Malati

Mathematics learning and teaching initiative

Geometry

Module 3

Polygons

Grades 8 and 9

Teacher document

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Overview of Module 3

The activities are designed to encourage leaners' movement to the van Hiele *ordering level*. Learners are given opportunities to explore the properties of polygons and to develop definitions and informal arguments. It is important that learners have rich experiences with activities of this nature and that they are not pushed into more formal geometry too soon.

Furthermore, as indicated in the teacher notes, some activities can be done on different van Hiele levels, depending on the challenges put to learners.

For the sake of clarity we have included activities in the Module that will give teachers ideas on how the basis laid in this Module can be developed in the higher grades.

We provide recommendations for the use of the Malati polygon activities in Grades 8 and 9. Use of the activities will of course depend on the needs of the class and of individual learners.

Additional activities have been provided at the end of the Module for use by teachers where appropriate.

This Module focuses on learners who are on the van Hiele analysis level, but we have provided additional exemplar materials at the end of the Module (Sec 03c) as suggestions to the teacher on how to proceed beyond this level and to encourage ordering level thinking.

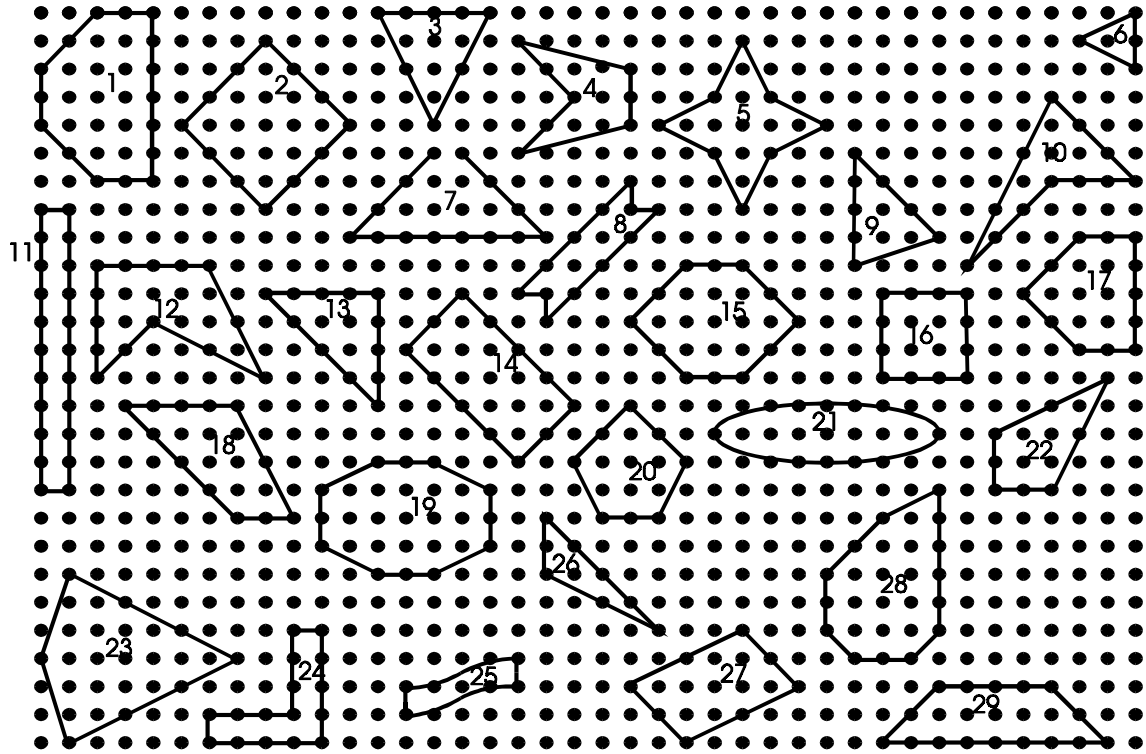
Suggested classroom use of the activities in Module 3 . . . PTO

Suggested classroom use of the activities in Module 3:

Grade 8	Grade 9
Core: Which Figures Belong Together? Grouping Figures 1 Grouping Figures 2 Mrs Sisulu's Maths Lesson Gurgles Tiling the Kitchen Matches Helping Lucas Helping Andile 1 Helping Andile 2 Find the Value Rulers Symmetry	**Core:** Grouping Figures 2 Helping Lucas Helping Andile 1 Helping Andile 2 Find the Value Symmetry Agatha's Rectangle Cut and Rearrange (and additional activities) Diagonals Guess My Quadrilateral Clues for Squares Families of Polygons Investigating Figures What is a Polygon? What is a Quadrilateral? Which are Rectangles? Making a Patchwork Quilt Family Picnic
Fortification: Creating New Polygons Quadrilaterals Exploring using Diagonals	**Fortification:** Creating New Polygons Quadrilaterals Exploring using Diagonals Drawing our own Triangles
Assessment: Notes on Polygons Notes on Quadrilaterals Reflecting Triangles Helping Tanya Helping Tema Agatha's Rectangle Which are Rectangles?	**Assessment:** Notes on Quadrilaterals Helping Tanya Helping Tema
Extension: Investigating Figures Clues for Squares Guess my Quadrilateral	**Extension:** How Many Lines of Symmetry 1? How Many Lines of Symmetry 2?
	Ordering level examples: Defining a Kite 1 Defining a Kite 2 Defining a Kite 3 Defining Rectangles New Shapes

Which Figures Belong Together?

1. Group the following shapes according to the way you think they belong together. Explain why you group them this way.

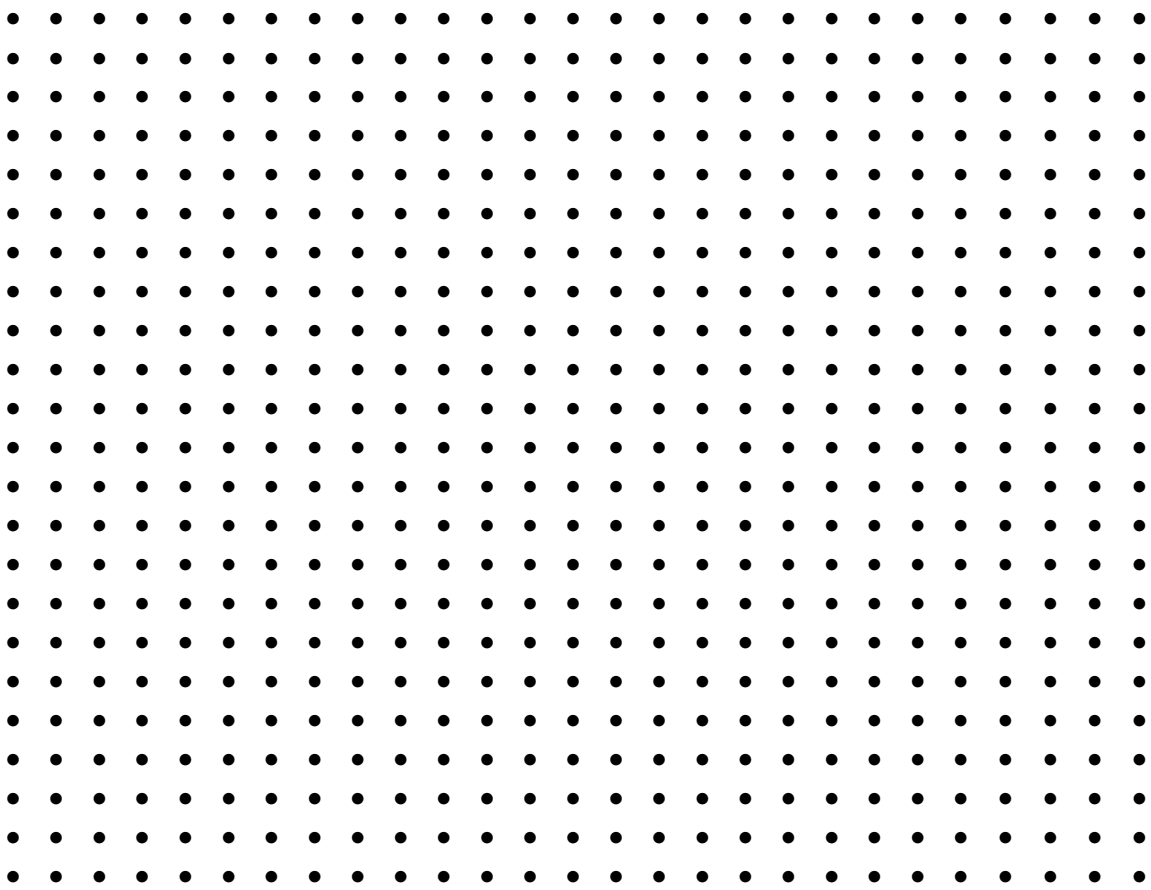


2. Now group these shapes in any different way.

3. Draw a different shape to each of your groups that fits the description of your groups on the dotty paper provided. Use a different notation to mark these shapes.

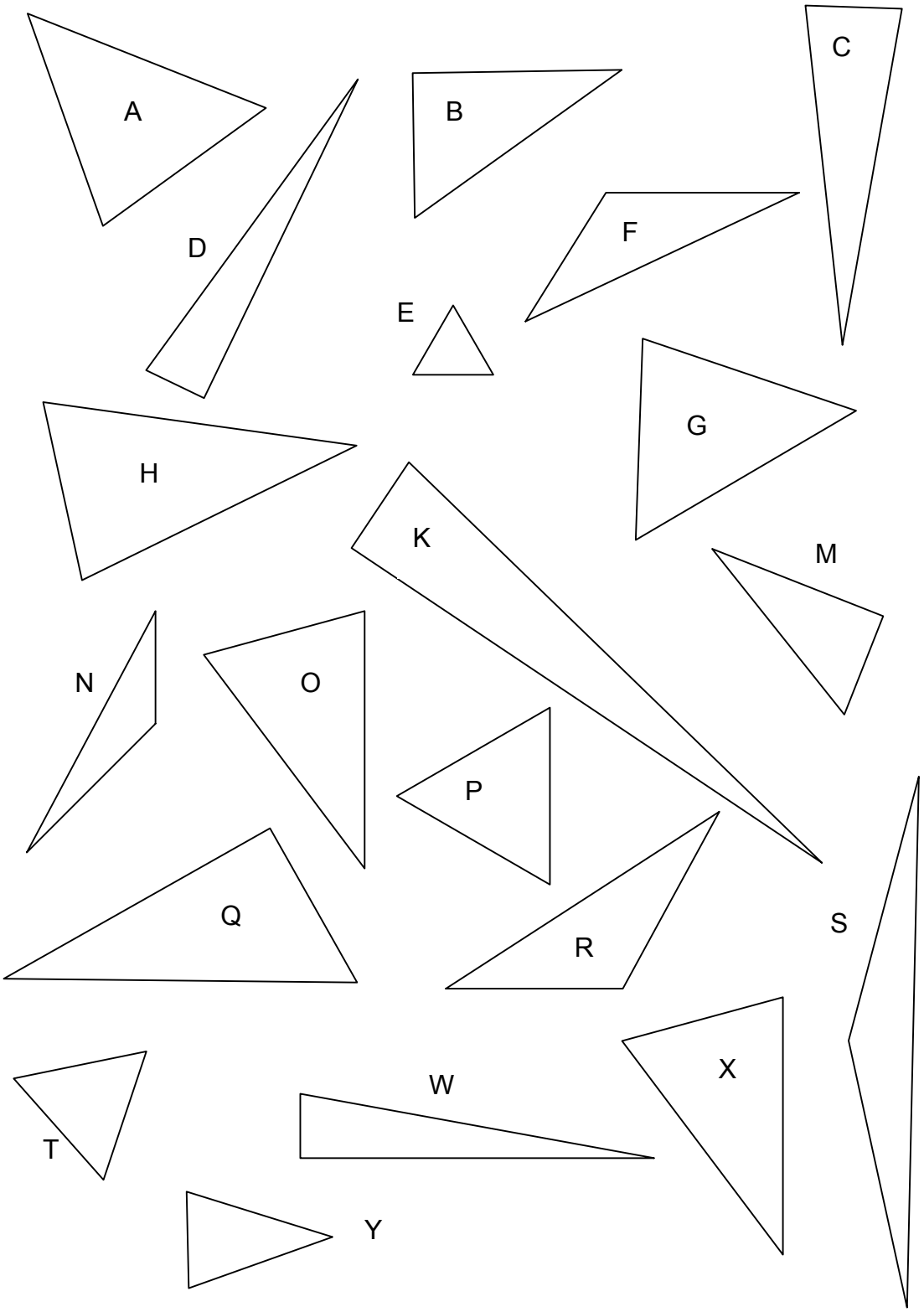
4. Sizwe classified shapes marked 4, 8 and 10 as belonging to one group. Explain why he classified them in this manner. Draw another shape from the list given above that fit the description of this group.

5. For each of your groups, draw **a different shape** that fits the description of your groups on the dotty paper provided below. Use a different notation (to describe the group where it belongs) to mark these shapes.



Grouping Figures 1

Look carefully at the following shapes and decide which shapes you think belong together. In each case give a reason for your answer.



Teacher Notes: Grouping Figures 1
Learners on the van Hiele analysis level will answer using the properties, for example, using the sides and angles. Learners on the van Hiele visual level will answer according to the shape of the triangles as a whole. Orientation might be a problem in this case.

Class Discussion:
Learners should be required to think of more than one possible grouping. Use of measuring instruments to compare shapes should be permitted where necessary.

Learners might group figures according to the angles, for example, acute-angled, obtuse-angled or right-angled triangles, and / or according to their sides (equilateral, isosceles, scalene triangles).

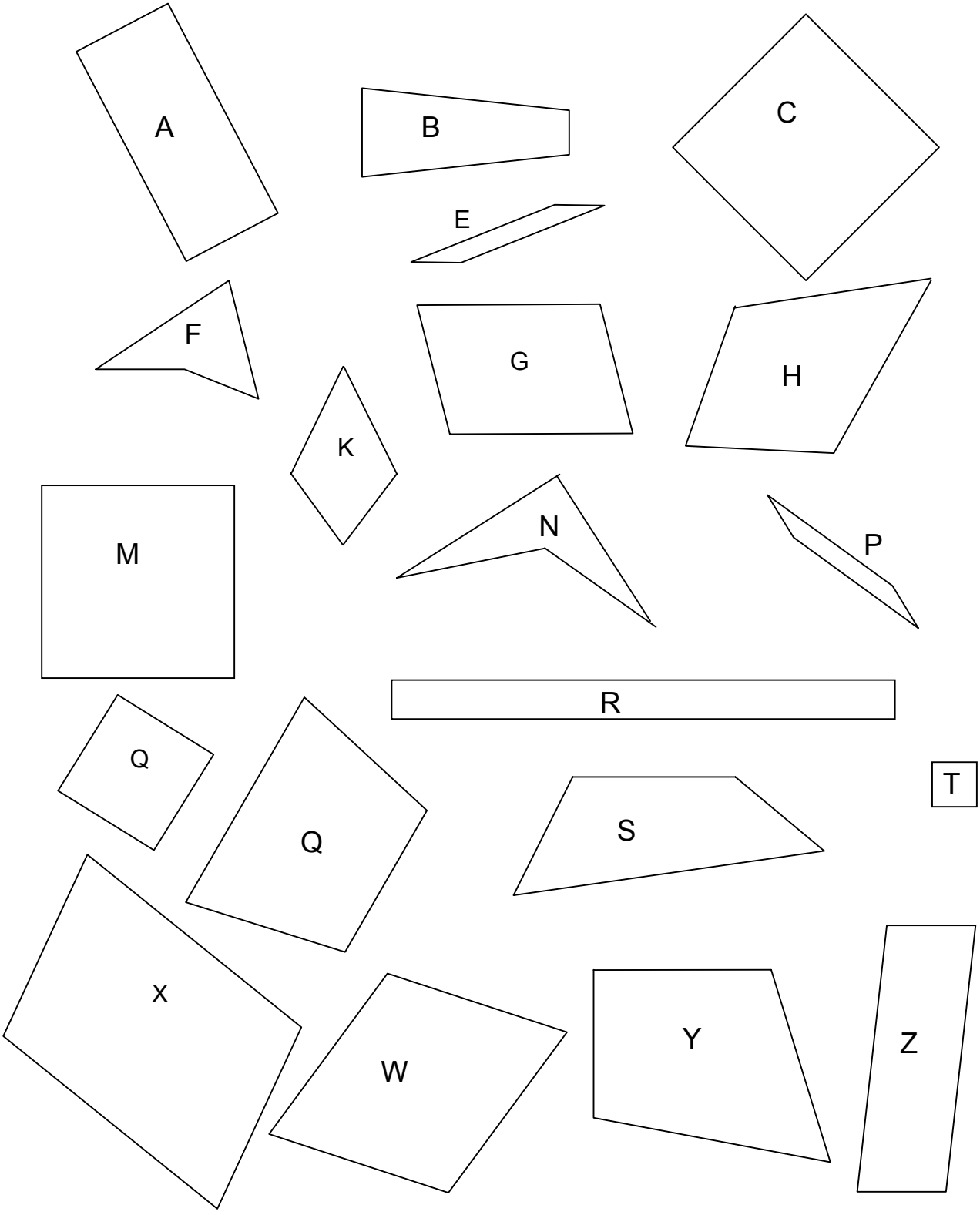
Note that some of the triangles are ***similar*** *(D and K; T and E) and some are congruent (D and W; O and X). Learners who have completed the Malati Similarity Module should explore these aspects.*

The teacher should introduce the correct mathematical vocabulary where necessary. Conventions for indicating equal angles and sides can also be discussed.

Further Activities:
Learners can be encouraged to investigate the figures for symmetry (line and rotational).

Grouping Figures 2

Look carefully at the following shapes and decide which shapes you think belong together. In each case give a reason for your answer.



Teacher Notes: Grouping Figures 2

Learners on the van Hiele analysis level should answer using the properties, for example, using the sides and angles. Learners on the van Hiele visual level will answer according to the shape of the figures as a whole. Orientation might be a problem in this case.

Learners on these two levels will not include a square in a group of rectangles or a rectangle in a group of parallelograms. This is because they will not be comparing figures at this stage. Class inclusion (for example, recognising that a square is a special kind of rectangle) should not be forced: movement towards this kind of thinking (on the Informal Deduction Level) can, however, be facilitated by providing activities that require that learners compare the properties of different figures.

<u>Class Discussion:</u>

Learners should be required to think of more than one possible grouping. Use of measuring instruments to compare shapes should be permitted where necessary.

Some possible groupings are:

- *Convex /concave figures*
- *Figures with right angles / Figures with no right angles*
- *According to the number parallel lines*
- *According to the number of equal sides.*
- *By naming different quadrilaterals: In this case learners can be asked how they identified the different quadrilaterals.*

The teacher should introduce the correct vocabulary, for example, trapezium, rhombus, where necessary. Conventions for indicating equal angles and sides can also be discussed.

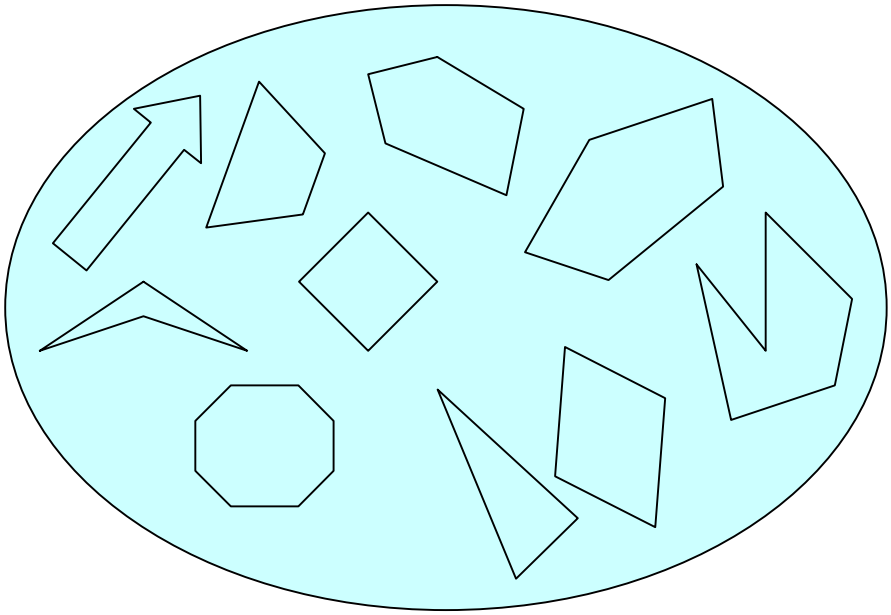
Some of these quadrilaterals are similar (T, C and Q; T, M and Q; K and H; X and G) and some are congruent (C and M; E and P). Learners who have completed the Malati Similarity Module should explore these aspects. Are all the squares similar? And the rectangles? Why?

<u>Further Activities:</u>

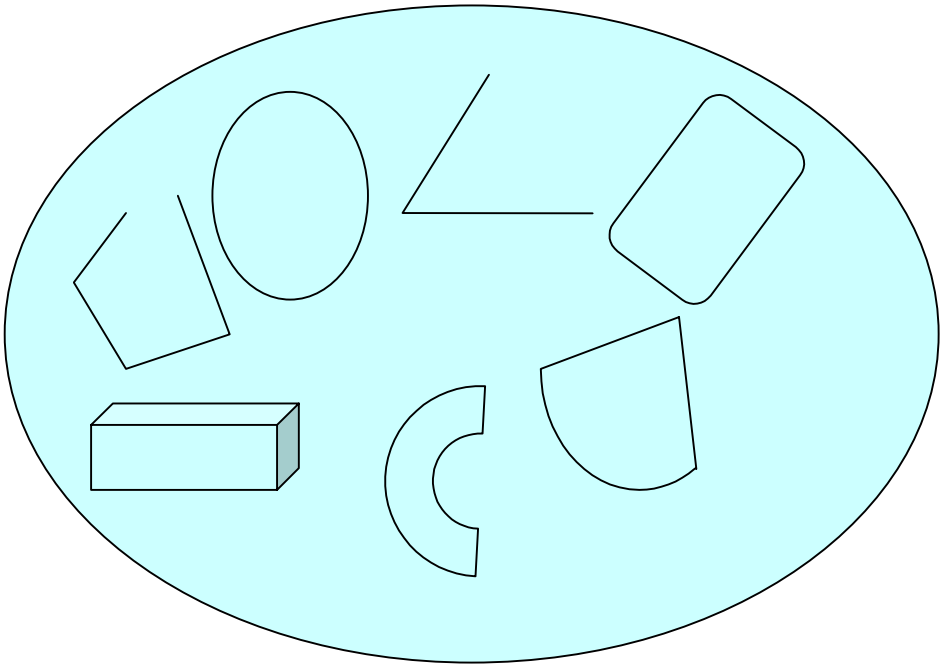
Learners can be encouraged to investigate the figures for symmetry (line and rotational).

Mrs Sisulu's Maths Lesson

Mrs Sisulu's class is learning about polygons.
She provides the class with the following examples of polygons:



Mrs Sisulu says the following figures are NOT polygons:



1. What, according to Mrs Sisulu, are the properties of a polygon?
2. What is Mrs Sisulu's definition of a polygon?

Teacher Notes: Mrs Sisulu's Maths Lesson
Learners are required to reflect on the properties of the polygons.

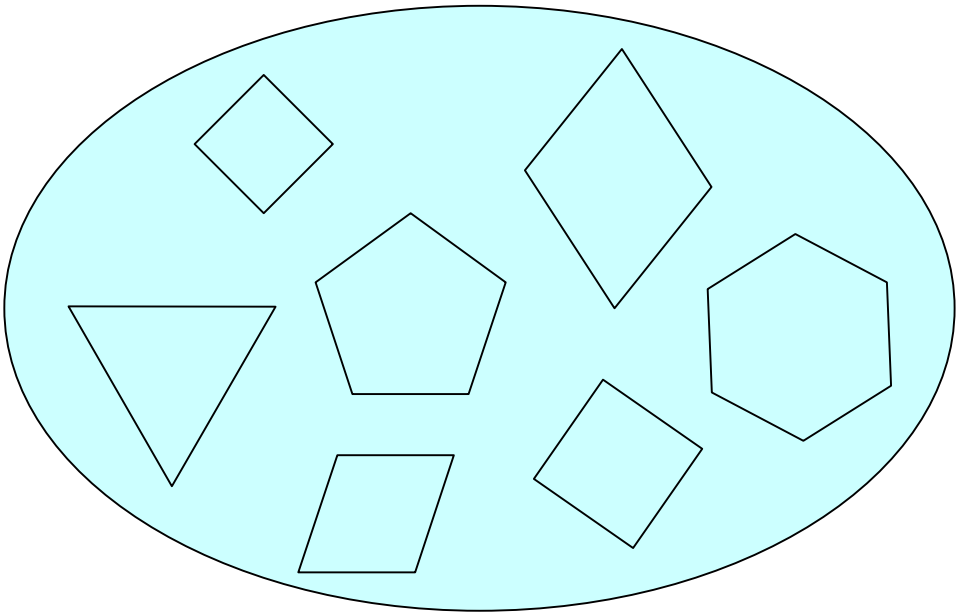
Class Discussion:

Learners should note that the polygons are closed two-dimensional figures with only straight sides.
Definitions at this level are likely to include extraneous information.

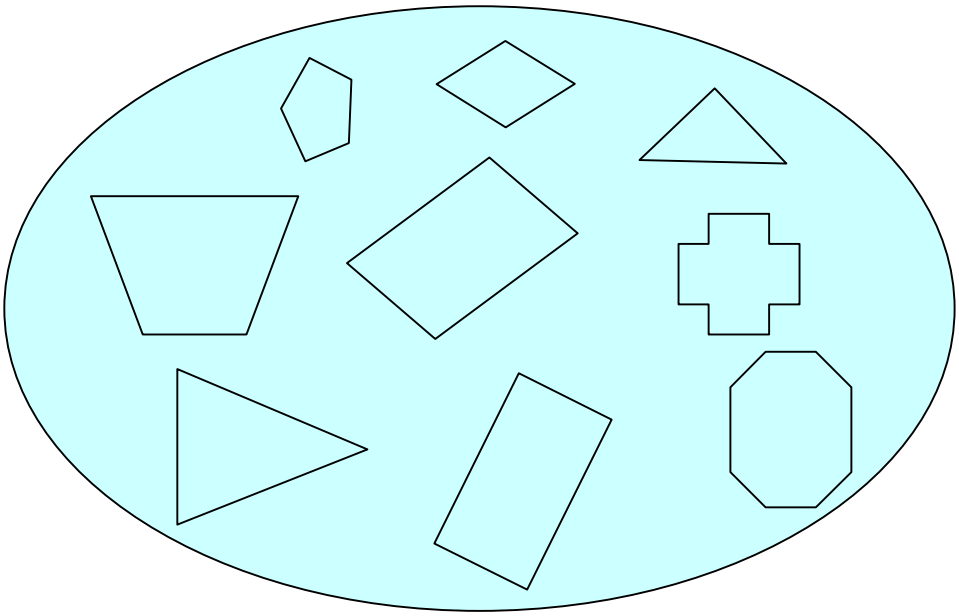
<u>Further Activities:</u>
The teacher can create additional activities of this type dealing with general polygons, quadrilaterals and triangles.
Learners can be encouraged to construct their own collections of examples and non-examples of familiar classes of shapes. Presentation of figures in different orientations should be encouraged.
Definition-making can be encouraged by encouraging learners to make up their own classes of figures.
Learners in transition to Informal Deduction should be encouraged to consider the minimum properties required for the definition.

Gurgles

Mukoni has created a new class of polygons which he calls “gurgles”.
He has provided the following examples:



Mukoni says that the following polygons are NOT “gurgles”:



1. Draw two more “gurgles”.
2. What, according to Mukoni, are the properties of a “gurgle”?
3. Write down what you think is Mukoni’s definition of a “gurgle”.

Teacher Notes: Gurgles

Class Discussion:

Learners can use measuring instruments: they should note that the gurgles are actually "equi-sided" polygons, that is, all sides are equal. The teacher should introduce this vocabulary once the properties have been noted.

Further Activities:

The teacher can ask how the examples and non-examples can be changed so that gurgles are regular polygons, that is, all sides are equal and all angles are equal.

Learners can be required to construct their own sets of examples and non-examples for certain classes of figures and to set similar questions for one another.

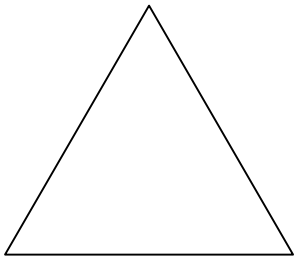
Learners in transition to Informal Deduction should be encouraged to consider the minimum properties required for the definition. For example, is it necessary to mention in the definition that all angles and all sides are equal?

Tiling the Kitchen

The Zungu family is having tiles put on the floor of the kitchen in their new house.
The tiles must cover the floor and there must be no gaps in between the tiles.
The family cannot decided which shape tile to choose. Can you help them?

1. Joshua says that square tiles can be used to cover the floor but he must convince the rest of the family. Can you help him to explain why square tiles will work?

2. Lindiwe says that triangles like this will work:



   (a) Is she correct? Explain.
   (b) Can any other kinds of triangles be used for the tiling?

3. Explain to the Zungu family what other shaped tiles could be used.

Teacher Notes: Tiling the Kitchen

Learners on the van Hiele visual level could answer the questions by simply manipulating the cut-out figures. Learners on the van Hiele analysis will use the properties to explain.

Class Discussion:

Learners should use the interior angles of the polygons to explain why certain figures tessellate.

Further Activities:

Learners can be encouraged to consider which combinations of regular polygons will tessellate (semi-regular tessellations). The design of floor patterns can be set as a project.

Matches

1. You have 7 matches. How many different triangles can you build using all 7 matches?

   Draw your triangles in the space provided. In each case show the number of matches in each side.

2. Now do the same as in Question 1, but use 12 matches to build your triangles.

3. Use your observations in Questions 1 and 2 to formulate a conjecture about the lengths of the sides of a triangle. Write down anything you notice about the angles too.

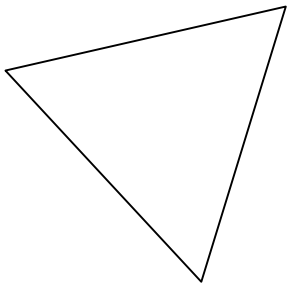
Teacher Notes: Matches
Learners are required to work with the properties of the figures and have to generalise from their empirical work.

Class Discussion:
Learners should be permitted to use matches or sticks of equal length. After some practice they might only need to draw the matches.
Learners should be encouraged to be systematic when recording their results
Learners should note that the sum of the lengths of the smaller two sides must be greater than the length of the longer side. They should be encouraged to note the position of the angles in relation to sides of different length.

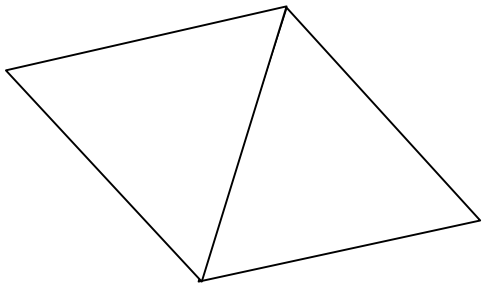
Helping Lucas

Lucas has to make polygons out of triangles. He may only use congruent triangles for each polygon.

He begins with this equilateral triangle:



He can place two congruent equilateral triangles together to make this polygon:



1. What type of polygon is this? What are the properties of this polygon?

2. Help Lucas to make some more polygons with this equilateral triangle. Name and write down the properties of each triangle.

3. What type of polygons can Lucas make if he uses each of the following triangles? In each case write down the properties of the polygon.
   (a) an isosceles triangle
   (b) a scalene triangle
   (c) a right-angled triangle.

Teacher Notes: Helping Lucas

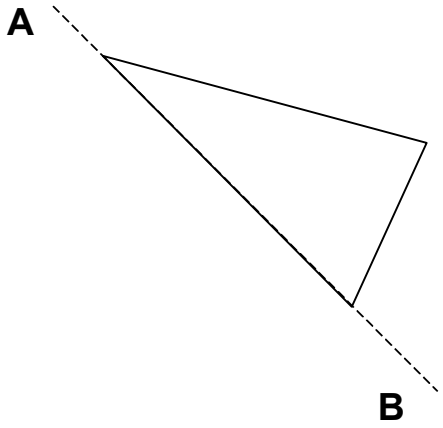
On the van Hiele visual level learners can physically maneuver the triangles and identify the polygons by "looking". On the van Hiele analysis level learners will have to work with the properties of the triangles to decide what kind of polygon results.

Class Discussion:
Learners can be encouraged to use vocabulary of transformations to describe the construction of the figures.

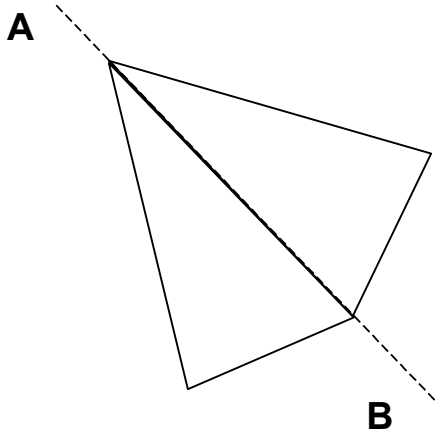
Further Activities:
This activity could be restricted to quadrilaterals only.
Learners could be required to consider what triangles would be required to form particular polygons, for example, an isosceles triangle, a kite, a rectangle etc. They can also be asked what polygons cannot be constructed in this way, for example, a trapezium.
The area of each polygon in terms of the area of the original triangle can be explored.
The use of triangles to construct the figures can be used to explore the sum of the interior angles of polygons (reinforcement after the activity "Tiling the Kitchen").

Helping Andile 1

Andile has **one** triangle as shown below:



She **reflects** the triangle in the dotted line AB to create the following figure:



1. What type of polygon has she created? What are the properties of this polygon? Explain how you know.
2. Now help Andile to make polygons by reflecting the triangle in each of the other sides. In each case write down the properties and the name of the polygon you have formed.
3. What if Andile's triangle is an isosceles triangle? Write down the properties **and** name of the figures you can make by reflecting the triangles in its sides in this way.
4. What if Andile's triangle is a right-angled triangle?
5. What if Andile's triangle is an obtuse-angled triangle?
6. What if Andile's triangle is a right-angled isosceles triangle?

Teacher Notes: Helping Andile 1

Class Discussion:

Learners will need to be familiar with different kinds of triangles and with performing reflections.

Learners will need to reflect actual copies of the triangle if necessary. Some learners should be able to visualise the figure formed by reflection.

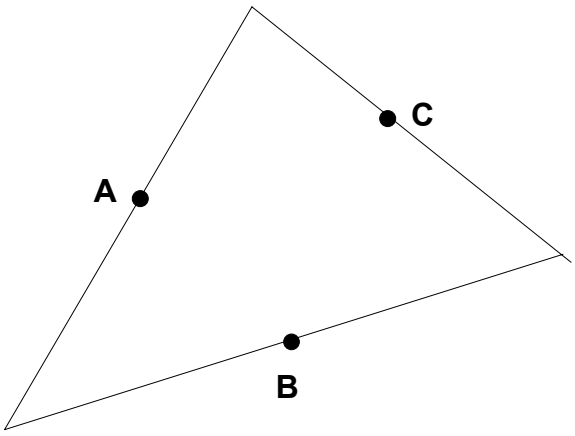
Further Activities:

Learners could be required to use other polygons, for example quadrilaterals as the template.

When pushing towards the Informal Deduction Level, learners can be asked questions such as, "Kathy claims that all squares are kites. Do you agree?" This will require that they compare the properties of the figures obtained in questions 2 to 5.

Helping Andile 2

1. Rotate the following triangle 180° about each of the points A, B and C.



   What kind of quadrilateral do you get in each case? In each case explain how you used the rotation to decide on the shape.

2. The points A, B and C have been specially chosen. Can you explain?

3. Will you get the same results using an obtuse angled triangle?

4. Investigate whether it is possible to create other kinds of quadrilaterals using rotations in this way. Explain your answers.

Teacher Notes: Helping Andile 2

Question 1: Learners will need to use the properties of the shapes derived from the rotations to decide on the shapes. Learners working on the van Hiele analysis level will list properties, some of which will be extraneous. Those on the ordering level should choose certain properties or refer to the definition when explaining.

Question 4: Learners will need to use their knowledge of the properties of the quadrilaterals to decide on the kind of triangle required. Learners on the ordering level should be able to use if..then arguments as indicated below.

Class Discussion:

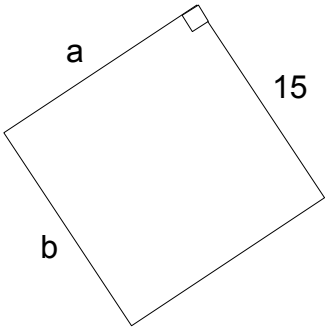
- *Question 1: The rotation of the scalene triangle will result in a parallelogram.*
- *Question 2: Learners should note that the points must be midpoints of the sides in order to create a quadrilateral.*
- *Question 3: Rotations about the midpoints of the sides of an obtuse angled triangle will also result in parallelograms.*
- *Question 4: Learners will have to consider, firstly, which quadrilaterals can be created in this way, and secondly, what kind of triangle one needs to start with to create the shapes. they should note the following:*
  - *(a) It is not possible to create a trapezium because the opposite sides of the quadrilateral will always be parallel. Learners on the ordering level should be able to explain this using the angles in the quadrilateral.*
  - *(b) To create a rhombus one needs to start with an isosceles triangle. Learners should also note that it is not possible to create a rhombus using any side of the triangle and should indicate which side should be used.*
  - *(c) To create a square a right-angled isosceles triangle is required. Again learners should indicate the point of rotation.*
  - *(d) A right-angle triangle is required to create a rectangle. The point of rotation is the midpoint of the hypotenuse.*
  - *(e) Learners on the ordering level should use inclusion to note that the rhombus and the square are the only kites that can be created using rotation in this way. Other kites can only be created by reflecting the triangle in one of its sides. Learners on the analysis level are likely to deal with the kite separately.*

Find the Value

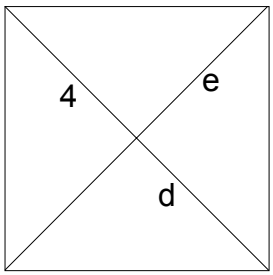
Use **transformations** to write down the value of the letters a to p.

Justify your answers.

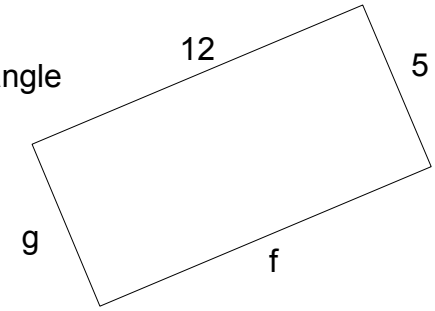
1. square



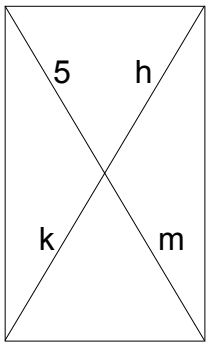
2. square



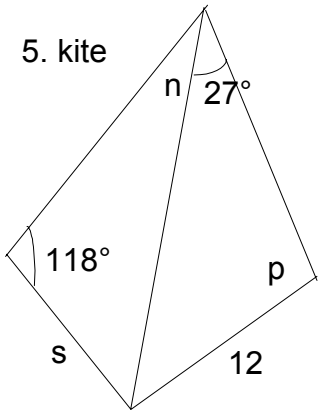
3. rectangle



4. rectangle



5. kite



Teacher Notes: Find The Value

Class Discussion:

- *Learner's explanations must indicate that they have used transformations to find the values, and not other knowledge of quadrilaterals.*
- *They should be encouraged to use more than one method where possible, for example, using line or rotational symmetry.*

Further Activities:

The teacher can use textbook examples involving quadrilaterals, but insist that explanations be given ***in terms of transformations****.*

BACK TO BEGINNING OF MODULE

CONTINUE TO NEXT SECTION SEC03B