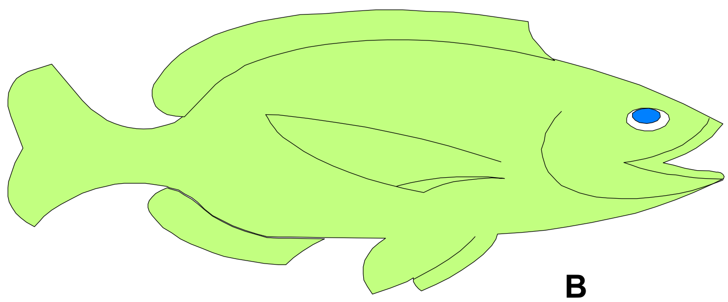


***The original picture of fish***

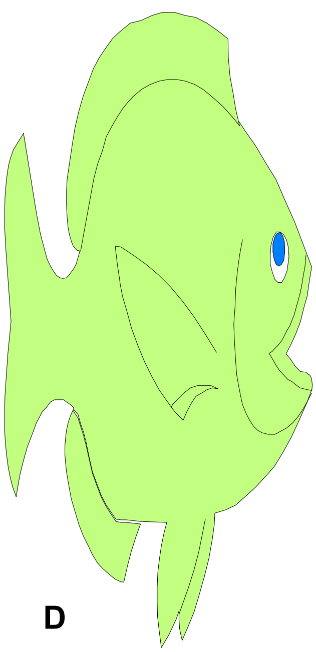
Decide which of the pictures are enlargements or reductions of the original picture.
How did you decide?
By how much have the fishes been enlarged or reduced?



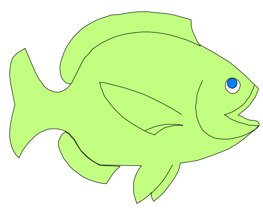
**A**



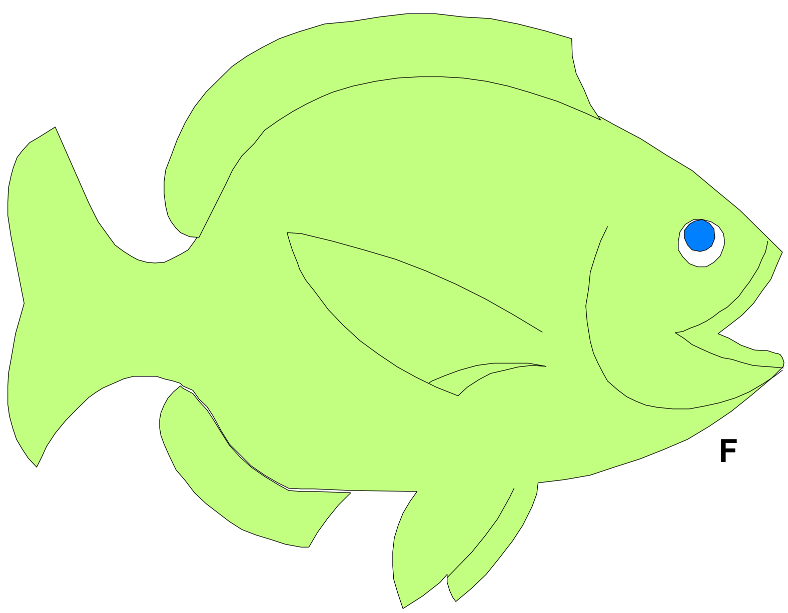
**B**



**D**



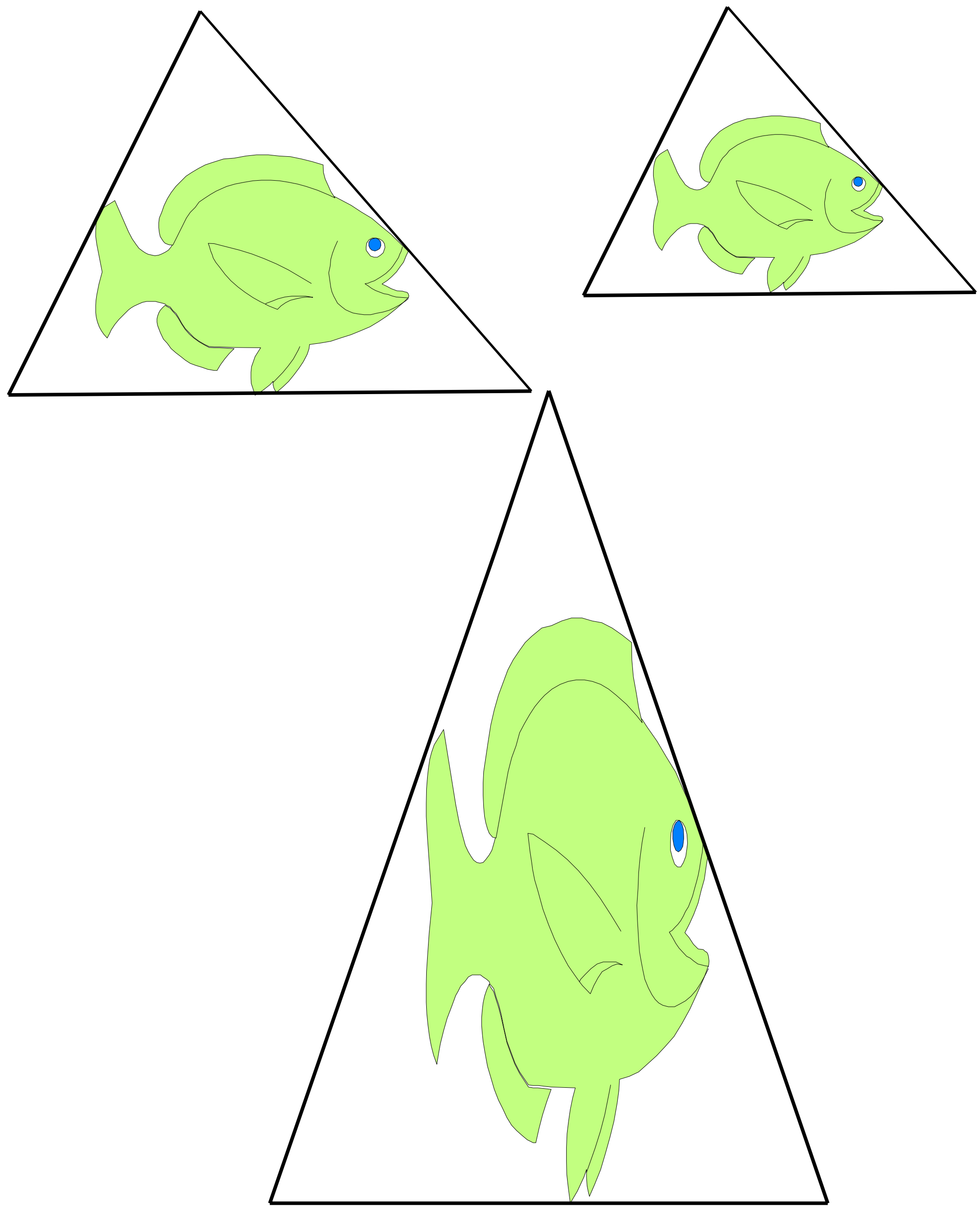
**E**



**F**

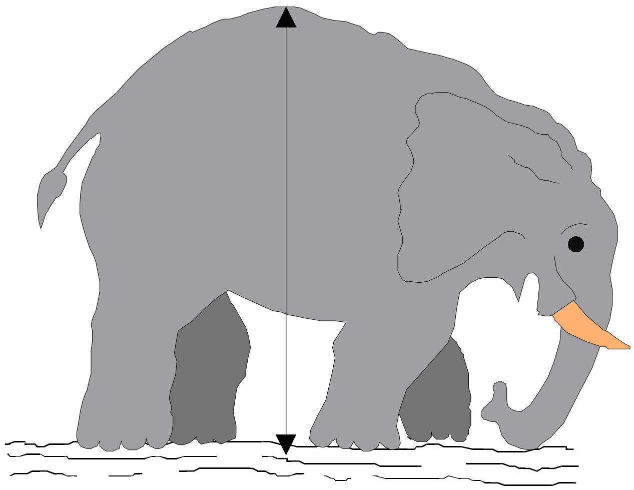
***Teacher Notes: Well-scaled Fishes***

*The aim of this activity is to determine the scale factor in non-rectilinear similar figures. In the whole-class discussion we can again focus on the conditions for similarity. Visually the learners will recognise how the "curvature" changes in those figures that are not enlargements or reductions. We can suggest to the learners that a rectilinear frame be drawn around the figures to see the "angular" changes, as shown below:*

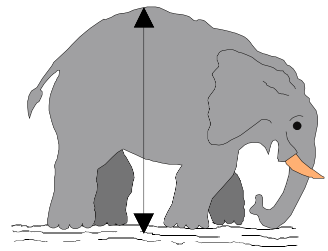


*In determining the scale factor of the enlargements and the reductions, emphasise that* ***corresponding*** *parts of the fish must be measured to see by how many times the original part must be multiplied to give the measure of the enlargement or the reduction.*

# PARTS OF THE ELEPHANT



Original drawing



New drawing

The new drawing of the elephant above is a reduction of the original drawing of the elephant.

Let us measure some of the corresponding parts of the elephant:

Complete the table by measuring other body parts of the elephant.

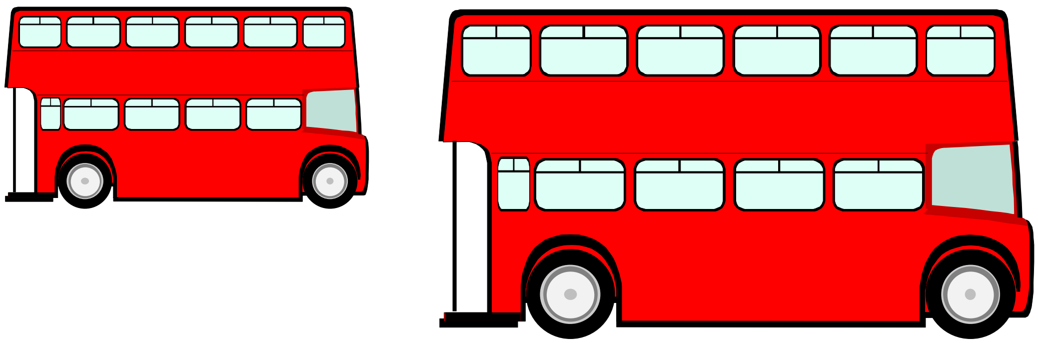
| **body part** | **original drawing** | **new drawing** | **difference in size of the body parts** | **how many times as small is the body part of the new drawing?** |
|---|---|---|---|---|
| height | | | | |
| length of the ear | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

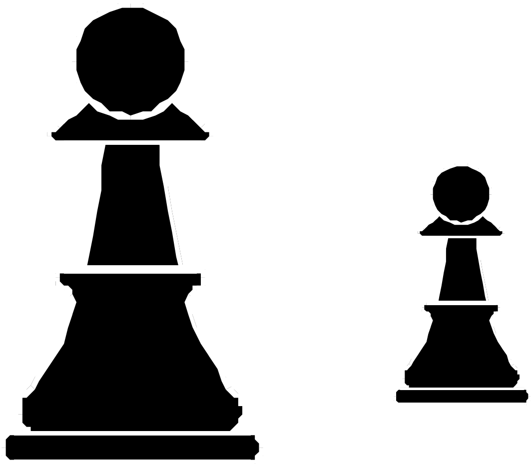
Are there patterns in the measurements? Discuss.

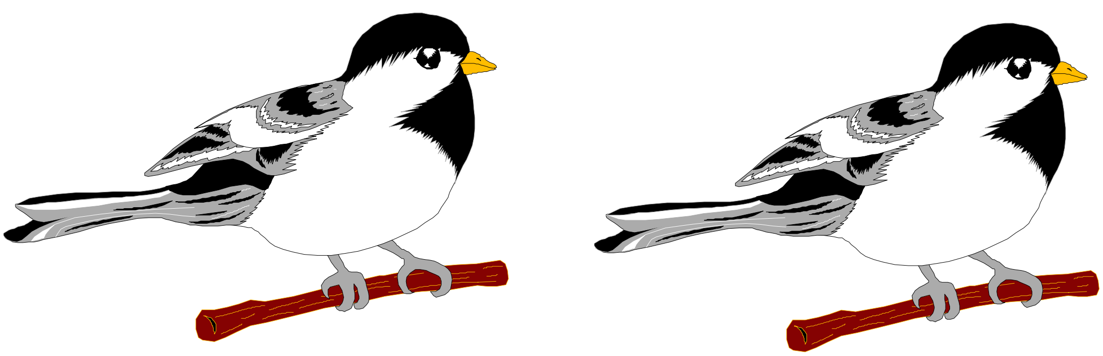
By what factor has the new drawing been scaled?

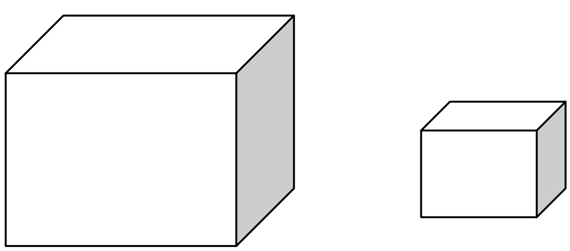
## SCALED PICTURES

What scale factor will transform the picture on the left into its scaled copy on the right?









***Teacher Notes: Scaled Pictures***

*In this activity the learners are given further experiences in determining the scale factor of different kinds of objects.*
*Remind the learner that a scaled copy is a similar figure of the original figure.*
*In order to find the scale factor, corresponding parts in the figure need to be measured first.*

*Revisit the definition of the scale factor in the previous activities (see "*Mark's Reduction*")*

*In the case of the birds the learners will recognise that they are congruent and hence the scale factor is one.*

*Other aspects that can be raised in the whole-class discussion:*

*Ask the learners what the scale factor will be if the picture on the right is the original one.*

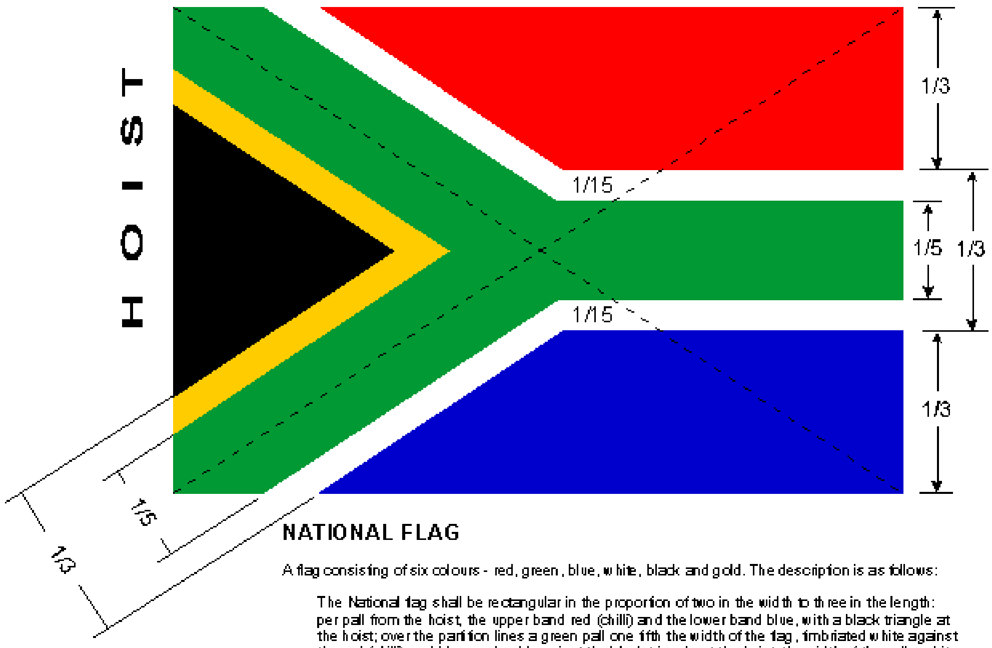
*What is the relationship between these numbers?*

*What kinds of numbers can scale factors be?*

*A figure is scaled by the following values: 4; 1;0,33; 1,75;* $\frac{3}{4}$.
*Will the new figure be larger or smaller? The same size?*

# Our Flag: The Right Shape 1

The *shape* of our flag, but not its *size*, is determined by law, as in this sketch:




**NATIONAL FLAG**

A flag consisting of six colours - red, green, blue, white, black and gold. The description is as follows:

> The National flag shall be rectangular in the proportion of two in the width to three in the length: per pall from the hoist, the upper band red (chilli) and the lower band blue, with a black triangle at the hoist; over the partition lines a green pall one fifth the width of the flag, fimbriated white against the red (chilli) and blue, and gold against the black triangle at the hoist; the width of the pall and its fimbriations is one third of the width of the flag.

Which of these flags do you think have the right shape?
Why do you say that? Does everyone in your group agree?
How can you be sure?

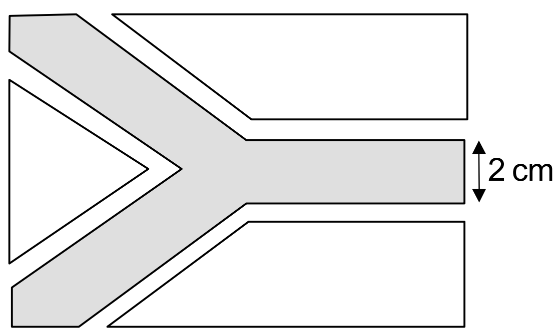
# Our Flag: The Right Shape 2

1. Draw a nice big South African flag in your workbook.

   Convince a classmate that it is the right shape.

2. Jane wants to draw a flag so that the Y-part is 2 cm wide.

   Calculate the dimensions of all the other parts of the flag.




RETURN TO PART A OF SIMILARITY PACKAGE