Malati

Mathematics learning and teaching initiative

Geometry

Module 1

Similarity

Grade 8

Teacher Document

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THE RATIONALE FOR THE SIMILARITY MODULE

Similarity is certainly an important geometric and spatial concept. In everyday life the learner is involved with visual materials such as maps, scale models, photocopies and photographs all of which are contexts for similarity.

The teaching of similarity can also facilitate the learners understanding of proportional reasoning. Fuson (1978) points out the importance of similarity in various aspects of the school curriculum and hence the need to develop this concept:

> Some models for rational number concepts are based on similarity; thus, part of student's difficulty with rationals may stem from problems with similarity ideas. Ratio and proportion present many difficulties to students. Similar geometric shapes would seem to provide a helpful mental image for other types of proportion analogy situations. (Fuson, 1978, p.259)

In this Module we explore similarity within the framework of the van Hiele Theory. According to the van Hiele Theory when a new concept is introduced to learners it is important that they are provided with experiences on the visual level even though they might be at a higher level in the development of other concepts.

We begin working with the learners' everyday understandings of the notion of *similar*. The images connected with this term in everyday language do not however belong to the geometrical concept. In the process of developing this concept we need to point out to the learners that mathematicians use the term *similar* to describe objects that have the same shape. The notion of "same shape" is not, however, a clearly defined mathematical notion. The learners need to realise that mathematicians are very specific as to what they mean when they refer to *same shape.* Most school textbook definitions provide a definition for similar figures as follows:

> Figures that have the *same shape* but *not necessarily* the same size are **similar.**

This is not, however, a precise definition for similarity and it is important in the process of developing this concept that we focus carefully on the following aspects of the definition:

- the qualifier "*not necessarily*" that includes *congruent figures* as a special case of similar figures
- The notion of same shape and the *necessary conditions* for figures to have the same shape:
  1. Corresponding angles of the figures must be congruent (equal)
  2. Corresponding sides of the figures must be proportional.

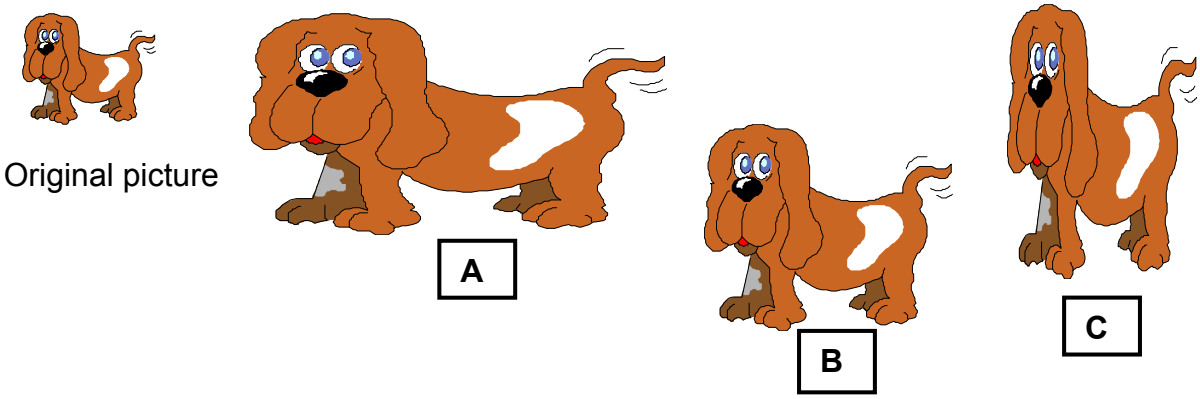
Instead of providing pupils with a definition at the start, our aim is for the learners to build a definition of similarity. The experiences that we provide for the learners in the process of developing an understanding of similarity could possibly lead to other definitions of similarity, for example:

> Two figures are *similar* if you can rotate, translate and/or reflect one of them so that it can be enlarged or reduced onto another.

In the Malati Spatial Module 5 we deal with *transformations* (*translations, reflections, rotations*) that only change the position of figures in a plane. Revisit these notions as a form of introduction. In developing the concept of similarity it needs to made clear to the learners that we are dealing with a particular kind of *transformation*, *enlargement and reduction*.

> Note that we will also use the word 'enlargement' to refer to similarity transformations that make a figure bigger **or** smaller. A 'reduction' will therefore be regarded as an enlargement with a scale factor between 0 and 1.

We therefore explore the concept of similarity by providing all learners with experiences on the **van Hiele visual level.** The activities for this level involve classification tasks in which the learners explore the notion of *same shape* as part of the process in developing the concept of similarity. At this level the learners focus on the figures as a whole and describe the differences in figures using language such as bigger, smaller, stretched or wider. Some of the learners at this level only recognise the differences in figures as either "bigger' or 'smaller', for example, the learners will consider all the pictures of the dogs below to be bigger than the original picture of the dog:



The learners must be challenged to find differences between the "bigger" pictures and the original picture. In this way these learners will realise that another *transformation*, stretching is involved in creating the pictures A and C.

At the **van Hiele analysis level** the learners will focus on specific aspects of figures such as the lengths of the sides and the sizes of the angles so that they can deduce what the necessary conditions are for figures to be similar. The *transformations* used in similarity, the enlargements and reductions, is a focus in all the activities in which the learners create similar figures.

At the **van Hiele deductive level** the learners are able to logically order figures and relationships among properties of figures, for example, they may deduce that if two triangles are equiangular then the lengths of the corresponding sides will be proportional.
Hence at this level the pupils distinguish between *necessary* and *sufficient* conditions for the different polygons.

Links With Curriculum 2005

In this Module we address the following specific outcomes:

Specific Outcome 7: *Describe and represent experiences with shape, space, time and motion, using all available senses.*

Specific Outcome 8: *Analyse natural forms, cultural products and processes as representations of shape, space and time.*

We deal explicitly with the following **assessment criteria, range statements** and **performance indicators** in the intermediate and senior phases:

Intermediate Phase:

SO 7		
Assessment Criteria	**Range Statement**	**Performance Indicator**
Descriptions in changes in shape of an object	Transform and tessellate shapes	identify enlargement carry out specified enlargements
SO 8		
Assessment Criteria	**Range Statement**	**Performance Indicator**
Generation of ideas through natural forms, cultural products and processes	Use representations to generate new ideas	classify figures in terms of congruencies and similarity deduce properties of, and relationships between figures

Senior Phase

SO 7		
Assessment Criteria	**Range Statement**	**Performance Indicator**
Descriptions in changes in shape of an object	Transform and tessellate shapes	display some transformation geometry skills on objects describe tessellations on shapes: • identifying similarity and congruency • identifying and classifying shapes
SO 8		
Assessment Criteria	**Range Statement**	**Performance Indicator**
Generation of ideas through natural forms, cultural products and processes	Use representations to generate new ideas	make links between cultural mores, social conditions, natural forms and cultural products and some of the mathematics used within the community

Many of the notions dealt with in similarity, such as the notion of scale factors, address some aspects of Specific Outcome 4: *Critically analyse how numerical relationships are used in social, political and economic relations*. See in the table below.

Senior Phase

SO 4		
Assessment Criteria	**Range Statement**	**Performance Indicator**
Demonstration of knowledge of the use of mathematics in determining location	working with mapping scales	interpret plans in terms of real structures draw a plan of a structure(e.g. classroom) use scale to interpret the distances between points. compare the distances calculated using scales to distances the learners know from daily life occurrences

References and Sources:

Fuson, K.C. (1978). Analysis of research needs in projective, affine and similarity geometries, including an evaluation of Piaget's results in this area. In R. Lesh (Ed.), **Recent Research concerning the Development of Spatial and Geometric Concepts** (pp. 243– 260). Columbus, Ohio: ERIC/SMEAC.

Fendel, D., Resek, D., Alper, L. & Fraser, S. (1997). **Shadows: Teacher's guide**. Interactive Mathematics Program (Year 1). California: Key Curriculum Press.

Connected Geometry (1997). **A matter of scale: Pathways to similarity and trigonometry**. Education Development Centre.

LIST OF ACTIVITIES:

Visual Level:
Communication
Under the Magnifying Glass
Vusi's Photos

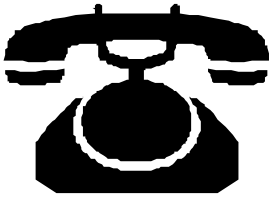
The activities below are in the Sec 01b document:
Cycles
Rectangles 1
Traffic signs
Triangles1
Triangles 2
Polygons

Analysis Level:
Drawing Houses
Lucas's Similar Figures
Bingo's Figures
Mark's Enlargement
Mark's Reduction
Hexagons
Well-scaled Fishes
Parts Of The Elephant
Scaled Pictures
Scaled Figures
How To Shrink It
Horses
Rectangles 2
Wayne's Triangles
Renata's Arrow
Scaling Polygons
Thabo and Tembi's Triangles
Can You See the Triangles?
Terry's Problem
Problems At The Press
Mike's Triangles

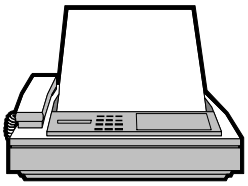
Project / Extension:
Our Flag: the Right Shape 1
Our Flag: the Right Shape 2
Slicing Off Similar Triangles

COMMUNICATION

Which of the following pictures of communication objects are similar? Explain why you have decided that the objects are similar.



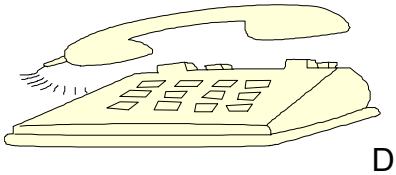
A



C

B

F

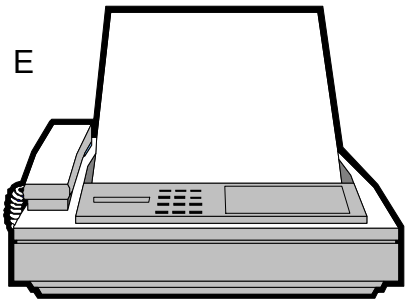


D

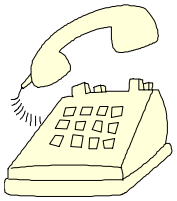
G

H

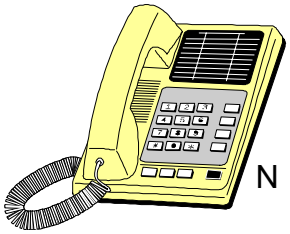
I



E



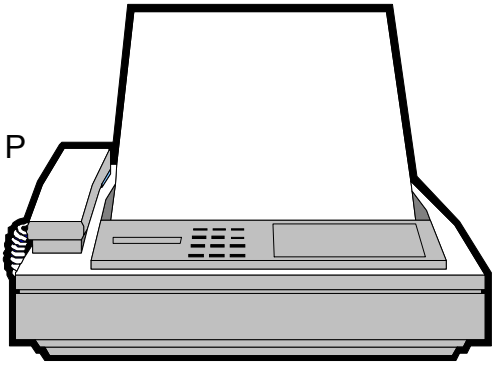
J



N

O

K L M

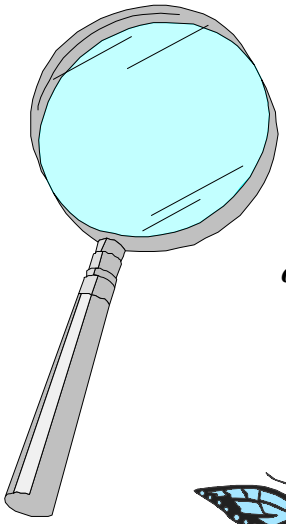


P

Teacher Notes: Communication

The aim of this activity is to establish the intuitive understanding that the learner's have of the term similarity. Most of the learner's we observed referred to the objects that have the same features, for example, A, D, I, J, M, N, O were grouped as similar objects as they were all telephones. B, F, G, H, K, L and M were grouped as similar objects as they were all cellular phones. It needs to be made explicit in the whole-class discussion that the images connected with this term in everyday language do not however belong to the ***mathematical*** *concept of* ***similarity****. Point out to the learners that the mathematical concept involves the notion of "same shape". This activity can be revisited again once the notion of "same shape" has been explored.*

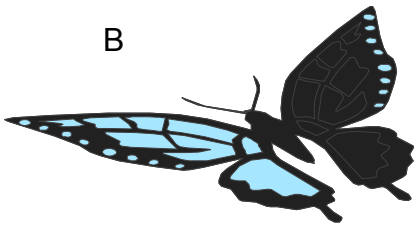
UNDER THE MAGNIFYING GLASS



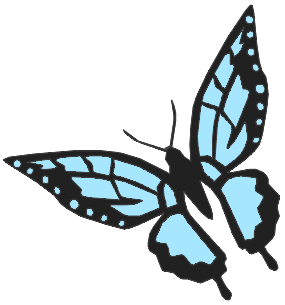
original butterfly

Belinda looks at this butterfly under a magnifying glass.
Which of the butterflies below will she see?

A



B

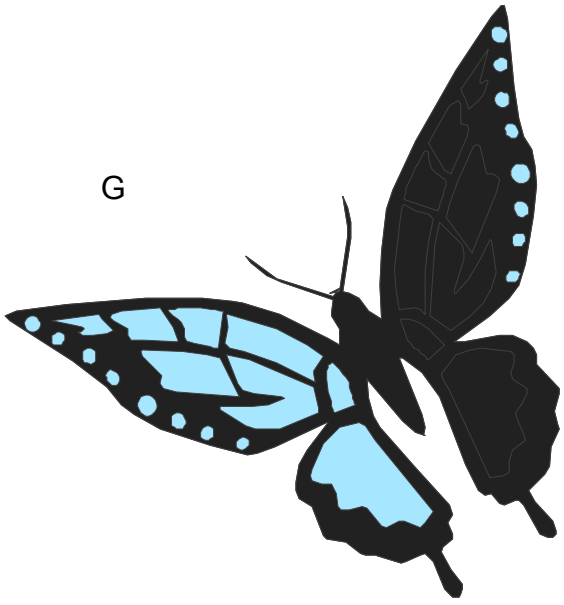


D

C

E

F



G

Teacher Notes: Under The Magnifying Glass

This activity requires that the learners visualise how the butterfly is transformed when looked at through a magnifying glass. Provide the learners with magnifying glasses.
In the discussion the following issues can be raised;

- *How the magnifying glass has been held to see the possible representations of the butterfly. If the learners insist that only the butterflies A, C, D and G can be seen, ask them what would happen if the butterfly were viewed through the magnifying glass at an angle.*
- *How objects are transformed in the real world. For example, the photocopier is a technological device that enables us to make copies of pictures.*
- *How the butterflies have changed.*

Listen carefully to the language used by the learners in describing the different butterflies in relation to the original butterfly. Where learners say, for example, that butterflies have the same shape, get them to explain what they mean by this. Here are some explanations of learners in attempting to clarify what meant they by "same shape" in the context of this activity:

"Butterfly G is the same as the original butterfly, it is just bigger."

"Butterfly G and the original butterfly both have both pair of wings the same size. In butterfly F, for example, the two of wings on top is definitely not the same size."

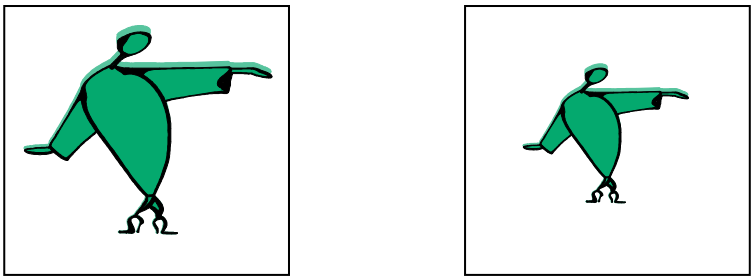
Most learners will focus on the global features, namely the size or shape of the butterflies in comparing the different butterflies. This is evident in the responses above. The second response involves a comparison of the relative components (the wings of the butterfly). One of the learners described the situation for butterfly G as "In butterfly G everything is in proportion." The learner has an intuitive visual understanding of proportion.

VUSI'S PHOTOS

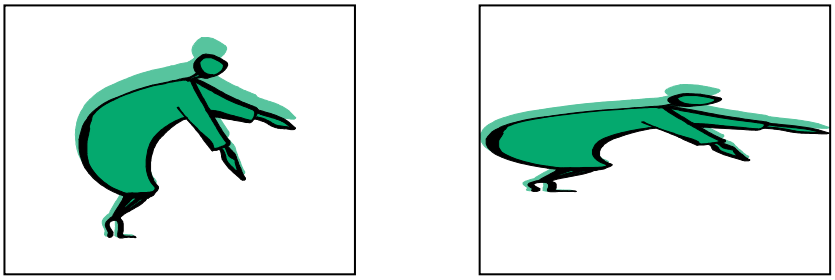
Look carefully at the following pairs of photographs taken by Vusi at an art gallery

In each case, state whether you consider the figures in the photographs to have the same shape or not and *why*.

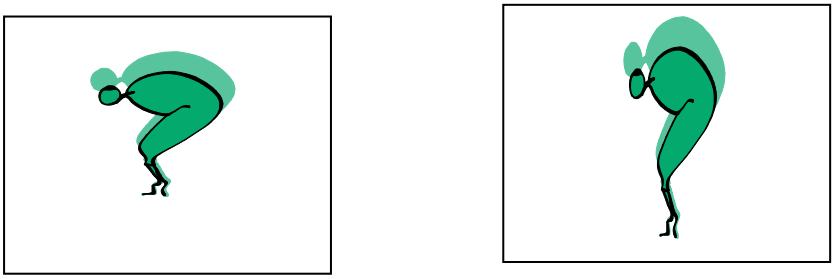
A



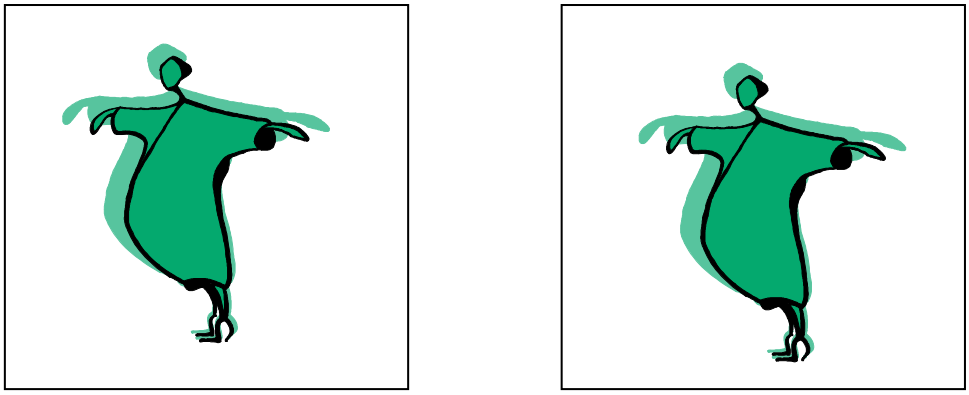
B



C



D



FIGURES THAT HAVE THE **SAME SHAPE** ARE CALLED **SIMILAR FIGURES**

Teacher Notes: Vusi's Photos

In this activity we focus on a pair of figures, where the one is a specific transformation of the other. The learners will be able to visually determine which figures are similar in shape. In the whole-class discussion ask the learners to describe for example how the figure on the left was changed to create the figure on the right. At this stage the learners may be asked to classify the transformations into those that conserve the shape of the figure and those that do not. The special case, in picture D, of congruent figures as similar figures need to be made explicit. The transformations that result in congruent figures can be revisited, namely, rotations, translations and reflections.

The term similar can now be defined to mean figures that have the same shape.

GO TO PART B OF SIMILARITY MODULE