

## **MODULE 3C:**

### **Exemplar materials for learners on the van Hiele ordering level.**

We provide here a selection of activities as examples for the teacher on how to take the activities provided in the polygons module further and to encourage ordering level thinking. Activities of this nature are only appropriate for learners who have proceeded through the [van Hiele](#) visual and analysis level activities, and the use of these activities need not be hurried. Detailed teacher notes are provided and the activities can be varied by varying the figures studies.

Once learners have had rich experiences exploring properties of figures in a concrete way, they will be able to make informal, logical arguments about **the relationships within and between these figures**, for example, “the opposite sides of a parallelogram are parallel, so the opposites angles are equal”, or “a rectangle is a parallelogram because it has all the properties of a parallelogram as well as the special properties of right angles.” New properties can be also be determined using deduction. The learner can also formulate and use formal definitions.

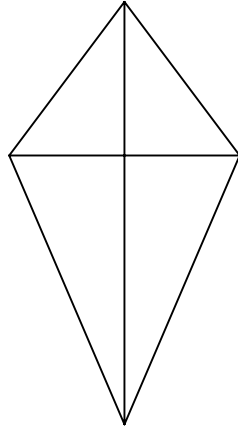
#### **List of exemplar activities:**

- Defining a Kite 1
- Defining a Kite 2
- Defining a Kite 3
- Defining Rectangles
- New Shapes

#### **Reference:**

De Villiers, M.D. (1998). To teach definitions in geometry or to teach to define? In Olivier, A. & Newstead, K. (Eds.), **Proceedings of the 22<sup>nd</sup> Conference of the International Group for the Psychology of Mathematics Education**, 2, 248-255.

## DEFINING A KITE 1



Balki discovered the following properties of the kites:

1. two pairs of equal adjacent sides
2. perpendicular diagonals
3. one diagonal bisects the other
4. one pair of opposite angles is bisected by a diagonal
5. one diagonal is a line of symmetry
6. one pair of equal opposite angles

Can you help Balki to write a definition for a kite?

**Teacher Notes: Defining a Kite 1**

*In this activity we introduce the pupils to a process of defining referred to as “descriptive defining” where a set of properties of a concept is known and a subset of the properties serves as the definition. From this subset the other properties are logically derived using theorems.*

*The aim of this activity to get the pupils to select a subset of the total set of properties. In the following activity we will discuss the subsets chosen to serve as the definition for the kite.*

*In the class discussion write down the definitions of the pupils. Based on the Van Hiele analysis three categories of definitions have been identified.*

*Van Hiele 1: Visual definitions*

*Van Hiele 2: Uneconomical definitions*

*Van Hiele 3: Correct economical definitions*

*It has been observed that pupil who provide definitions 1 and 2(also referred to as **partitional definitions**, as they do not allow the inclusion of other figures) have difficulty with inclusion hence their choice of definition. The economical definitions also referred to as **hierarchical definitions**, on the other hand, allow for inclusion.*

## DEFINING A KITE 2

Balki comes up with the following definitions for a kite:

1. A kite is a quadrilateral with perpendicular diagonals.
2. A kite is a quadrilateral with two pairs of equal adjacent sides and one pair of opposite angles equal.
3. A kite is a quadrilateral with perpendicular diagonals with one being bisected by the other.

Accurately construct quadrilaterals that comply with each of these definitions.

Now analyse and discuss each definition given by Balki.

## **Teacher Notes: Defining a Kite 2**

*The aim of this activity is to help the pupils realise that it is common practice to keep a definition as short as possible. In the discussion the following aspects of this process of defining needs to be considered:*

- 1. Did the definition have **sufficient** information to construct the kite?*
- 2. Was it **necessary** to use all the information in the definition? Which information was used and why?*

*The first definition given is not correct since it does not contain sufficient properties. It needs to be emphasised here that the statement that “a kite is a quadrilateral with perpendicular diagonals” is a correct statement about a property of kites but it contains **too little** information to be used as a **definition**.*

*The second definition is correct, but contains **too much** information. This is an example of an uneconomical definition. Which information can be left out?*

*What happens if we leave out “two pairs of adjacent sides”? In the class discussion allow the pupils to explore and to see that it is possible to construct a quadrilateral with one pair of opposite sides equal which is not a kite. On the other hand the condition “two pairs of adjacent sides equal” logically implies that “one pair of opposite sides are equal”.*

*The third definition is an example of an economical definition since we cannot leave out any of the properties and we can construct a kite from it.*

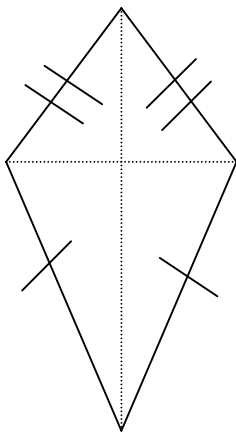
## DEFINING A KITE 3

Balki is trying to find the most economical definition for a kite.

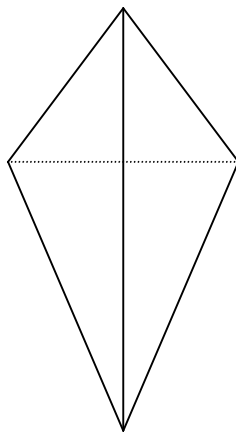
He comes up with the following definitions:

1. A kite is a quadrilateral with two pairs of adjacent sides equal
2. A kite is a quadrilateral with one pair of opposite angles bisected by a diagonal.
3. A kite is a quadrilateral with one diagonal line a line of symmetry.

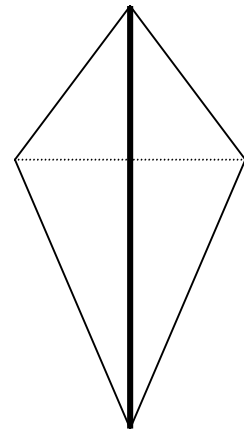
Use the diagrams below and the corresponding definition to deduce the other properties of the kite.



Definition 1



Definition 2



Definition 3

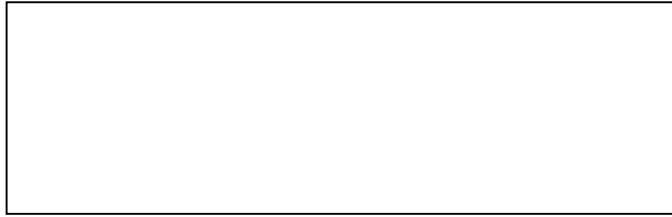
**Which definition is deductively the most economical? Discuss!**

### **Teacher Notes: Defining a Kite 3**

*The aim of this activity is to focus on the proofs that are necessary to deduce the properties that are not contained in the definition.*

*Definitions 1 and 2 both require the proof of the congruency of the triangles. Definition 3, from the definition of line symmetry, all the other properties follow immediately. Definition 3 is therefore deductively the most economical.*

## DEFINING RECTANGLES



Write down all the properties of rectangles.

A large, empty rectangular box with a thick black border, intended for the student to write down the properties of rectangles. The box is oriented horizontally and occupies most of the page's width and a significant portion of its height.

Now write down a definition for a rectangle.



### **Teacher Notes: Defining Rectangles**

See Teacher Notes “Defining A Kites 1”. In the class discussion get the pupils to draw the quadrilaterals that comply with the definition they have written. Drip the notion partition and inclusion in the discussion.

This process can be extended to other concepts with which the pupils are familiar: Parallelograms, Trapeziums, Rhombi, Squares.

It needs to be made clear to the pupils that both the partitional and hierachical definitions are mathematically correct.

The role and advantages of the hierachical definitions need to be discussed in order to understand why partitional definitions are not used. For example: Consider the following definition of a parallelogram:

**A parallelogram is a quadrilateral with both pairs of opposite sides parallel.**

Note this includes the special cases; rectangles; squares; and rhombi.

If we now prove the theorem that the diagonals of a parallelogram bisect each other, it is immediately true for the special cases as well.

If on the other hand the parallelogram was defined as

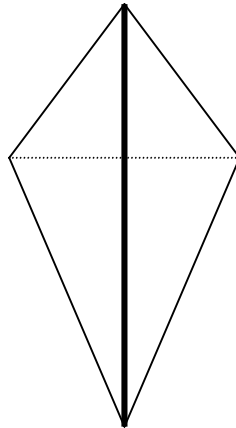
**A parallelogram is a quadrilateral with both pairs of opposite sides parallel, but not all angles or sides equal,**

we would need to prove separately for each case that the diagonals bisect each other.

## NEW SHAPES

A kite is defined as:

A QUADRILATERAL WITH A LINE OF SYMMETRY THROUGH ONE PAIR OF OPPOSITE ANGLES.



Balki decides to add the following constraint to the above definition of the kite:  
that it must have **two** such lines of symmetry.

Draw the new figure and deduce the properties of this figure.