Malati

Mathematics learning and teaching initiative

Geometry

Module 2

Transformations

Grades 8 and 9

Teacher document

Malati staff involved in developing these materials:

Kate Bennie
Zonia Jooste
Dumisani Mdlalose
Rolene Liebenberg
Piet Human
Sarie Smit

We acknowledge the assistance of Zain Davis, Shaheeda Jaffer, Mthunzi Nxawe and Raymond Smith in shaping our perspectives.

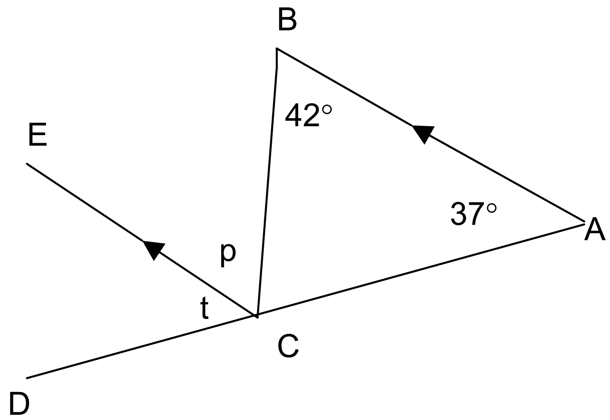
December 1999

Malati Geometry: The Transformation Approach

A Common Approach to the Study of Plane Geometry:

This problem is commonly used in geometry in the senior phase.

Find the value of each of the angles p and t. Give reasons for your answers



The mathematical figure in this problem is presented as being static. In order to solve the problem, learners are required to recognise the pairs of corresponding and alternate angles (this is usually done by recognising the "F" and the "Z" in the diagram. 'Proofs' in two column format are acceptable. The function of proof in such a case is verification or systematisation (Bell, 1976).

Transformation Geometry in the School Curriculum:
Traditionally isometric transformations have formed part of the geometry curriculum in South Africa:

- In the study of tessellations (although the transformation aspect is seldom made explicit).
- As a separate topic in Grade 9, sometimes linked to co-ordinate geometry.

Much of this work on transformations has, however, been restricted to the *perceptual* level, that is, pupils have been given opportunities to physically manipulate figures using cut-outs, paper folding, geoboards, tracings, etc. Learners are seldom challenged to transfer their perceptual understanding to a representational level, that is, to perform the operations *mentally*. It is this ability to perform transformations mentally that is valuable in mathematics.

Similarity is studied informally in Grade 9 and more formally in Grade 12, but the transformation dimension (enlargement) is not explored.

A Broader View of Transformations:
Transformations are not, however, restricted to isometric transformations and enlargements. Light (1993) uses the work of Klein to classify the different transformations as shown in the table that follows. Examples of the use of each transformation and the geometrical context in which each occurs are also given:

Transformation	Invariants	What Changes	Examples	Geometrical Context
Isometric (translation, reflection, rotation)	shape, distance	position, orientation	tessellations, symmetry,	metric
Dilation (shrinking or enlargement)	shape	size	maps, plans, toys, models, trigonometry, shadow geometry from a point light source	similarity
Shears / Stretches	parallelism	shape	shadow geometry using sun as a light source	affine
Oblique projection	cross ratio / straightness	parallelism	perspective drawings, photographs	perspective
Transformations that do not tear space	closeness of points	Straightness of lines	networks, underground maps	topology

MALATI believes that the range of transformations studied at school can be widened – the examples mentioned above can be used to study the different transformations on the Van Hiele visual and analysis levels in the intermediate and senior phases. The actual differences between the transformations need not be made explicit in these phases.

Furthermore, certain concepts traditionally taught at school level can be *redefined* and studied in terms of transformations:

- Congruence: Two figures are congruent if there is a line reflection, translation, rotation or glide reflection that maps one triangle onto the other
- Similarity: Two figures are similar if there is dilation that maps one onto the other.

Sanders and Dennis (1968) give the following definitions:

- Parallel lines: Two lines are parallel if there is a translation that maps one line onto another
- Perpendicular lines: Two lines are perpendicular whenever one is invariant under reflection about the other
- Line symmetry: A figure has line symmetry if it is invariant under a reflection about some line.

When points in a plane are described in terms of co-ordinates, transformations can be expressed algebraically. Matrix representations can also be used to study transformations in their own right.

MALATI Transformations:
MALATI has identified a number of ways in which the use of transformations in the teaching and learning of geometry can be valuable:

- As a means to develop spatial *skills*.
- As one *method* for studying plane geometry.
- As a means to *integrate* mathematical topics which have traditionally been studied separately, for example, the study of plane figures using co-ordinate geometry.
- As a topic of study in its own right – the transformations themselves can be regarded as the *objects of study*.

These factors are reflected in the Malati approach to transformations and in the design of our materials:

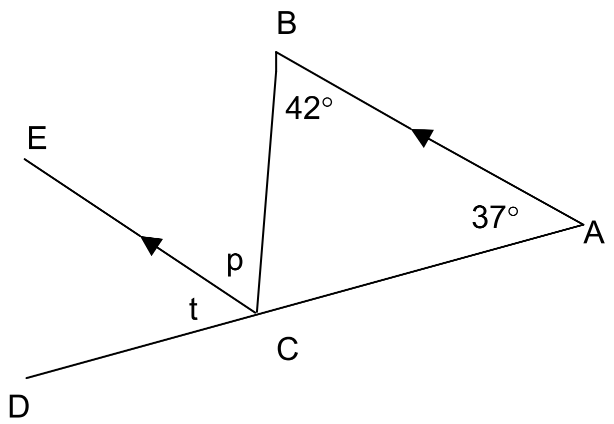
Intermediate Phase:
Learners explore transformations on the visual van Hiele level. They are required to perform and identify isometric transformations. Enlargements and projections are explored informally. It is important that learners at this level have rich experiences physically manipulating objects, as this forms a foundation for movement to the van Hiele analysis level and the use of visual skills.

Senior Phase:
In the MALATI activities for this phase, transformations are regarded as a *method* for studying plane figures.

As mentioned above, certain concepts used in the study of plane figures can be redefined using transformations. For example, rather than learners having to use a formal 'Side-Angle-Side" argument to show two triangles congruent, they could simply describe the relevant transformation.

Transformations can be used to study the properties of plane figures. Consider the problem given earlier:

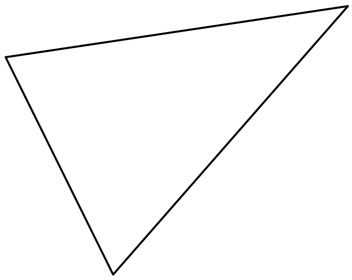
Find the value of each of the angles p and t. Give reasons for your answers



Pupils could argue that, if one translates ∠ECD along line AD so that it lies on ∠BAC, then t = 37°.
∠ECB can be rotated 180° about the midpoint of CB so that it lies on ∠ABC. Hence p = 42°.

Transformations can also be used to identify line and rotational symmetry in polygons. These examples are from the MALATI materials:

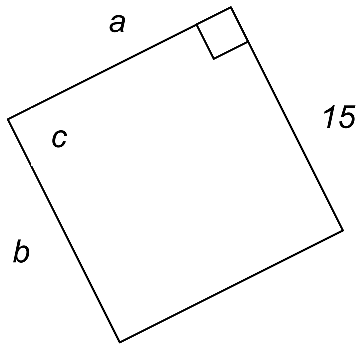
A. *Consider the triangle below:*



1. *Reflect the triangle in its sides to make as many different quadrilaterals as possible.*
   In each case write down the name of the quadrilateral formed, list its properties and explain how you got the properties.

2. *What if Andile's triangle is an isosceles triangle?*

B. Write down the value of each of the letters (a) to (c) in this square. Use transformations to explain your answers



The question might be asked: "At what stage do learners begin formal deductive geometry?". According to the van Hiele theory, learners need a range of opportunities to explore geometric figures from which they will be able to generalise their observations. This activity on the van Hiele analysis level is necessary before learners can make deductive arguments on the van Hiele ordering level. In the activities for the senior phase, learners are given an opportunity to explore the properties of geometric figures using transformations. After experiences on this level, they should come to recognise that the corresponding angles in parallel lines are equal. When such a generalisation has been made, they will be able to proceed with more formal, deductive arguments. The foundation for this has, however, been laid using transformations.

In the above examples transformations are being used as a vehicle to explore other concepts, but the properties of the transformations can be explored in their own right. For example, learners can be encouraged to find equivalent transformations and the composition of certain transformations.

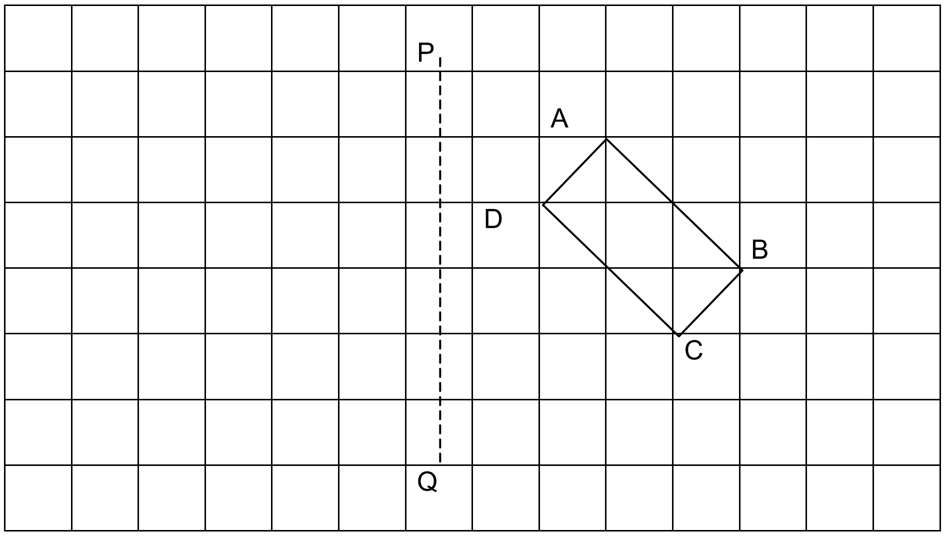
Shaughnessy (1995) has placed the study of *isometric transformations* in the van Hiele framework.

Van Hiele Level	Characteristic of Thinking	Teaching Objective
Recognition	Pupil has a global, non-mathematical view of transformations.	To recognise each isometry as a movement, and to perform isometries using appropriate manipulatives. To recognise the invariance of shape and space under an isometry.
Analysis	Pupils consider the properties of an isometry.	To explicitly use the properties that characterise an isometry. To discover new properties of isometries by experimentation. To discover compositions of isometries. To use mathematical notation and vocabulary for isometries.
Ordering	Students can discover and use properties of and relations between isometries; can follow mathematical reasoning and make informal arguments.	To justify and use properties of isometries. To use and understand formal definitions of isometries. To understand and use the intersection of perpendicular bisectors to determine the centre of rotation. To understand simple proofs.

The MALATI transformations module includes activities for the study of isometric transformations at the *analysis level*. Enlargements (dilations) are explored in depth in the Malati similarity module (module 2).

Consider this activity:

Reflect rectangle ABCD in the line PQ. What is the relationship between ABCD and A'B'C'D'?



Of course a learner could solve this by using physical manipulation, but if it is done visually the learner will need to know that the line PQ will be the perpendicular bisector of the line joining a point on ABCD with its image on A'B'C'D'.

Learners can also be required to consider the relationships between the transformations:

- *Can a combination of two translations always be replaced with one transformation? And two reflections? Two rotations?*
- *Can a combination of a translation and a reflection always be replaced with one transformation? And a translation and a rotation? And a reflection and a rotation?*
- *Is the combination of two transformations always commutative?*

Activities of this nature are classified as *enrichment activities* in the MALATI materials.

See the summary of activities overleaf.

Transformations and Curriculum 2005:
Curriculum 2005 recognises different transformations in mathematics and requires that learners study the changes in shapes over time. The performance indicators for the Intermediate phase are as follows:

- Identify translation
- Identify rotation
- Identify reflection
- Identify enlargement
- Carry out specified translation, rotation, reflection and enlargement
- Realise and show that certain objects change shape and the way these occupy space when stretched or contracted for a specific length of time
- Classify figures in terms of congruencies and similarities

Senior phase learners are required to

- display some transformation geometry skills on objects
- describe tessellations on shapes (identifying symmetry, similarity and congruency)

Learners are also required to work with mapping scales, an aspect of similarity. The requirement that learners in the senior phase work with a co-ordinate system increases the scope for the study of transformations.

References:

Bell, A. W. (1976). A study of pupils' proof-explanations in mathematical situations. **Educational Studies in Mathematics, 7,** 23-40.

Light, R. (1993). Which shape, which space? **Mathematics Teacher,145,** 30-33.

Sanders, W.J. & Dennis, J.R. (1968). Congruence geometry for junior high school. **Mathematics Teacher, 61,** 354-369.

Shaughnessy (1995). Source unknown.

Content of MALATI Module 2 (Transformations)

This module has an introductory section designed for learners who have not studied isometric transformations (translation, rotation and reflection) at the primary school. Learners who have completed these activities should be able to transform geometric figures as well as recognise given transformations. Initially, some learners might need to make copies of the figures and to physically manipulate these, but visualisation of the movements should be encouraged. Computer software packages such as Sketchpad can be used to study transformations. Some reflection on the actual properties of the transformations is required if learners are to perform the transformations.

The teacher should determine whether learners have developed the above-mentioned skills and provide learners with relevant experiences and remediation *before* proceeding to the rest of the module.

In the remaining activities in the module, geometric configurations are presented as dynamic entities and pupils are given an opportunity to explore the properties using transformations (van Hiele analysis level activities). The teacher can encourage movement towards the van Hiele ordering level by requiring that the pupils generalise the properties for particular classes of figures (using the correct mathematical vocabulary).

The Transformation Activities:

Core activities: Strange Pictures!
Patterns
Isometric Transformations
More Isometric Transformations

Consolidation: With Your Partner 1
With Your Partner 2
Glide Reflections

Enrichment: Looking at Transformations
Combinations of Transformations
Enlargements
Finding the Centre of Rotation

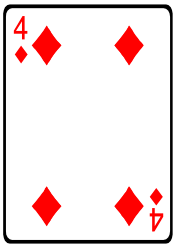
Plane Geometry Activities:

Core activities: Just Points and Lines
Creating New Figures
Tracey's Transformations
Angles!
Moving Lines 1
Moving Lines 2

Consolidation: Use textbook examples

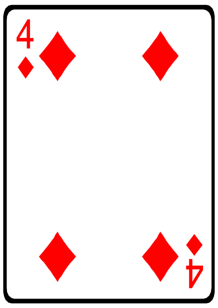
Strange Pictures!

Look carefully at this picture of a playing card:

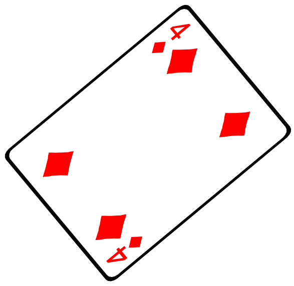


What has happened to these cards? In each case describe the changes.

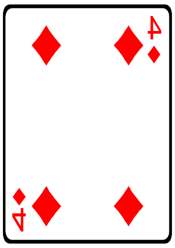
A.



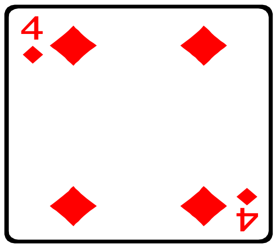
B.



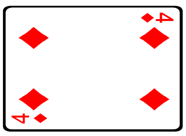
C.



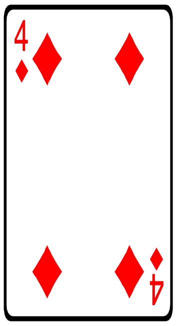
D.



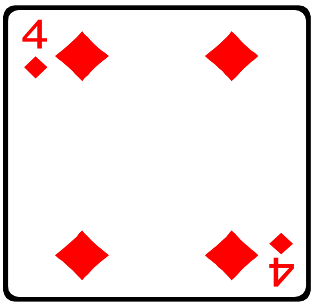
E.



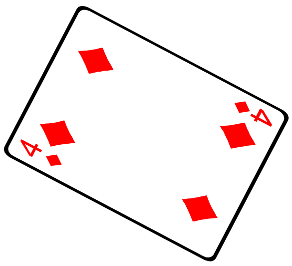
F.



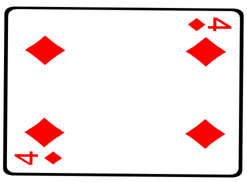
G.



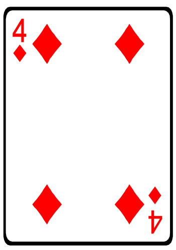
H.



K.



M.



Teacher Notes: Strange Pictures!

This activity builds on those activities in the Similarity module and revisits the concepts of congruence and similarity. The aim is to explore the different transformations, that is, the different changes that can take place in a shape. Learners can respond by commenting on the visual appearance of the figures.
The following responses can be expected:
Figure A is an enlargement, it has the same shape as the original, but differs in size.
Figure B is the same size and shape (congruent), but the figure is facing a different way. It has been rotated / turned.
Figure C: Is the same shape and size as the original, but has been reflected / flipped. vertically. (Learners can also be challenged to consider what a horizontal reflection / flip will look like.)
Figure D: Figure D has been stretched horizontally (not vertically).
Figure E: This is a reduction of the original – the two figures are the same shape and size. The figure has also been rotated / turned.
Figure F: Figure F has been stretched vertically (not horizontally).
Figure G: The figure has been stretched vertically and horizontally. Shape has not been preserved.
Figure H is the same size and shape (congruent), but the figure is facing a different way. It has been rotated / turned.
Figure K is congruent to the original, but has been rotated / turned.
Figure M is congruent to the original figure and has the same orientation on the page.

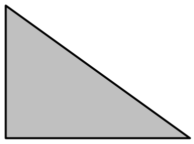
Learners should be encouraged to reflect on the different types of changes:

- *Those in which the shape and size is preserved, but the orientation or position is changed. These are called isometric transformations and the figures are said to be congruent.*
- *Those in which the shape, but not the size is preserved. These are called enlargements and the figures are said to be similar.*
- *Those in which the shape changes, but parallelism is preserved (shears or stretches).*

The isometric transformations are explored further in the activities that follow.

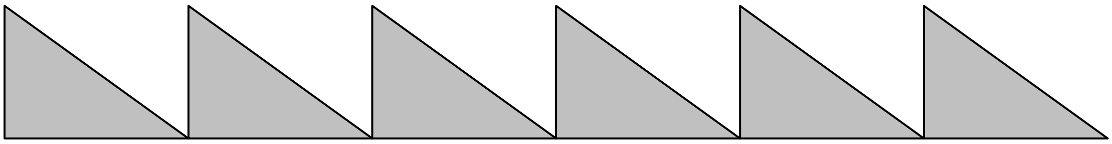
Patterns

Linda is making patterns with a cardboard triangle like this:

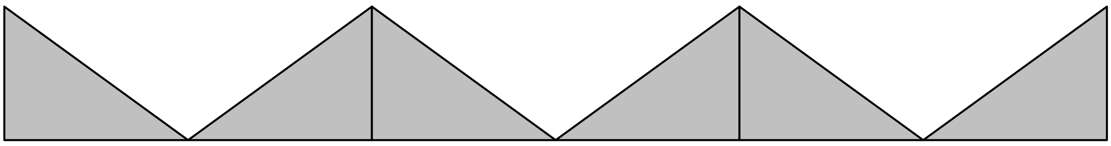


Each times she moves the triangle in a certain way and traces around the cardboard shape as she goes. In this way she can make **regular** patterns. So far she has made these patterns:

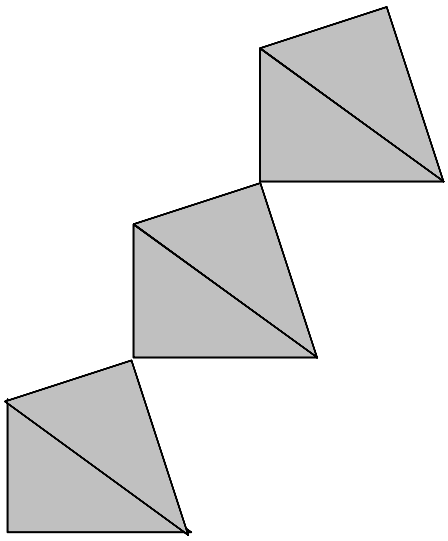
A.



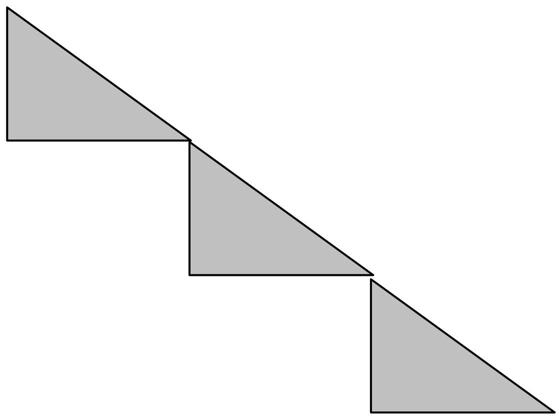
B.



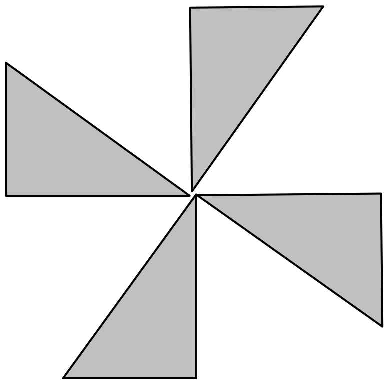
C.



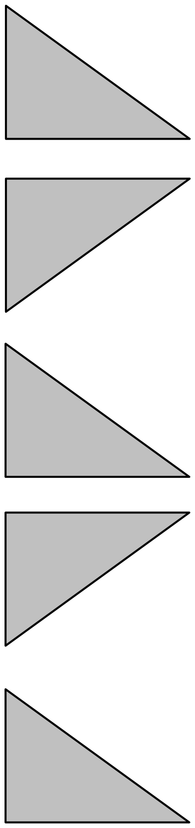
D.



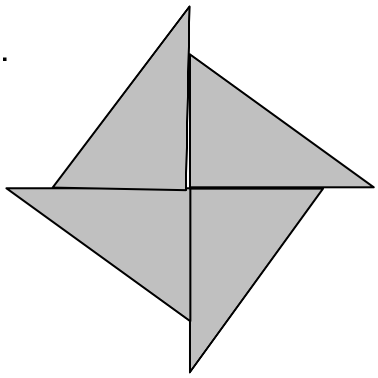
E.



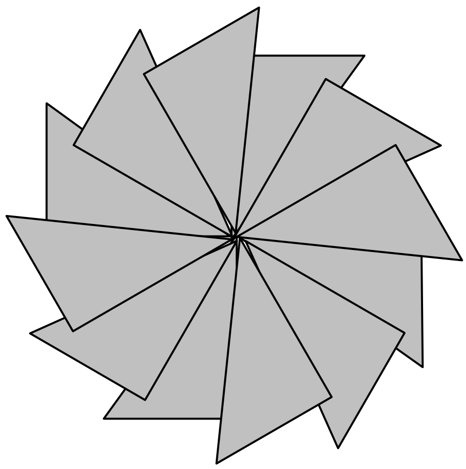
F.



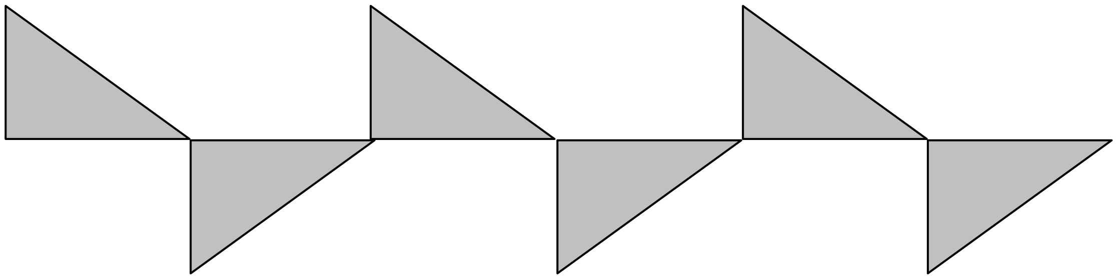
G.



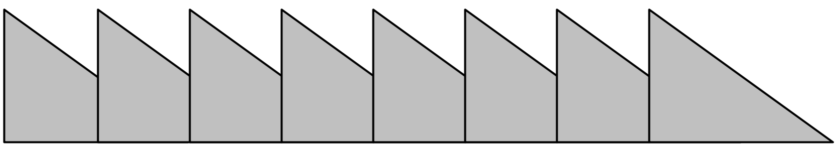
H.



K.



M.



1. Linda is very excited about these patterns and phones a friend to tell her about them. Can you help Linda to describe to her friend how she moved the triangle to make each pattern? Is there more than one way to describe the movement?

2. Are there any other patterns she can make by moving this triangle? Draw these patterns. In each case describe how you moved the triangle.

3. Now choose a different figure and make your own patterns. In each case describe how you moved the figure.

Teacher Notes: Patterns

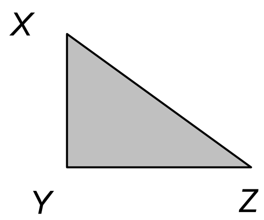
*This activity is designed to introduce pupils to the different isometric transformations (translation, rotation, reflection and glide reflection). It is important to note that when a figure is transformed, it is moved. These border patterns are thus created by combining **all the copies of the figure**.*

In question 1 pupils are required to identify the movements and describe these precisely. Some learners might find it useful to make a copy of the triangle and to move this template along the pattern, while others will be able to visualise the movements.

Learners might use everyday vocabulary such as "slide'', "shift". "turn"' and "flip", and the teacher should introduce the appropriate mathematical vocabulary where possible. The descriptions of the transformations should be precise: Learners should be encouraged to test their descriptions on one another to make sure that the same pattern is created. For example,

- *Some learners might indicate in (a) that the triangle has been "shifted" – but the case of a translation, the direction and distance in which the object is moved also needs to be given.*
- *In the case of reflection the line in which line the triangle has been reflected should be described*
- *In the case of a rotation the point and angle of rotation should be given.*

Pupils might find it helpful to label the vertices of the triangle:



The questions have been designed to illustrate a number of different movements in an attempt to prevent learners from forming limiting conceptions. For example,

- *A translation does not have to be either horizontal or vertical, but can be oblique - pattern D is included to challenge this view.*
- *The distance a figure is translated varies – in patterns A and D the figure is translated the length of the different sides of the triangle, but it pattern M the figure is translated half the length of the side YZ.*
- *The line of reflection can vary – it can be vertical or horizontal line (vertical in the case of pattern B, it can be oblique as in the case of pattern C where the triangle is reflected through the line XZ, and it does not have to lie on the figure itself (as in pattern F). In the last case the learners should be encouraged to identify the line reflection.*
- *In a rotation the point of rotation as well as the angle of rotation can vary – n pattern E the triangle is rotated 90º about point Z, in figure G the triangle is reflected 90º about point Y, and in pattern H the figure is rotated 30º about point Z. The point about which a figure is rotated is called the **centre of rotation**. Learners should be encouraged to identify the rotational symmetry in these three*

patterns – patterns E and G have rotational symmetry of order 4, whereas pattern H has rotational symmetry of order 12.

Pattern K is a combination of isometric transformations. There are three possibilities:

- *A reflection over the line YZ followed by a translation the length of YZ to the right.*
- *A translation the length of YZ to the right, followed by reflection over the line YZ.*

(These two transformations are called ***glide reflections****. This type of transformation is not usually classified separately as it can be described in terms of the other isometric transformations (a translation and a reflection).)*

- *Two rotations, these being, a rotation of 180º about the point Z, followed by a rotation of 180º about the new point Y'.*

Transformations such as these which give the same patterns are called ***equivalent transformations****.*
There are also equivalent transformations for creating pattern B – successive vertical reflections or one vertical reflection followed by a translation a distance of 2YZ of the two triangles to the right.

In question 2 learners are required to create their own patterns using the given triangle. The rectangular grid paper can be used.

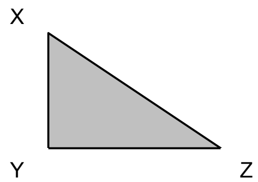
In question 3 learners should make their own border patterns. Rectangular or square grid paper can be used.

<u>Further Activities:</u>

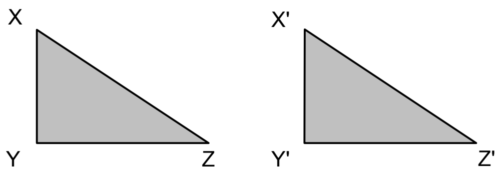
1. *Patterns A, B, and K are border patterns. Learners can be encourage to investigate how many* ***different*** *border patterns can be generated using the three isometric transformations. Learners should note that different sets of transformations can be used to generate the same pattern.*
2. *The teacher can provide or encourage the learners to bring examples of regular patterns to class. Examples of logos, beadwork, house painting, fabric and lace are useful for discussion. It is important that learners identify and describe the transformations used in creating the patterns. This is appropriate for project work.*

Isometric Transformations

In the activity *Patterns* we transformed triangles like this to make patterns:

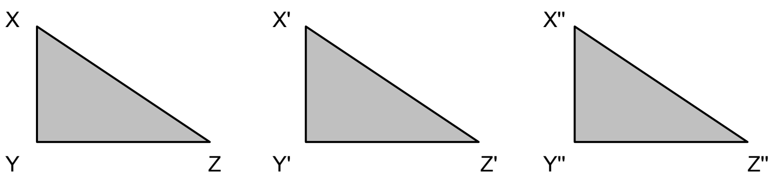


In each case we made a copy of ΔXYZ and then moved it to a different position where we made a new copy. For example,



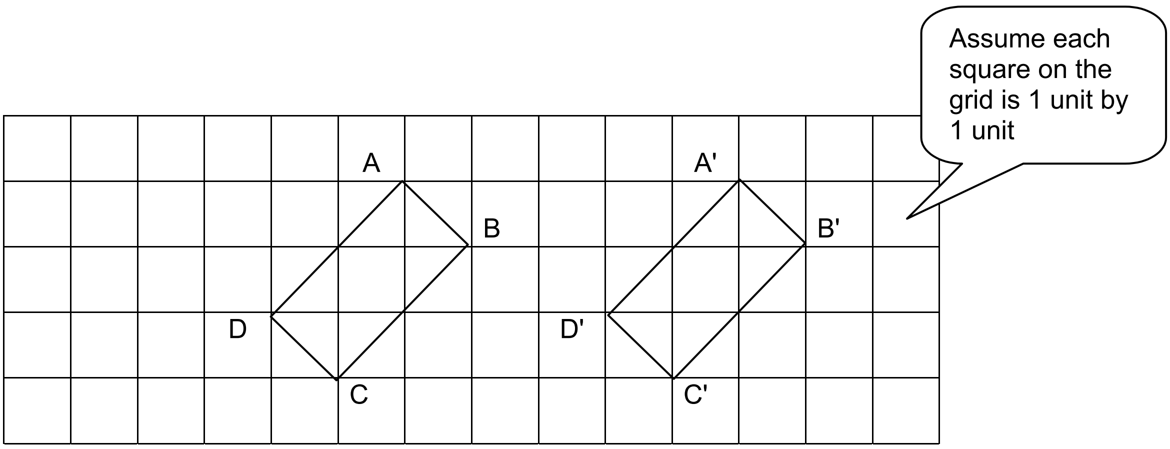
The new figure is called the *image* of ΔXYZ and we label it ΔX'Y'Z'.

We can now translate ΔX'Y'Z' and its image will be called ΔX''Y''Z'':

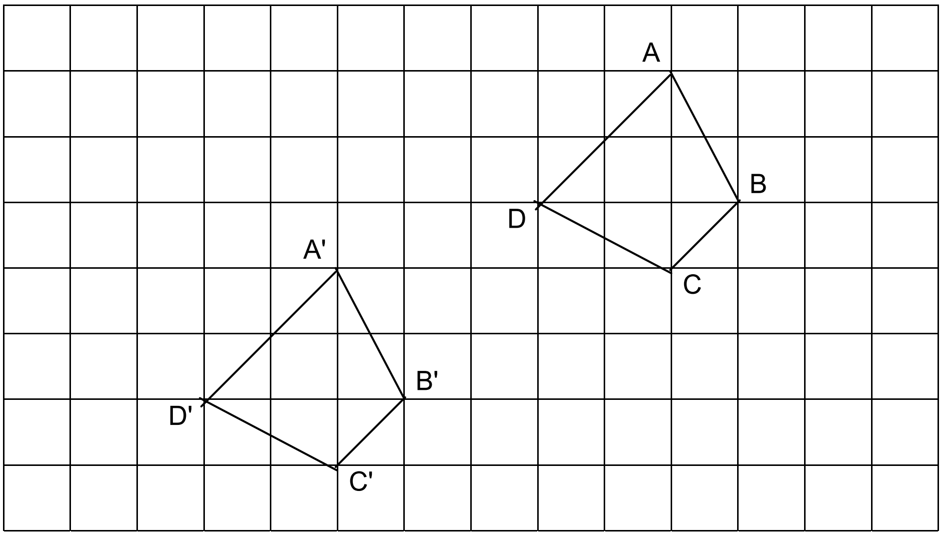


In each of the following diagrams the given figure has been transformed. In each case name and describe the transformation. Is there more than one possible description? Remember to give **all** the necessary information about the transformation.

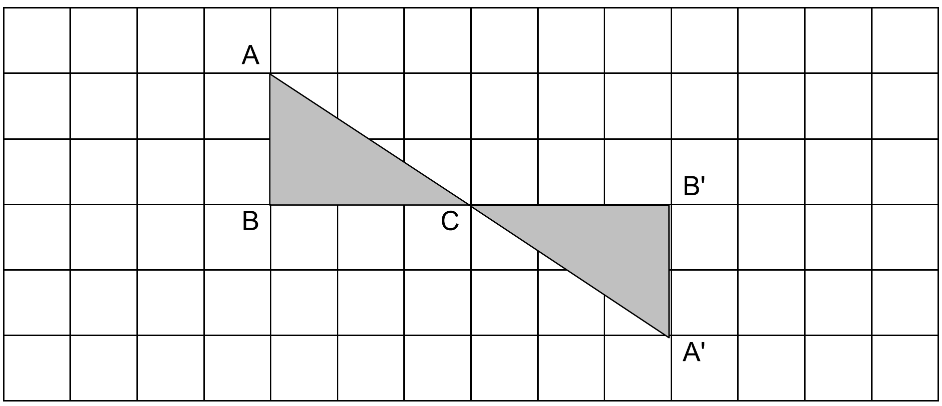
1.



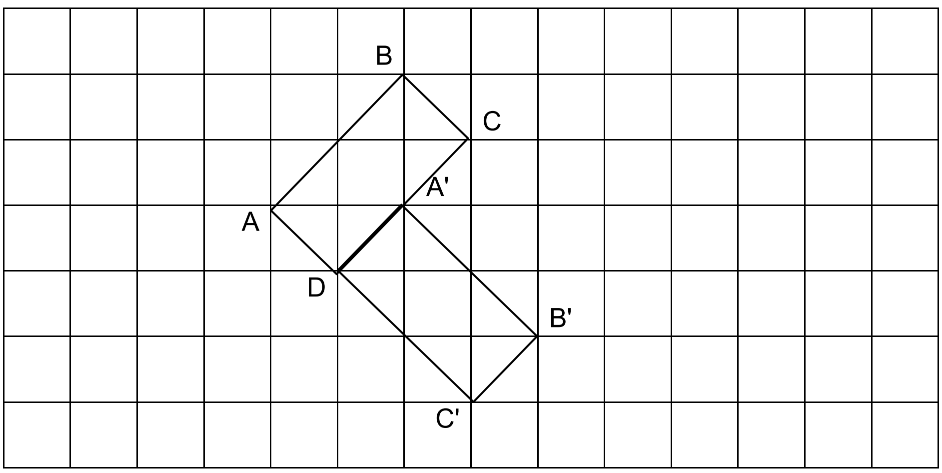
2.



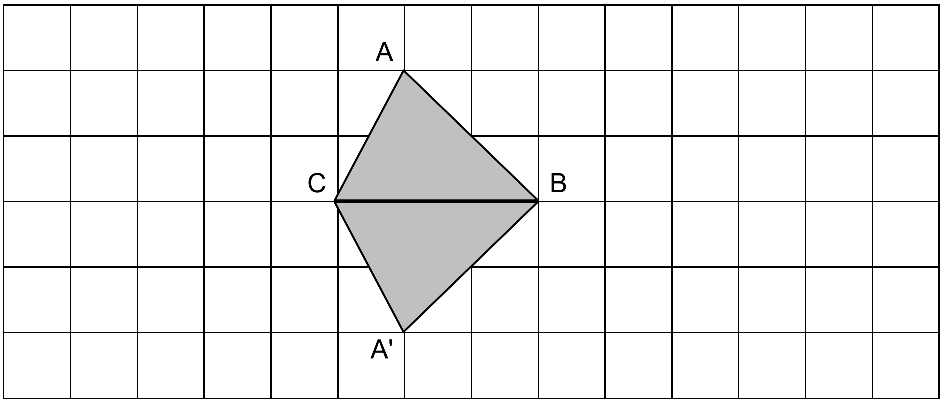
3.



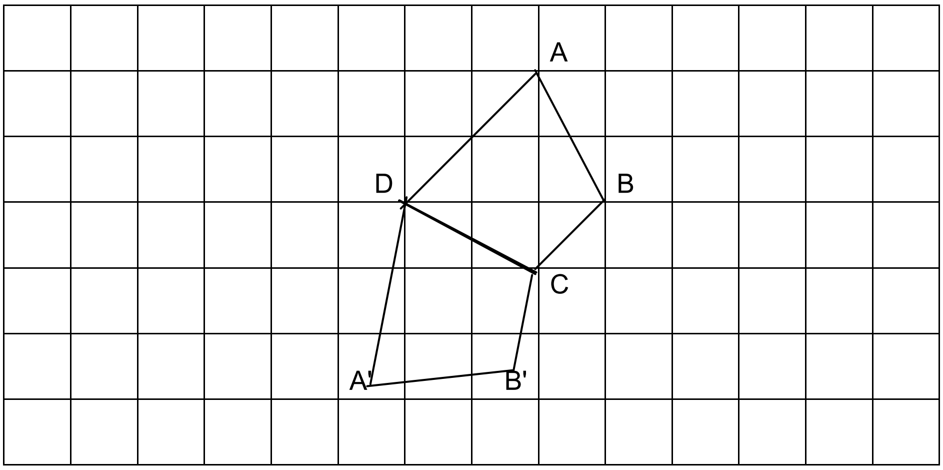
4.



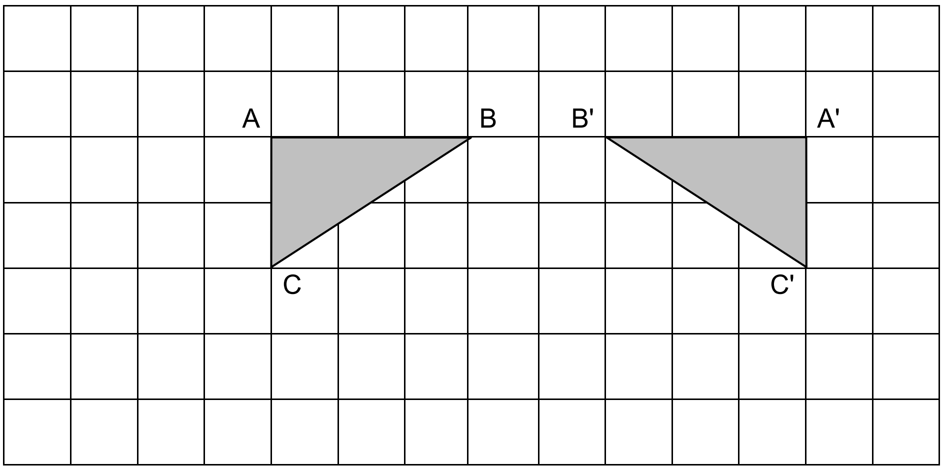
5.



6.



7.



8. When we do transformations we would like to be able to do these **without having to move an actual triangle**. Look carefully at the translations in question 1. Explain how you could find the image **without using a copy of the figure**. And for reflections? And rotations?

Teacher Notes: Isometric Transformations

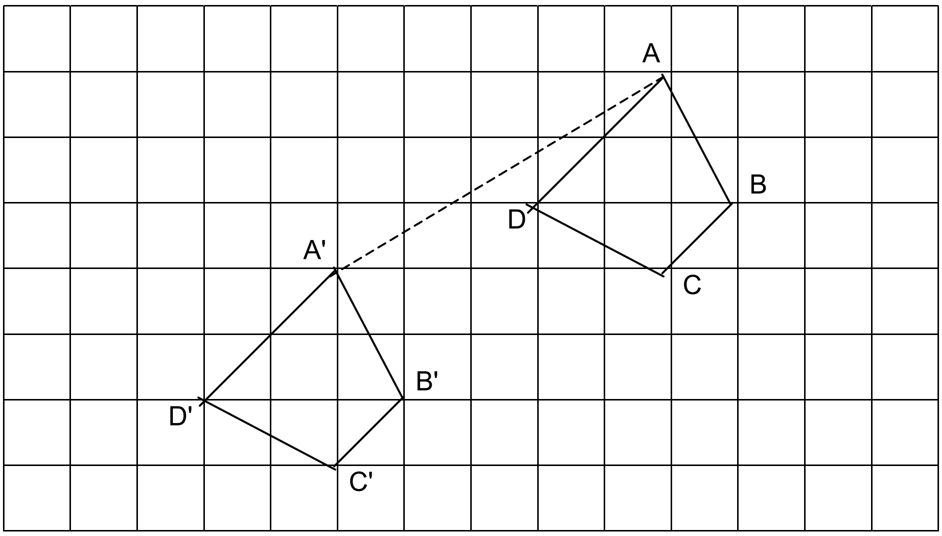
This activity has two purposes:

- *As consolidation to give learners practice in recognising the transformations in introduced in the activity "Patterns". In this case the transformations of the various mathematical figures are performed on square paper.*
- *To encourage learners to reflect on the properties of the transformations, for example, that in a translation every point in the figure is moved the same distance in the same direction. A knowledge of the properties is required if learners are to be able to use visualisation (rather than physical manipulation) to perform the transformations. If necessary, learners should be encouraged to consider each point and its image under a transformation. For example, what is the relationship between A and A' and B and B' in a translation?*

Solutions:

Question 1: Rectangle ABCD has been translated 5 units to the right.

Question 2: Trapezium ABCD has been translated 5 units to the left and three units down. The line along which the figure has been translated can also be identified as shown below. Each point on the figure is translated along a line which is the same length as and parallel to line AA'.



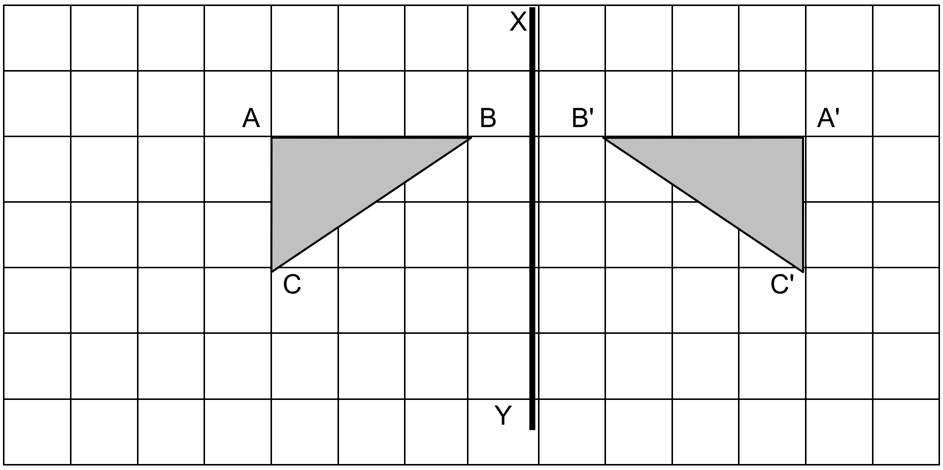
Question 3: ΔABC has been rotated 180° about point C (the direction does not make a difference in this case). Learners should not that the lines AA' and BB' are straight lines (the sum of the angles is 180º).

Question 4: Rectangle ABCD has been rotated an angle of 90º about the point D. (In mathematics n anti-clockwise rotation is said to be positive and a clockwise rotation is negative. This is social knowledge which must be told to learners) Learners should note this could also be described as a rotation of -270º about the point D. Each side of the image is at 90º to the side of the original figure ABCD, for example, AD⊥A'D'.

Question 5: ΔABC has been reflected in the line CB.

Question 6: Trapezium ABCD has been reflected in the line DC.

Question 7: ΔABC has been reflected in the line XY as shown below:



Learners should be encouraged to draw in the lines of reflection and then to consider the relationship between this line and the lines AA', BB' and CC' (the line of symmetry is the perpendicular bisector of each these lines.

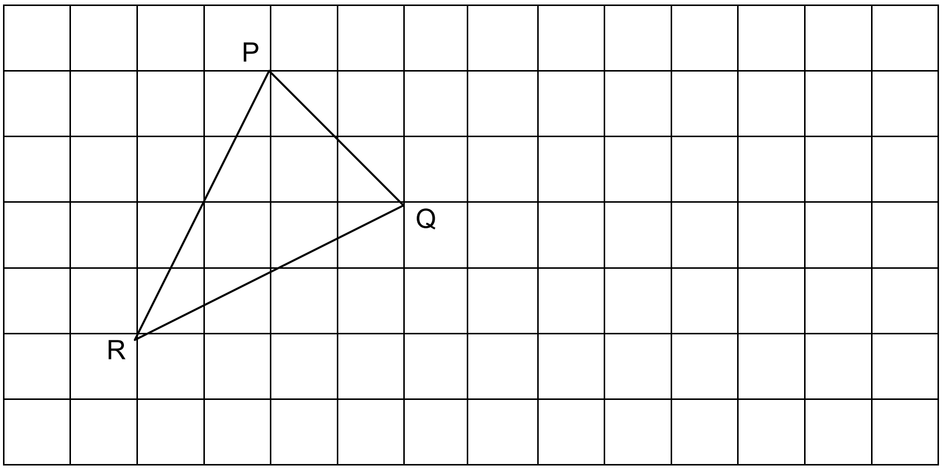
Question 8 requires that learners write down the observations they have made.

More Isometric Transformations

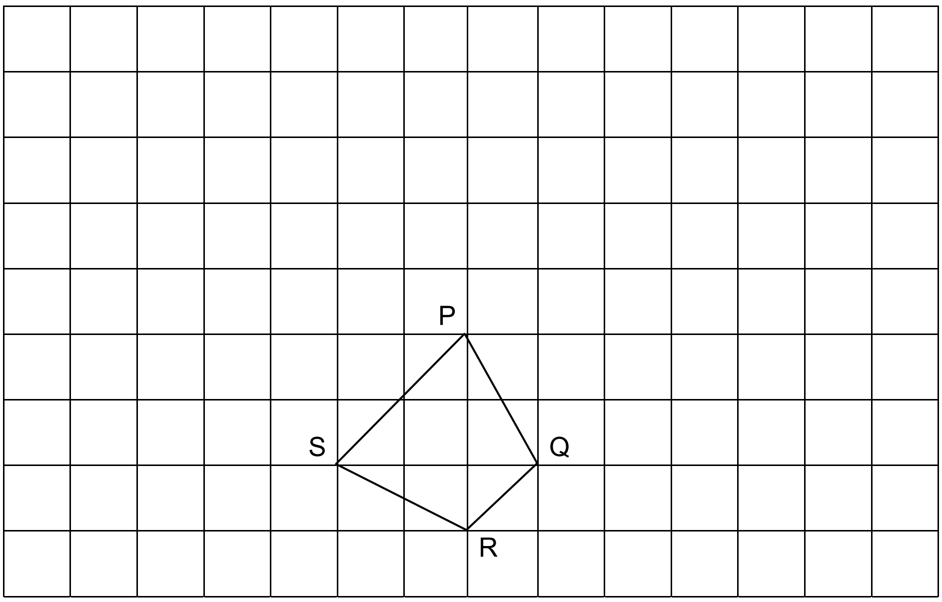
Now do these transformations. Try to use what you learnt in the previous activity to perform each transformation **without actually moving the figure**. Remember to label the image correctly.

- The size of each square on the grid is 1 unit by 1 unit.
- An anti-clockwise rotation is given as positive and a clockwise rotation is given as negative.

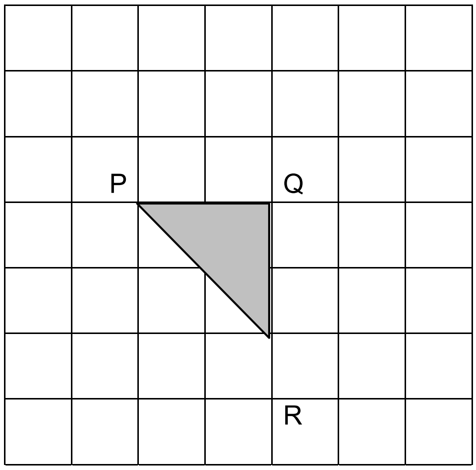
1. Translate ∆PQR four units to the right:



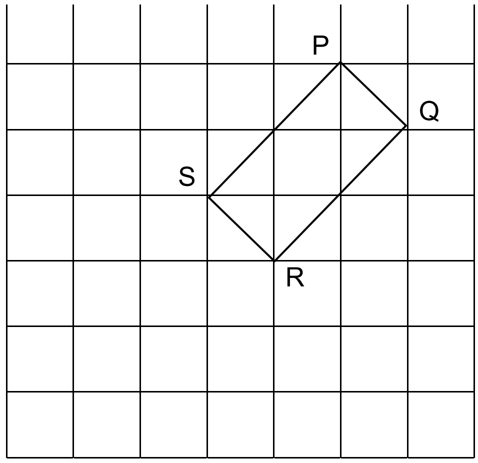
2. Translate trapezium PQRS three units to the left and five units up:



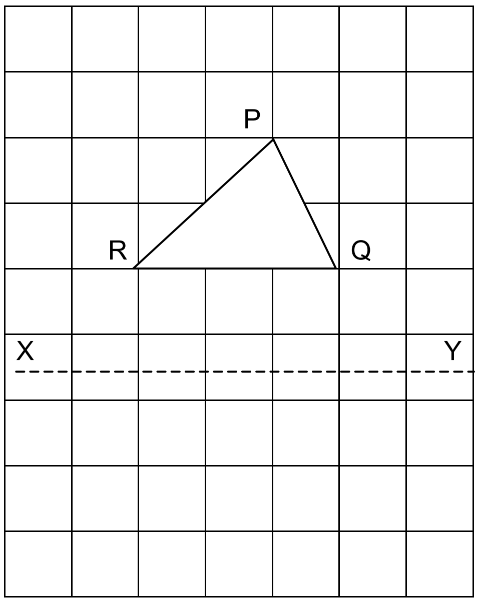
3. Reflect ∆PQR over the line PQ:



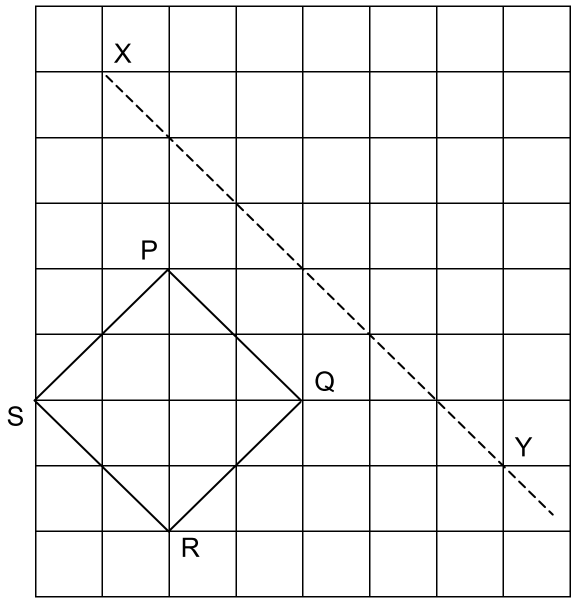
4. Reflect figure PQRS over the line SR:



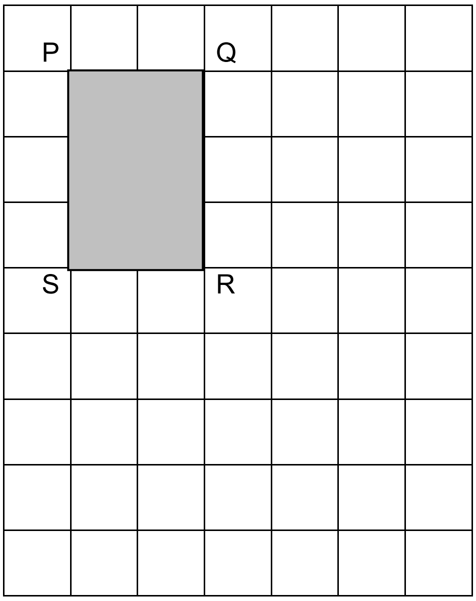
5. Reflect ∆PQR over the line XY:



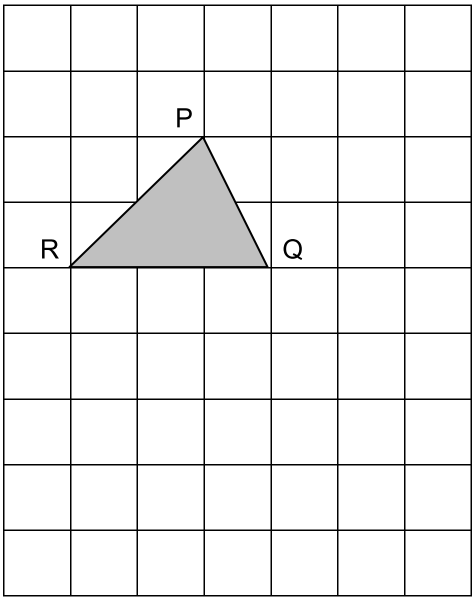
6. Reflect figure PQRS over the line XY:



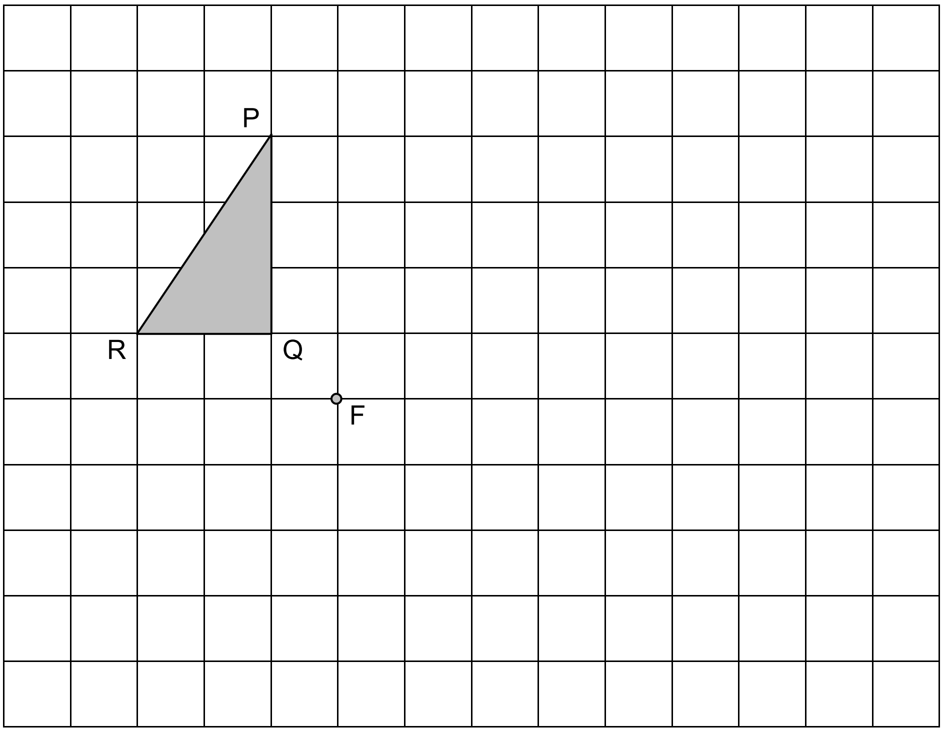
7. Rotate figure PQRS 180° about point R:



8. Rotate ΔPQR 90° (clockwise) about point Q:



9. Rotate ΔPQR 180° about point F



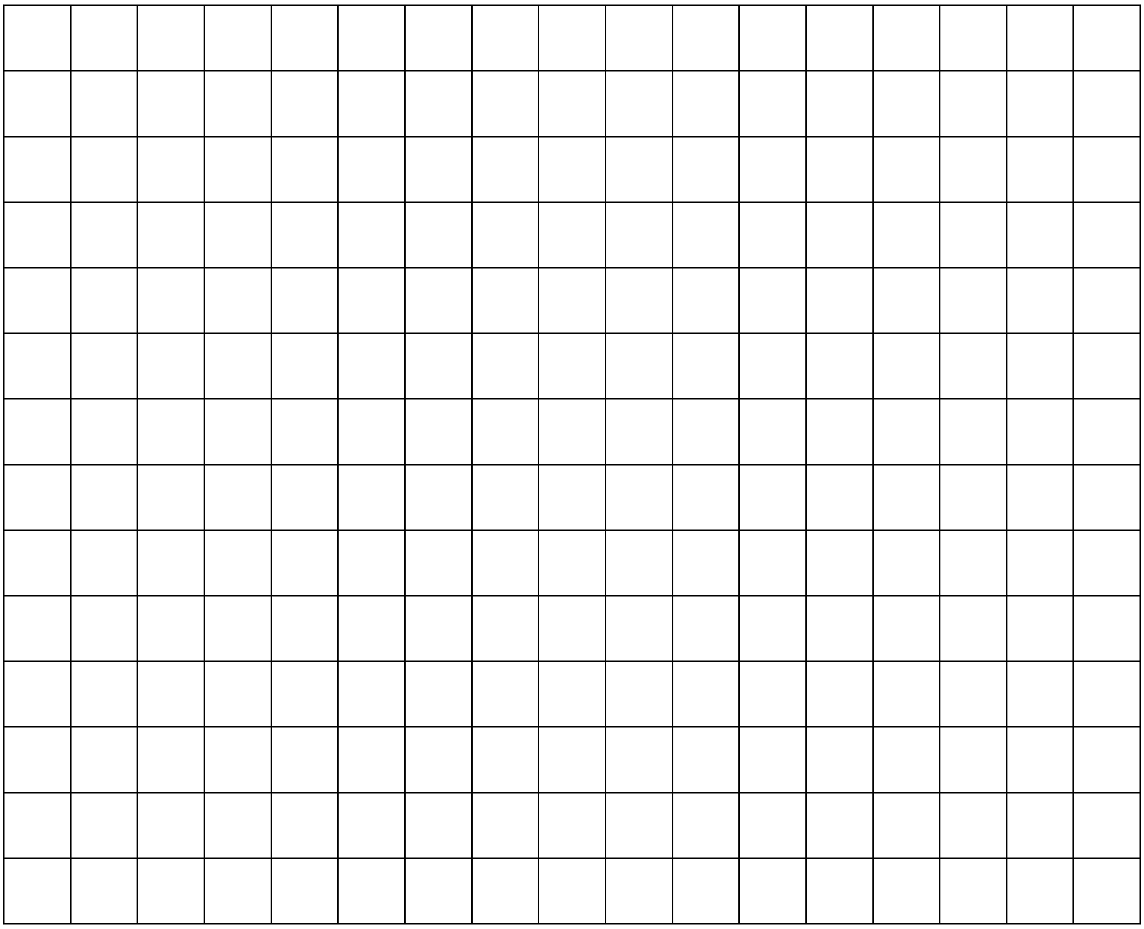
Teacher Notes: More Isometric Transformations

This activity requires that learners use what they learned in the activity "Isometric Transformations" to perform a variety of transformations. Learners should be permitted to physically manipulate the triangles in order to check their answers. Some learners will need to go back to using the physical manipulation. The teacher can provide learners with additional practice if necessary – geometric figures can be drawn on the square paper provided.

In question 9 learners encounter a centre of rotation which is not on the figure for the first time. This can be done by making a copy of the point and the figure on plastic / a transparency and then rotating this as required. Additional practice should be provided where necessary. The extension activity explores the properties of such a rotation in more detail.

Work With Your Partner 1 (Consolidation)

Draw a geometric figure on this grid. Label the figure.



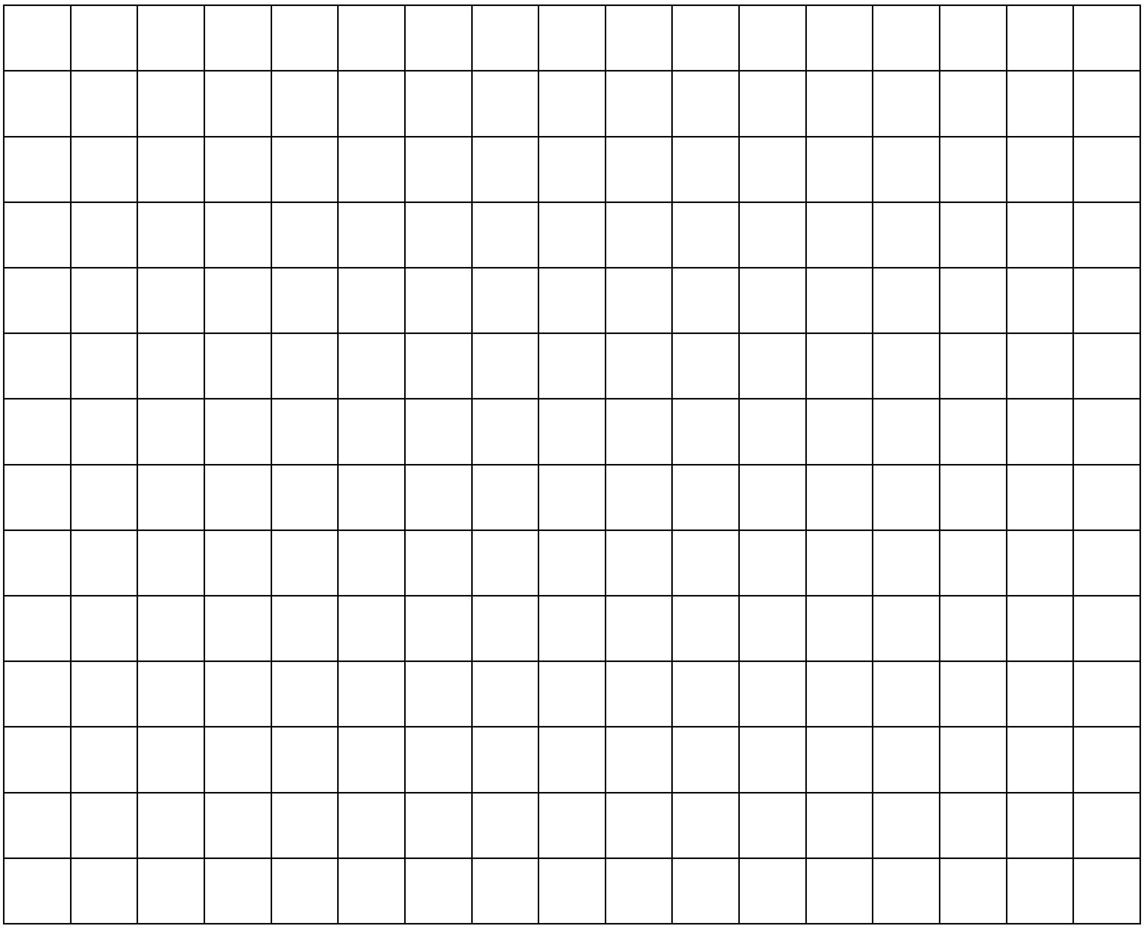
Write a sequence of **three** transformations, for example, a translation, followed by a reflection and then a rotation.

Ask your partner to perform these transformations on the figure your have drawn.

Check that she is correct.

Work With Your Partner 2 (Consolidation)

Draw a geometric figure on this grid. Label the figure.



Now use one transformation to move the figure. Draw the image of the figure and label it correctly.

Ask your partner to identify the transformation you have used. Has she described it accurately?

Teacher Notes: Work With Your Partner 1 and 2 (Consolidation)

These activities provide learners with additional practice in performing and accurately describing isometric transformations. They should be encouraged to perform the transformations mentally, using the rules they formulated in the activity "Isometric Transformations".

In the first activity the number of transformations in the sequence can be increased or decreased.

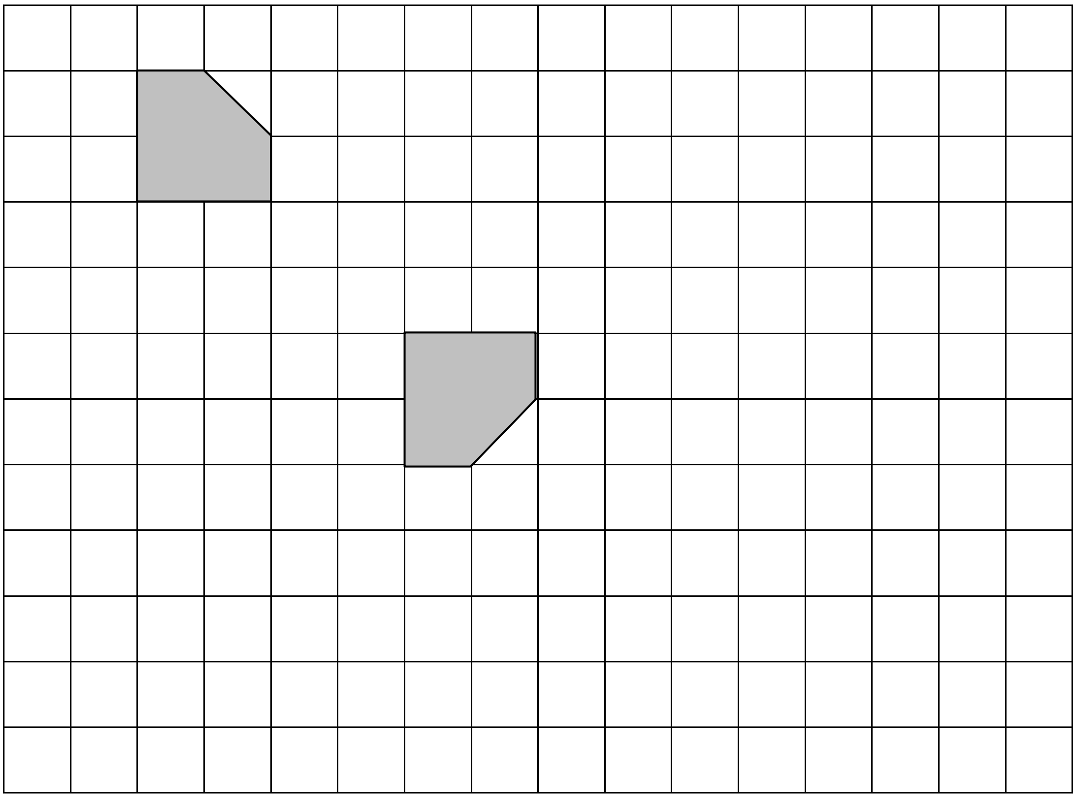
In the second activity can be extended by requiring that learners identify the composition of transformations, for example, a translation followed by a reflection.

Glide Reflections

A **glide reflection** is a combination of a translation and a reflection.

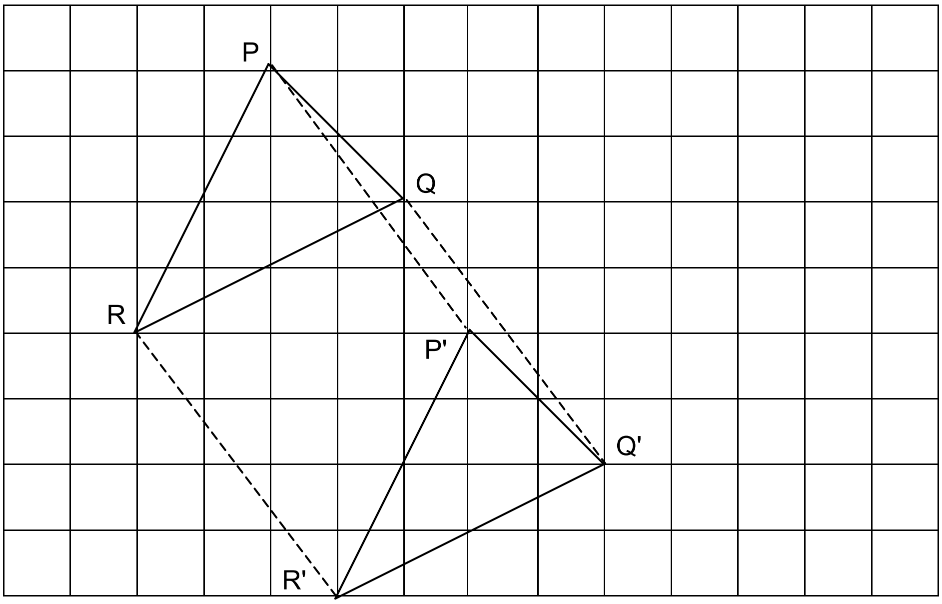
Pattern K in the activity *Patterns* is an example of a glide reflection.

The transformation below is a glide reflection. Continue the pattern with two more glide reflections. Describe the transformation.



Looking at Translations

Consider the following transformation: *Translate ∆PQR 3 units to the right and 4 units down*.

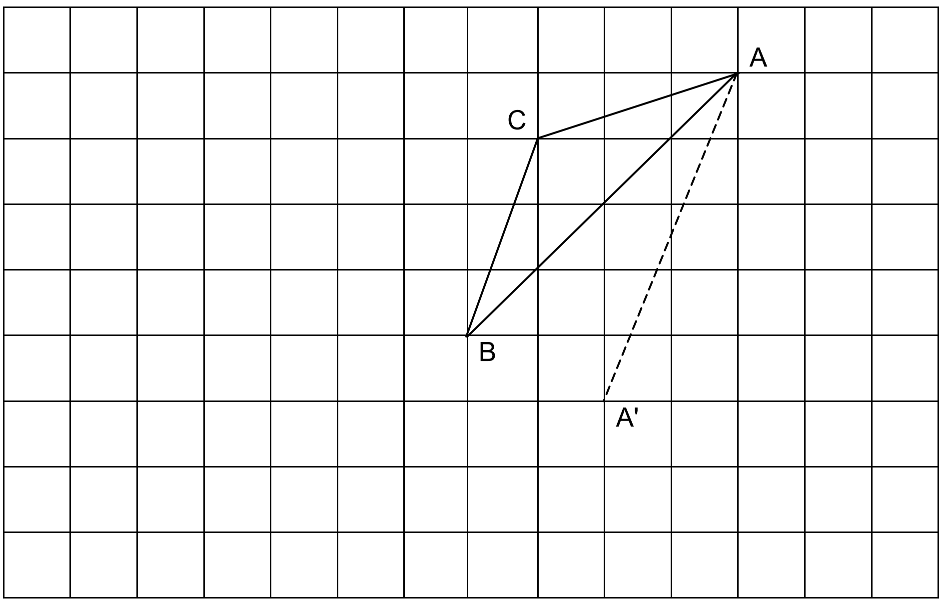


Note that:

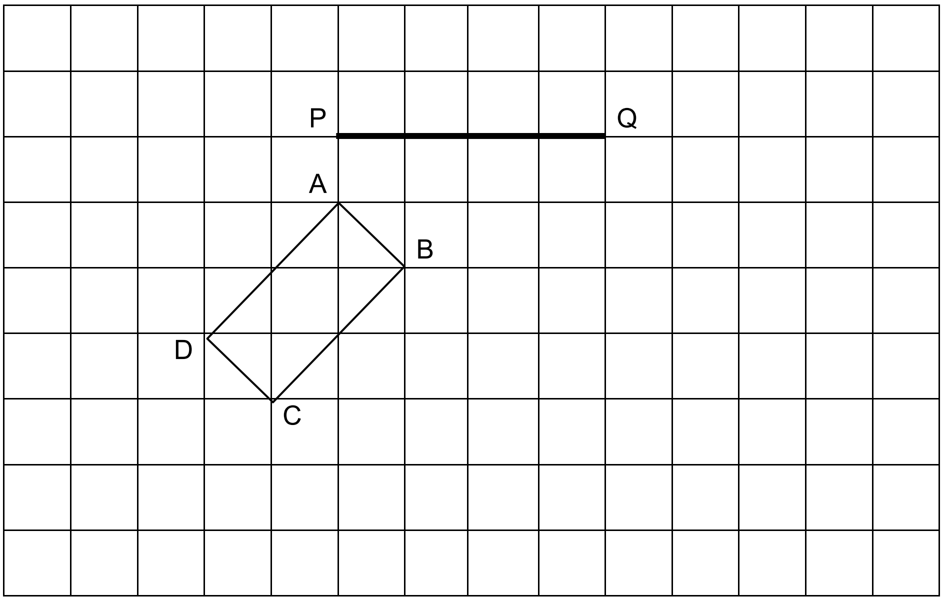
point P has been translated along the line PP',
Q has been translated along the line QQ', and
R has been translated along the line RR'.

So, rather than specifying the distance we must translate a figure to the left or right, we can specify **the line** along which we must translate it.

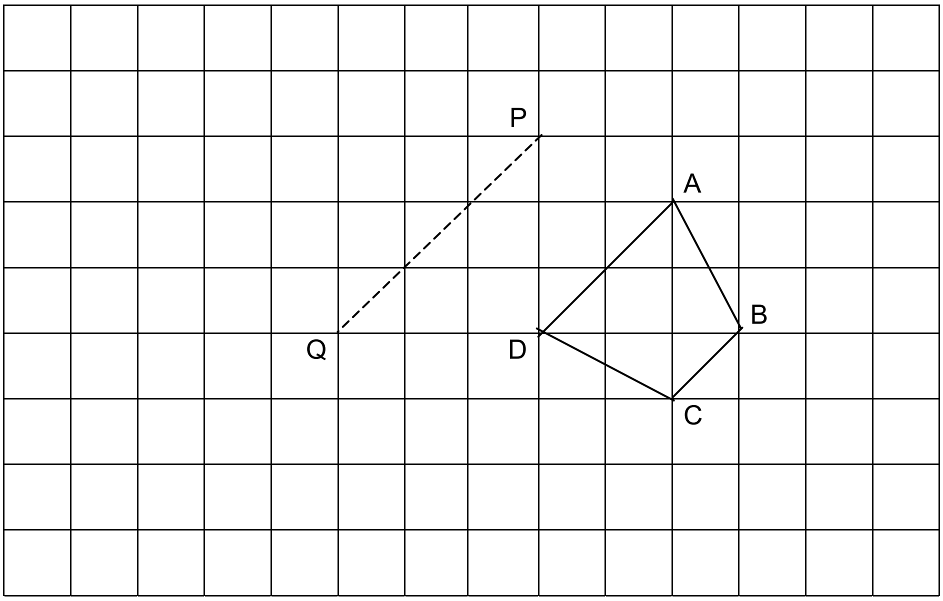
1. Translate ∆ABC along the line AA'. Explain how you translated the figure.



2. Translate rectangle ABCD along the line PQ. Explain how you translated the figure.



3. Translate trapezium ABCD along the line PQ



Teacher Notes: Looking at Translations

This activity introduces another way of describing a translation, that is, by describing the vector along which the figure must be translated (this is the same as translating the figure vertically and then horizontally). Learners should note that the line joining a point and its image under the translation is parallel to the translation vector. So each point is translated along a line parallel and congruent to the translation vector. This approach to describing a translation is used in the software Sketchpad.
Co-ordinates can also be used to explore this further.

Combinations of Transformations

Use square paper and figures of your choice to investigate each of the following. In each case explain your answer.

1. Can a combination of two **translations** be replaced with one transformation? And two **reflections**? And two **rotations**?

2. Can a combination of a translation and a reflection always be replaced with one transformation? And a rotation and a reflection? And a reflection and a rotation?

3. Is it always possible to transform a figure to coincide with a second congruent figure by performing one of the following?
   - a single translation
   - a single reflection
   - a single reflection followed by a single translation
   - a single reflection followed by a single rotation.

4. Is it possible to transform a figure to coincide with a second congruent figure by a sequence of reflections only? Explain.

5. Is the combination of two transformations always commutative?

6. Which transformations are their own inverses?

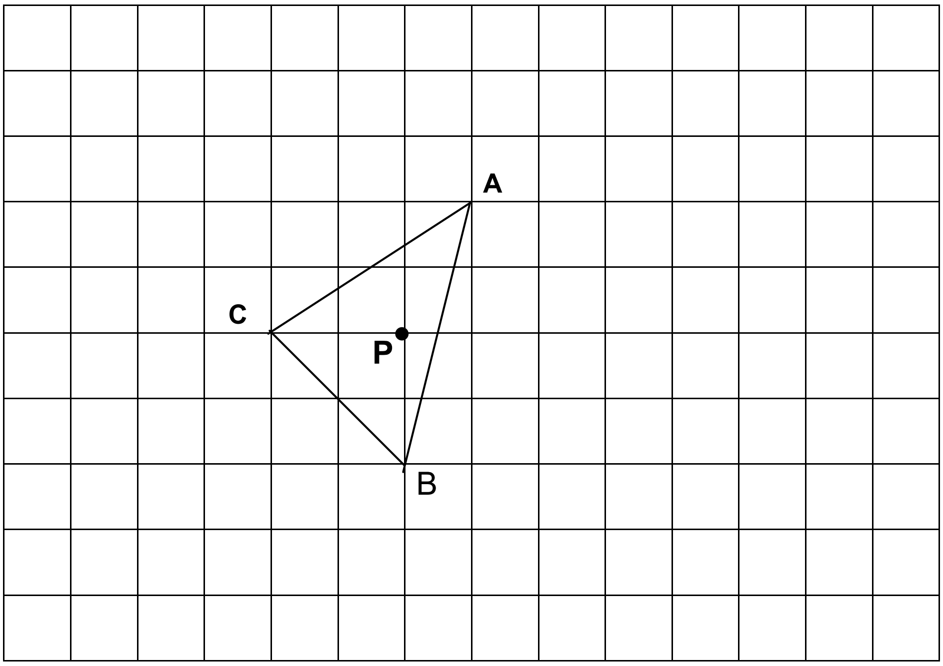
7. Discuss the notion of an identity transformation.

Teacher Notes: Combinations of Transformations

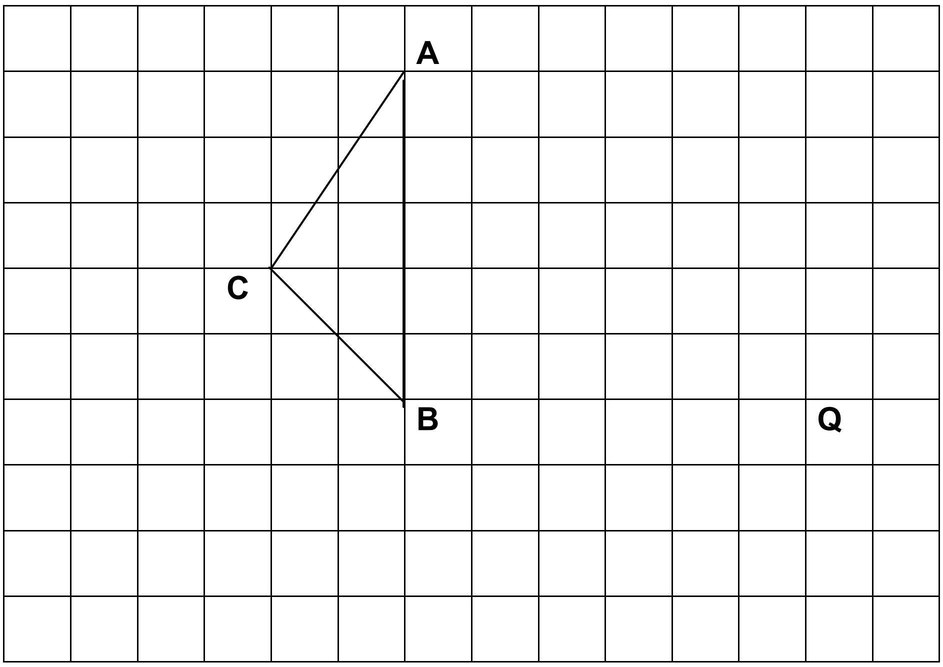
In this activity learners should be encouraged to use the properties of the transformations to explain their answers. The use of co-ordinates can be of assistance here.

Enlargements

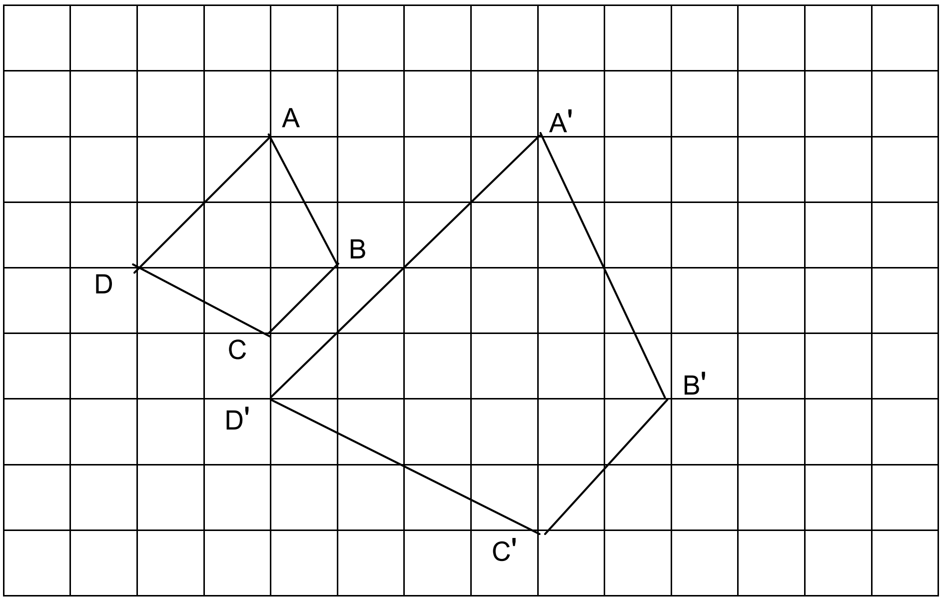
1. Enlarge ΔABC by a factor of 2 using the point P as the centre of enlargement. What is the relationship between ΔABC and ΔA'B'C'?



2. Now enlarge ΔABC by a factor of $\frac{1}{2}$, but using Q as the point of enlargement:



3. Identify and describe the transformation used to map ABCD onto A'B'C'D':



Teacher Notes: Enlargements

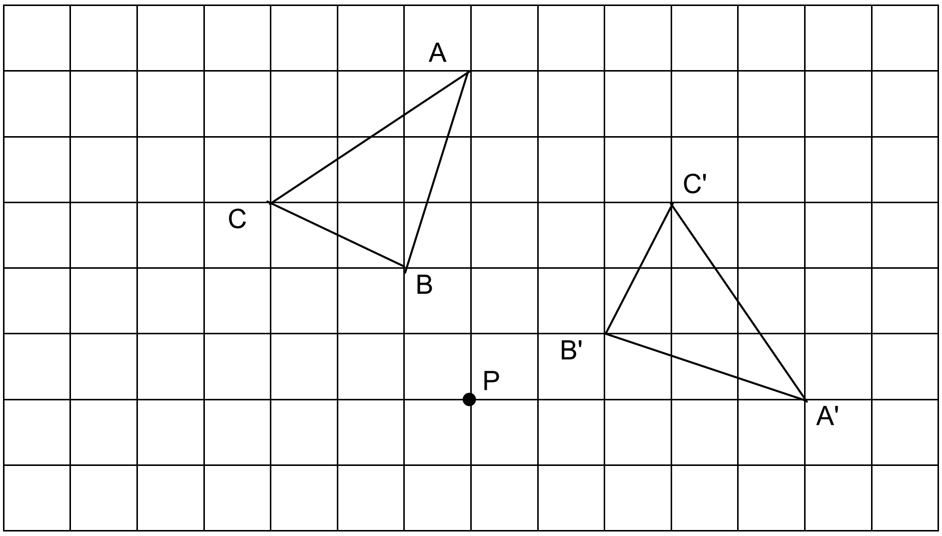
This activity provides learners with an opportunity to revisit another type of transformation, namely the enlargement, which was explored in the Malati Similarity Module.

Finding the Centre of Rotation

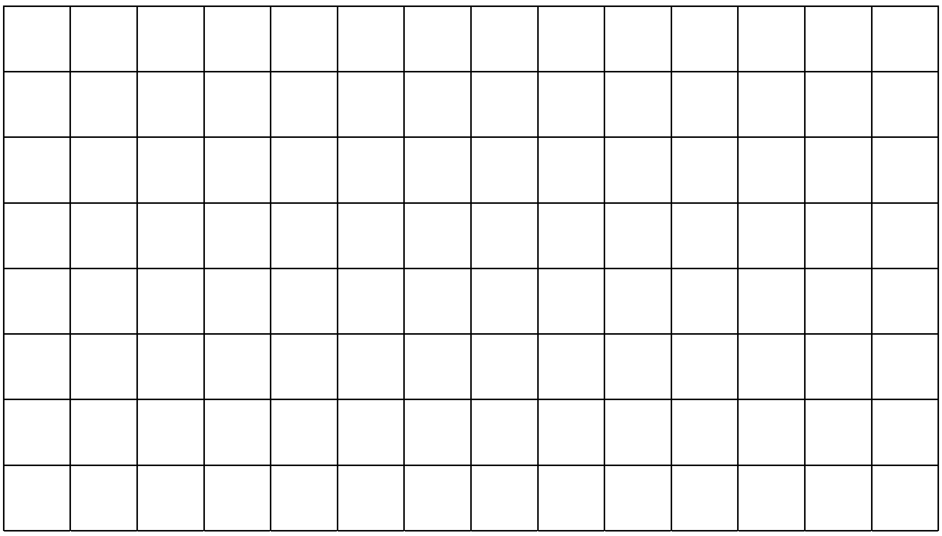
In the activity *Isometric Transformations* we identified some translations, rotations and reflection. In the case of the rotation we only used a centre of rotation that was on a vertex of the figure. But what if the centre of rotation is **not** on the figure?

We are going to look at an example of rotations to help us:

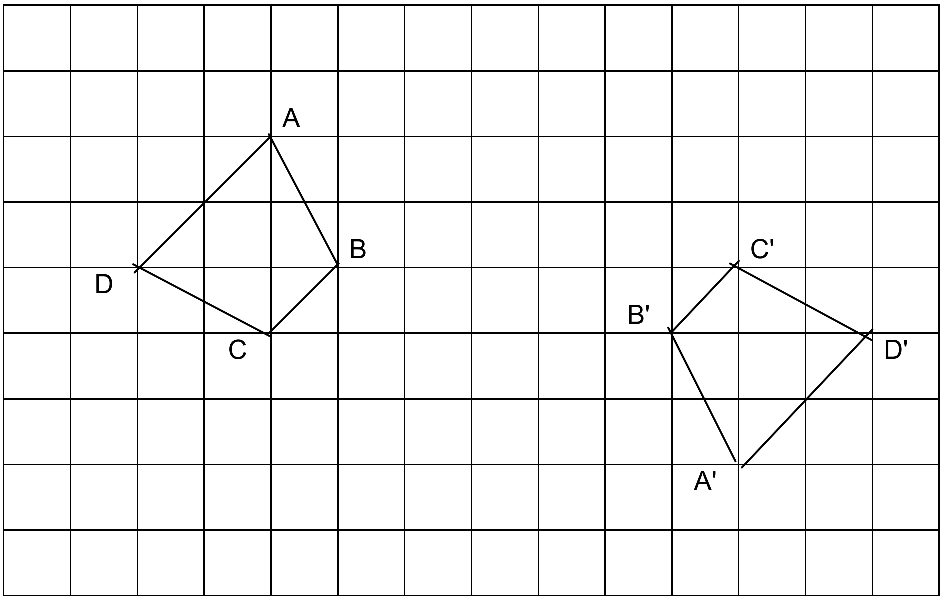
In this example ΔABC has been rotated 90° about the point P. look carefully at the relationship between each point, its image and the point P.



Now investigate your own example using a different angle of rotation:

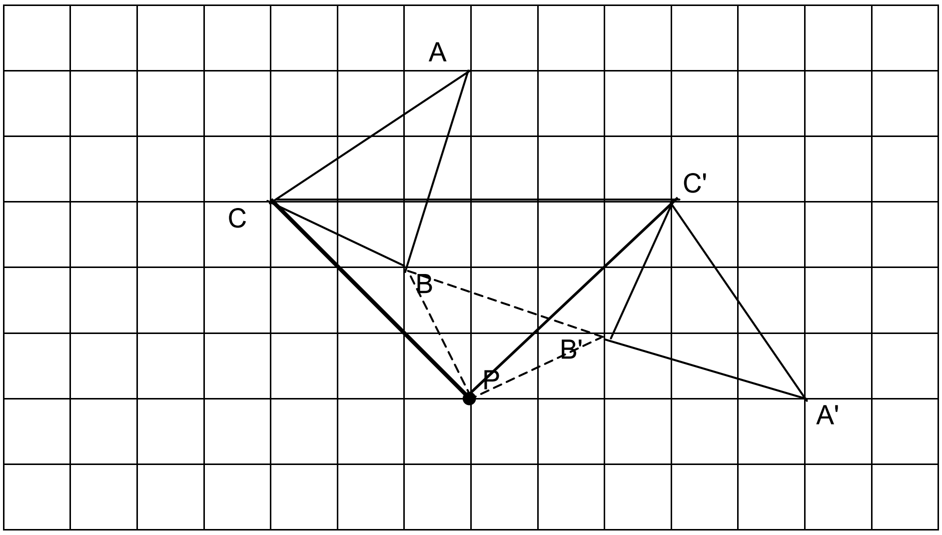


Use your observations in the above two examples to find the point of rotation in this transformation:



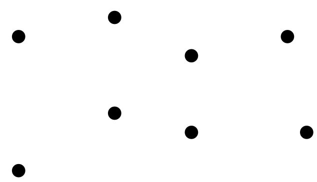
Teacher Notes: Finding the Centre of Rotation

Learners can join each vertex with its image and the centre of rotation. In each case an isosceles triangle is formed. ∆BPB' and ∆CPC' are shown below. The angle at the vertex P of each triangle gives the angle of rotation. The point P is also the vertex of three isosceles triangles. Thus the perpendicular bisectors of the lines AA', BB' and CC' all meet at point P. So the point of rotation can be found be finding the intersection of the perpendicular bisectors of each of these sides.

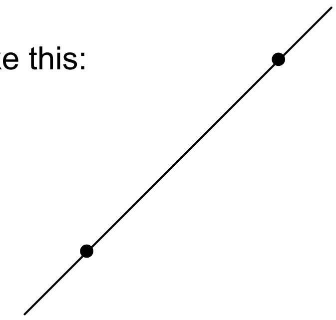


Just Points and Lines

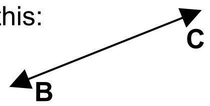
Consider points like this:



I can join two points to form a line segment like this:

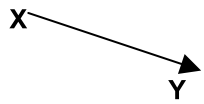


In mathematics we represent a **line** like this:

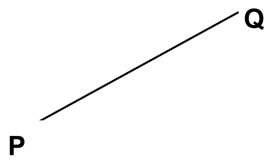


The arrows show that the line goes on forever in both directions. We use capital letters to name a line – this line is called line BC or line CB.

A line with one endpoint is called a **ray**.
This is ray XY:



A part of a line with two endpoints is called a **line segment**.
This is line segment PQ:



1. How many geometrical figures can you make with **three** points? Name and use letters to label each figure.

2. How many geometrical figures can you make with **four** points? Name and use letters to label each figure.

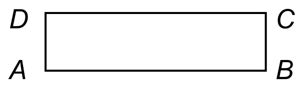
3. How many geometrical figures can you make with **five** points? Name and use letters to label each figure.

4. Now investigate how many different figures you can make with **two** lines. Label each figure and, where possible, name the figure.

5. What if you have **three** lines? And four lines? And five lines?

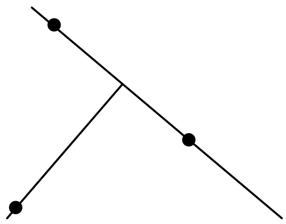
Teacher Notes: Just Points and Lines

This activity requires that learners explore and create a variety of geometrical figures: lines, general polygons, open figures, triangles, quadrilaterals etc. They should be encouraged to create as many different figures as possible with the given number of points or lines, increasing this number if possible. This will require a systematic approach. The requirement that learners name and label their figures provides the teacher with an opportunity to reinforce terminology and conventions used for the naming of geometric figures. For example, we name a rectangle such as this ABCD and not ACBD:

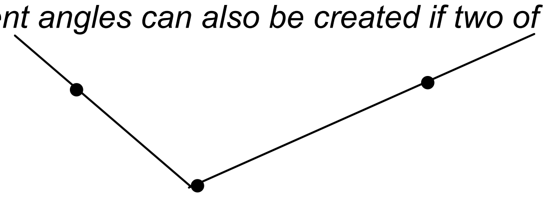


The following figures could be created in question 1:

- *A straight line (this can be used to discuss collinearity – three or more points are collinear if they can be joined by a straight line.*
- *Perpendicular lines:*

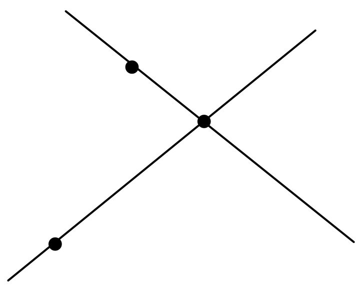


- *Different kinds of triangles – learners should consider how the points need to be positioned to create different triangles eg equilateral, obtuse-angled, isosceles triangles*
- *Different angles can also be created if two of the points are not joined:*

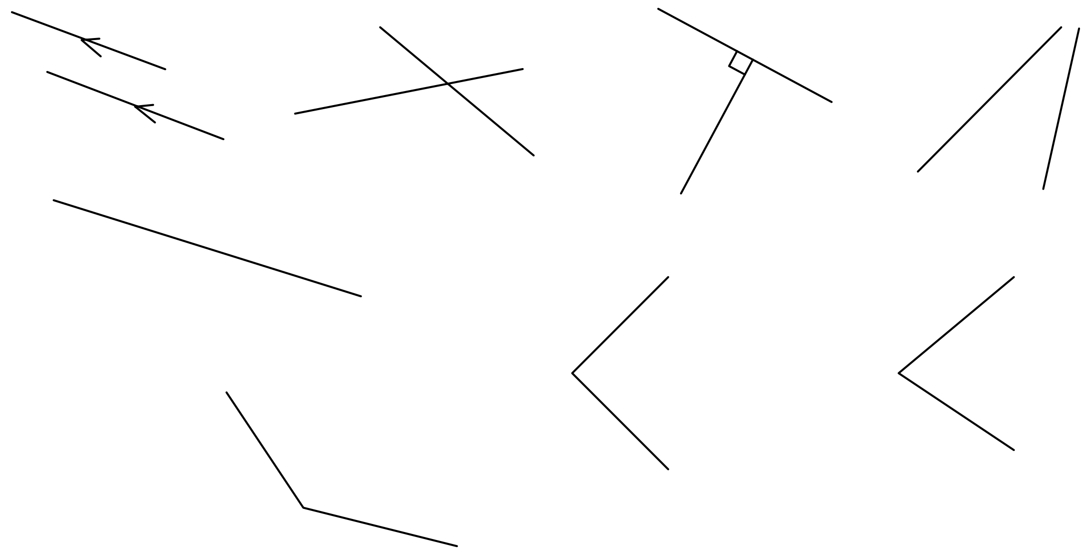


Again learners should consider how many different kinds of angles can be formed.

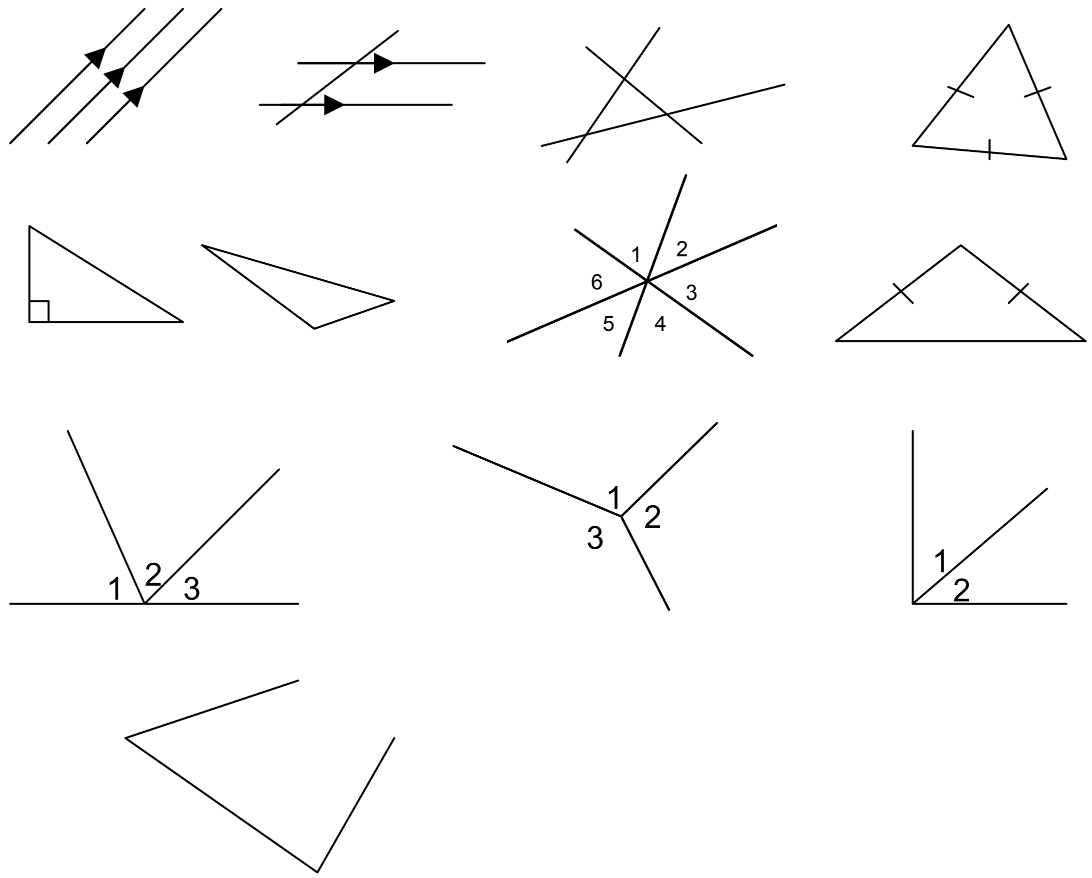
- *Intersecting lines can also be formed:*



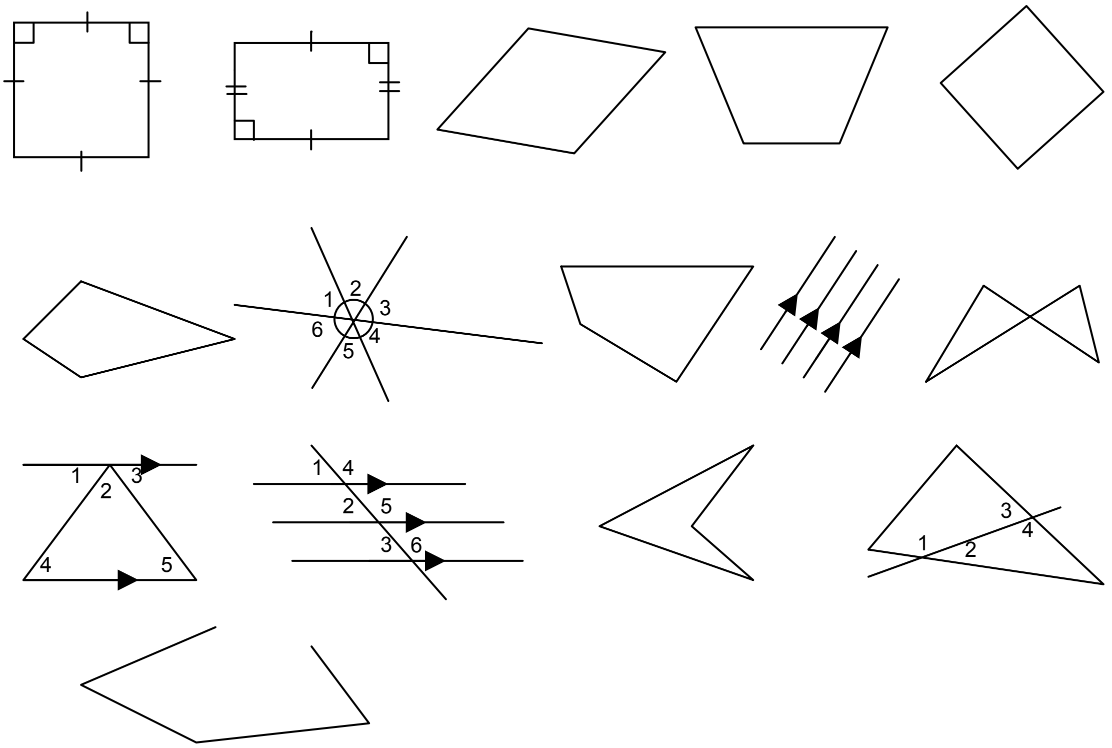
In question 4 the two lines could be placed as follows:



Here are some possibilities for arranging three lines:



And some examples of figures made with four lines:



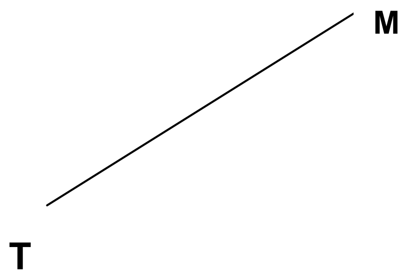
This activity can be used to explore the following geometrical concepts and figures:

- *collinearity*
- *parallel and perpendicular lines*
- *intersecting lines*
- *angles*
- *triangles*
- *quadrilaterals*
- *polygons in general (regular / not regular)*
- *open figures.*

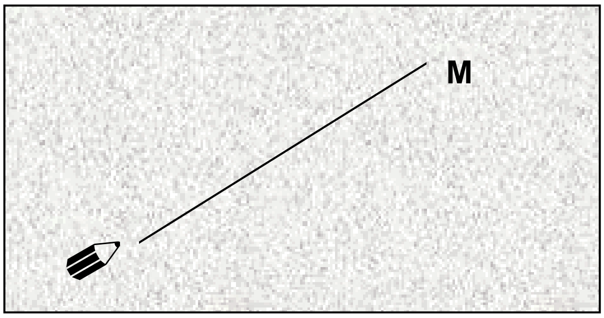
It is important that the teacher introduce / reinforce the required terminology.

Creating New Geometric Figures

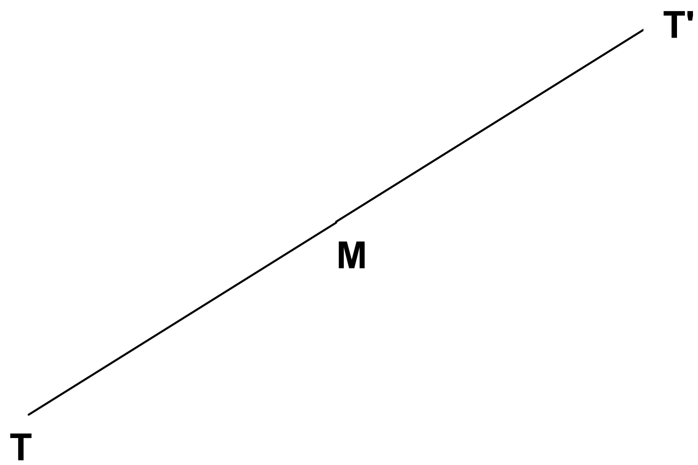
1. Bingo draws a line segment TM:



Bingo places a piece of transparency over the line segment, and traces a copy of TM:

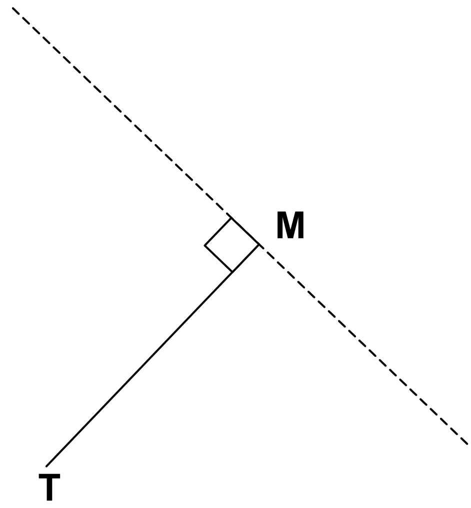


Bingo rotates the transparency 180° about the point M. The original line segment TM **and** the image of TM under the rotation form a new line segment, called TT':



Write down what you can about the new line segment TT'

2. Therine says she can create a line segment congruent to line segment TT' using **reflection**. She says she will copy line segment MT onto the transparency and then reflect the copy in the dotted line. She says the original line **and** its image under the reflection will form a line congruent to TT'.

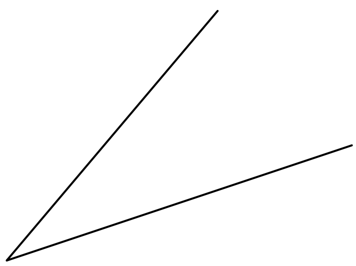


Do you agree with Therine? Discuss.

3. Godfrey says he can create a line segment congruent to TT' using a different method. He says that he will **translate** the copy so that the original line segment **and** its image will form a line segment congruent to TT'.

Do you agree with Godfrey? Discuss.

4. Hayley says that she can transform the copy of TM so that the image **and** the original line segment TM make geometric figures **other than straight lines**. She creates this angle:



Explain how Tracey transformed the copy of TM to make this figure.

What other figures can you make in this way? In each case describe the transformations you used write **and** down the properties of the new figure.

Creating New Geometric Figures: Teacher Notes
Pieces of overhead transparency or plastic for covering books can be used effectively in this activity.

Pupils should be encouraged to visualise the movement wherever possible. Some pupils might, however, need to use transparency to follow the explanation and/or create the new figures. Others could use the transparency to check only.

When a line is transformed it is actually moved. For example, when a line is translated, it is moved into a new position. In this activity, however, pupils have to work with new geometric figures which are created by using the original line ***and*** *the image.*

Class Discussion:
Pupils should be encouraged to reflect on the properties of the new figure, for example, the length of the new line segment is twice as long as that of line segment TM. Point M is the midpoint of line TT'. This is because the transformations are isometric, that is, length is preserved. This type of transformation is the basis of Euclidean geometry.

Pupils should also be asked why Godfrey and Therine's line segments are congruent to the first line segment TT' made by Bingo.

The following geometric figures can be created by transforming the line TM:
- *acute, obtuse, right-angled, reflex and straight angles*
- *parallel lines*
- *intersecting lines*

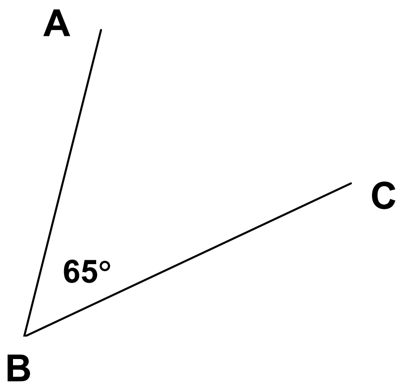
Learners should name and label these figures.

*The teacher should use the figures created by pupils to reflect on the fact that the same figure can be created by different sets of transformations (These are called "****equivalent transformations****"). Ask pupils to find equivalent transformations for other figures they have created, for example, for a right angle. How many different transformations can be found to make the same figure (congruent figures)?*

Pupils should be encouraged to use the figures that they have already created, for example, the acute angle, to create new figures using transformations (see Tracey's Transformations below).

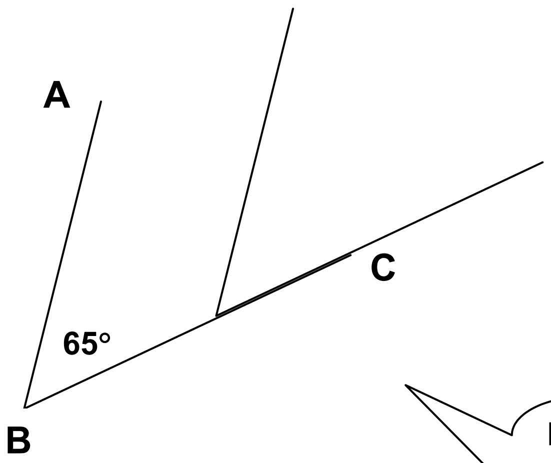
Tracey's Transformations

Tracey has drawn ∠ABC = 65°:



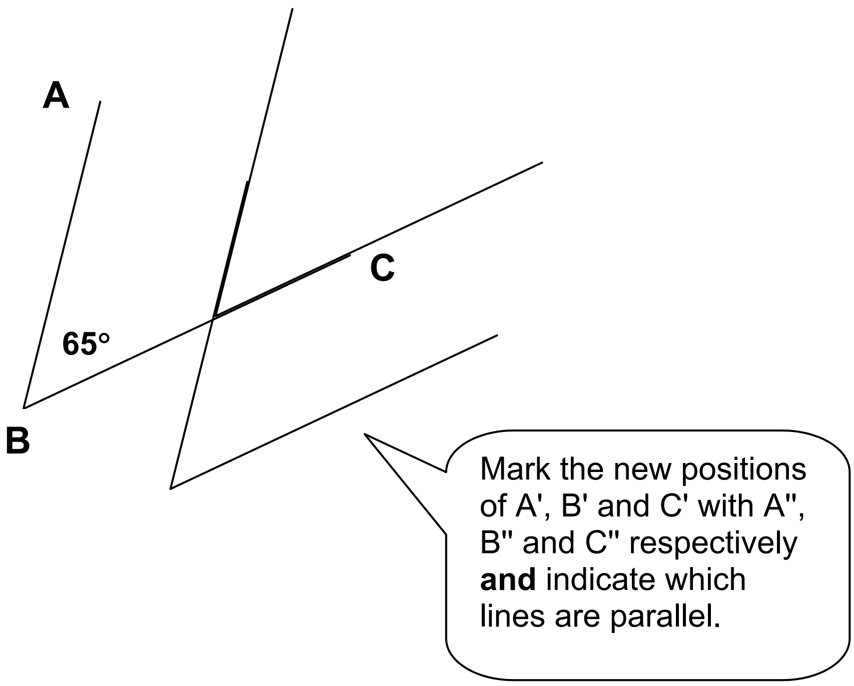
She decides to create a new geometric figure by using transformations.

She **translates ∠ABC to the right along the line BC**. The original ∠ABC and its image under the translation form a new figure that looks like this:



Mark the new positions of A, B and C with A', B' and C' respectively **and** indicate which lines are parallel.

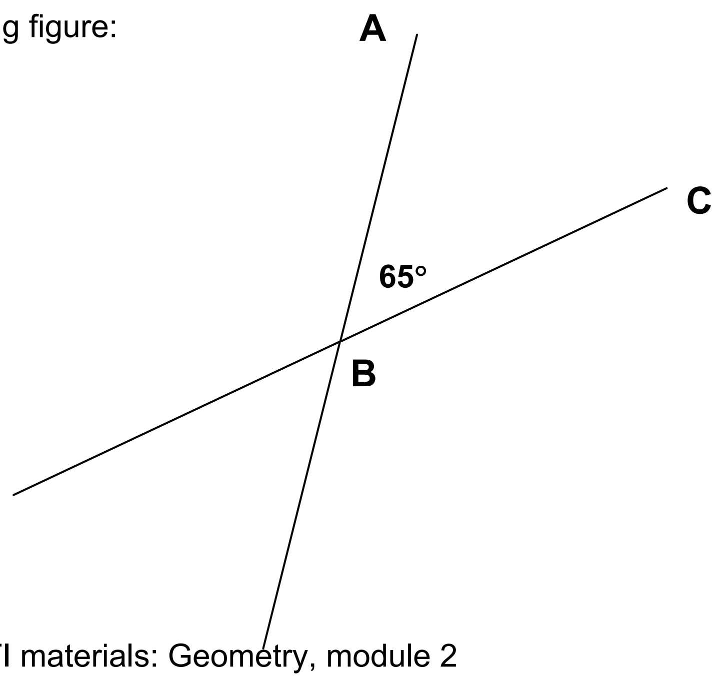
Tracey then translates the image of ∠ABC along the new AB. The three angles now form this figure:



1. Fill in the size of each angle in the new figure. In each case explain how you got your answer.

2. Try to find a set of transformations that is equivalent to Tracey's.

Tracey says she can create a different geometric figure using ∠ABC, but using a **rotation** this time. She says the original and the image under the rotation will form the following figure:



3. What rotation has Tracey used?

4. Label the appropriate points on the diagram and fill in the sizes of the angles.

5. What other geometric figures could Tracey make using transformations? Remember that the figure must be made up of $\angle$ABC and its image under a transformation. In each case describe the transformation you have used **and** fill in the sizes of the angles.

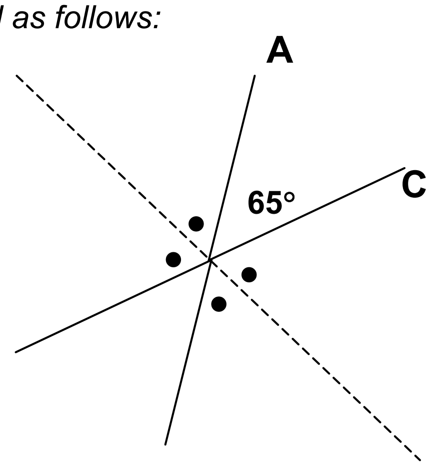
Teacher Notes: Tracey's Transformations

Some learners will need to follow the descriptions of Tracey's transformations using transparencies, whereas others will be able to visualise the movements. The same applies to actually performing the transformations to make new figures in question 5.

As in the activity "Making New Geometric Figures", the figures formed consist of the original angle and its image under a transformation.

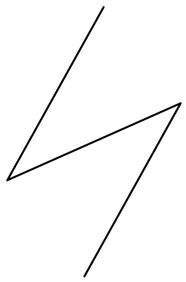
*The teacher should check that the learners are describing the Tracey's transformations correctly and are providing **all** the necessary information:*

- *In question 2 an equivalent transformation would be a translation along the line AB followed by a translation along the new line B'C'.*
- *In question 3 the point of rotation (point B) and the angle of rotation (180º) should be given. Learners can find the angle of rotation by noticing that the lines AA' and CC' are straight. Some learners might suggest that a reflection could also have been performed to create this figure. In such a case the line of reflection would have to be identified as follows:*

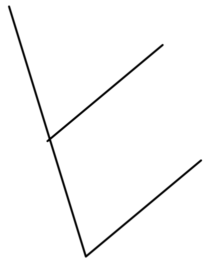


The teacher can introduce the term vertically opposite angles at this point.

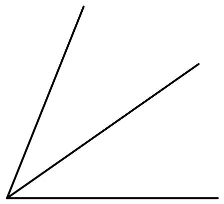
Below are some figures that could be created in question 5:



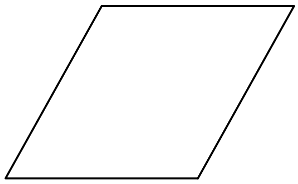
Rotation 180° about the midpoint of BC



Translation along AB



Reflection in BC



Reflection in the line joining AC

It is important that learners label and name the new figures correctly.

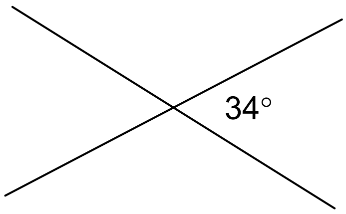
Learners should also compare the transformations they have used: Which are equivalent?

ANGLES!

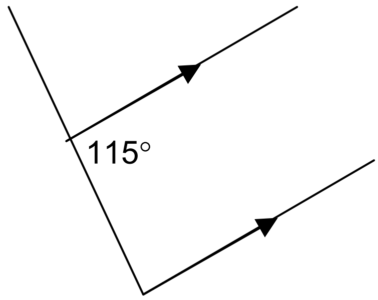
Each of the following figures was created by transforming angles. In each case the original angle and its image under the transformation has been drawn.

In each case fill in the sizes of the angles and use transformations to explain your answer.

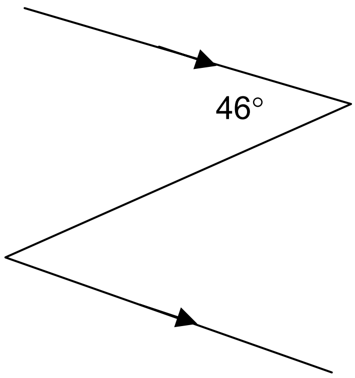
1.



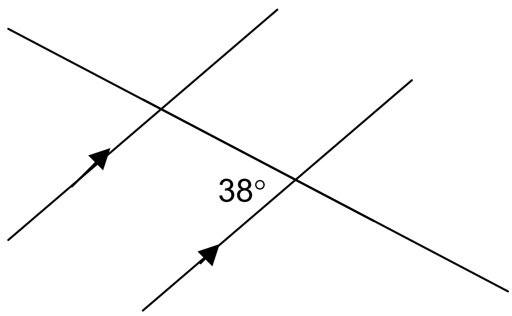
2.



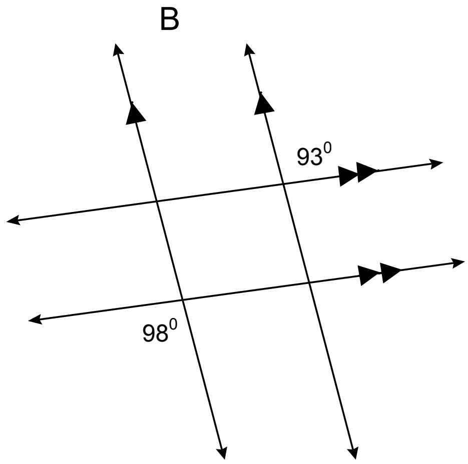
3.



4.



What is wrong with this figure?

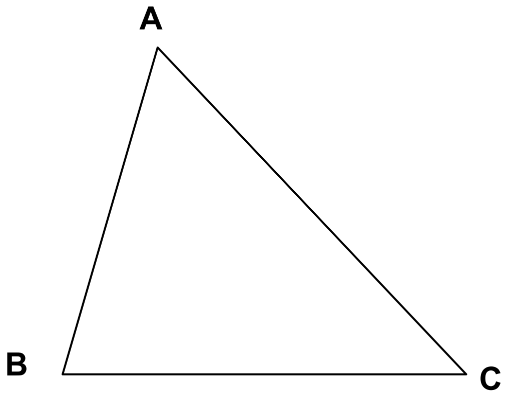


Teacher Notes: Angles!

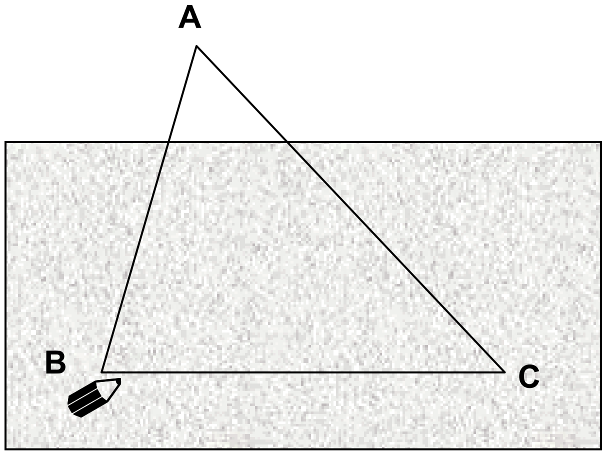
The teacher can select similar textbook examples for consolidation work.

Moving Lines 1

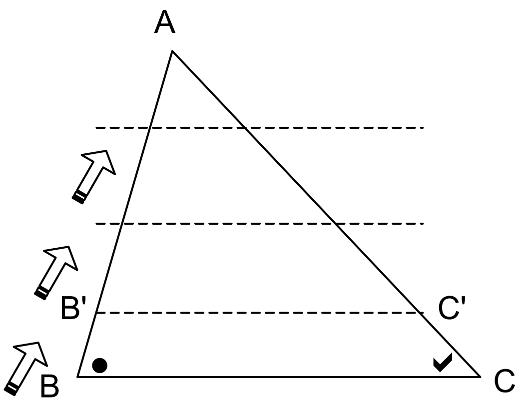
Tracey draws a triangle like this:



She places a transparency over the triangle and makes a copy of line segment BC:

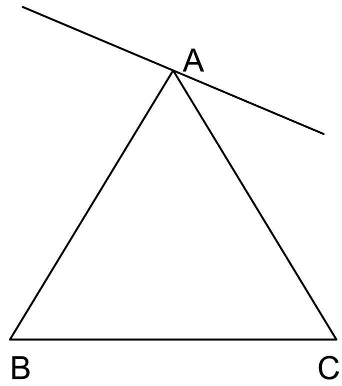


Then Tracey translates the line BC like this so that it is moving towards the vertex A:



1. What can you say about the angles formed by copy of the line and the sides of the triangle as it moves from BC to point A?

2. What if Tracey moves the copy of line segment BC like this?

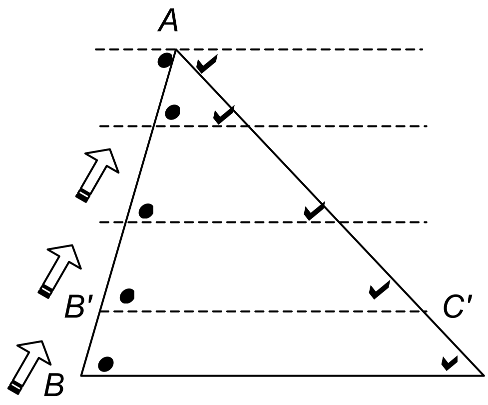


3. What if ∆ABC is an obtuse angled triangle?

4. Make a conjecture about the interior angles of a triangle.

Teacher Notes: Moving Lines 1

This activity enables learners to use transformations and their knowledge of corresponding and alternate angles to explore the sum of the interior angles of a triangle.



Learners should note the corresponding and alternate angles marked above. At the point A it can be seen that

• + ∠BAC + ✔ = 180°

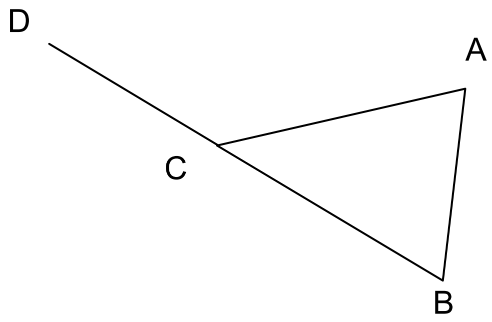
So the sum of the angles of ΔACB is 180°.

In this case we can use the corresponding and alternate angles because the property of translation ensures that the dotted lines are parallel.

*In question 2 one cannot use this argument as BC and then line moved are **not** parallel.*

Moving Lines 2

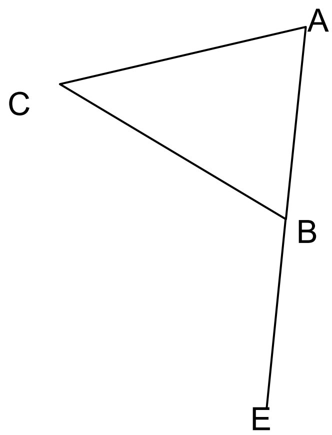
When one side of a triangle is produced, the angle formed between this line and the side of the triangle is called an **exterior angle** of the triangle:



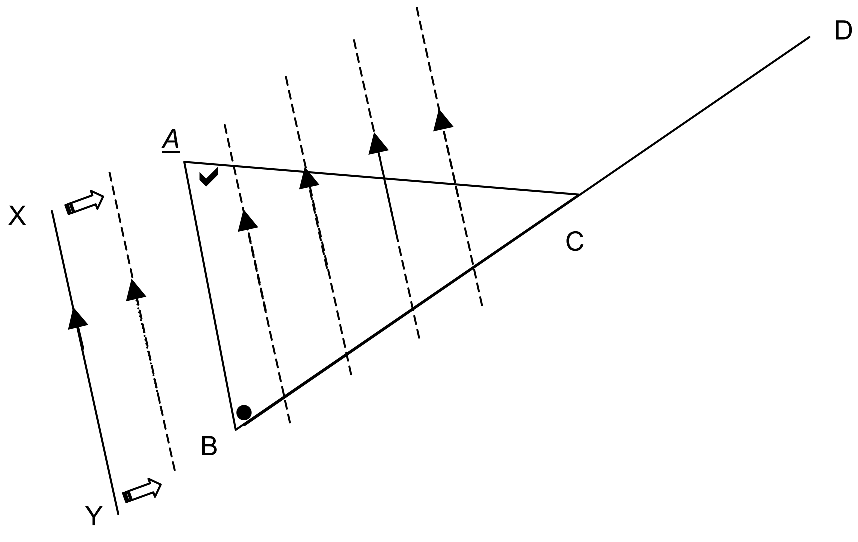
∠ACD is an exterior angle of ∆ABC.

We can also extend AB or CA to form exterior angles:

In this case ∠CBE is an exterior angle of ∆ABC



The line XY, parallel to AB, is translated and moves across the triangle like this:



Use transformations to answer these questions:

1. What can you say about the sizes of the angles in the diagram as XY is translated.

2. What happens when XY intersects BD at C?

3. What happens when XY moves beyond C, towards D?

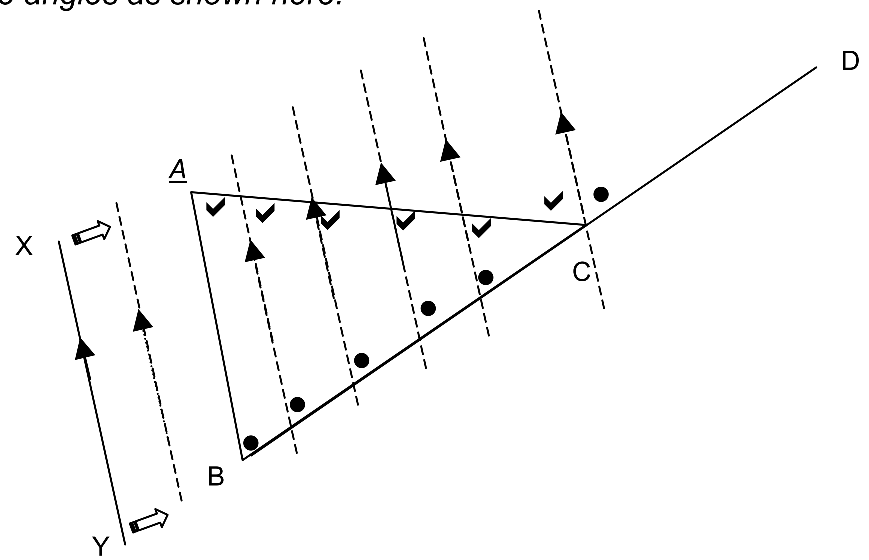
4. What if the line XY is **not** parallel to AB?

5. What if △ABC is an obtuse-angles triangle?

6. Make a conjecture about the relationship between an exterior angle and the interior angles of a triangle.

Teacher Notes: Moving Lines 2

As in the activity "Moving Lines 1" the learners should note the corresponding and alternate angles as shown here:



Where the line cuts Bd at C one can see that the sum of the exterior angles is equal to the interior opposite angles.

When XY moves beyond point C there is no particular relationship between the angles.

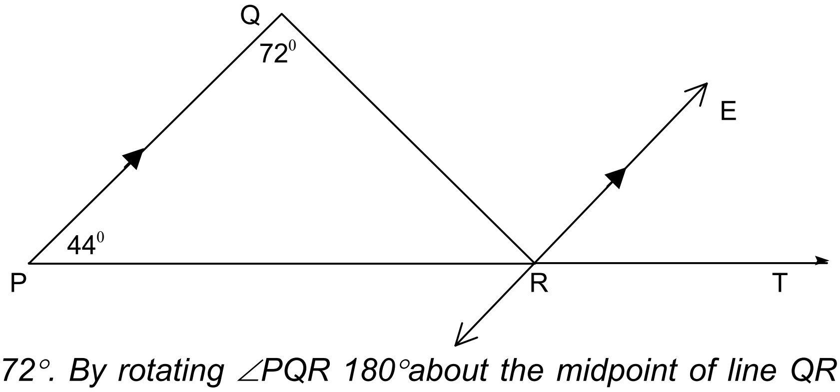
Learners should note that this argument only works if the line XY is parallel to AB.

Additional Activities:

The teacher can now provide learners with practice finding the size of different angles – these can be found in a Grade 8 or 9 textbook. Learners can use transformations as explanations.

For example:

In the figure below, PQ // RE. Find the size of ∠QRE, ∠ERT and ∠QRT. Explain your method.



∠QRE = 72°. By rotating ∠PQR 180°about the midpoint of line QR it can be shown that ∠QRE = ∠PQR = 72°.

∠ERT = 44°. By translating ∠QPR a distance of PR along the line PT it can be shown that ∠QPR = ∠ERT = 44°.

∠QRT can be found by adding ∠QRE = 72° and ∠ERT = 44° to get 116°.

Assessment Activity:

Use the figure below to describe transformations that are necessary to show that the sum of angles of any triangle PQR add up to 180°.