Rationale for teaching and learning *Spatial Sense*

Geometry in the Primary School

Traditionally, the geometry studied in the primary school has been in preparation for the study of geometry in the secondary school. Learners have had opportunities to classify and explore the properties of plane figures as a basis for the systematic study of these figures within a specific axiomatic system that follows at high school. Consequently, geometry at the primary school is often simply regarded as “triangles, circles and squares” or “ lines and angles”.

Research in South Africa and elsewhere sheds light on the nature of geometry instruction experienced by learners. De Villiers and Njisane (1987), Smith (1987), Senk (1989) and Usiskin (1982) have indicated that many secondary learners are on van Hiele visual or analysis levels. In order for a learner to cope with the demands of an axiomatic system as required in secondary school, however, s/he needs to be on the van Hiele ordering level (see MALATI Van Hiele Theory Document). Learners who have not received adequate experience on the visual and analysis levels resort to memorisation to cope with the demands of formal school geometry. It is in the primary school that the learners require experiences on the visual and analysis levels in preparation for activity on the van Hiele ordering level. In the MALATI Geometry Vision document we provide examples of low level responses from learners at MALATI project schools.

Trends in Geometry Curriculum Development

While recognising the importance of the study of plane figures and the need for the provision of appropriate experiences in this regard in the primary school, it is important to note that there is more to the study of geometry than the study of this particular area of content. An additional aspect of geometry is the view that geometry is a body of knowledge that can support interaction in space. Different ways in which people interact in physical space may be distinguished:

1. Observing spatial objects in a discriminating way, that is, two- and three-dimensional figures and the properties of these figures
2. Generating information that cannot be directly observed, for example, determining distances, elevations, area and volumes
3. Designing spatial objects and configurations, for example, gardens, furniture arrangements, furniture, buildings and artistic designs
4. Representing spatial configurations with plane drawings
5. Interpreting plane representations of spatial configurations.

Reflection on the traditional school geometry curriculum reveals that this fails to address aspects (4) and (5). This is surprising considering the extent to which both our interaction in space and our study of mathematics requires an understanding of plane representations.

This recognition of the importance of the study of space is reflected in recent curriculum innovations in South Africa and elsewhere. For example, in Curriculum 2005 in South Africa, one of the ten Specific Outcomes for Mathematical Literacy, Mathematics and Mathematical Sciences claims that

Mathematics enhances and helps to formalise the ability to be able to grasp, visualise and represent the space in which we live. In the real world, space and shape do not exist in isolation from motion and time. Learners should be able to display an understanding of spatial sense and motion in time.

The outcome requires that learners:

…describe and represent experiences with shape, space, time and motion, using all available senses.

Department of Education, 1997

Standard 3 of the NCTM Draft “Standards 2000” Document (1998) suggests that mathematics instruction programmes should pay attention to geometry and spatial sense so that all students, among other things, “use visualisation and spatial reasoning to solve problems both within and outside of mathematics”. One of the six strands in the Western Australian Curriculum focuses on “the visualisation, analysis, representation and interpretation of shapes and objects in space” (Student Outcome Statements, Working Edition, 1994).

But what is the value of the study of space and the development of spatial sense as suggested in the above curricula?

Learners are surrounded by spatial settings and the ability to perceive spatial relations is regarded as important for **everyday interaction in space**. For example, Smit (1998) stresses the importance of these skills:

Without spatial sense it would be difficult to exist in this world – we would not be able to communicate about position, relationships between objects, giving and receiving directions or imagine changes taking place regarding the changes in position and size of shapes.

While recognising the importance of this interaction in physical space, we need to consider why spatial sense should form such an important part of the **mathematics** curriculum itself. Research on spatial sense since the late 1950’s has focused on the relationship between spatial sense and other aspects of mathematics. Firstly, this quote from the work of Van Niekerk (1995) suggests the value of spatial sense for the study of formal **geometry**:

The Geometry curriculum for the primary school should start with the real world of the child. The intuitive notions that children reveal when exposed to spatial situations should be capitalised on (van Hiele, 1982). Once the child has experienced these situations he/she must be able to reflect on them. It is only possible to reflect if there is an underlying relationship between the experiences he/she is exposed to….Geometry does not start with the formulation of definitions and theorems. It already starts when the child has to orientate him/herself in the everyday surroundings. This familiarisation with the physical environment will eventually lead to more experiences that pave the way for developing these definitions and theorems (Freudenthal, 1991).

Recent research has studied links between spatial sense and performance in geometry. Clements and Battista (1992) provide a summary of the work done in this area and quote the work of Gardner, Hadamard, Krutetskii, Fennema and Sherman, Guay and McDaniel etc.

Furthermore, some research has suggested a link between spatial sense **and general performance in mathematics** itself. For example, Presmeg (1992) stresses the importance of visual imagery in general reasoning skills in mathematics and Guay and McDaniel (1977) suggest that high mathematics achievers at elementary school have greater spatial ability than low achievers and that there is a relationship between mathematical and spatial thinking for pupils with high as well as low spatial ability.[1] Clements and Battista (1992), however, warn that the relationship between spatial sense and the learning of non-geometric concepts is not straightforward. They note a study of foundation year engineering students in Papua New Guinea by Lean and Clements (1981) in which it was found that spatial sense and knowledge of spatial conventions had less influence on mathematical performance than the literature seemed to suggest.

Recent curriculum innovations that stress the importance of learners developing spatial sense appear, therefore, to reflect a recognition of the importance of spatial sense in mathematics as well as a recognition that learners have not received the necessary experiences in this regard in the past.

What is spatial sense?

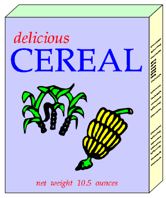
There is little consensus on the definition of spatial sense and this is complicated by the use of a variety of terms to describe the phenomenon, for example, “spatial reasoning”, “spatial intuition”, “spatial perception”, “spatial ability”, ‘spatial orientation” and “spatial visualisation”. At MALATI we have chosen to use the word “spatial sense” to describe a learner’s interaction in space, both concrete and visual. In general one could describe spatial sense as the ability **to interact in a spatial environment** and **to work with visual images**. In the discussion that follows we use examples from our work in MALATI project schools to illustrate the complexities of the phenomenon we call spatial sense.

Learners in Grade 5 and 6 were required to respond to this activity:

[1] Guay and McDaniel (1977 : 211) define “low-level spatial abilities” as those requiring the visualisation of two-dimensional configurations, but no mental transformations of these visual images. “High-level spatial abilities” are characterised as requiring the visualisation of three-dimensional configurations, and the mental manipulation of these images.

Draw What You See

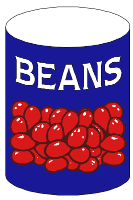
1. *Look carefully at this box:*



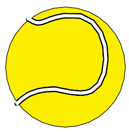
(a) Draw what you would see if you were looking at the box from behind.
(b) Draw what you would see if you were looking at the box from above.
(c) Draw what you would see if you were looking at the box from the side.

Now answer these questions for each of the objects shown below:
(a) Draw what you would see if you were looking at the box from behind.
(b) Draw what you would see if you were looking at the box from above.
(c) Draw what you would see if you were looking at the box from the side.

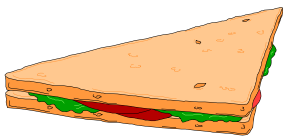
2.



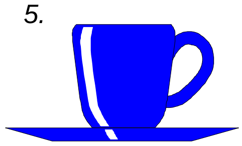
3.



4.



5.



Firstly, it should be noted that this activity requires that learners recognise the three-dimensional object represented in the two-dimensional representation. If we consider the cereal box, it should be noted that this box is represented using parallel projection, a form of representation commonly used in mathematics and having particular conventions.

Furthermore, these responses from learners suggest the mental processes that might be required in responding to this question:

Trevor (Grade 6) describes how he turns the box in his mind:

> *…Miss, I picture the box in my mind, Miss. Then I just turn the box…put the box down, Miss, and look at it from the top.*

Ashton (Grade 5) suggests that he visualises a movement in his own position in relation to the image of the object:

> *I'm not standing there…I'm standing a little further…standing on that line there. I'm standing there, so I am imagining that I can see this part…so I can see the 3-d.*

Some learners appear to rely on visual memory, that is, their memories of when they have encountered these objects in other situations:

Trevor: *…whenever I play cricket I look at the ball.*

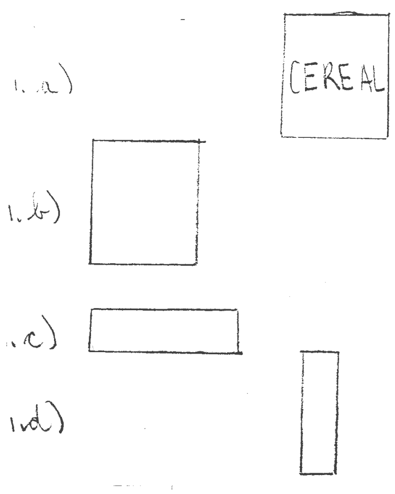
Researcher: (Referring to the teacup representation in Activity 1(a)) *How did you know that it would look like that?*

Belinda: *Miss, because I always make tea.*

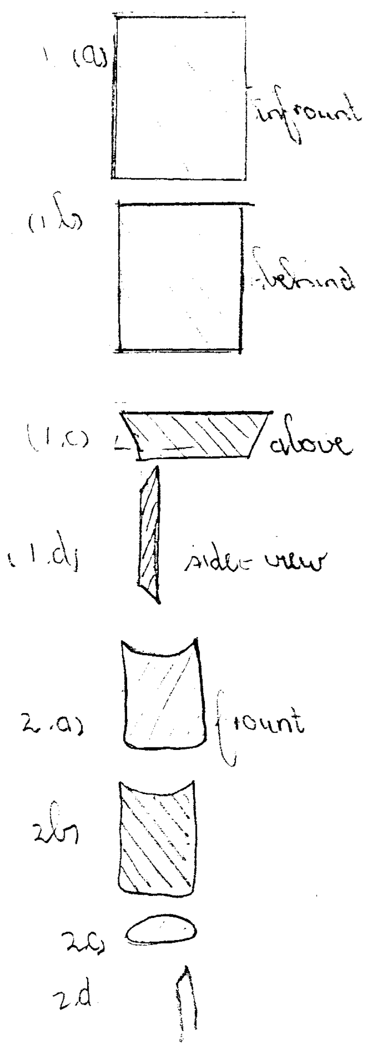
It is important to note that a learner's communication, whether verbal or in the from of a drawing, is a means for the teacher to gain access to the spatial thought of the learner. The learners described above can clearly describe their mental processes in words, but this is not always the case. Consider, for example, this comment from Cindy (Grade 6):

…I can't tell you what goes on in my mind, Miss.

Another common way of gaining access to learners' spatial thought is through their drawings. Consider these responses to the above activity:



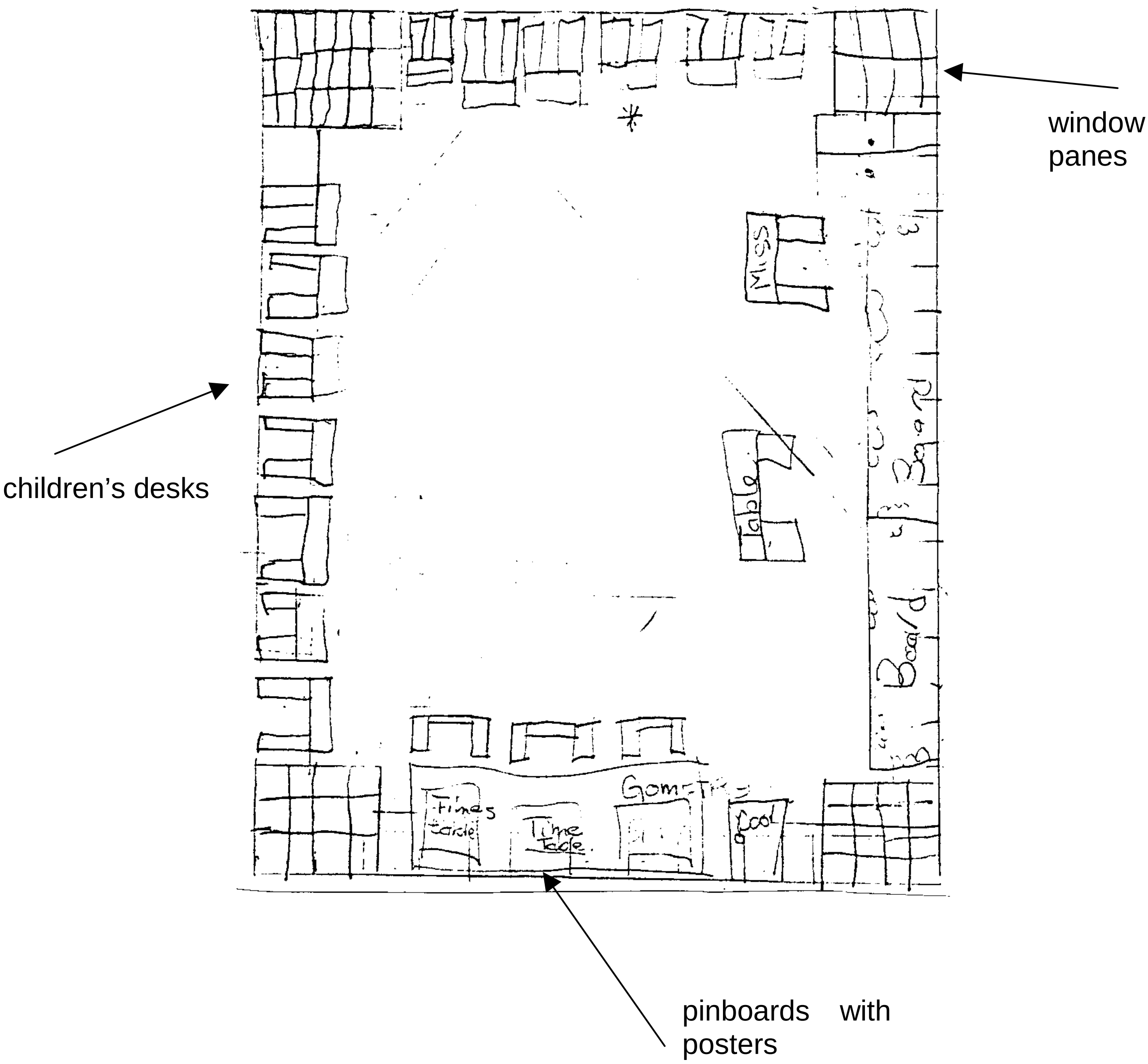
response 1



response 2

Response 1 gives the “direct view” of the box, that is by viewing the box from directly above, behind or the side. Drawings 1(c), 1(d) and 2(b) in response 2 give the view from an angle. In response 1, a particular mathematical convention is being used – if we require learners to use this form of communication we must make the conventions explicit. The learner who has given response 2 is not wrong, but has drawn the objects as s/he sees them. In such a case there is a danger of a teacher regarding such a response as incorrect, highlighting the importance of the teacher discussing responses with learners.

The importance of drawing conventions in communicating spatial thought is also illustrated in the work of Cindy. This drawing represents the aerial view of Cindy’s classroom. If one looks at the desks one can see that she has not drawn them directly from above as we would expect, but has shown the top as well as the legs of each desk:



It is clear form this drawing that Cindy is not using the conventions for drawing the aerial view of a scene. The question is whether she possibly has the visual skills, but cannot communicate these in the form of drawing. Stuart (Grade 5) is an example of a learner who can describe his mental processes in words, but struggles to draw his visual images: When responding to a question, he claimed that he had a picture in his mind of what to draw, but "I couldn't like draw the shapes".

Mitchelmore (1980) suggests that the relationship between a learner's drawings skills and spatial sense is complicated and that the one does not simply "reflect" the other. He distinguishes between "representational ability" and "spatial-perceptual development" and his research suggests that the former lags behind the latter. Thus although a learner might have developed the necessary visual skills, s/he might not be able to represent this thinking in a drawing. Once again the need to talk to learners about their work is apparent.

Furthermore, the above drawings by learners indicate that there are certain conventions associated with different forms of representation, for example, an aerial view. Certain MALATI activities are designed to provide the necessary social knowledge in this regard.

When reflecting on Cindy's map of the classroom, one also needs to consider whether a question requiring that learners draw the aerial view of the classroom or of the school grounds places different demands on a learner than the activity using the cereal box as given above. We find it useful to distinguish between "large-scale" space which is the space that surrounds the individual and requires multiple vantage points to be comprehended completely, and "small scale" space, that is the space that does not surround the learner (although multiple vantage points might be required for comprehended). In our classification of "small-scale" space we include scaled-down versions of "large-scale" space, for example, maps and scale drawings.

Herman and Siegal (1978) note that many assessments of spatial knowledge have focused on "small-scale" environments only, and that "large-scale" environments "which seem more closely to simulate the real world" are used infrequently. We could add, too, that diagrams used in mathematics are often presented in scaled-down form. Furthermore, Herman and Siegal suggest that in studies in which learners are tested on their knowledge in a particular environment, this is tested in a "small-scale" environment, that is, the solutions have to be presented in a "scaled down" form. A learner's mapping ability can thus be confounded with the ability to translate and represent his / her knowledge on a "small-scale".

Again we can use the verbal responses of learners at MALATI project schools to illustrate some of the mental processes that might be involved in translating a visual image of "large-scale" space into an aerial view.

In the case of the classroom, learners looked around the scene from their position within the classroom:

Researcher: *How do you know that this is what it is going to look like?*

Ashton: *If you look from here you see the desks.*

But then it was still necessary to visualise what the furniture would look like from above:

Gaylene: (To Trevor): *If you are looking from a helicopter above, there are some bits you won't see like the edges of the pinboards.*
And you can like picture yourself in a helicopter, Miss, and then you can see like down, Miss.

The process of drawing an aerial view of the school grounds appears to be more demanding, as the learner cannot see the entire scene, as is the case with the classroom. It appears that learners rely on being able to see some of the scene:

Researcher: *…so what do you do if you can't see the whole school, how do you know to draw that?*

Belinda: pause*…I can see some parts.*

When it comes to visualising the parts of the school grounds that cannot be seen from the classroom, learners appear to rely on visual memory:

Researcher: *How do you know it is going to look like that?*

Trevor: *How do I know, Miss…I am in Grade 6…it is six years now that I have been in this school. Every day, Miss…I've seen it so many times.*

Thus when designing materials for developing spatial ability it is important that we provide learners with opportunities to work in both small- and large- scale space, as well as opportunities to translate between the two sized-spaces.

The responses of the learners at MALATI project schools to the spatial activities described suggest that spatial sense is a complex phenomenon and possibly involves more than simply being able to form a visual image and to manipulate this image in problem solving.

Using data form the MALATI project schools as described above, as well as a review of the literature on spatial sense, Bennie (1999) identified four aspects of spatial sense:

- the ability to form and retain a visual image
- the ability to use mental manipulations for problem solving
- the ability to reflect on the mental thought processes used in problem solving
- the ability to communicate visual images and processes to others in a meaningful way (both verbally and visually).

She notes, too, that communication in the form of drawing requires a knowledge of the social conventions associated with making representations.

Wessels and van Niekerk (1998) also suggest there is more to spatial sense than simply having the visual skills. They identify four skills that are said to “enable” spatial thinking, namely,

- *visual skills*: this includes all the abilities and competencies to view objects from different points (lines / angles) and understand their characteristics as a whole
- *verbal skills*: the ability to talk about different views and interpret what is observed, including the mastery of the use of and understanding of the terminology
- *tactile skills*: the ability to “build, cut and paste, to sew, to construct etc according to a specific plan or manual”
- *mental skills*: this is described as “the ability to mentally manipulate spatial images and thus to understand the interconnectedness between these four skills…”.

Spatial sense and mathematics

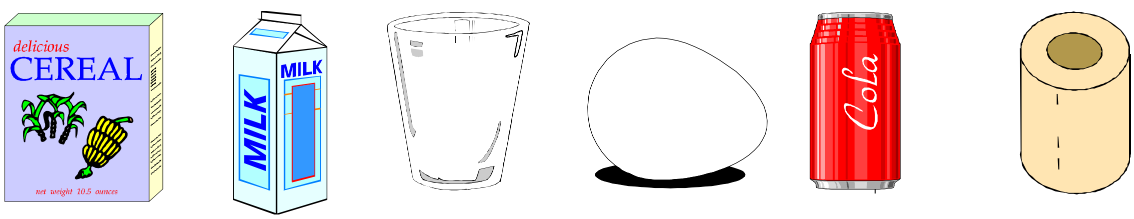
When discussing spatial sense as described above, the topic is frequently linked to other school subjects traditionally studied at school, for example, geography, woodwork or art. The question is why should the topic be studied in mathematics if it is also explored in other subjects? We provide two responses to this challenge.

Firstly, the acknowledgement of the importance of spatial sense in the study of geometry and mathematics in general has been noted, along with the recognition that the traditional geometry curriculum has not provided learners with the spatial sense required. It appears, too, that learners experiences in the other school subjects mentioned above have not necessarily prepared them with the skills required specifically for the study of mathematics.

In considering what is required in mathematics, one needs to reflect on precisely how the notion of space is studied in mathematics itself. Bishop (1983) has attempted to answer this question:

> *Geometry is the mathematics of space, and mathematicians approach space differently form artists, designers, geographers, or architects. They search for mathematical interpretations of space. Mathematics educators, therefore, are concerned with helping pupils gain knowledge and skills in the mathematical interpretations of space.*

It appears, therefore, that mathematicians cast a particular gaze on space, which is different from the gaze cast by artists, geographers or architects. Consider for example, the way mathematicians classify space. This is a common activity used in primary mathematics in which learners are required to classify different objects:



object 1 | object 2 | object 3 | object 4 | object 5 | object 6

Learners could classify objects 1 to 4 together as they could be used for breakfast. But in mathematics we would classify object 1 and object 2 as different - object 1 is a rectangular prism. In mathematics we certainly would not place the egg with these objects - in fact, in mathematics we do not even have a name for an object shaped like an egg. Learners might place objects 3 and 6 together as they are hollow inside. But in mathematics we would class figures 5 and 6 together as cylinders as each has two circular faces that are congruent. We would ignore the fact that the objects might be hollow or solid. The point is that in when studying space within the field of mathematics we recognise certain features, and ignore others.

Lehrer, Jacobson, Vera & Strom (1999) have suggested that spatial experience does not necessarily lead spontaneously to mathematical knowledge, but only lays a foundation for this development. This emphasises the importance of providing learners with the necessary experiences on the van Hiele visual level where spatial sense can be developed, and movement through the van Hiele levels as preparation for formal geometry. (See MALATI Van Hiele Theory Document). The activities in the MALATI primary geometry packages are designed to facilitate the development of **spatial sense** as well as **conceptual development in geometry**.

The MALATI Primary Geometry Packages
With the above discussion in mind, we have developed and trialled six packages for the primary school. The features of these packages are described below.

A child's experience of space begins long before s/he enters school and this is an experience of **three-dimensional space**. Del Grande (1987) notes that, since a child's early behaviour is prelinguistic, this behaviour is essentially "spatial". A child's thinking at this stage is dominated by experiences of seeing, touching, hearing, moving etc. These initial experiences lay the foundation for children's understandings of space and geometric concepts. The MALATI activities for the primary school thus begin in the three-dimensional world of the learner. Learners experiences of spatial and geometric relationships are used as a basis for developing spatial and geometric knowledge. These activities are designed to provide learners with rich experiences on the van Hiele visual level where spatial skills can be developed, and to assist learners in progressing to the van Hiele analysis level.

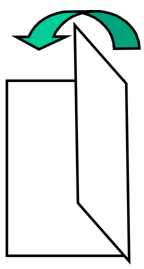
Consider for example an activity such as this:

Only half of each of the following pictures has been drawn. Draw the rest of the picture:

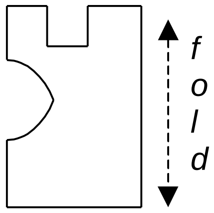
This activity requires visual skills (reflecting a figure) **and** is informally developing the geometric concept of symmetry.

The activities reflect the skills for "spatial thinking" (visual, verbal, tactile and mental) (Wessels & van Niekerk, 1998) and Hoffer's (1981) five geometric skills of visual, verbal, drawing, logical and application. Consider this activity:

Temba folded a piece of paper like this:



He made cuts like this:



1. *Draw what the paper will look like when Themba unfolds the paper. Explain how you know it will look like this.*

2. *Compare your drawing with your classmates' drawings.*

This activity requires that learners use visual and drawing skills in deciding on and presenting the solution. In justifying their answers, learners must use logical and verbal skills. In comparing the figures of learners in the class analytic skills will be used.

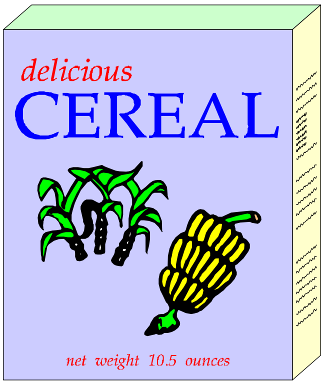
As noted above, the development of spatial sense is thought to depend largely on the experiences learners have had in this field. Consequently learners in Grades 4 to 7 are likely to have different needs, for example, some learners might be able to visualise the appearance of the nets of a variety of three-dimensional figures, whereas others will still need to cut open actual boxes to see the nets. Furthermore, as noted in the discussion of the van Hiele theory above, some activities can be used for learners on different levels (for example the tangram activities) – learners on different levels will respond differently and the teacher can pose questions suitable for the different levels. We have thus developed general packages for Grades 4 to 7 and make suggestions regarding activities suitable for different grades and provide guidelines on how the same activity can be used for different levels of development. The teacher has an important role to play in identifying the needs of learners.

Both Bishop (1983) and Clements (1983) list a number of researchers whose work supports the idiosyncratic nature of performance on spatial activities, that is, learners will solve the activities in different ways. In her work in a MALATI project school, Bennie (1999) noted that different learners use different strategies or combinations of strategies on the same task and that a learner varied the use of strategies across tasks. The MALATI activities have been designed to cater for the range of approaches used by learners and to encourage the development of a range of skills.

For example, some activities involve “small-scale” space, that is, the space that does not surround the learner (although it might require multiple vantage points to be comprehended), while others focus on “large-scale” space ,that is, space that surrounds the individual and requires multiple vantage points to be apprehended (Clements, 1983).

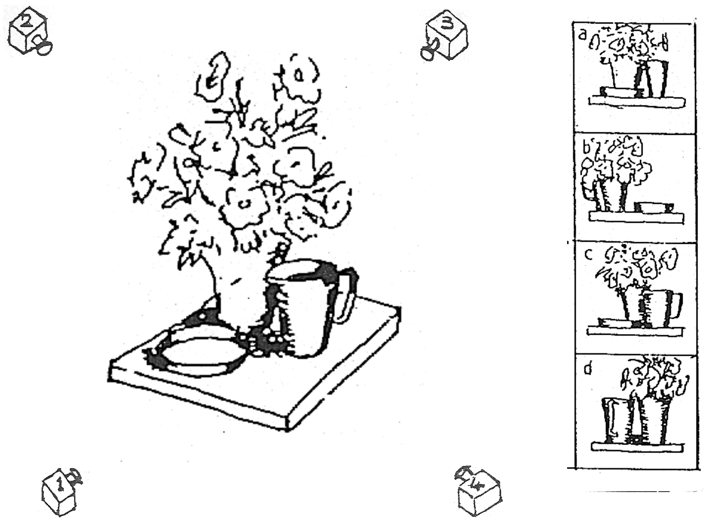
We have also attempted to vary the nature of presentation of the activities to encourage the use of different skills. Consider activities 1 and 2 below.

1. *Look carefully at this box:*



(a) Draw what you would see if you were looking at the box from ***behind****.*
(b) Draw what you would see if you were looking at the box from ***above****.*
(c) Draw what you would see if you were looking at the box from ***the side***

2. *Which camera sees which picture?*



Write your answer like this:
1 sees ….
2 sees ….
3 sees ….
4 sees ….

In each case, explain why you chose each picture.

In activity 1 the learner has to **visualise** the different views and then **draw** the mental image. But in activity 2 learners are provided with possible solutions. In such a case the learner can “match” the answers with the different views, possibly without having to form a visual image. In explaining their choices, learners will have to describe in words the relative position of the objects in the picture.

Catering for Diversity

In attempting to cater for the different needs of learners in the primary school it should be noted that the range of developmental levels of learners in the foundation and intermediate phases can be pronounced. Our experiences in trialing the MALATI materials in primary schools has shown that differences in the learner performance on spatial activities can become apparent very soon. This might be acceptable if it occurs in localised sections, for example, the drawing of nets, but it can be problematic if it occurs more generally. For if the gap between learners becomes too large, we are reducing the opportunities for learners to develop “taken-as-shared” knowledge (Cobb, 1996) required for meaningful interaction in heterogeneous groups.

We recommend, therefore, that the teacher use core materials which are both accessible and challenging for **all** learners, for example, activities requiring physical exploration. All the learners can benefit from core activities of this nature and, in fact, the van Hiele theory has suggested that learners need to work on lower levels on new topics. Diversification can then take place towards the end of the section for a few activities.

Assessment

The developmental nature of geometrical thought and spatial sense has implications for assessment. If the teacher is to provide the activities necessary for the development of a learner in these areas, a knowledge of the needs of the learner is required. Furthermore, if the learner’s development depends on having the necessary experiences, we cannot penalise a child for not having developed to a particular level.

In keeping with assessment policy for Curriculum 2005, we recommend that assessment be descriptive and used to **determine what a learner knows and can do**. The description of a learner’s performance at a particular point can be used for planning subsequent instruction.

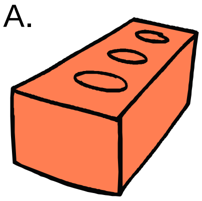
Researchers have suggested that written tests are not appropriate for the assessment of spatial sense and that this should be done using a range of assessment tools (Bishop, 1989; Johnson & Meade, 1987). Teachers in MALATI project schools have found that what can be assessed in a time-limited test does not reflect the diversity of the activity that has taken place in their classrooms. When looking at a written or drawn response, a teacher has often found a need to talk to the learner in order to clarify the response.

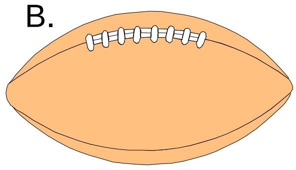
Bearing in mind that we want to capture a range of skills (visual, verbal, drawing, mental, tactile, logical and application), it is necessary to design tools that will capture learners’ performance in these areas and to develop methods of recording this performance.

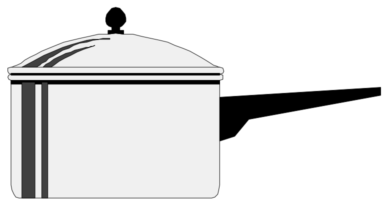
In trialling the MALATI materials in classrooms we have found it useful to assess learners during class-time, rather than in a time-limited test or examination. This provides the teacher with an opportunity to discuss responses with learners where necessary. We have also developed and used short projects for assessing the development of spatial sense. An example of a grid for recording learner performance which was developed by MALATI staff and teachers in a MALATI project school is shown on the next page. This grid provides a description of the nature of the skills required to complete an activity. Teachers appear to value this form of assessment as it reflects what their learners can do and "paints a picture" of each learner in the class.

We give an example of an assessment activity below and the accompanying grid on the page that follows.

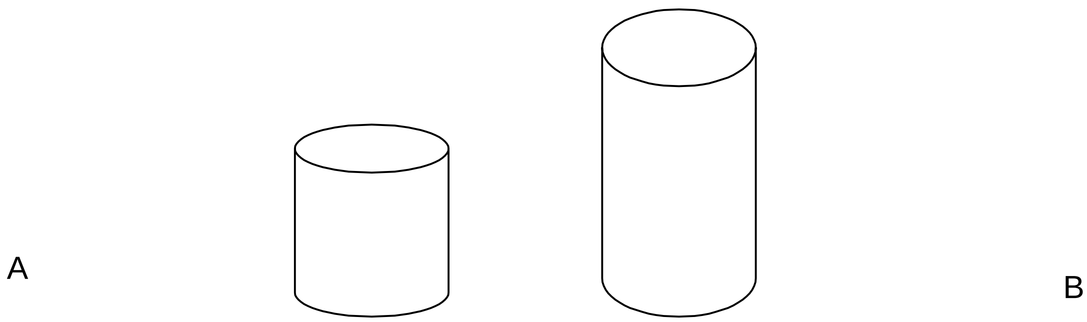
1. Look carefully at these pictures of objects. In each case draw the following:
   (a) the **front view** of the object
   (b) the **rear view** of the object
   (c) the **aerial view** of the object
   (d) the **side view** of the object.

A.


B.


C.


2. This is a picture of two cylinders. Draw what you would see if you were looking at the cylinders:
   (a) directly from the front
   (b) directly from behind
   (c) from point A on the side
   (d) from point B on the side.



Class: **Date:**

NAME	Understands notions of “aerial view”, “side view”, “front view”	Visualises different views of objects (represented in pictures)		
		One Object (in picture)	Two Objects (recognises from given picture or description)	Left/Right Discrimination

Not yet attained	1
Partially Attained	2
Attained	3
Attained with Merit	4

MALATI Primary Geometry and Curriculum 2005

As noted above, Curriculum 2005 presents a broader notion of geometry than has traditionally been studied in the primary school. The MALATI materials and methods of assessment reflect this new trend. We have found it useful to organise the geometry in the curriculum in broad categories, namely, shape, motion, vision and position. These should not, however, be regarded as distinct, unrelated categories. The table below shows the organisation of the Specific Outcomes into these categories.

Specific Outcome	Assessment Criteria	Category
SO7	1. Descriptions of the position of an object in space	Position
	2. Descriptions of changes in shape of an object	Motion, Shape
	3. Descriptions of the orientation of an object	Position, Vision
	4. Demonstrate an understanding of the interconnectedness between shape, space and time	Position, Vision, Motion, Shape
SO8	2: Representation of natural forms, cultural products and processes in a mathematical form	Shape
	3: Generation of ideas through natural forms, cultural products and processes	Shape
SO2	2: Evidence that number patterns and geometric patterns are recognised and identified using a variety of media	Shape, Motion
	4: Exploration of patterns in abstract and natural contexts using mathematical processes	Shape Motion
SO4	4. Demonstrate knowledge of the use of mathematics in determining location	Position, Vision

In addition it should be noted that important mathematical thinking skills (SO10) are developed and required in the MALATI activities and geometrical terminology (SO9) is introduced and reinforced throughout the packages.

References

Bennie, C. J. (1999) **Building a model of spatial ability: An analysis of Grade 5 and 6 learners' strategies for solving 'spatial' activities**. Unpublished master's dissertation, University of Cape Town, Cape Town, South Africa.

Bishop, A. J. (1983) Space and geometry. In Lesh, R. & Landau, M. (Eds.) **Acquisition of mathematics concepts and processes**. New York: Academic Press.

Bishop, A. J. (1989) Review of research on visualisation in mathematics education. **Focus on learning problems in mathematics, 11,** 7-16.

Cobb, P. (1996) Accounting for mathematical learning in the social context of the classroom. In Alsina, C., Alvarez, J. M., Hodgson, B., Laborde, C. & Pérez, A. (Eds.), **Selected lectures from the 8th International Congress on Mathematical Education**, 85-100. Sevilla, Spain.

Clements, D. H. & Battista, M. T. (1992) Geometry and spatial reasoning. In Grouws, D. A. (Ed.), **Handbook of research on mathematics teaching and learning**. New York: MacMillan.

Clements, M. A. (Ken) (1983) The question of how spatial ability is defined and its relevance to mathematics education. **Zentralblatt für Didaktiek der Mathematik, 15,** 8-20.

Del Grande, J. (1987) Spatial perception and primary geometry. In Lindquist, M. M. & Schulte, A. P. (Eds.), **Learning and teaching geometry, K-12** [1987 Yearbook]. Reston, VA: NCTM.

Department of Education (1997) **Curriculum 2005 policy document**. Pretoria, South Africa

De Villiers, M. D. & Njisane, R. M (1987) The development of geometric thinking among black high school pupils in KwaZulu (Republic of South Africa). Bergeron, J. C, Herscovics, N. & Kieren, C. (Eds.), **Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education, 3,** 117-123.

Education Department of Western Australia (1994) **Student outcome statements** [working edition].

Herman, J. F. and Siegal, A. W. (1978) The development of cognitive mapping of the large-scale environment. **Journal of Experimental Child Psychology, vol 26,** 389-406.

Hoffer, A. (1981) Geometry is more than proof. **The Mathematics Teacher, 74,** 11-18.

Guay, R. B. & McDaniel, E. D. (1977) The relationship between mathematics achievement and spatial ability among elementary school children. **Journal for Research in Mathematics Education, 8,** 211-215.

Johnson, E. S. & Meade, A. C. (1987) Developmental patterns of spatial ability: An early sex difference. **Child Development, 58,** 725-740.

Lean, G. and Clements, M. A. (1981) Spatial ability, visual imagery and mathematical performance. **Educational Studies in Mathematics, 12,** 267-299

Lehrer, R. Jacobson, C., Kemeny, V. and Strom, D. (1999) Building on Children's intuitions to Develop Mathematical Understadning of Space. In Fennema, E. and Romberg, T. A. (Eds), **Mathematics classrooms that promote understanding** (pp. 63-88). Mahwah, New Jersey: Lawrance Erlbaum Associates.

Mitchelmore, M. C. (1980) Prediction of developmental stages in the representation of regular space figures. **Journal for Research in Mathematics Education, 11,** 83-93.

National Council or Teachers of Mathematics (1998) **Principals and standards for school mathematics (Draft Document)**. Reston, VA.

Presmeg, N. C. (1992) Prototypes, metaphors, metonymies and imaginative rationality in high school mathematics. **Educational Studies in Mathematics, 23,** 595-610.

Senk, S. L. (1989) Van Hiele levels and achievement in writing geometry proofs. **Journal for Research in Mathematics Education, 20,** 309-321.

Smit, S. (1998) **An introduction to geometry through shape, vision and position**. Unpublished manuscript, University of Stellenbosch, Stellenbosch, South Africa.

Smith, E. C. (1987) **'n Kwaliatiewe en kwantitatiewe vergelyking van enkele van Hiele-toetsinstrumente.** Unpublished master's thesis, University of Stellenbosch, Stellenbosch.

Van Niekerk, R. (1995) From spatial orientation to spatial insight: A geometry curriculum for the primary school. **Pythagoras, 36,** 7-12.

Wessels, D. & Van Niekerk, R. (1998**) Semiotic models and the development of secondary spatial knowledge**, Short oral communication at the 22nd Conference of the International Group for the Psychology of Mathematics Education, Stellenbosch, South Africa.

Usiskin, Z. (1982*)* **Van Hiele levels of achievement in secondary school geometry**. *CDASSG Project,* Chicago University.