Malati

Mathematics learning and teaching initiative

ALGEBRA

Module 5

The linear function

Grades 8 and 9

TEACHER DOCUMENT

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OVERVIEW OF MODULE 5

In this Module learners explore and analyse the characteristics of the linear function $y = \mathrm{a}x + \mathrm{b}$ and the effect of the parameters a and b on the behaviour of the function and on the form of the graph of the function. Learners will reflect on functions that they have encountered in the previous modules.

This Module has been designed to use the (TI-82) graphing calculator. We enclose a useful manual with key procedures for the TI-82 graphing calculator.

However, if learners have access to a computer teachers may use
- the *Graphmatica* computer graphing program as an alternative. You can install Graphmatica from our software page. Here are some guidelines and activities using Graphmatica as a teaching aid (it is a PDF file).
- the enclosed *polynomial graphing java applet* as an alternative. You can install the graphing applet from our software page.
- a graphing applet on the world-wide web as alternative, for example *Function Grapher* at http://147.4.150.5/~matsrc/Graf/Graf.html

We also include some Excel activities that teachers may use as optional extra activities.

Teachers should also note that they can install Graph Paper Printer from our software page so that learners can draw some point-by-point graphs on graph paper.

What is important about the dynamic nature of the variable is not only the change in the values of variables but the effects of those changes on the value of other (dependent) variables. Families of functions like the linear, quadratic, exponential, etc. are studied because they depict different kinds of ways variables effect each other, and they are reasonable models for relationships among real world quantities.

Rather than allowing learners to roam freely in problem solving explorations, the suggested sequence of activities intends to enhance learners' development of the particular concepts.

Learners are not expected to master each concept and procedure when they *first* encounter it , but rather to *develop* their mathematical understandings continually. The activities aim to clearly expose the relationship between a linear graph and its expression.

At the end of this module the learners should be able to draw the graph of a linear function from the algebraic expression without the table as an intermediary step and also be able to construct the algebraic expression from the graph.
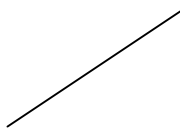
ACTIVITY 1   CLASSIFYING FUNCTIONS BY THEIR GRAPHS

One by one, enter each of the algebraic expressions given below into the graphing calculator and look at the shape of each graph. Decide whether the graph is a straight line or some other curve. (Remember that you are only looking at a part of the graph). Then rewrite the algebraic expression in the correct column in the table.

(a) $10 - x^2$

(b) $5x - 15$

(c) x

(d) x^3

(e) $-4x^2$

(f) $\dfrac{x^2}{5}$

(g) $\dfrac{x}{5}$

(h) $-6x$

(i) $(x + 6)^2$

(j) 15

(k) $x^2 + 18$

(l) $2x - x^2$

(m) $50 - 5x$

(n) $10x$

(o) x^2

(p) $x^3 + 9x$

(q) -20

(r) $x + 8$

(s) $10 - x$

(t) $-3x + 18$

(u) $-3x^2 + 5$

(v) $-4x - 8$

(w) $x^4 - 5$

(x) $x^2 - 5$

(y) $\dfrac{5}{x}$

Makes a straight line	Makes another curve

Look at the expressions that you have put in the column "makes a straight line". The functions that they represent are called **linear functions**.

What do these expressions have in common?

Try to create *three* expressions that make *straight lines*.
Check your suggestions with the help of the graphing calculator.

Teacher Note: Activities 1 and 2

The main purpose of Activities 1 and 2 is to categorize functions into linear and non-linear functions by the shape of their graphs and by noting differences in their expressions.

In Activity 1 the learners should enter the expressions one by one into the graphing calculator and classify the functions according to the shape of the graph. They need to be reminded that the graph that they see is only a portion of the function and that they may change the window setting to see more of the function.

Teachers are reminded that some children have a concept image of straight lines as only those that are horizontal or vertical, e.g. they see the following as straight lines

Yet, they do not see the following as a straight line:

In the whole class discussion teachers could ask the children why do they think we call these functions **linear** (LINEar).

ACTIVITY 2   CLASSIFYING FUNCTIONS BY THEIR EXPRESSIONS

1. Look carefully at the following expressions for functions. Just by looking at the expression, try to decide whether their graph will be a straight line (linear) or some other curve. Write the expression in the column of the table that you think corresponds to the shape of the function.

   (a) $3x - 7$

   (b) $3x^2 - 7$

   (c) $5x^2$

   (d) $100 + 2x$

   (e) $55x$

   (f) $10 - x^3$

   (g) $-7x^2 + 4x$

   (h) $-3x - 7$

Makes a straight line	Makes another curve

2. Write down what you were looking for when you made your decisions.

ACTIVITY 3
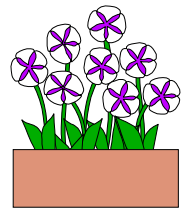
1. Mr Bean buys a plant at the nursery when it is 8 cm high. He measures the height of the plant at the end of every week and finds that it grows at a rate of 2 cm per week.

    (a) Complete the table showing the growth of the plant over time if the plant continues to grow at the same rate.

Week number	Height of plant (in cm)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

    (b) Write down the algebraic rule that you used to calculate the height of the plant.

    (c) Represent the information from the table on the Cartesian plane.

    (d) Can the points that you plotted be joined? Explain.

    (e) Can the graph be extended farther to the right? Explain.

    (f) Can the graph be extended farther to the left? Explain.

    (g) If you think that the points can be joined, join them.

2. Sam buys a plant at the nursery when it is 12 cm high. He measures the height of the plant at the end of every week and finds that it also grows at a rate of 2 cm per week.

    (a) Make a table to show the growth of Sam's plant.

    (b) Write a formula to calculate the height of the plant.

    (c) Draw a graph to show the height of Sam's plant on the same Cartesian plane as the graph of Mr Bean's plant.

3. Carla bought plant seeds that must be planted and completely covered with soil. She also measured it every week and observed that it grew 2 cm per week.

   (a) Make a table to show the height of Carla's plant over time.

   (b) Write a formula to calculate the height of the plant.

   (c) Draw a graph to show the height of Carla's plant on the same Cartesian plane as the graph of Mr Bean and Sam's plants.

4. Write down the height of Mr Bean, Sam and Carla's plants at zero weeks.

5. Study the three graphs, their tables and formulae.
   Write down what is the same and what is different.

> The formula or algebraic rule for a linear function is often written as:
>
> $$f(x) = ax + b \qquad \text{or} \qquad y = ax + b \qquad \text{or} \qquad y = mx + c$$

ACTIVITY 4

Consider the following linear functions:

$$y = 2x + 7$$

$$y = -\frac{1}{2}x - 3$$

$$y = \frac{4}{3}x + 2{,}5$$

$$y = 2x - 1{,}5$$

$$y = 4 - \frac{3}{2}x$$

$$y = -5 + 3x$$

Draw each of the above functions on the graphing calculator and say where each graph crosses the vertical axis.

Try to formulate a rule about the point where a linear graph crosses the vertical axes.

Check your rule with some examples you have made.

Teacher Note: Activities 4, 5, 6 and 7

The main aim of Activities 4, 5, 6 and 7 is to analyse the characteristics of a linear function and the effect of the parameters on the behaviour of the linear function represented by the algebraic formulae:

$$f(x) = ax + b$$
$$y = ax + b$$
$$y = mx + c$$
$$ax + by + c = 0$$

In the whole class discussion on the similarities and differences of the linear graphs, the concept of slope will necessarily arise. The teacher needs to ensure that the term 'slope' is understood (i.e. steepness).

In their groups they should examine the effect on the expression if the slope is the same and if it varies. They should then be able to identify which parameter effects the slope of the linear graph.

Similarly, they should be able to identify which parameter effects the point where the graph intersects the y-axis. In the discussion the teacher needs to point out that this point is called the 'y – intercept'.

Activities 3 and 5 could be extended so that learners explore and are able to identify a point on the line by its co-ordinates.

The understanding of the concepts domain, range, continuity which were 'dripped' in Module 3, could also be further developed in this Module when the following questions in Activity 3 are discussed:

(h) Can the points that you plotted be joined? Explain.
(i) Can the graph be extended farther to the right? Explain.
(j) Can the graph be extended farther to the left? Explain.

Learners should be provided with more examples to *consolidate* the ideas in this Module.

It might be a good idea to have a short assessment at this point to identify learners' needs and then to provide them with appropriate tasks to cater for their *individual* needs.

ACTIVITY 5
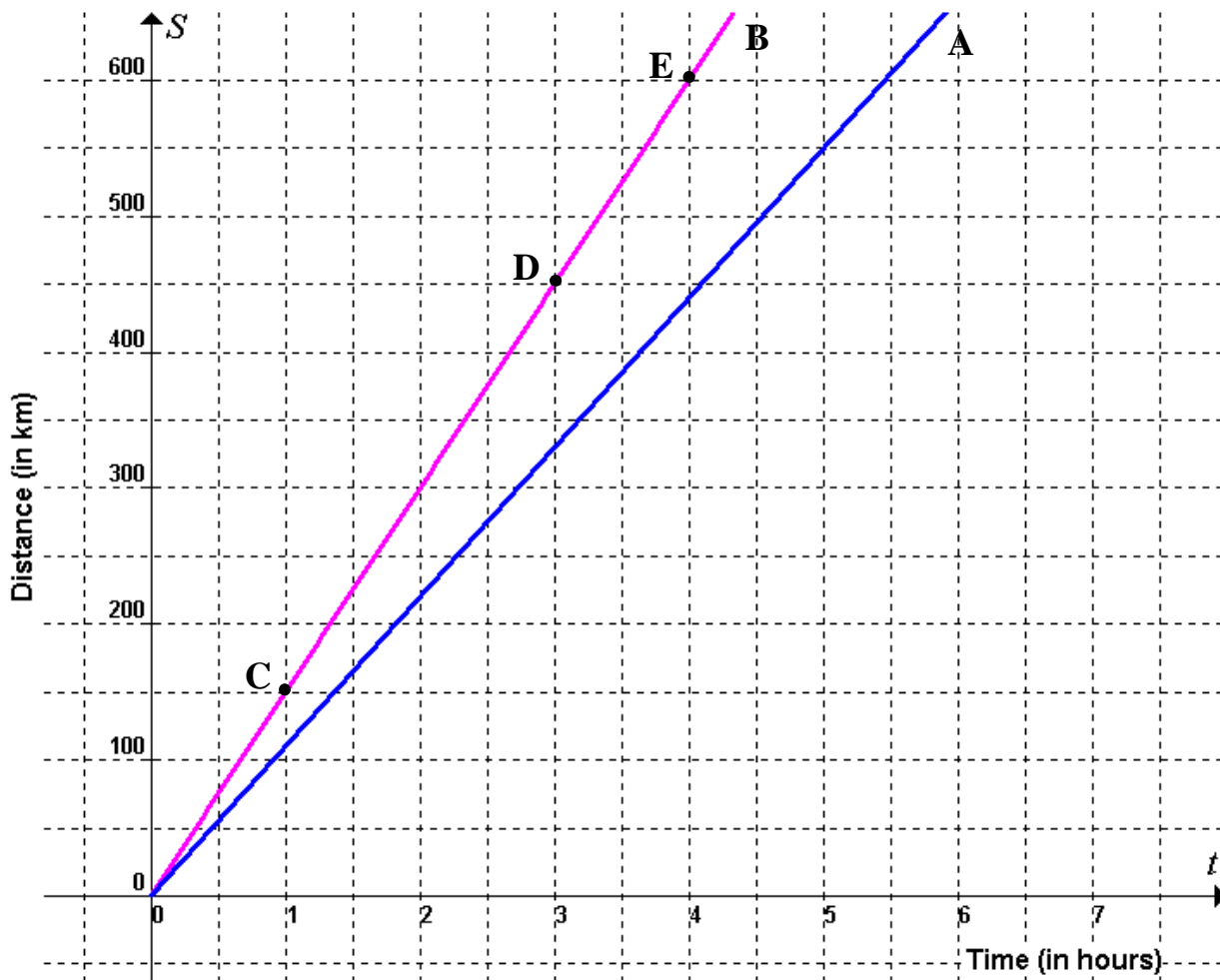
Two motorists A and B each traveled at a constant speed. The distances (*S* km) they covered after *t* hours are represented in the graphs above.

1. Use the graphs to make a table of time-distance values for each motorist.

2. What is the speed of motorist B between points
    (a)   C and D
    (b)   C and E
    (c)   D and E
    Explain your answers. What do you notice?

3. What is the speed of motorist A?

4. Write an algebraic formula that represents the relationship between time and distance for each motorist.

5. Which part of the algebraic *formulae* represents the speed of each motorist?

6. Explain how you can tell from the *tables* at which speed each motorist is driving.

7. Explain how you can tell from the *graphs* at which speed each motorist is driving.

ACTIVITY 6
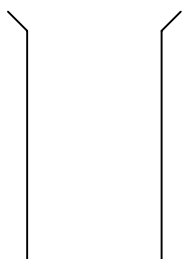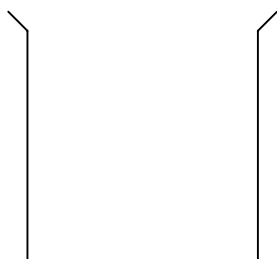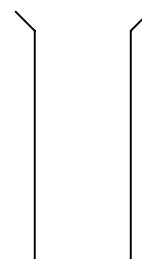
Beaker X   Beaker A   Beaker B

In order to calibrate the above beakers, it is necessary to know how the height of the liquid depends on the volume in the beaker.

The sketch graphs below represent the height-volume relationship for each beaker on the same system of axes.

In your group discuss which graph best represents the height-volume relationship for each beaker. Explain your choice in each case.
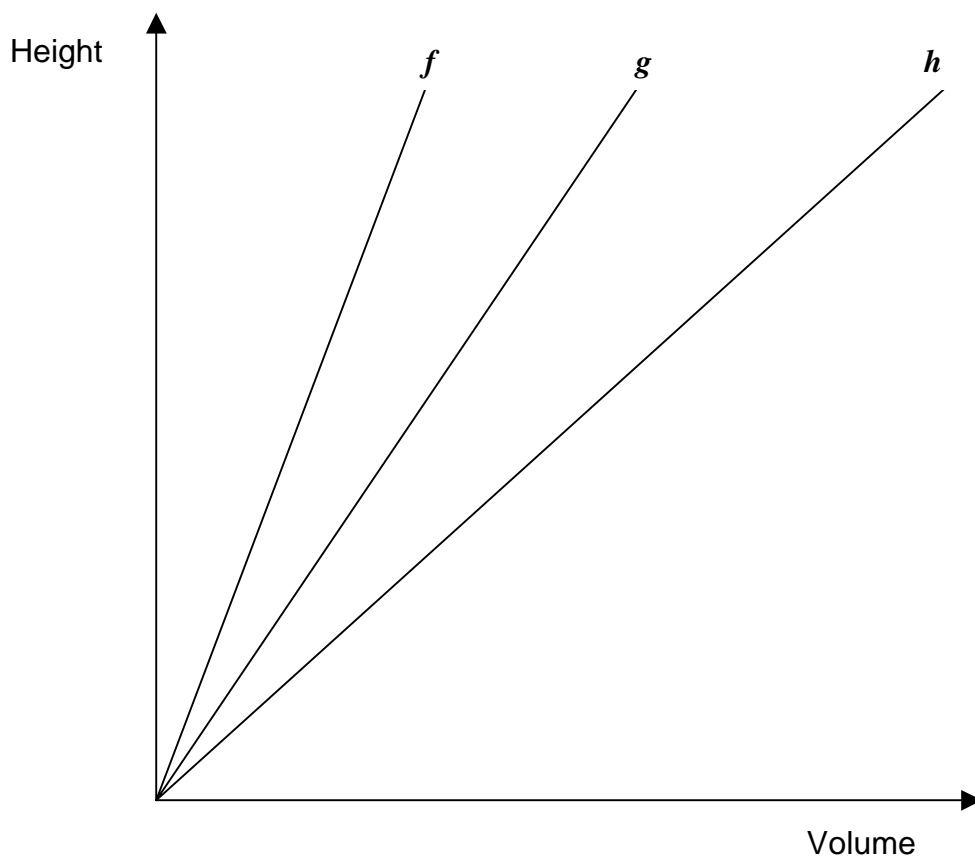
ACTIVITY 7
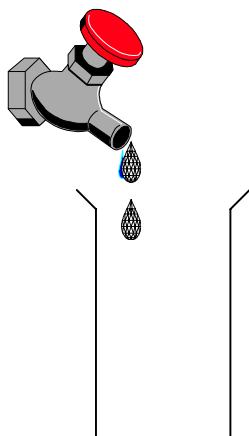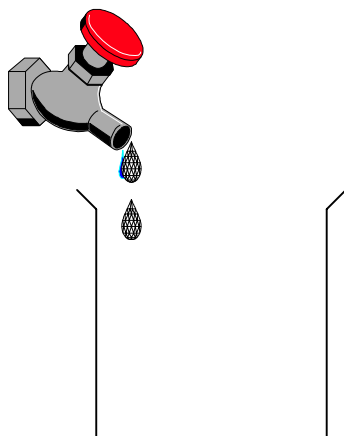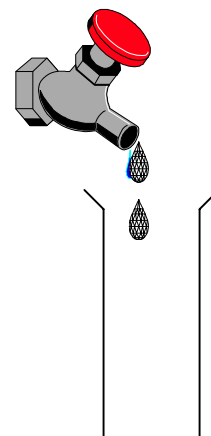
Beaker X   Beaker A   Beaker B

Water drips simultaneously into the above beakers from three different taps which deliver water at the *same* constant rate (millilitres per second).

On the same system of axes sketch graphs to show how the height of the water in each beaker varies with time.
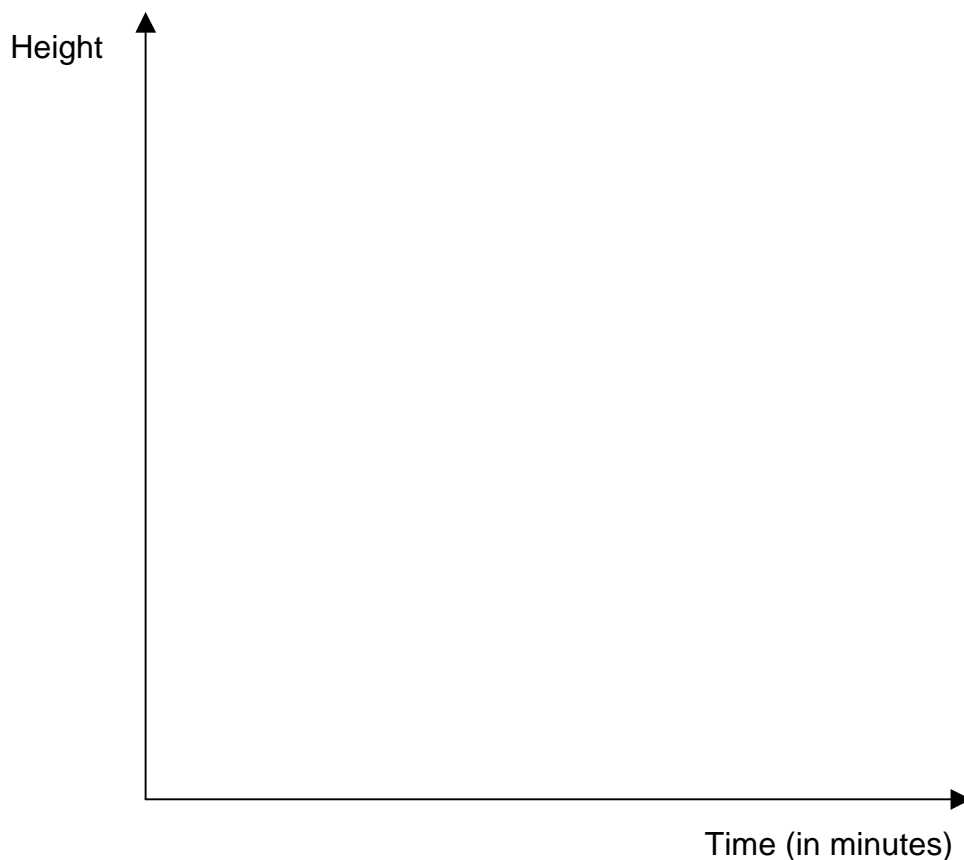
ACTIVITY 8   THE GRADIENT
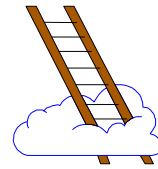
1. Complete the table. What do you notice?

x	-2	-1	0	1	2	3	4	5	6	Difference between consecutive output values
$2x$										
$2x+1$										
$\frac{1}{2}x-2$										
x^2										
$\frac{4}{x}$										

The amount by which the function value changes when the input variable is increased by 1 is called the **gradient** of the function.

2. On the graph paper below draw the graphs of

$f(x) = 2x$, $g(x) = 2x + 1$, $h(x) = \dfrac{1}{2}x - 2$, $p(x) = x^2$ and $k(x) = \dfrac{4}{x}$

What do you notice?
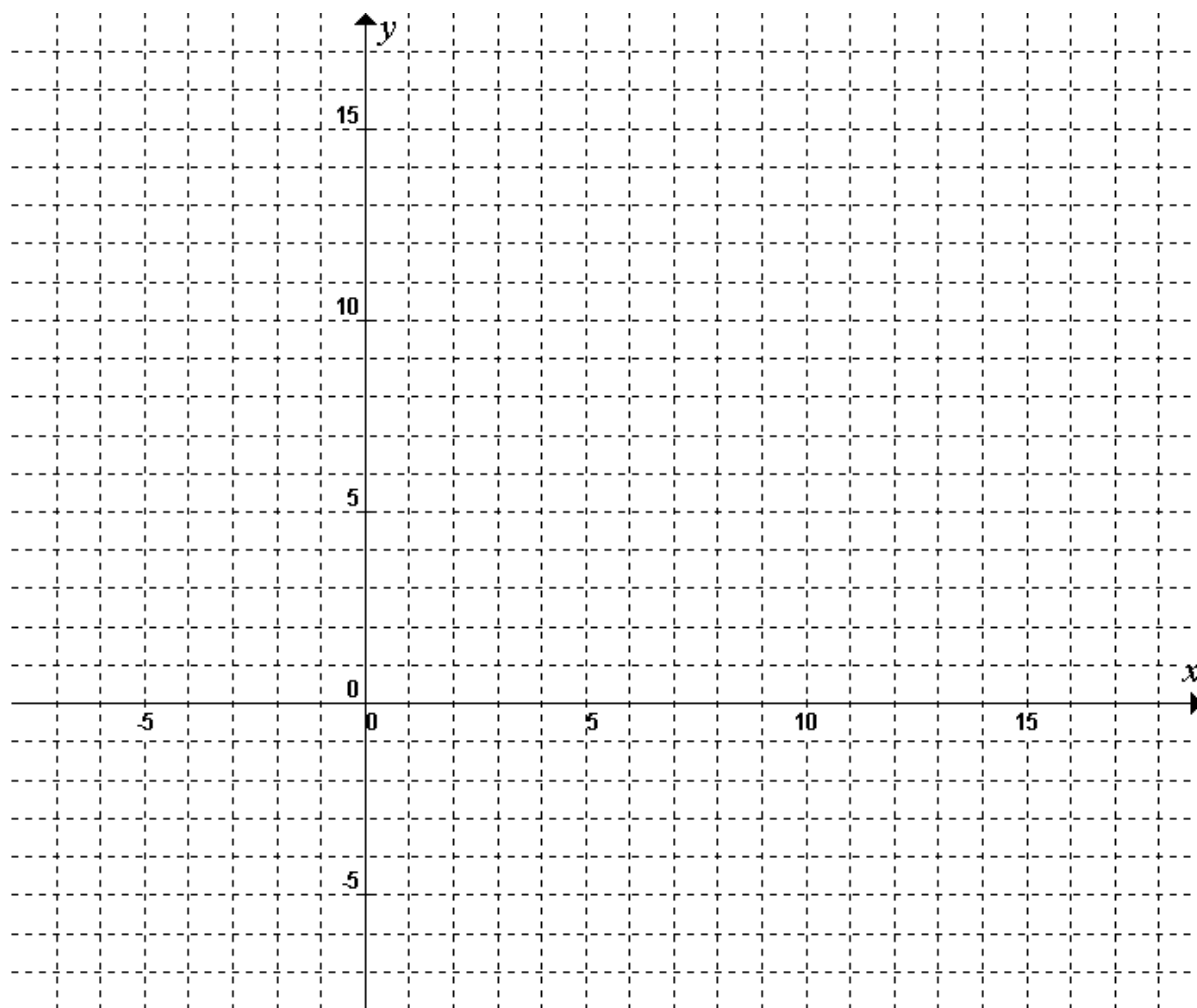
3. Some language about functions:

> g is a linear function with gradient 2,5. When the value of the input variable is 4, the value of g is 22,5.

What is the value of g when the input variable is 5?

What is the value of g when the input variable is 1?

What is the value of g when the input variable is 0?

What is the value of g when the input variable is 100?

What is the value of g when the input variable is 2,4?

Here is a shorter way to ask questions like those above:

The value of g when the input variable is 5 is indicated by the symbol $g(5)$.

$g(5)$ is read as "g of 5" or "the value of g at 5".

So instead of asking "What is the value of g when the input variable is 5?",
one
may simply ask "What is the value of $g(5)$?" or "Calculate $g(5)$".

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4. If $f(x) = -4x + 5$ and $h(x) = -4x^2 + 5$, find the value of :

(a) $f(3)$ and $h(3)$

(b) $f(-2)$ and $h(-2)$

(c) $f(100)$ and $h(100)$

(d) $h(4) - h(0)$

(e) $\dfrac{f(7) - f(2)}{7 - 2}$

(f) $\dfrac{h(2) - h(-1)}{2 - (-1)}$

5. Consider the graph of $f(x) = 2x + 1$ that you drew in question 2.
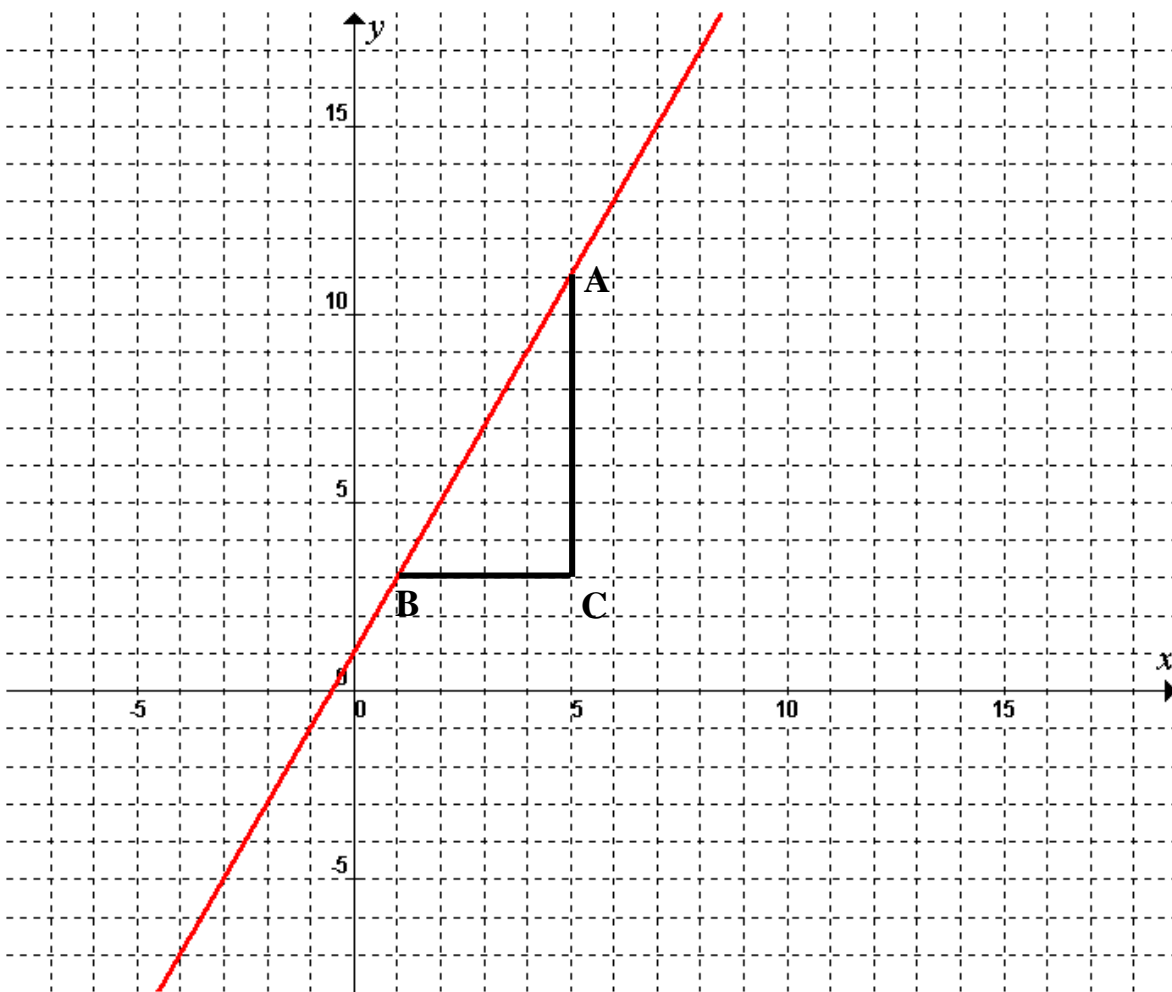
The length of line-segment BC represents an increase in the input values x, from $x = 1$ to $x = 5$ $(5 - 1 = 4)$ while the length of CA represents a corresponding increase in the output values $[\, f(5) - f(1) = 11 - 3 = 8\,]$.

Thus: $\dfrac{CA}{BC} = \dfrac{f5) - f(1)}{5 - 1} = \dfrac{11 - 3}{4} = \dfrac{8}{4} = 2$

(a) Use the graph and in a similar way calculate the following:

(i) $\quad \dfrac{f(2) - f(1)}{2 - 1}$

(ii) $\quad \dfrac{f(5) - f(2)}{5 - 2}$

(iii) $\quad \dfrac{f(0) - f(-2)}{0 - (-2)}$

What do you notice?

(b) Also do the calculations in (a) for the functions $p(x) = x^2$ and $k(x) = \dfrac{4}{x}$.

What do you notice?

Teacher Notes: Activity 8

It is important that learners understand slope (gradient) not only from a graphical perspective but also numerically (algebraically). The purpose of this activity is to make a link between the graphic notion of gradient and the algebraic notion.

The graphing calculator can also be very useful in this regard, for example in question 1 learners can easily compare the formula, the graph and the table representations of the functions:

Using Y=   Using GRAPH   Using TABLE
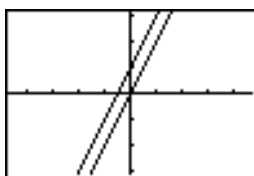
ACTIVITY 9

1. p is a linear function with a gradient of 4.
    Complete the following table of values of p.

Input variable	0	1	2	3	4	5	6	10
Value of p			21					

2. The gradient of the non-linear function q is 4 when the input variable increases from 2 to 3. When the input variable increases from 3 to 4, the gradient is 6, and when the input variable increases from 4 to 5 the gradient is 8. When the input variable increases further, the gradient itself continues to increase by 2 for every increase of 1 in the value of the input variable. Complete the following table of values of q.

Input variable	1	2	3	4	5	6	7	8
Value of q								

3. Find the gradient of each of the linear functions represented in the tables, and complete the tables:

Input variable	1	3	5	77	9	11	13	15
Value of g	5	7	9	11	13	15	17	

Input variable	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7
Value of g								

Input variable	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
Value of k	5	7	9	11	13	15	17	

Input variable	1	2	3	4	5	6	7	8
Value of k								

4.   Here is a table of values of the function m:

Input variable	1	3	3,5	5	8	12	20	30
Value of m	5	11	12,5	17	26	38	62	92

   (a) Is m a linear function?

   (b) If it is a linear function, what is its gradient?

5.   The area of a square can be calculated by multiplying the length of a side by itself, i.e.

$$area = (length\ of\ side)^2$$

   Clearly, *when the length of the side changes, the area changes too*, as is clear from the following table:

Length of side (in cm)	1	2	3	4	5	6
Area (in cm^2)	1	4	9	16	25	36

   The relationship between the area of a square and the length of a side is a function.

   We say: the area *is a function of* the side length.

   (a) Is this a linear function?

   (b) Describe the way in which the gradient of this function changes.

   (c) Complete the following table:

Length of side (in cm)	1,5	2	2,5	3	3,5	4
Area (in cm^2)						

   (d) Consider your table as well as the table given above and answer the following questions:

   Is it true to say that when the side length is increased from 2 to 3, the gradient of the area function is 5?

   What is the gradient of the area function when the side length increases from 2 to 2,5?

   What is the gradient of the area function when the side length increases from 2,5 to 3?

Teacher Note: Activity 9

Learners are given more opportunities to explore and develop their understanding of the concept of gradient numerically.
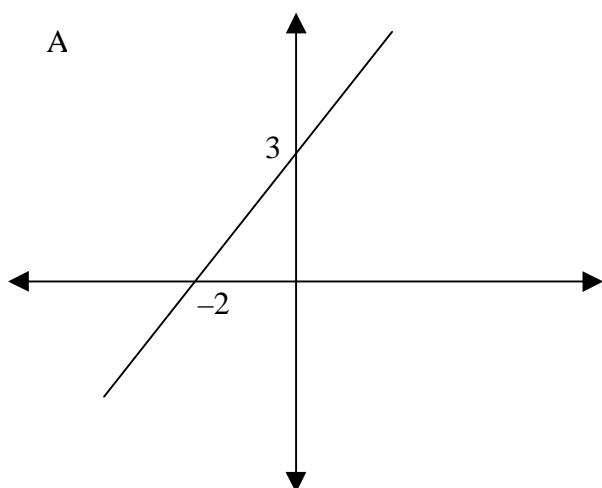
It is important that they understand the concept of gradient irrespective of whether they are working with a graph, a table of values or a formula and that they are able to make the necessary links between these different representations.

ACTIVITY 10

Express in words what information is given about the gradients of each of the following situations.

ACTIVITY 11

A — graph with y-intercept at 3 and x-intercept at −2, line rising to the right.

B — graph with y-intercept at 4 and x-intercept at 4, line falling to the right.

C — horizontal line at −3.

D — vertical line at 5.
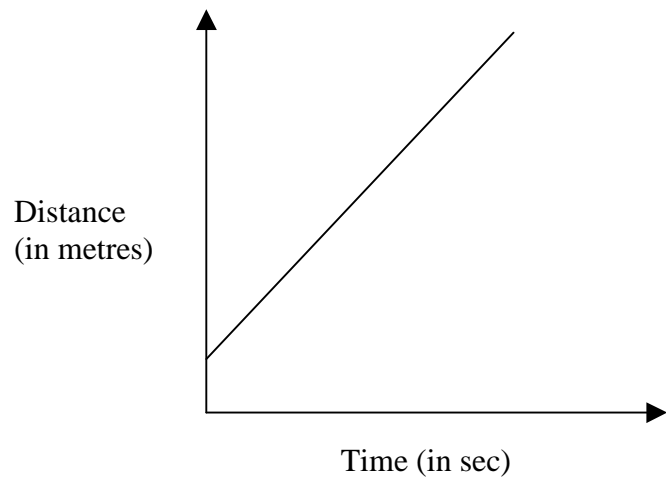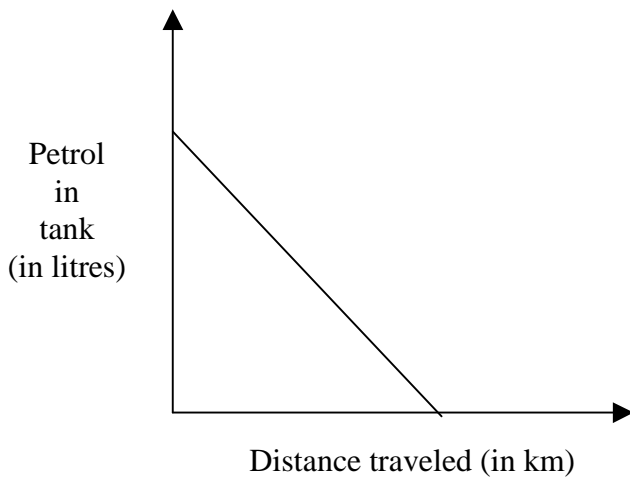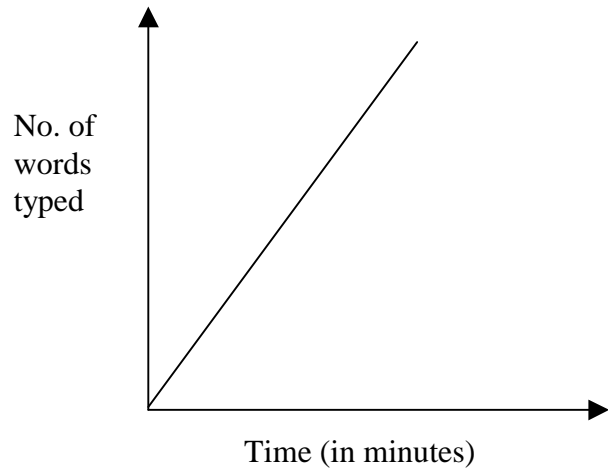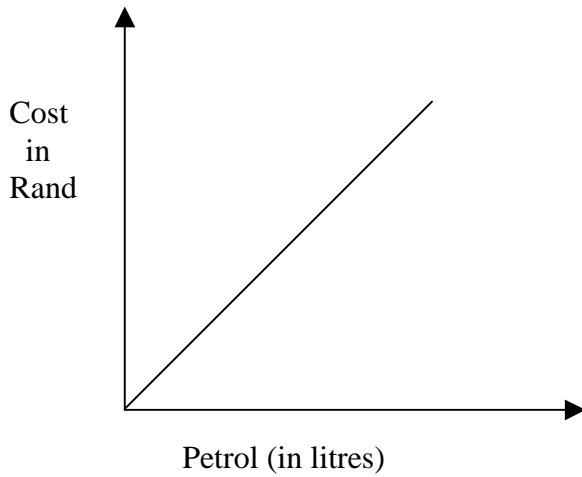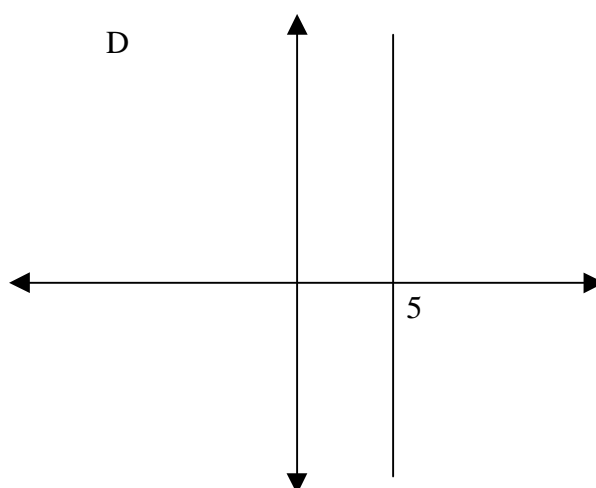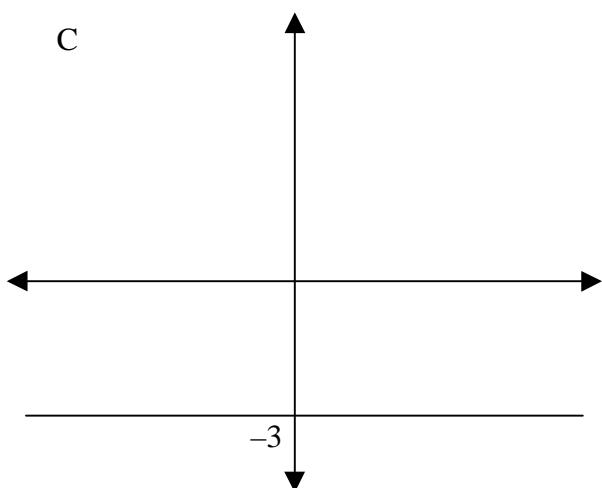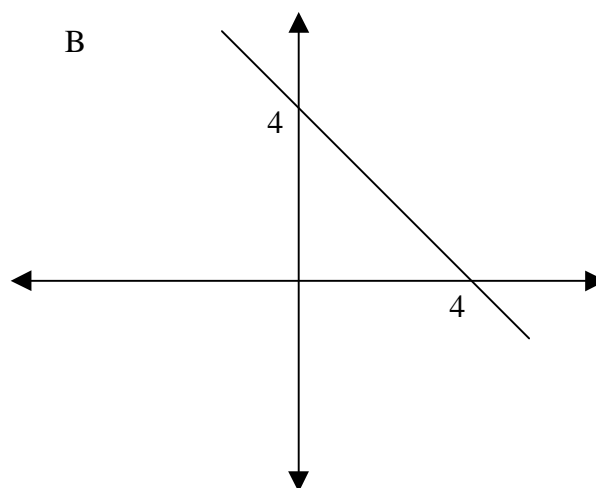
1. Where does the graph cut the y-axis in each of the above graphs?

2. What is the gradient of each of the above graphs?

3. Write an algebraic formula for each of the above graphs.
   Can you *see* the y-intercept and the gradient in the formula?

ACTIVITY 12

Let's consider $f(x) = 2x - 1$ and $g(x) = -x + 2$.
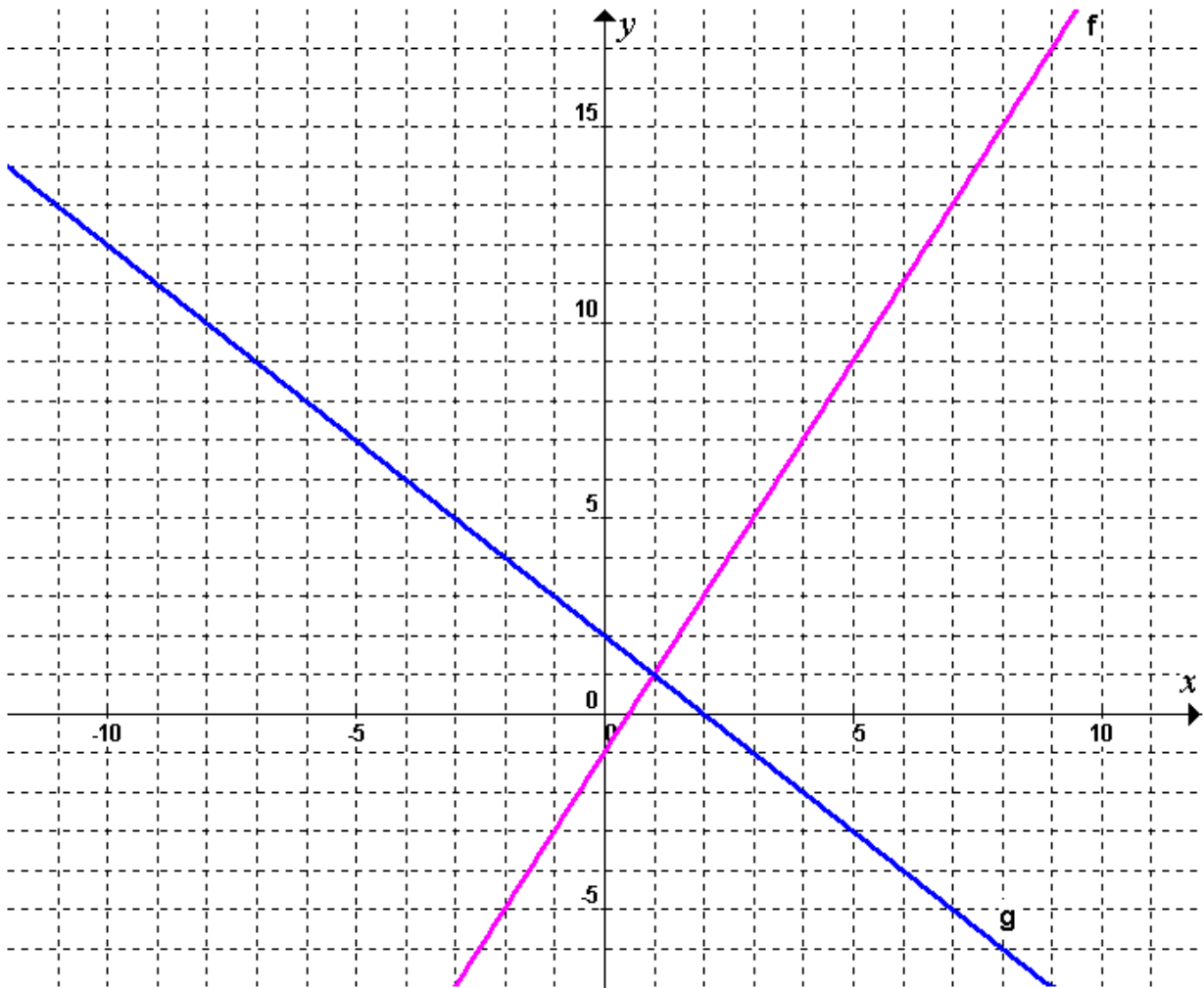
Calculate each of the following. Use the graphs to check your answer in each case. What do you notice?

$\dfrac{f(6) - f(2)}{6 - 2} =$ $\dfrac{g(6) - g(2)}{6 - 2} =$

$\dfrac{f(0) - f(-2)}{0 - (-2)} =$ $\dfrac{g(0) - g(-2)}{0 - (-2)} =$

$\dfrac{f(5) - f(1)}{5 - 1} =$ $\dfrac{g(5) - g(1)}{5 - 1} =$

$\dfrac{f(4) - f(0)}{4 - 0} =$ $\dfrac{g(4) - g(0)}{4 - 0} =$

$\dfrac{f(-1) - f(3)}{(-1) - 3} =$ $\dfrac{g(-1) - g(3)}{(-1) - 3} =$

Formulate a rule to determine the gradient of a linear function.

Teacher Note: Activity 12

The main purpose of this activity is to analyse the numerical calculations and to establish an algebraic formula for calculating the gradient of a linear function.

ACTIVITY 13

(a)
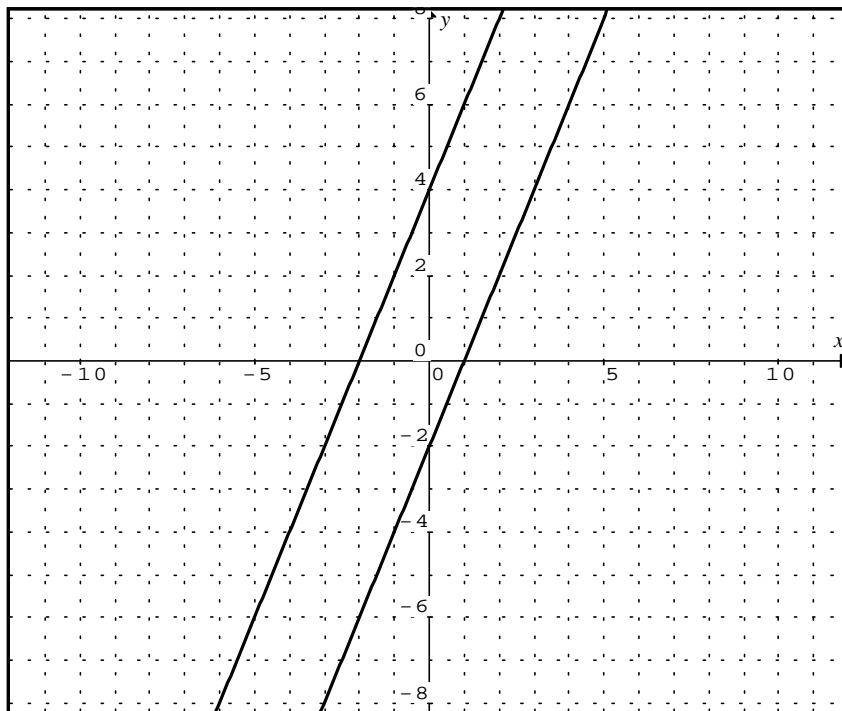
(b)
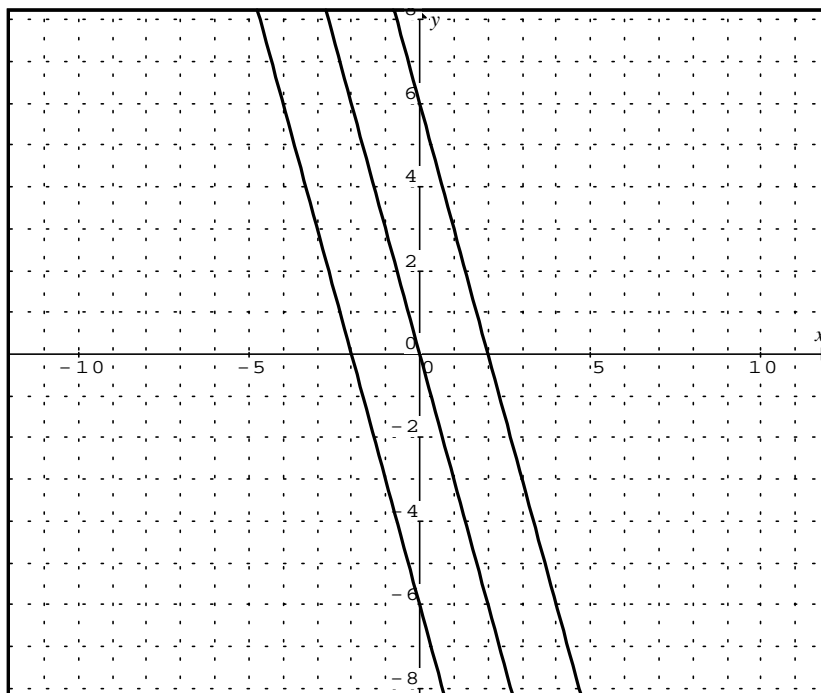
What are the gradients of the lines in (a) and (b).

Try to formulate a rule about parallel lines.

ACTIVITY 14
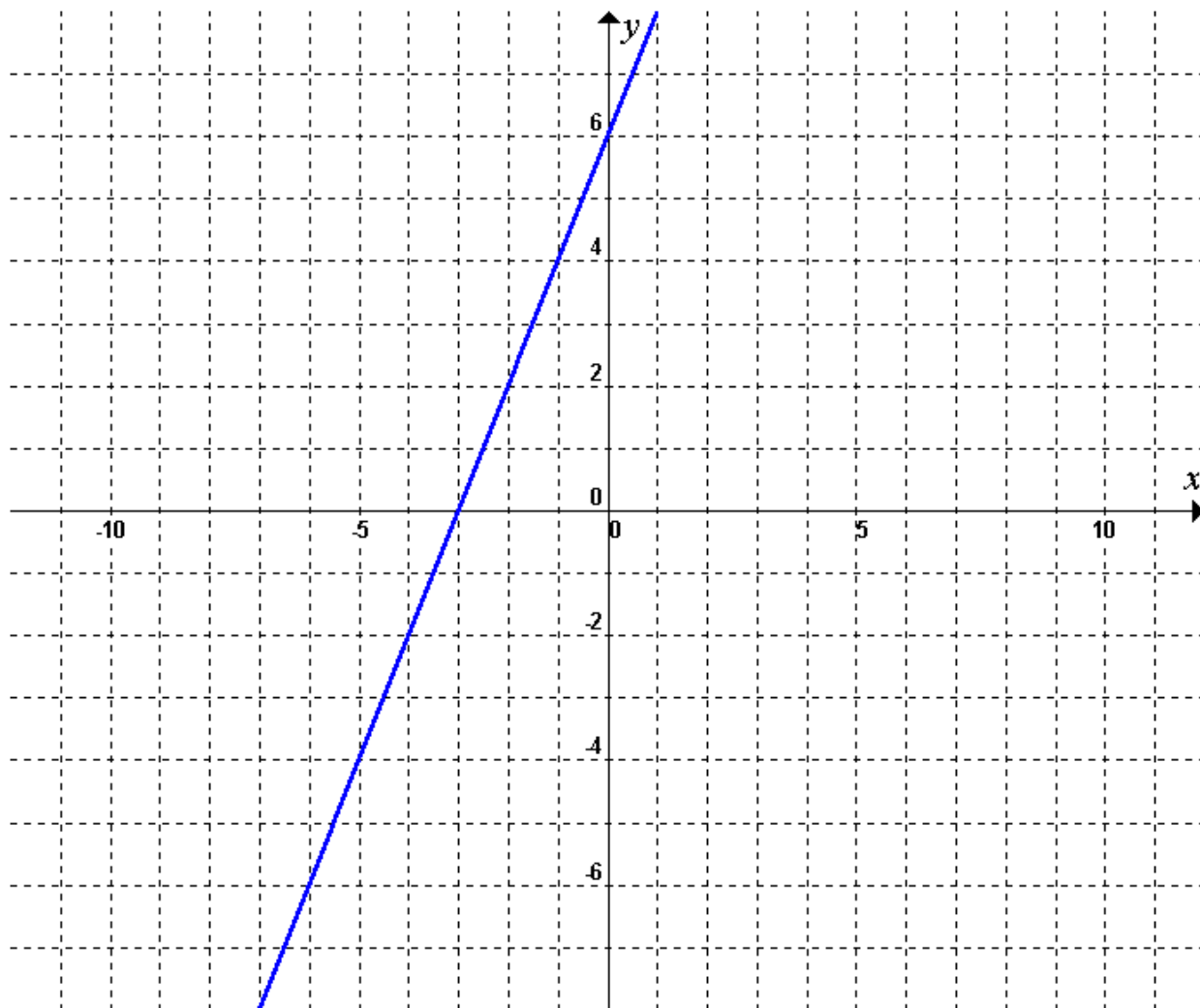
1. Write an algebraic expression for the graph above.

2. Write a story that would fit the graph above.

3. Draw the graph on your graphing calculator.

ACTIVITY 15

1. Write algebraic expressions for each of the following graphs and draw each graph on your graphing calculator.

1
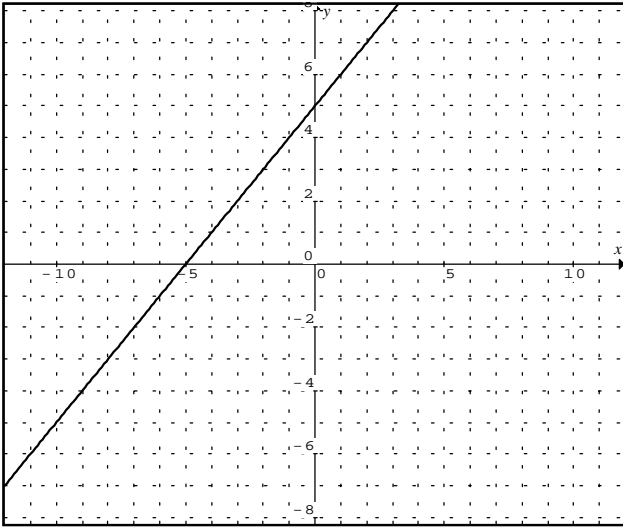
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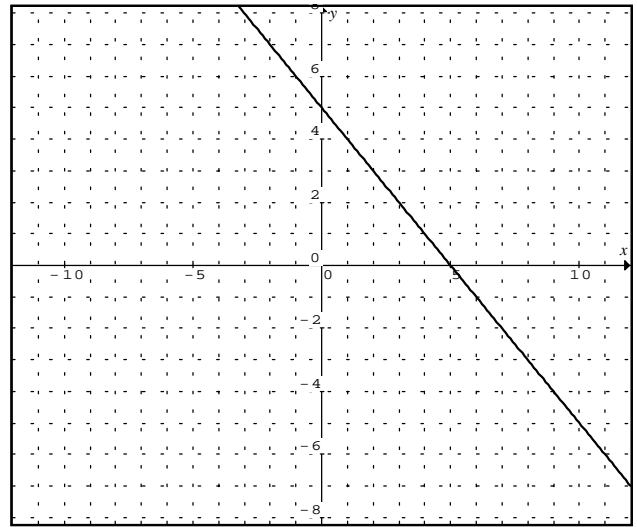
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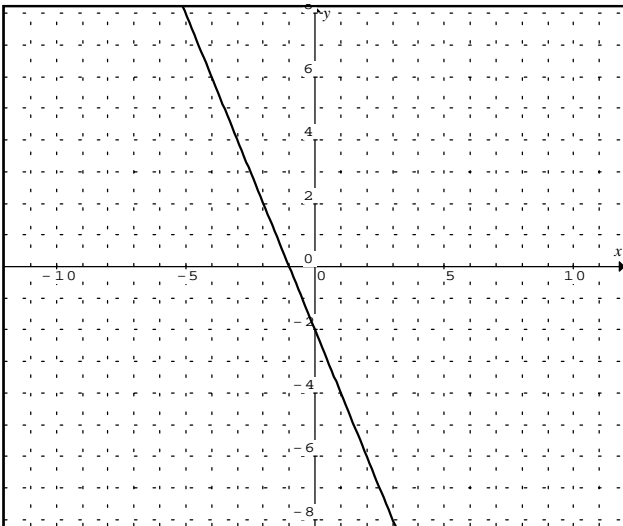
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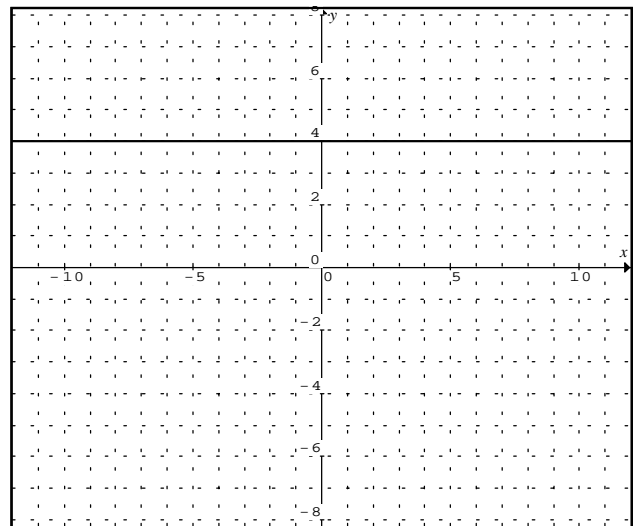
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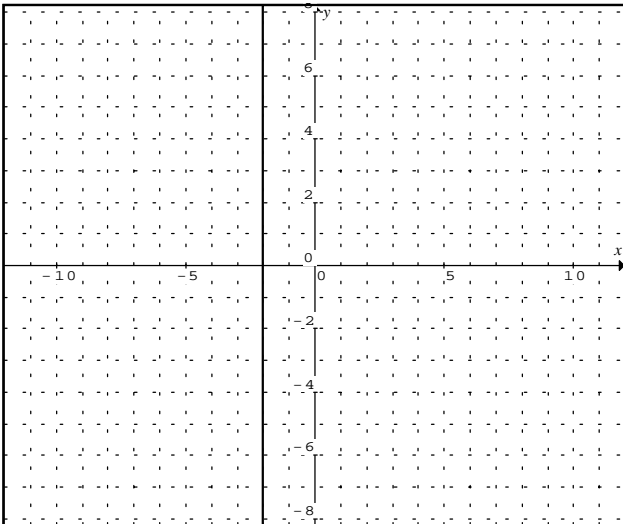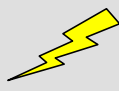
6
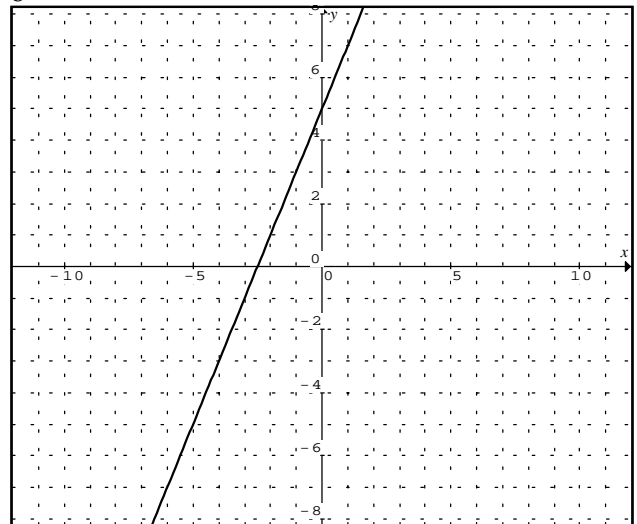
ACTIVITY 16

1. The following is Mr Tsabala's household electricity account for the month of April:

Newtown City Council Electricity account			April
	UNIT	TARIFF	AMOUNT*
Basic Fee	1 kwh	R30,00	R30,00
Units used	700,00 kwh	R0,20	R140,00
*VAT included		**Electricity Payable:**	**R170,00**

    (a) In February the Mr Tsabalas used 572 kwh of electricity. What is the amount payable?
    (b) In August the Mr Tsabala used 987 kwh of electricity. What is the amount payable?
    (c) Write down a formula that you can use to calculate the amount payable for any amount of electricity used per month.
    (d) Draw a graph to illustrate the information in (c).

2. The municipality offers households an alternative scheme to obtain electricity. They offer to install metering devices in houses. Householders then have to buy a certain amount of electricity beforehand. The tariff for this system is 25 cents per kwh for the first 500 kwh and 20 cents per kwh for each additional kwh.

    (a) How much would it cost if a household with a meter uses 700 kwh?
    (b) Write down a formula that you can use to calculate the cost for any amount of electricity used per month with the metering system.
    (c) Draw a graph illustrating the information in 2(b) on the same system of axes as 1(d).
    (d) Which system do you prefer? Give reasons for your answer.

3. In some communities people used to pay a *flat rate*, but this has mostly been changed to a *metered rate*. A *flat rate* means that a person pays the same amount every month, no matter how much electricity is used.

    (a) John pays a flat rate of R50 per month to the city council for electricity. David pays the metered rate of R20 per month for the service plus R1,25 per kwh. Complete the table to show what they pay for different usages per month.

Units used (kwh)	0	10	20	30	40	50	60	70
John (flat rate)								
David (metered rate)								

    (b) Draw a graph illustrating John and David's electricity bill for different amounts of electricity used.
    (c) Which system (model) do you prefer? Is it fair? Discuss!

Teacher Note: Activities 16, 17 and 18

Activities 16, 17 and 18 give learners opportunities to explore relationships in real-life that could be represented by a linear model.

ACTIVITY 17

AMANDA'S BISCUIT BAKES

Amanda has started a biscuit bakery. She sells her biscuits in boxes. She has employed you to calculate the costs for the various orders. Amanda charges R8,00 per box plus an administration fee of R15,00 per order.

1. How much would you charge for an order of 3 boxes? 10 boxes? 100 boxes? Write a sentence describing how you found these answers.

2. How many boxes could a customer purchase with R111? R500? R2000? Write a sentence describing how you found these answers.

3. A competitor advertises that they will sell the same boxes of biscuits for R5,50 per box plus an administration fee of R49,95 per order. When Amanda sees this advertisement she tells you to figure out how this might affect her business.

   In order to give Amanda good answer, clearly identify the problem situation, produce tables of values, graphs and algebraic rules.

ACTIVITY 18 (Enrichment)

Heart rate
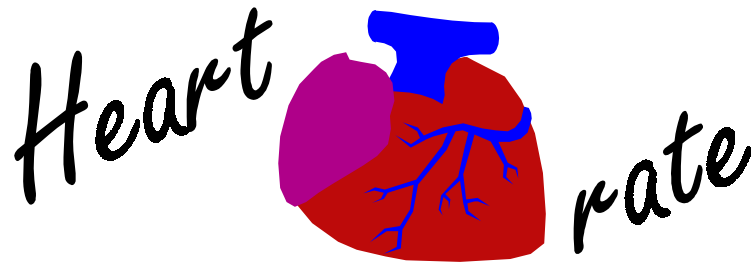
Blood carries oxygen from the lungs to the rest of the body. The blood is pumped through the body by the heart. If your heart beats faster the blood circulates around the body and back to the lungs more quickly and thus more oxygen reaches your muscles so they can work harder. We measure this heart rate in beats per minute (bpm).

1. Find your heart rate by placing your figures on your wrist and feeling the beat. Count how many times your heart beats in 1 minute. Record the result.

2. Now run energetically on the spot for 2 minutes and measure your heart rate again. What do you notice.

3. Run for another minute and measure you heart rate again. Is there any change?

Good sports people have to train hard so that their hearts get used to pumping lots of blood around their bodies. However they must be careful that they don't strain their hearts as this can lead to a heart attack. Below is a graph that sports people use to determine how hard they are exercising by measuring their heart rate. The line shows what 60% of their maximum heart rate is.
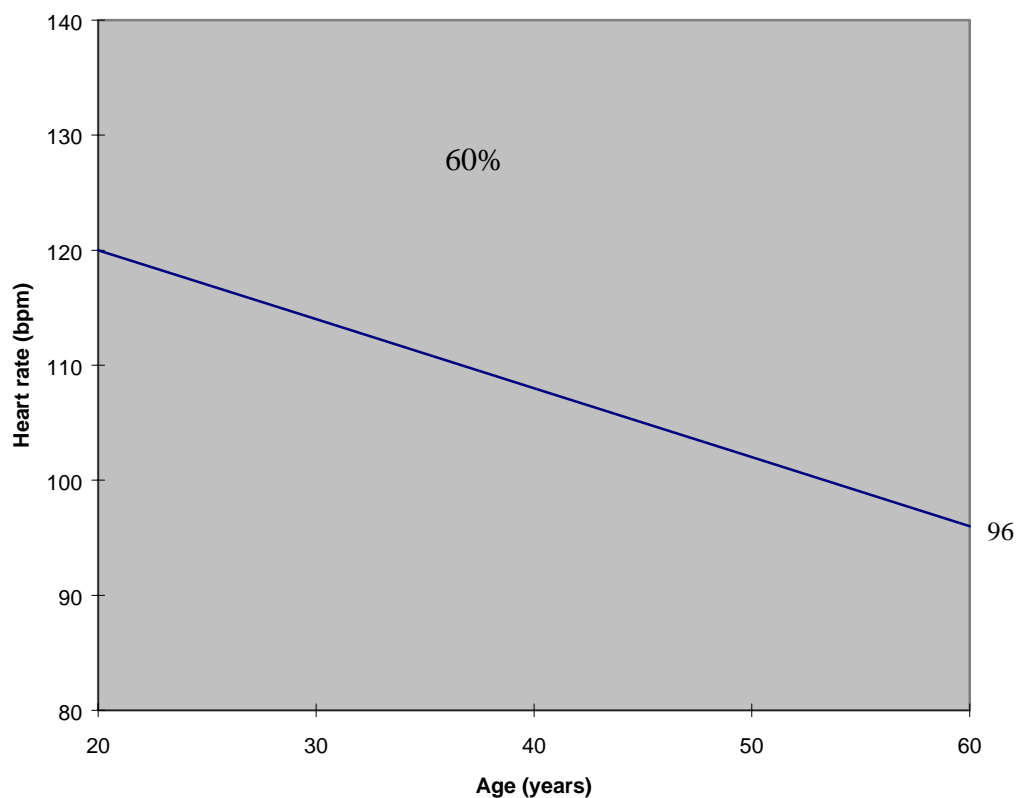
4. As you get older what happens to your heart rate?

5. Calculate the maximum heart rate of a 20 year old.

6. If an athlete exercises at 60% of his maximum heart rate, his heart rate is 115 bpm, estimate the age of the athlete.

7. Estimate what 60% of your maximum heart rate should be. Explain how you calculated this.

8. When your grandfather is 80 years old what will 60% of his maximum heart rate be?

9. Elana Meyer is 30 years old, what is 60% of her maximum heart rate? During a race her heart rate goes to 95% of her maximum heart rate, how many times per minute will her heart beat?

This was in 1997! What about now?

10. Paul Adams is 20 years old. When he is bowling in a cricket match his heart rate is 80% of his maximum heart rate, what will his heart rate be in beats per minute?
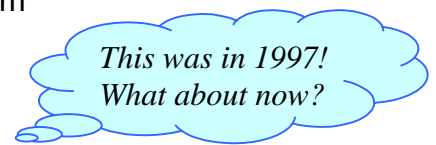
ACTIVITY 19

Graph Art

Make each of the designs below in your calculator window.

In each case *describe* the pattern in *words* and explain to others exactly how you designed the pattern (write down the functions you used).
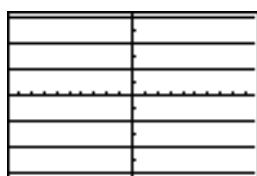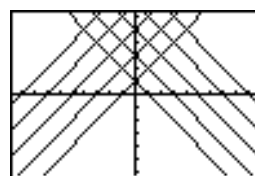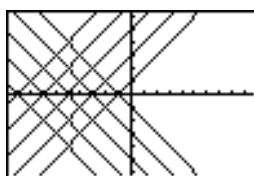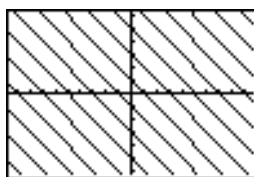
Note:

To begin, set the **WINDOW** on your TI-82 graphing calculator as shown.

Also clear all functions in **Y=**

```
WINDOW FORMAT
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Xmax=9.6
Xscl=1
Ymin=-6.2
Ymax=6.2
Yscl=1
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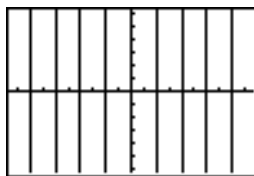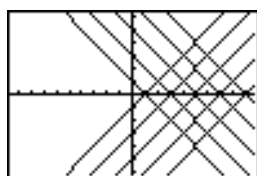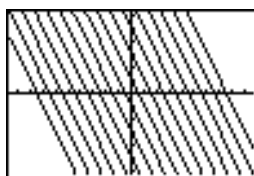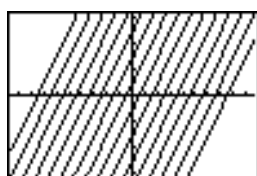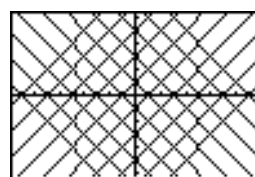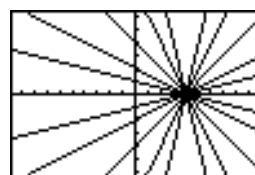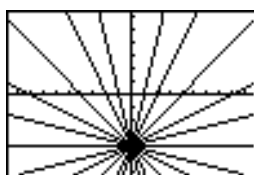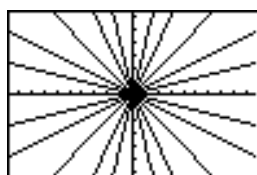
Teacher Note: Activity 19

Activity 19 has been designed for use with the graphing calculator.
Its purpose is to reinforce learners' understanding of the connection between the graph of a linear function, and the parameters m and b in the formula $y = mx + c$. Learners are given the set of designs, and they must create them on their calculator. They should do this by entering functions of the form $y = mx + c$.

Learners need not solve the problems in any particular order. As they work on one figure, they may accidentally create another one. It is important that learners keep records of how each image was achieved, and that there be some small-group and/or whole-class discussion of these records. Without such discussions, learners may get through the activity, and not learn all they can from it, even if it appears to "go well". Some of the questions that you may ask during these discussions are:

- How do you make lines steeper? less steep?
- How do you make lines that go "uphill"? "downhill"?
- How do you make lines horizontal? vertical?
- How do you make parallel lines?
- How do you make a parallel to an uphill line to the left of the original? to the right?
- How do you make a parallel to a downhill line higher up? lower down?

Note also that learners are asked to graph vertical lines. It is of course not possible to do it using the "y=" format. (It can be done with the Draw Vertical feature.)

This activity is an example of an approach that offers both access and depth in the same lesson. Access, because no one is frozen out of the activity: all learners can understand the question, get started, and find a challenge to stretch their own understanding. Depth because there are many ways to increase the mathematical payoff and to keep even the strongest learners challenged.

Whether a given design is reproduced accurately is largely a matter of opinion. The teacher may ask the learners to capture the general look of a given figure, or for a nearly-exact replica. The former can be done by trial and error, and helps learners develop a feel for the effect of m and b in a general sort of way.

The activity can easily be extended. For example, learners can create their own designs, which can be printed or shown on the overhead for others to emulate.
Note: There is a similar activity in Module 7 on quadratic functions called Graph Art.

Source:
http://www.picciotto.org/math-ed/calculator/make-these/

Note: See the teacher notes in Module 7 on handling families of functions and symmetry on the T1-82 graphing calculator.

ACTIVITY 20   Entrepreneur

An entrepreneur starts a new business.

He produces gadgets at a cost of $Y_1 = 4x + 200$ rands and then sells it at $Y_2 = 5x$ rands, where x is the number of gadgets.

Note: He assumes that he can sell all the gadgets he produces.

> **Profit = Selling price – Production cost**

1. Calculate his profit for different numbers of gadgets.

2. Draw graphs of the *cost-function* Y_1 and the *selling price-function* Y_2 on the same system of axes.

3. How many gadgets must he produce and sell to make a profit?
    Can you show this on the graphs?

4. Show how he can increase his profit by reducing his production costs and/or increasing his selling price.

Challenge:

Draw the graph for the *profit-function* by entering $Y_3 = Y_2 - Y_1$.

Can you deduce the same information as above from this graph?

Note: to enter Y_2 and Y_1 directly into the equation for Y_3 you must press
2nd Y-VARS , choose FUNCTION and then select the appropriate function:

Can you write down a formula (in simplest form) for Y_3 using x?
How can you *check* that you are right?

Teacher Note: Activities 19 and 20

Activities 19 and 20 have been designed for use with the graphing calculator.
They facilitate explorations of linear functions by providing learners with continual access to numerical, graphical and symbolic representations of functions.

Teachers could use these activities as projects or as further development and enrichment.

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GO TO ALGEBRA INDEX